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Involuntary Unemployment and Micro-Foundations for Inflexible Aggregate Investment in Diamond-Type Overlapping Generations Models

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Abstract

It is the aim of this paper to investigate the micro-foundations for inflexible aggregate investment in Diamond-type overlapping generations (OLG) models of involuntary unemployment. As is well-known, in Diamond's (1965) seminal OLG model, aggregate investment is macro-founded in that aggregate savings govern perfectly flexible aggregate investment. Perfect flexibility of aggregate investment precludes, however, involuntary unemployment of the labor force: any lack of aggregate demand in comparison to full-employment output is instantaneously compensated by flexible aggregate investment. In contrast, inflexible aggregate investment can cause involuntary unemployment through aggregate demand remaining below full-employment output. However, to date, there has been no attempt in the literature to micro-found inflexible aggregate investment in Diamond-type OLG models of involuntary unemployment. After reviewing several approaches to micro-founding aggregate investment in intertemporal equilibrium models with both full and underemployment, a deterministic OLG model with production and physical capital accumulation à la Magill and Quinzii (2003) is set up in which optimally indeterminate firm investment and Keynes (1936) like "animal spirits" of investors are compatible. Sufficient conditions for the existence and dynamic stability of a Golden Rule steady state with involuntary unemployment are then presented and the comparative dynamics of this steady state is investigated. While an increase in investor optimism decreases unemployment in the short and long run, a smaller savings rate does this only temporarily.

Keywords

Involuntary Unemployment, Micro-Foundations for Aggregate Investment,

Diamond-Type OLG Model, Existence, Dynamic Stability and Comparative Dynamics of Steady States

1. Introduction

The concept of "under-employment equilibrium" with "involuntary unemployment" figures prominently in Keynes' (1936) renowned "General Theory of Employment, Interest, and Money". Here, as is common knowledge, a lack of aggregate demand (aggregate demand failures) is the root cause of involuntary unemployment. There is, however, despite decades of debate in this field, still no agreement among macro-economists as to why aggregate demand in a perfectly operating market economy continues to be below full-employment output. Currently, mainstream macro-economists adhere to the New Keynesian approach of micro-founded, dynamic general equilibrium models. Here, prices and wages are thought to adapt sluggishly to market imbalances due to price adjustment costs, imperfect competition, and other forms of market failure (for a survey see Dixon (2000)). The New Keynesian approach of dynamic stochastic general equilibrium (DSGE) models, which originates with Smets and Wouters (2003) and Woodford (2003), is now widely used by applied macro-economists in central banks and public administrations.

A minority of macro-oriented general equilibrium modelers employ stylized, dynamic, intertemporal general equilibrium with perfect competition in factor and output markets in order to feature involuntary unemployment. Implicitly based on Morishima's (1977) seminal insight that involuntary unemployment under perfect competition in output and factor markets ought to be related to the existence of an aggregate investment function independent of aggregate savings, Magnani (2015) modeled involuntary unemployment in a Solow (1956)-type neo-classical growth model. Following the lead of Magnani (2015), Farmer and Kuplen (2018), and more recently, Farmer (2022) modeled growth and involuntary unemployment in a Diamond (1965)-type overlapping generations (OLG) economy, with production, physical and human capital accumulation, by simply assuming the existence of an aggregate investment function. This is in line with Magnani's (2015) claim that there is no need to micro-found the independent investment function since it is macro-founded as in Solow's (1956) seminal growth model.

In contrast, there is Lucas's (1972) magisterial claim that all macro-economic relationships should be micro- or general-equilibrium-founded. This is obviously not the case with an independent aggregate investment function which is, however, decisive for the occurrence of involuntary unemployment in intertemporal equilibrium. As mentioned above, involuntary unemployment is traced back to a lack of aggregate demand relative to full employment output. A gap between full employment output and aggregate demand cannot occur, however, with perfectly flexible aggregate investment in Solow's (1956) and Diamond's (1965) neo-classical growth models. Perfectly flexible aggregate investment features as a buffer between aggregate full employment output and aggregate demand such that aggregate output never turns out to be demand-constrained and hence labor demand never falls short of labor supply. If, in contrast, aggregate investment does not adapt passively to aggregate savings, a gap between full-employment output and aggregate demand might pop up such that aggregate output stays below full-employment output and labor demand then remains below labor supply despite prices and wage rates being perfectly flexible.

