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The Magnetic Penetration Depth Calculated with the Mechanism of "Close-Shell Inversion"

Wangzuo Shangguan[1,2], Xusheng Wang[1], Ziqian Huang[1]

[1]School of Pharmacy, Guangxi University of Chinese Medicine, Nanning, China
[2]School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore City, Singapore
Email: shanggwz@gxtcmu.edu.cn

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Abstract

Meissner effect is one of the two fundamental properties of superconductors, it allows them to actively exclude external magnetic fields from their interior, leaving the field to decay quickly from the surface to the interior within a very thin layer whose thickness is characterized by the penetration depth λ. Based on the mechanism of "close-shell inversion" for superconductivity proposed earlier, we proceed in this paper to calculate the magnetic penetration depth. It is found that repelling the external magnetic field is just a spontaneous and dynamic response of conduction electrons in superconductors. Calculation results show that the net magnetic field decays exponentially, in consistent with the existing theories and experimental data.

Keywords

Meissner Effect, Magnetic Penetration Depth

1. Introduction

When a material enters into the superconducting state from normal state, it actively expels all magnetic fields from its interior, leaving zero magnetic field inside (for the type-I superconductor). This is known as the Meissner effect [1], one of the two fundamental properties of superconductors. It makes the externally applied magnetic field decay quickly from the surface of a superconductor to the interior, leaving only a thin layer penetrated by the magnetic field whose thickness is characterized by the so-called magnetic penetration length, usually denoted by λ.

For more than two decades, zero electrical resistance was considered to be the only fundamental—also superb—property of the superconductor, since its first discovery by Kamerlingh Onnes in 1911, as its name indicates: superconductor.

It was not realized that expelling magnetic field was another fundamental property of all superconductors until Meissner and Ochsenfeld measured the magnetic field of a superconductor in 1933. This amazing property of superconductors to expel external magnetic field and maintain zero field inside is distinct from the perfect diamagnetism that would arise from its zero resistance. If an external magnetic field is applied to a perfect conductor, which is zero resistance only, current loops would be generated to exactly cancel the imposed field. However, if an external magnetic field is already applied to the material in normal state before it is cooled to become superconducting state, the penetrated magnetic field should be expected to remain, because if there is no change in the applied magnetic field, nor relative movement between the magnetic field and the material, no voltage would be generated to drive currents, even in a perfect conductor. Therefore, the active exclusion of magnetic field must be considered to be an effect distinct from just zero resistance. On the other hand, just like the two impartible sides of a coin, these two fundamental properties of superconductors must be attributed to the same mechanism of superconductivity.

In 1935, London brothers developed the London equations to provide a mathematical description of the Meissner effect, demonstrating that the magnetic field decays exponentially inside the superconductor over a distance of 20 - 40 nm [2]. The decay rate was described in terms of a parameter called the London penetration depth. The microscopic theory for superconductivity, BCS theory, successfully explained the behaviors of the so-called type-I, or the so-called "conventional" superconductors [3] [4]. Its treatment of the Meissner effect was consistent with the London equations. However, neither London equations nor BCS theory provided a dynamic explanation. Some researchers argue that both BCS theory and London equations actually cannot physically explain the Meissner effect [5] [6].

Theoretically, so far the magnetic penetration depth can be calculated by two-fluid model [7], or by the above-mentioned London theory or the BCS theory, while experimentally many measurement techniques have been developed, such as muon-spin relaxation/rotation (μSR) [8], two-coil mutual inductance [9], Superconducting Quantum Interference Device (SQUID) technique [10], magnetic force microscope [11], Tunnel Diode Oscillator [12], etc. Each theoretical calculation method or experimental measurement technique has its own advantages and limitations. For example, the two-coil mutual inductance technique is particularly suitable for measurement with superconducting films.