Once Lucas's (1972) claim is accepted, the research challenge then consists in providing micro-foundations for an independent aggregate investment function such that aggregate-output demand and not labor supply governs employment. It is the objective of the present paper to meet this research challenge within the confines of intertemporal general equilibrium models, in particular Diamond-type OLG models. As will become clear from the review of the literature on micro-foundations for aggregate investment in the following section, the micro-foundations' approaches thus far cannot be simply incorporated in a Diamond-type OLG model of involuntary unemployment without further modifications: either they presuppose full employment of the labor force (e.g. Miyashita, 2000) or if involuntary unemployment is modeled investment is not micro-founded (e.g. Farmer, 2020).

Against this research background, our first contribution to literature is to set up a Diamond-type OLG model in which inflexible aggregate investment is micro-founded and involuntary unemployment occurs.[1] It turns out that Magill and Quinzii's (2003) modification of Diamond's (1965) original OLG model with production and capital accumulation contains investment micro-foundations which are compatible with the investment-quantity determination by a belief function à la Farmer (2020). To be able to model involuntary unemployment, the labor-market clearing condition in Magill and Quinzii's (2003) stock market OLG model will be cancelled, and the unemployment rate will be made endogenous in line with Magnani (2015).

Our second contribution to the literature is to show how the structure of the intertemporal equilibrium dynamics derived from households' and firms' optimization conditions, from government's budget constraint and the intertemporal market clearing conditions changes when firms' investment quantities are both optimally indeterminate and determined by a degenerate belief function in line with Farmer (2020). Moreover, the existence and the dynamic stability of a Golden-Rule steady state of the intertemporal equilibrium will be shown.

Our third contribution to the literature consists in deriving analytically the steady-state effects on the endogenous variables of main parameter changes. This is completed by a numerical calculation of the intertemporal equilibrium paths of the endogenous variables in response to small parameter changes.

[1]Our OLG model can be considered as complementary to Tanaka's (2020) three-period OLG model of involuntary unemployment without real capital and investment.

The structure of the paper is as follows. The next section presents the review of the literature on micro-foundations for aggregate investment. Then, our stock market OLG model with involuntary unemployment is set-up. This is followed by derivation of the intertemporal equilibrium dynamics and demonstration of sufficient conditions for the existence and dynamic stability of steady states. We then investigate analytically the comparative dynamics of the steady-state responses of the capital-output ratio, the equity price discount and the unemployment rate to the main parameter changes. A numerical specification of all model parameters is then used to calculate numerically the intertemporal equilibrium paths of these dynamic variables in response to small parameter changes. The main conclusions are drawn in the final section of the paper.

2. The Challenge of Providing Micro-Foundations for Aggregate Investment: A Literature Review

As mentioned above, Magnani (2015) rightly remarked that in Solow's (1956) famous neo-classical growth model, aggregate investment was not micro- but macro-founded. He thus concluded that there was also no need to provide micro-foundations for the aggregate investment function which he introduced to make the general equilibrium solution determinate. Following Magnani's (2015) lead Farmer and Kuplen (2018) and Farmer (2022) also made no effort to establish micro-foundations for the aggregate investment function in a modified Diamond-type OLG model.

However, since the new-classical revolution in macroeconomics initiated by Lucas (1972), all mainstream macro-economists agree that macroeconomic models ought to rest on firm general equilibrium foundations based on the following three assumptions: 1) All private agents maximize their objective functions subject to their respective budget constraints; 2) All markets clear in each period where market sessions are held; 3) All private agents form rational expectations with respect to future prices, wage and interest rates, i.e., agents' expectations of future prices, wage and interest rates turn out to be equal to the prices, wage and interest rates that actually occur in every conceivable state of nature.

While Farmer and Kuplen's (2018) and Farmer's (2022) OLG model, with the exception of the aggregate investment function, clearly accord with the first assumption, complying with the second assumption is more problematic since it automatically precludes involuntary unemployment. In principle, there are two ways of addressing this problem.

The first, is to make use of Keynesian search theory, as proposed by Farmer (2012, 2013, 2020), whereby any attempt to derive labor supply from utility-maximizing labor-leisure choice is abandoned, and employment is instead determined by labor demand and a search technology. The latter relates the number of unemployed workers, and the resources firms allocate to the process of filling vacancies, to the number of new employment relationships. Hereby, it is assumed that matching unemployed workers to firm job openings is costly, and that the search technology cannot be decentralized via markets. In such a setting, it is taken as given that due to the absence of market signals, workers and firms are not led to the correct point on the so-called Beveridge curve. While dominant search theory (Pissarides, 1990) deviates from the assumption of perfectly competitive workers and firms, and lets both bargain over the wages, Keynesian search theory retains the assumption of perfectly competitive firms and workers. As a consequence, the general-equilibrium solution in steady state becomes indeterminate, and it is this indeterminacy which provides the opportunity for the existence of involuntary unemployment in general equilibrium.

In order to make this verbal description more precise, we now reproduce the main equations of Farmer's (2020: p. 21) Keynesian search theory.

The first equation represents GDP market clearing for a closed economy:

$$Y_t = C_t + I_t + G_t, \tag{1}$$

where Y_t denotes GDP in period *t*, C_t is aggregate consumption in period *t*, I_t denotes aggregate investment in period *t*, and G_t is government expenditures in period *t*. For ease of exposition, we assume that G_t is time-stationary and equals the exogenous magnitude $\bar{G}$.

The second equation features the aggregate production function:

$$Y_t = S_t\left(K_{t-1}\right)^{\alpha}\left(X_t\right)^{1-\alpha}, \tag{2}$$

whereby S_t denotes a total productivity shock governed by an exogenous stochastic process, K_t is the aggregate capital stock, and X_t denotes aggregate employment. $0 < \alpha < 1$ is the production elasticity of capital (=capital income share in GDP).

Aggregate capital accumulation is well-known and is depicted by the third equation:

$$K_t = K_{t-1}\left(1-\delta\right) + I_t, \tag{3}$$

where $0 < \delta < 1$ denotes the depreciation rate of the capital stock.

The fourth equation represents the Euler equation for an intertemporal utility-maximizing, representative, infinite-lived consumer:

$$\frac{1}{C_t} = \beta E_t\left[\frac{1}{C_{t+1}}\left\{1-\delta+\alpha\frac{Y_{t+1}}{K_t}\right\}\right], \tag{4}$$

where $0 < \beta < 1$ denotes the exogenous time preference factor and E_t is the expectations operator.

Farmer (2020: p. 21) claims that Equations (1)-(4) exhibit four equations in five endogenous variables, Y_t, I_t, K_t, C_t and X_t. The system of Equations (1)-(4) becomes determinate by adding the equation governing aggregate investment:

$$I_t = \rho_I I_{t-1} + \left(1-\rho_I\right)\bar{I} + \varepsilon_t^I, \tag{5}$$

whereby the stochastic process ε_t^I exhibits zero mean, persistence is $0 < \rho_I < 1$, and $\bar{I}$ is the unconditional mean of aggregate investment in line with full employment. As Equation (5) shows, aggregate investment is inter alia governed by the stochastic beliefs ε_t^I of investors, which eventually determine aggregate employment. This is in stark contrast to real business cycle theory (Kydland & Prescott, 1982) in which aggregate labor supply determines aggregate employment via labor market clearing.

In contrast to the above, another version of Keynesian search theory proposed by Farmer (2012, 2013) comes closer to providing micro-foundations for aggregate investment. Instead of aggregate investor beliefs, as in Equation (5), we now have consumer beliefs about stock market value, denoted by the stochastic variable Z_t^B, which eventually determine aggregate employment:

$$Z_t^B = \rho_B Z_{t-1}^B + (1-\rho_B)\bar{Z}^B + \varepsilon_t^B, \tag{6}$$

whereby the belief shock ε_t^B again exhibits zero mean, $0 < \rho_B < 1$ is the persistence of Z_t^B and $\bar{Z}^B$ is its unconditional mean.

The value of the stocks is defined to be the discounted value of future returns to capital:

$$Z_t = E_t\left[(1/i_t)(\alpha Y_{t+1} + Z_{t+1})\right], \tag{7}$$

where $i_t = 1 - \delta + Y_{t+1}/K_t$ is the real interest rate between periods t and $t+1$.