Recently, we proposed a simple universal physical model of superconductivity based on the mechanism of "close-shell inversion" [13], with which the two fundamental properties of superconductors can be explained self-consistently. In principle, this model is applicable to all superconductors, no matter type-I or type-II, or the newly-emerging high-pressure superconductors, including the so-called "conventional" and "unconventional" superconductors. We believe that all superconductors must be due to the single mechanism of superconductivity. Based on this model of superconductivity, we proceed in the present work to calculate the magnetic penetration depth in a superconductor subject to an externally applied magnetic field, offering an alternative calculation method in addition to the afore-mentioned three ones. We show that repelling against the externally applied magnetic field is just a spontaneous and dynamic response of the conduction electrons in the superconductor. In other words, as long as the net local magnetic field remains non-zero, it will always give rise to a difference in the atomic magnetic moments of any two nearest-neighboring atomic sites, yielding a net repelling magnetic field against the external field. The calculation results show that the net intensity of the magnetic field decays exponentially from the surface to interior of the superconductor with respect to distance. Along with the analytical results, numerical results are also presented, which agree consistently with that from previous theories and experimental measurement data.

In the next section, the theoretical formulation of the magnetic penetration depth is presented, with which the numerical calculations are carried out in Section 3, where a comparison with some typical experimental measurement data is made. The concluding remarks are presented in Section 4.

2. Theoretical Formulation

To begin with, let us consider a simple case in which the material consists of single element only. For those compounds containing multiple elements, the basic principle remains the same, though the detailed calculation may be a bit more complicated. In this single-elemental superconductor crystal, we sample a line of 4 atoms from some layer of the lattice atom, as shown in **Figure 1**. Due to the so-called "close-shell inversion", the conduction electron always inverts its circulation direction around the nuclei between any two nearest-neighboring atomic sites [13]. If an external magnetic field is applied to the superconductor, these conduction electrons will acquire azimuthal force and their circulation radii will change accordingly. For example, supposing that there is one electron at point X in the atomic site B in some instant, this particular electron is now exerted by an additional Lorentz force F pointing to the center of the nucleus B, as indicated by the inset of **Figure 1**, where shown also are the direction of the externally applied magnetic field B and that of the velocity of the particular electron, V. As a result, a difference will be generated between the magnetic moments of any two nearest-neighboring atomic sites, leading to a net repelling magnetic field to neutralize the external field.

Before we continue to calculate the penetration depth, let us first make some simplifications. First of all the magnetic field associated with a conduction electron moving around a nucleus is treated as a current loop and its associated atomic magnetic moment is denoted in the center of respective atomic site. Here only one single outmost electron of each atomic site is considered. Regardless of that microscopically there may be some fluctuations, in the equilibrium case, *i.e.* no externally applied field (including magnetic and electric), it is no doubt that macroscopically the interior field vanishes. While in the case of an external

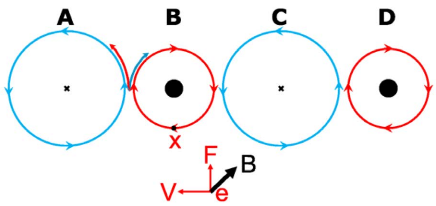

Figure 1. (Color online) mechanism of Meissner effect. With application of an external magnetic field pointing into the paper, electron moving clockwise moves in a smaller radius while that moving anticlockwise, in a bigger radius, resulting in a net induced atomic magnetic field pointing against the externally applied field.

magnetic field is applied, as is shown in **Figure 1**, an additional unbalanced magnetic field is induced, and the net local magnetic field intensity is thus equal to the sum of that of the externally applied field and that of the additional induced magnetic field. In view of that both the dimensions of atoms and distance between them are very small in superconductors, and also that the lattice atoms are in translational symmetry, we approximate the local induced additional magnetic field to the difference of the center intensities of any two nearest-neighbouring current loops.