"Beliefs about the future value of the stock market are connected to realizations of the stock market by the rational expectations assumption" (Farmer, 2020: p. 22):

$$Z_t^B = E_t[Z_{t+1}]. \tag{8}$$

Based on Equation (6), expectations of stock market value are exogenous, but they are also rational in the sense that beliefs are self-fulfilling (see Equation (8)). This apparent contradiction disappears once one realizes that, in contrast to dominant search theory, Keynesian search theory displays steady-state indeterminacy in which there is a continuum of labor-market equilibria and a continuum of steady-state unemployment rates.

The second way to address the problem of modeling involuntary unemployment in intertemporal equilibrium is that of Farmer and Kuplen (2018) and Farmer (2022). Their model features are the same as those found in Farmer's (2020) Keynesian search theory where, first, the animal spirits of investors or equity holders represent a fundamental element affecting aggregate demand, and second, involuntary unemployment may still prevail in the short and in the long run even when prices, wages and interest rates are assumed to be perfectly flexible. The main difference with respect to Farmer's Keynesian search theory is that in Farmer and Kuplen's (2018) and Farmer's (2022) OLG model there are no search costs and matching frictions. While this is not tantamount to saying that such frictions do not play an important role in explaining observed unemployment rates, it does make it clear that involuntary unemployment in general equilibrium still arises in the absence of such frictions, as is shown by Magnani's (2015) relatively simple specification of equilibrium unemployment. Thus, the challenge of establishing micro-foundations for the aggregate investment function remains. While Farmer's (2012, 2013) use of stock market foundations in his Keynesian search theory represents an important step in this direction, it remains open how investors make decisions concerning changes in physical and/or intangible capital over time.

Here a third approach to micro-found aggregate investment comes to mind: Magill and Quinzii's (2003) stock market OLG model with affine price expectations and non-shiftable capital. In contrast to Farmer's (2020) approach presented above, in Magill and Quinzii's (2003) OLG model there is no uncertainty regarding future prices, and firms, not households are investing in tangible capital, which is firm-specific and cannot be sold after use among firms. It is the stock market which assumes the role of transferring ownership of the long-lived firms to short-lived shareholders. The issuing of shares by firms enables the illiquid sum of past investments (=firm's current capital stock) to be subdivided into perfectly divisible amounts. In this way, the firm is kept intact over time and capital can be accumulated as in Equation (3) above, even though households (=shareholders) are short-lived.

For the formation of share prices in the stock market, Magill and Quinzii (2003) propose a two-part pricing rule, according to which the equity price Q_t^j of firm $j = 1, \cdots, J$ is equal to its replacement costs $(1-\delta)K_t^j$ minus a discount V_t^j:

$$Q_t^j\left((1-\delta)K_t^j\right) = (1-\delta)K_t^j - V_t^j, \tag{9}$$

and

$$V_{t+1}^j = (1+i_{t+1})V_t^j. \tag{10}$$

Shares are freely traded in the stock market. Thus, perfect competition among young shareholders, who are indifferent between holding shares or holding firm bonds, drives the equity price of firm *j* to the point where the return on holding shares is equal to the return on bonds, i_{t+1}:

$$\begin{aligned} Q_t^j\left((1-\delta)K_t^j\right) = \frac{1}{1+i_{t+1}} \max_{\left\{\left(I_t^j, N_{t+1}^j\right)\right\}} \Big\{ & \left(N_{t+1}^j\right)^{1-\alpha}\left((1-\delta)K_t^j + I_t^j\right)^{\alpha} - w_{t+1}N_{t+1}^j \\ & -(1+i_{t+1})I_t^j + Q_{t+1}^j(1-\delta)K_{t+1}^j \Big\}. \end{aligned} \tag{11}$$

To ensure that $I_t^j > 0$ in the optimal solution to Equation (11), Magill and Quinzii (2003: p. 244) show that the following inequality and equations ought to hold:

$$0 \le V_{t+1}^j \le (1-\delta)^2 K_t^j, \tag{12}$$

$$w_{t+1} = (1-\alpha)\left(k_{t+1}^j\right)^{\alpha}, \; k_{t+1}^j \equiv \frac{K_{t+1}^j}{N_{t+1}^j}, \tag{13}$$

$$\delta + i_{t+1} = \alpha\left(k_{t+1}^j\right)^{\alpha-1}. \tag{14}$$

The inequalities and equations of Magill and Quinzii's (2003) stock market model presented here come rather close to providing micro-foundations for the aggregate investment function in Farmer and Kuplen's (2018) and Farmer's (2022) Diamond-type OLG model. On the other hand, the attentive reader has probably noticed that the stock market model depicted by (9)-(14) describes a position of full employment. Thus, for the full integration of these equations into the following OLG model of involuntary unemployment, a belief function à la Farmer (2020) seems to be needed.

As mentioned above, such a belief function is present in Plotnikov's (2019) intertemporal general equilibrium, which employs a Keynesian search technology and physical capital accumulation within the household sector. Here, as in Farmer's (2020) models, households are not on their labor supply curve. Moreover, households are not two-periods, but infinitely lived.

The intertemporal equilibrium in Plotnikov (2019) is represented by the equations (1)-(4) of Farmer's (2020) model and the following closing equations:

$$C_t = \psi Y_t^P, \psi > 0, \tag{15}$$

$$\frac{Y_t^P}{w_t} = \left(\frac{Y_{t-1}^P}{w_{t-1}}\right)^{\chi} \left(\frac{Y_t}{w_t}\right)^{1-\chi} \exp\left(\varepsilon_t^P\right),\ \varepsilon_t^P \sim N\left(0, \sigma_P^2\right), 0 < \chi < 1, \tag{16}$$

$$w_t = (1-\alpha)\frac{Y_t}{X_t}. \tag{17}$$

In Equation (15), Y_t^P is expected real permanent income and $1-\chi$ measures the speed of adjustment of expected permanent income to new information. The term ε_t^P represents an independent shock to households' beliefs about expected permanent income. $\psi > 0$ is the marginal propensity of consumption out of permanent income. To ensure rational expectations, ψ stands in a specific functional relationship to the other model parameters (see Plotnikov 2019, equation (27)).

Disregarding the need for a belief function in micro-founding inflexible aggregate investment brings us to Miyashita's (2000) adjustment-cost modification of Diamond's (1965) OLG model. Assuming a log-linear intertemporal utility function, Miyashita (2000: p. 60) derives the following saving function s_t:

$$s_t = \frac{\beta}{1+\beta}\frac{w_t}{q_t}, 0 < \beta < 1, \tag{18}$$

where β is the utility discount factor, w_t is the real wage rate and q_t is the real market price of equity shares bought by the younger household.

Assuming a Cobb-Douglas production function $K_t^{\alpha} N_t^{1-\alpha}, 0 < \alpha < 1$, the representative firm maximizes dividends by choosing N_t and I_t as follows:

$$(1-\alpha)k_t^{\alpha} = w_t,\ k_t \equiv \frac{K_t}{N_t}, \tag{19}$$

$$1 + \left(2\gamma I_t / K_t\right) = q_t, \gamma > 0. \tag{20}$$

From Equation (20), there follows a sort of Tobin's (1969) q investment function:

$$I_t = \begin{cases} (2\gamma)^{-1}(q_t - 1)K_t > 0, & \text{if } q_t > 1, \\ 0, & \text{otherwise.} \end{cases} \tag{21}$$

Equation (21) represents the micro-founded aggregate investment function in the adjustment cost version of Diamond's (1965) seminal OLG model. Although the investment quantity is determined, no over-determinacy occurs in Miyashita's (2000) OLG model since the other dynamic variable present, q_t, is determined in intertemporal equilibrium. Miyashita (2000: p. 63) shows that the intertemporal equilibrium dynamics then read as follows:

$$k_{t+1} = \left[(2\gamma)^{-1}(q_t - 1) + 1\right]k_t, \tag{22}$$

$$q_t = \frac{\beta}{1+\beta}(1-\alpha)k_t^{\alpha}k_{t+1}^{-1}. \tag{23}$$

Obviously, there are no further primitives in addition to preference, technology and adjustment costs in the equilibrium dynamics (22) and (23). Moreover, (22) and (23) represent full-employment intertemporal dynamics despite a determinate and micro-founded investment function. Finally, the convex adjustment cost function which ensures determinate optimal investment presupposes full employment, at least in parts of the economy, which clearly contradicts the formation of widespread underemployment (see more extensively Ebel, 1978). Thus, it is questionable whether Miyashita's (2000) approach to micro-found determinate (inflexible) aggregate investment is useful in an OLG model of involuntary unemployment.