For an electron moving around a nucleus, it is equivalent to a current loop I and the magnetic field at the center of the current loop is given by

$$B = \mu_0 \frac{I}{2R} = \frac{\mu_0 ev}{4\pi R^2}, \tag{1}$$

where μ_0 is the permeability of free space, e the elementary charge, v the velocity of the electron moving around the nucleus with effective circulation radius R. Once an external magnetic field is applied to the superconductor, the net local field intensity on the surface will no longer be vanishing, this will in turn give rise to a difference in R for any two nearest-neighboring atomic sites, resulting in a difference in magnetic field and thus a net repelling field $\Delta B = B_B - B_A$ which is given by

$$\Delta B = \frac{\mu_0 ev}{4\pi}\left(\frac{1}{R_B^2} - \frac{1}{R_A^2}\right), \tag{2}$$

where B_i represents the magnetic field intensity of atomic site i ($i = A, B$).

Next, we have to calculate the difference of R_A and R_B, or the term within the bracket in the above equation, namely $1/R_B^2 - 1/R_A^2$. In the presence of an external magnetic field with intensity of B_a, the conduction electron acquires a Lorentz force which is given by $F = evB_a$. One will find later that here B_a is actually the net intensity of the local magnetic field. Neglecting the influence of the electrons from inner shells, for an electron moving around a nucleus, the centripetal force F_e is given by

$$F_e = m\frac{v^2}{R}, \tag{3}$$

where m is the mass of an electron. With the additional Lorentz force caused by the net non-vanishing magnetic field, the total force F_e now changes to

$$m\frac{v^2}{R_A} = m\frac{v^2}{R_0} - evB_a \tag{4}$$

$$m\frac{v^2}{R_B} = m\frac{v^2}{R_0} + evB_a \tag{5}$$

where R_0 is the effective circulation radius of electrons around nuclei at equilibrium, *i.e.* in the case no external magnetic or electric field is applied to the superconductor. Dividing both sides onto the above equations by mv^2 yields

$$\frac{1}{R_A} = \frac{1}{R_0} - \frac{eB_a}{mv}, \tag{6}$$

$$\frac{1}{R_B} = \frac{1}{R_0} + \frac{eB_a}{mv}. \tag{7}$$

It is obvious from the above equation group that as long as the net local magnetic field B_a remains non-zero, it will always give rise to a difference between R_A and R_B. Substituting Equations (6) and (7) into Equation (2) yields the difference of magnetic fields between any two nearest-neighboring atomic sites

$$\Delta B = \frac{\mu_0 e^2 B_a}{\pi m R_0}. \tag{8}$$

For notational simplicity, we rewrite the above equation as

$$\Delta B = aB_a, \tag{9}$$

where the prefactor *a*, referring to the attenuation rate of the magnetic field penetrating from the surface of a superconductor into its interior after a distance of one layer of lattice atom, is defined by

$$a \equiv \frac{\mu_0 e^2}{\pi m R_0}. \tag{10}$$

From the above equation, it is easy to verify that *a* is a dimensionless number, which is also required by and in consistent with Equation (9). In addition, from the above definition of *a*, one can see that the attenuation rate is determined only by the variable R_0, the effective circulation radius of electron around nuclei at equilibrium, since μ_0, *e*, and *m* are all constants for any material.

Equation (9) implies that, once an external magnetic field with intensity B_a is applied to a superconductor at equilibrium, this external field will be reduced by a factor of *a* inside the superconductor after one layer of lattice atom from the surface, leaving a net field intensity of $(1-a)B_a$ at this particular point. Assuming Simple Cubic (SC) lattice structure and going further one more layer of lattice atom into the interior of the superconductor, this net field intensity of $(1-a)B_a$ will be further reduced by a factor of *a*, leaving a net field intensity of $(1-a)^2 B_a$ at the position where is two atomic layers deep from the surface of the superconductor. Going on this way recursively and setting $B_a = B_0$, one can write:

$$B_1 = B_0 - aB_0 = (1-a)B_0, \tag{11}$$

and after second layer of atom, the net intensity of magnetic field becomes

$$B_2 = B_1 - aB_1 = (1-a)^2 B_0 \tag{12}$$

and so on, thus

$$\vdots$$

$$B_n = (1-a)^n B_0 \tag{13}$$

where B_n represents the net magnetic field intensity after n layers of lattice atom from the surface into the superconductor, which has been attenuated from B_0 at the surface, by a factor of $(1-a)^n$, with a being defined by Equation (10). From its definition, it is interesting to note that a does not depends on B_a, which is reasonable because the attenuation rate is material-dependent instead of external-field dependent.

However, if $B_a = 0$, there will be no difference produced between any two nearest-neighboring atomic magnetic moments, as indicated by Equation (8), which means that the superconductor is at equilibrium macroscopically without any additional net field, so there will be no decay of field. Hence, B_a is actually the net magnetic field intensity. On the other hand, as long as the local net magnetic field remains non-zero, it will always give rise to a difference in the effective electron circulation radii, e.g. R_A and R_B. According to Equation (2), this difference in radii will in turn lead to a repelling magnetic field ΔB, reducing the net magnetic field from B_0 to $(1-a)B_0$ after going through one atomic layer from the surface into the superconductor. In this way, the externally applied magnetic field is gradually neutralized from the surface of the superconductor to its interior. A remarkable feature of this screening mechanism of Meissner effect is that it is dynamic, spontaneous, fully physical instead of mathematical. We conclude that repelling externally applied field is just a spontaneous and dynamic response of the superconducting electrons.

3. Numerical Results and Discussion

3.1. Variation of the Net Magnetic Field Intensity

Since the attenuation rate of the penetrated magnetic field with respect to that on the surface, according to Equation (13), is exponential, the externally applied magnetic field decays very quickly from the surface of the superconductor to a negligible intensity in its interior, which is virtually zero [14]. **Figure 2** illustrates how the net intensity of the magnetic field inside a superconductor varies with distance starting from the surface. Here, the equilibrium effective circulation radius R_0 is set to be 0.2 Å, which is reasonable for most single-element superconductors. It is obvious from **Figure 2** that the net intensity of the magnetic field relative to that of externally applied field, namely $B_n / B_0 (\equiv B)$, decreases exponentially with respect to the distance from surface, and the penetration depth λ in this case is about 70 nm, where the net relative magnetic field intensity reduces to $1/e$ with e the Euler's number so that $1/e \simeq 0.3679$. The inset shows a straight line for the same data but the y-axis has been changed to logarithmic.

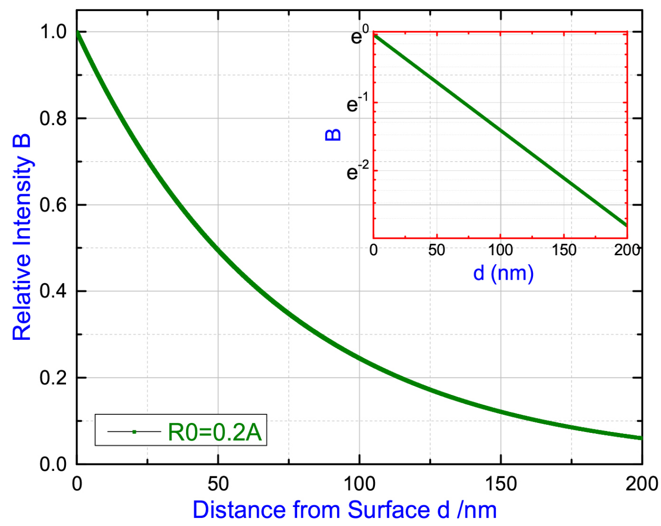

Figure 2. (Color online) variation of the net magnetic field intensity vs distance from the surface of a superconductor. Inset: same set of data in semi-log scale.