3. The Set-Up of the Stock Market OLG Model with Involuntary Unemployment

As in Magill and Quinzii (2003), we consider an economy of infinite horizon which is composed of infinitely lived firms and finitely lived households. In addition to Magill and Quinzii (2003), we also assume an infinitely lived government with a balanced budget from period to period. In each period $t = 0, 1, 2, \cdots$ a new generation, called generation *t*, enters the economy. A continuum of $L_t > 0$ units of identical agents comprises generation *t*.

As in Diamond's (1965) seminal OLG model, and in line with Magill and Quinzii (2003), we assume exogenous growth of the population $g^L > -1$ which implies the following dynamics of population L_t: $L_{t+1} = G^L L_t$, $G^L \equiv 1 + g^L$, $L_0 = \underline{L} > 0$. In addition to Magill and Quinzii (2003), we also assume exogenous growth of labor productivity denoted by $g^a > -1$ which implies the following dynamics of labor productivity a_t: $a_{t+1} = G^a a_t$, $G^a \equiv 1 + g^a$, $a_0 = \underline{a} > 0$.

Each household consists of one agent and the agent is intergenerationally egoistic: The old agent has no concern for the young agent and the young agent has no concern for the old agent. They live two periods long, namely youth (adult) and old age. In contradistinction to the original Diamond (1965) OLG model and to Magill and Quinzii's (2003) full-employment, stock-market model, in our model economy there are also employed and (involuntarily) unemployed households. All households are endowed with one unit of labor but only the employed households are able to sell it inelastically to firms. In exchange for the labor supply each employed household of generation *t* obtains the real wage rate w_t, which denotes the units of the produced good per unit of labor. Thus, the labor supply in period *t* is not equal to L_t, but only to $(1-u_t)L_t$, where $0 \le u_t < 1$ denotes the unemployment rate. The number of unemployed households (=people) is thus $u_t L_t$. Since the unemployed are unable to obtain any labor income from the market, they are supported by the government through the unemployment benefit ς_t (per household) in each period.

In order to finance the unemployment benefit, the government collects taxes on wages, quoted as a fixed proportion of wage income, $\tau_t w_t h_t$, $0 < \tau_t < 1$. The unemployed do not pay any taxes. Young, employed agents, denoted by superscript *E*, split the net wage income $(1-\tau_t)w_t$ each period between current consumption $c_t^{1,E}$ and savings s_t^E. Savings of the employed are invested in the shares of firms, where a share $\theta_t^{j,E}$ of firm $j = 1,\cdots,J$ in period *t* is bought in the stock market at price Q_t^j by the younger households from the older households. Moreover, the younger households also invest their savings in bonds emitted by firms $j(=1,\cdots,J)$, denoted by $b_{t+1}^{j,E}$, with a rate of return i_{t+1}.

In old age, the employed household sells the shares at the price $Q_{t+1}^{j,E}$ to the then younger household in period $t+1$. The revenues from asset sales and the returns from holding assets one period long, $(1+i_{t+1})\sum_{j=1}^{J} b_{t+1}^{j,E} + \sum_{j=1}^{J} \theta_t^{j,E}\left(D_{t+1}^j + Q_{t+1}^j\right)$, are used to finance retirement consumption $c_{t+1}^{2,E}$, where D_t^j denotes the dividend paid by firm *j* in period *t*. In old age, the previously young employed households consume their gross return on assets: $c_{t+1}^{2,E} = (1+i_{t+1})\sum_{j=1}^{J} b_{t+1}^{j,E} + \sum_{j=1}^{J} \theta_t^{j,E}\left(D_{t+1}^j + Q_{t+1}^j\right)$. This is also true for the unemployed households who finance their retirement consumption through the returns on equity purchases and firm bonds in youth financed by unemployment benefits:
$c_{t+1}^{2,U} = (1+i_{t+1})\sum_{j=1}^{J} b_{t+1}^{j,U} + \sum_{j=1}^{J} \theta_t^{j,U}\left(D_{t+1}^j + Q_{t+1}^j\right)$, where $c_{t+1}^{2,U}$, represents consumption of the unemployed in old age. To keep it all as simple as possible, we assume that the revenues from equity sales and dividends are not taxed.