For experimental measurement on single-element superconductors, as far as we know, Desirant and Shoenberg gave a penetration depth data for mercury in 1947, which ranged between 80 - 112 nm [15]. In another work by Laurmann and Shoenberg they found λ to be 43 nm for mercury by the Casimir method [16], which was only half of the result as in Ref. [15]. For tin, the measured penetration depth is 52 nm in Ref. [16] while 57 nm in Ref. [17] by Pippard. Unlike mercury, these two measurement values for tin match very well. Instead of single-element metals, more data is available for compounds. Recently, Loudon *et al.* measured the penetration depth of MgB_2 in all directions using transmission electron microscopy [18]. MgB_2 is a binary compound, and it is a rare two-band superconductor discovered in 2001 with a transition temperature $T_c = 39$ K. It is uniaxial with a hexagonal crystal structure composed of alternating layers of magnesium and boron with lattice parameters $a = b = 3.086$ Å and $c = 3.542$ Å. Their results show that the penetration depths $\lambda_{ab} = 100 \pm 8$ nm and $\lambda_c = 120 \pm 15$ nm at 10.8 K in a field of 4.8 mT. Another binary compound that has been studied recently is $PdTe_2$ by Salis *et al.* by tunnel diode oscillator technique [19]. The magnetic penetration depth $\lambda(0)$ of ~500 nm for $T \to 0$ was reported by them for $PdTe_2$, which is a superconductor with a T_c at 1.7 K, and with lattice constants $a = b = 4.024$ Å and $c = 5.113$ Å [20] [21].

Determining experimentally the magnetic penetration depth λ in superconductors proved to be quite challenging, because it is rather small and inside the superconductors. Over the years many methods or techniques have been developed, most of them rely upon measuring the diamagnetic response of the superconductor [22], for example, the mutual inductance measurement was used by Logvenov *et al.* for single copper-oxygen plane [23] and by Zhang *et al.* [24] on monolayer FeSe films. Except from the afore-mentioned binary compounds, there are also measurements done on complicated compounds, such NbCN, $YBa_2Cu_3O_{7-\delta}$ and many others (see, for example, Ref. [25]). For example, with the variable spacing parallel plate resonator, Talanov *et al.* have carried out measurements on $YBa_2Cu_3O_7$ and $GdBa_2Cu_3O_7$, respectively, with magnetic penetration depth 257 nm and 400 nm, while in Ref. [26], $\lambda(0) = 0.9 \sim 1$ μm is reported for $Sr_{3-x}SnO$ with $T_c = 6$ K using the muon-spin relaxation/rotation (μSR) technique. In overall these data diversify from method to method, even for the same superconductor sample, say mercury film. However, a common qualitative characteristic must be possessed, *i.e.* the penetrated magnetic field decays exponentially inside the superconductor, as in our numerical results in **Figure 2**.

3.2. Penetration Depth

Next, we try to figure out the dependence of the penetration depth λ on the equilibrium effective electron circulation radius R_0. Referring to Equation (13), on the right-hand side of the equation, we set the prefactor before B_0 to be $1/e$, *i.e.* the relative magnetic field intensity is given by

$$(1-a)^{n_\lambda} = \frac{1}{e} \tag{14}$$

where the exponential factor n has been rewritten as n_λ to denote the corresponding number of atomic layers at the penetration depth, and e is the Euler's number. Rewriting the above equation, one finds that n_λ is given by

$$n_\lambda = -\frac{1}{\ln(1-a)}. \tag{15}$$

In view of that the attenuation rate a is a very small positive dimensionless number, for example, taking a typical effective electron circulation radius to be 0.2 Å, *i.e.* $R_0 = 0.2 \times 10^{-10}$ m, based on Equation (10), one finds that

$$a = \frac{4\pi \times 10^{-7} \times (1.6 \times 10^{-19})^2}{\pi \times 9.1 \times 10^{-31} \times 0.2 \times 10^{-10}} = 5.626 \times 10^{-4}, \tag{16}$$

which is a very small number compare to 1, it is pretty safe to make the following approximation

$$\ln(1-a) \simeq -a. \tag{17}$$

Substituting the above equation into Equation (15) yields

$$n_\lambda \simeq \frac{1}{a}. \tag{18}$$

Thus the number a, the attenuation rate of the magnetic field penetrating into a superconductor before and after one layer of lattice atom, acquires an additional physical meaning: its inverse amounts to the layer number of the lattice atom from the surface of a superconductor to an interior position corresponding to the magnetic penetration depth of the superconductor. Therefore the penetration depth can be readily calculated and is given by

$$\lambda \simeq n_\lambda \cdot 2R_0 = \frac{2\pi m R_0^2}{\mu_0 e^2}. \tag{19}$$

Figure 3 illustrates variation of the penetration depth λ vs. the effective circulation radius R_0 based on the above equation. As R_0 increases from 0 to

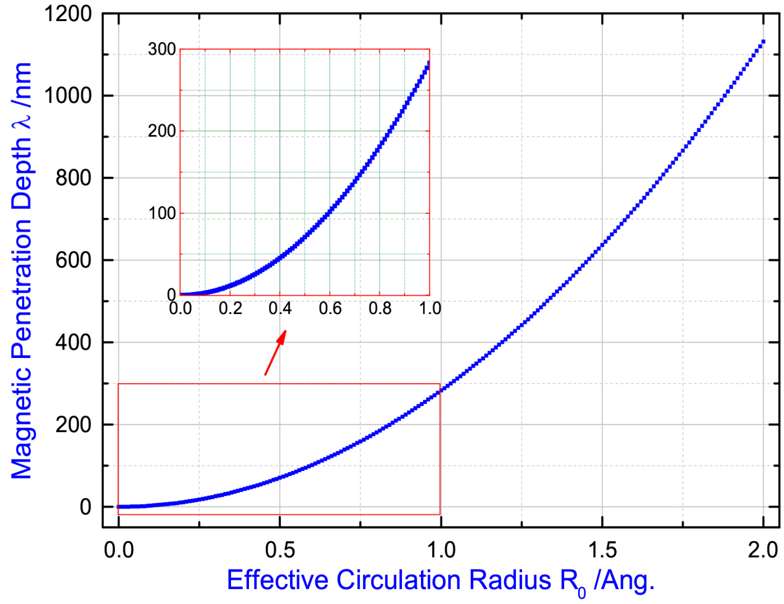

Figure 3. (Color online) variation of the magnetic penetration depth λ with respect to the effective circulation radius R_0. Inset shows in smaller scale with R_0 within 1 Å.

3 Å, the magnetic penetration depth expands parabolically from 0 to 2500 nm. The inset shows the zoom-in view of the curve within the range of 1 Å, where the penetration depth is about 290 nm at $R_0 = 1$ Å. As an example, the afore-mentioned λ of around 55 nm for tin in Refs. [16] [17], according to Equation (19) or inset of **Figure 3**, amounts to 0.44 Å, which is a reasonable number.

With the numerical data presented in **Figure 2** and **Figure 3**, we conclude that our results are in consistent with the experimental data. There might be some quantitative discrepancy, which could be due to some approximations, we have made during the calculation. First of all, only the single outermost electron is concerned in our model, while contribution from other conduction electrons is ignored. In addition, lattice structure and crystal orientation can also affect the measurement data, whereas in our model the simplest SC lattice structure has been assumed.

4. Conclusion

In summary, we have evaluated the magnetic penetration depth based on the mechanism of "close-shell inversion" for superconductivity, presenting an alternative theoretical evaluation method for λ. The results show that the externally applied magnetic field decays exponentially from the surface to interior of superconductors, in consistent with the previous theories and the experimental measurement data. It is demonstrated that the decay rate of the net local magnetic field, *a*, and the magnetic penetration depth λ are related decisively to R_0, the equilibrium circulation radius of conduction electron around nucleus, which can be affected by a number of factors, such as the material's temperature, ambient pressure, electronic density of states, mass and charge of nuclei, etc., and thus in turn influence both a and λ indirectly, so that vary from material to material. However, how these factors affect R_0, or the inter-atomic distances in a material and herein its physical properties, as the question we posed in the previous work [13], remains unsolved and entails further investigation.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Meissner, W. and Ochsenfeld, R. (1933) Ein neuer Effekt bei Eintritt der Supraleitfähigkeit. *Naturwissenschaften*, **21**, 787-788. https://doi.org/10.1007/BF01504252