The typical younger, employed household maximizes the following intertemporal utility function subject to the budget constraints of the active period (i) and of the retirement period (ii):

$$\text{Max} \to \varepsilon \ln c_t^{1,E} + \beta \ln c_{t+1}^{2,E}$$

subject to:

1) $c_t^{1,E} + \sum_{j=1}^{J} b_{t+1}^{j,E} + \sum_{j=1}^{J} Q_t^J \theta_t^{j,E} = w_t(1-\tau_t)$,

2) $c_{t+1}^{2,E} = (1+i_{t+1})\sum_{j=1}^{J} b_{t+1}^{j,U} + \sum_{j=1}^{J} \theta_t^{j,U}\left(D_{t+1}^j + Q_{t+1}^j\right)$.

Here, $0<\varepsilon\leq 1$ depicts the utility elasticity of employed household's consumption in youth and $0<\beta<1$ denotes the subjective future utility discount factor. Needless to say, the intertemporally additive utility function involves the natural logarithm of employed household's consumption in youth weighted by ε, and the natural logarithm of employed household's consumption in old age weighted by $0<\beta<1$.

In order to obtain the first-order conditions for a maximum of the intertemporal utility function subject to the constraints (i) and (ii), we form the following Lagrangian:

$$L_t^E \equiv \varepsilon \ln c_t^{1,E} + \beta \ln c_{t+1}^{2,E} - \lambda_t^E \left(c_t^{1,E} + \sum_{j=1}^{J} b_{t+1}^{j,E} + \sum_{j=1}^{J} Q_t^j \theta_t^{j,E} - w_t \left(1-\tau_t\right) \right)$$
$$- \lambda_{t+1}^E \left(c_{t+1}^{2,E} - \left(1+i_{t+1}\right) \sum_{j=1}^{J} b_{t+1}^{j,E} - \sum_{j=1}^{J} \theta_t^{j,E} \left(D_{t+1}^j + Q_{t+1}^j \right) \right).$$

Differentiating the Lagrangian with respect to $c_t^{1,E}, c_{t+1}^{2,E}, b_{t+1}^{j,E}, \theta_t^{j,E}$, $j=1,\cdots,J$ yields the following first-order conditions for an intertemporal utility maximum:

$$c_t^{1,E} = \frac{\varepsilon}{\varepsilon+\beta}\left(1-\tau_t\right) w_t, \tag{24}$$

$$\frac{D_{t+1}^j + Q_{t+1}^j}{Q_t^j} = 1 + i_{t+1},\ j=1,\cdots,J, \tag{25}$$

$$c_{t+1}^{2,E} = \frac{\beta}{\varepsilon+\beta}\left(1+i_{t+1}\right)\left(1-\tau_t\right) w_t, \tag{26}$$

$$s_t^E = \frac{\beta}{\varepsilon+\beta} w_t \left(1-\tau_t\right),\ s_t^E \equiv \sum_{j=1}^{J} b_{t+1}^{j,E} + \sum_{j=1}^{J} \theta_t^{j,E} Q_t^j. \tag{27}$$

The typical younger, unemployed household maximizes the following intertemporal utility function subject to the budget constraints of the active period (i) and the retirement period (ii):

$$\text{Max} \rightarrow \varepsilon \ln c_t^{1,U} + \beta \ln_t c_{t+1}^{2,U}$$

subject to:

(i) $c_t^{1,U} + \sum_{j=1}^{J} b_{t+1}^{j,U} + \sum_{j=1}^{J} Q_t^J \theta_t^{j,U} = \varsigma_t$,

(ii) $c_{t+1}^{2,U} = \left(1+i_{t+1}\right) \sum_{j=1}^{J} b_{t+1}^{j,U} + \sum_{j=1}^{J} \theta_t^{j,U} \left(D_{t+1}^j + Q_{t+1}^j \right)$.

Again, $0<\varepsilon\leq 1$ denotes the utility elasticity of consumption in unemployed youth, while $0<\beta<1$ depicts the subjective future utility discount factor and ς_t denotes the unemployment benefit per capita unemployed.