[2] London, F. and London, H. (1935) The Electromagnetic Equations of the Supraconductor. *Proceedings of the Royal Society of London*, *Series A—Mathematical and Physical Sciences*, **149**, 71-88. https://doi.org/10.1098/rspa.1935.0048

[3] Bardeen, J., Cooper, L.N. and Schrieffer, J.R. (1957) Microscopic Theory of Superconductivity. *Physical Review Journals Archive*, **106**, 162-164. https://doi.org/10.1103/PhysRev.106.162

[4] Bardeen, J., Cooper, L.N. and Schrieffer, J.R. (1957) Theory of Superconductivity. *Physical Review Journals Archive*, **108**, 1175-1204. https://doi.org/10.1103/PhysRev.108.1175

[5] Hirsch, J.E. (2010) Explanation of the Meissner Effect and Prediction of a Spin Meissner Effect in Low and High T_c Superconductors. *Physica C*: *Superconductivity and Its Applications*, **470**, S955-S956. https://doi.org/10.1016/j.physc.2009.10.066

[6] Hirsch, J.E. (2012) The Origin of the Meissner Effect in New and Old Superconductors. *Physica Scripta*, **85**, Article ID: 035704. https://doi.org/10.1088/0031-8949/85/03/035704

[7] Gorter, C. and Casimir, H. (1934) On Supraconductivity I. *Physica*, **1**, 306-320. https://doi.org/10.1016/S0031-8914(34)90037-9

[8] Sonier, J.E., Brewer, J.H. and Kiefl, R.F. (2000) μSR Studies of the Vortex State in Type-II Superconductors. *Reviews OF Modern Physics*, **72**, 769-811. https://doi.org/10.1103/RevModPhys.72.769

[9] Fiory, A.T., Hebard, A.F., Mankiewich, P.M. and Howard, R.E. (1988) Penetration Depths of High T_c Films Measured by Two-Coil Mutual Inductances. *Applied Physics Letters*, **52**, 2165-2167. https://doi.org/10.1063/1.99757

[10] Gross, F., Chandrasekhar, B.S., Einzel, D., Andres, K., Hirschfeld, P.J., Ott, H.R., Beuers, J., Fisk, Z. and Smith, J.L. (1986) Anomalous Temperature Dependence of the Magnetic Field Penetration Depth in Superconducting UBe_{13}. *Zeitschrift für Physik B Condensed Matter*, **64**, 175-188. https://doi.org/10.1007/BF01303700

[11] Lan, L. (2011) Magnetic Force Microscopy Studies of Unconventional Superconductors: Single Vortex Manipulation and Magnetic Penetration Depth Studies. Ph.D. Thesis, Stanford University, Stanford.

[12] Hardy, W.N., Bonn, D.A., Morgan, D.C., Liang, R. and Zhang, K. (1993) Precision Measurements of the Temperature Dependence of λ in $YBa_2Cu_3O_{6.95}$: Strong Evidence for Nodes in the Gap Function. *Physical Review Letters*, **70**, 3999-4002. https://doi.org/10.1103/PhysRevLett.70.3999

[13] Shangguan, W.Z. (2019) A Physical Picture of Superconductivity: Close-Shell Inversion. *Modern Physics Letters B*, **33**, Article ID: 1950393. https://doi.org/10.1142/S0217984919503937

[14] Bányai, L.A. (2021) Mean-Field BCS Theory of the Meissner Effect in Bulk Revisited. *The European Physical Journal B*, **94**, Article No. 24. https://doi.org/10.1140/epjb/s10051-020-00034-2

[15] Désirant, M. and Shoenberg, D. (1947) Penetration of Magnetic Field into Superconducting Mercury. *Nature*, **159**, 201-202. https://doi.org/10.1038/159201b0

[16] Laurmann, E. and Shoenberg, D. (1949) Penetration of Magnetic Field into Superconductors. II. Measurements by the Casimir Method. *Proceedings of the Royal Society of London*, *Series A—Mathematical and Physical Sciences*, **198**, 201-202. https://doi.org/10.1098/rspa.1949.0119

[17] Pippard, A.B. (1950) Field Variation of the Superconducting Penetration Depth. *Proceedings of the Royal Society of London*, *Series A—Mathematical and Physical Sciences*, **203**, 210-223. https://doi.org/10.1098/rspa.1950.0135

[18] Loudon, J.C., Yazdi, S., Kasama, T., Zhigadlo, N.D. and Karpinski, J. (2015) Measurement of the Penetration Depth and Coherence Length of MgB_2 in All Directions Using Transmission Electron Microscopy. *Physical Review B*, **91**, Article ID: 054505. https://doi.org/10.1103/PhysRevB.91.054505

[19] Salis, M., Rodière, P., Leng, H., Huang, Y. and de Visser, A. (2018) Penetration Depth Study of the Type-I Superconductor $PdTe_2$. *Journal of Physics*: *Condensed Matter*, **30**, Article ID: 505602. https://doi.org/10.1088/1361-648X/aaed31

[20] Pell, M., Mironov, Y. and Ibers, J. (1996) $PdTe_2$. *Acta Crystallographica Section C*: *Structural Chemistry*, **52**, 1331-1332. https://doi.org/10.1107/S0108270195016246

[21] Liu, Y., Zhao, J., Yu, L., Lin, C., Liang, A., Hu, C., Ding, Y., Xu, Y., He, S., Zhao, L., Liu, G., Dong, X., Zhang, J., Chen, C., Xu, Z., Weng, H., Dai, X., Fang, Z. and Zhou, X.J. (2015) Identification of Topological Surface State in $PdTe_2$ Superconductor by Angle-Resolved Photoemission Spectroscopy. *Chinese Physics Letters*, **1505**, Article ID: 06642. https://doi.org/10.1088/0256-307X/32/6/067303

[22] Langley, B., Anlage, S.M., Pease, R.F.W. and Beasley, M.R. (1991) Magnetic Penetration Depth Measurements of Superconducting Thin Films by a Microstrip Resonator Technique. *Review of Scientific Instruments*, **62**, 1801-1812. https://doi.org/10.1063/1.1142425

[23] Logvenov, G., Gozar, A. and Bozovic, I. (2009) High-Temperature Superconductivity in a Single Copper-Oxygen Plane. *Science*, **326**, 699-702. https://doi.org/10.1126/science.1178863

[24] Zhang, Z., Wang, Y.-H., Song, Q., Liu, C., Peng, R., Moler, K.A., Feng, D. and Wang, Y. (2015) Onset of the Meissner Effect at 65 K in FeSe Thin Film Grown on Nb-Doped $SrTiO_3$ Substrate. *Science Bulletin*, **60**, 1301-1304. https://doi.org/10.1007/s11434-015-0842-8

[25] Prozorov, R. and Giannetta, R.W. (2006) Magnetic Penetration Depth in Unconventional Superconductors. *Superconductor Science and Technology*, **19**, R41. https://doi.org/10.1088/0953-2048/19/8/R01

[26] Ikeda, A., Guguchia, Z., Oudah, M., Koibuchi, S., Yonezawa, S., Das, D., Shiroka, T.,

Luetkens, H. and Maeno, Y. (2020) Penetration Depth and Gap Structure in the Antiperovskite Oxide Superconductor $Sr_{3-x}SnO$ Revealed by μSR. *Physical Review B*, **101**, Article ID: 174503. https://doi.org/10.1103/PhysRevB.101.174503