Performing similar intermediate steps as above with respect to the younger, employed household yields the following first-order conditions for a constrained intertemporal utility maximum:

$$c_t^{1,U} = \frac{\varepsilon}{\varepsilon+\beta}\varsigma_t, \tag{28}$$

$$\frac{D_{t+1}^{j}+E_{t}Q_{t+1}^{j}\left(s\right)}{Q_{t}^{j}}=1+i_{t+1},\ j=1,\cdots,J, \tag{29}$$

$$c_{t+1}^{2,U}=\frac{\beta}{\varepsilon+\beta}\left(1+i_{t+1}\right)\varsigma_t, \tag{30}$$

$$s_t^U=\frac{\beta}{\varepsilon+\beta}\varsigma_t,\ s_t^U\equiv\sum_{j=1}^{J}b_{t+1}^{j,U}+\sum_{j=1}^{J}\theta_t^{j,U}Q_t^j. \tag{31}$$

All firms are endowed with an identical (linear-homogeneous) Cobb-Douglas production function which reads as follows:

$$Y_t^j=M\left(a_tN_t^j\right)^{1-\alpha}\left(K_t^j\right)^{\alpha},\ j=1,\cdots,J,\ 0<\alpha<1, M>0. \tag{32}$$

Here, Y_t^j denotes production output of firm $j=1,\cdots,J$, $M>0$ stands for total factor productivity (equal for all firms), N_t^j represents the number of employed laborers with firm *j*, with productivity of a_t each, while K_t^j denotes the input of capital services of firm *j*, all in period *t*, and $1-\alpha$ (α) depicts the production elasticity (=production share) of labor (capital) services, also equal for all firms. In line with the seminal paper of Magill and Quinzii (2003), we assume that (physical) capital is durable, depreciates at the rate $0<\delta<1$, and needs to be installed one period before it is used. Thus capital K_t^j used by firm *j* is the capital stock that has been carried over from the period before, i.e. period $t-1$. Moreover, we assume "that capital once installed in a firm cannot be 'unbolted' and transformed back into the homogeneous current output or transferred to another firm, without incurring significant adjustment costs, which for simplicity we take to be infinite" (Magill & Quinzii 2003: p. 242). As a consequence, such firm-specific capital has limited value in a resale market. In the extreme, it is completely firm-specific, so that no part of it has a positive value in the second-hand market.

"In such an economy capital accumulation will only take place if the market structure permits firms to be infinitely lived. Invested capital has … value only if the firm retains its identity as income generating unit in the economy. The natural market structure which permits short-lived agents to transfer ownership of long-lived firms from one generation to the next is an equity market for ownership shares of firms" (Magill & Quinzii 2003: p. 243). Consistent with the firm specificity of capital is that each firm is a corporation with an infinite life where ownership shares are transmitted from one generation to the next through the stock market. As already introduced above, Q_t^j denotes the equity price of firm *j* at date *t*.

Firms are owned by the equity holders and are managed so as to maximize the payoff of their current owners. These are the younger households who buy the shares of firm *j* endowed with a capital of $\left(1-\delta\right)K_t^j$, from the older households for the price Q_t^j, and decide on the investment $I_t^j\geq 0$ to be made. Magill and Quinzii (2003: pp. 244-245) show that an investment quantity larger than zero is chosen such that the net present value of the investment is maximized:

$$\underbrace{\max}_{\{I_t^j, N_{t+1}^j\}} \left\{ -I_t^j + \frac{1}{1+i_{t+1}} \left[\left(K_{t+1}^j\right)^\alpha \left(a_{t+1} N_{t+1}^j\right)^{1-\alpha} - w_{t+1} N_{t+1}^j + Q_{t+1}^j \left((1-\delta) K_{t+1}^j \right) \right] \right\}$$

$$\Leftrightarrow \underbrace{\max}_{\{I_t^j, N_{t+1}^j\}} \left\{ -I_t^j + \frac{1}{1+i_{t+1}} \left[\left((1-\delta) K_t^j + I_t^j \right)^\alpha \left(a_{t+1} N_{t+1}^j\right)^{1-\alpha} - w_{t+1} N_{t+1}^j \right.\right. \tag{33}$$

$$\left.\left. + (1-\delta)^2 K_t^j + (1-\delta) I_t^j - V_{t+1}^j \right] \right\}.$$

Here, the equivalence between the first and the second line comes from Magill and Quinzii's (2003: p. 244) insight that in an intertemporal equilibrium shareholders expect an affine (linear) relationship between the expected equity price of non-depreciated capital in period $t+1$, $Q_{t+1}^j \left((1-\delta) K_{t+1}^j \right)$, and non-depreciated capital stock at that time, i.e.:

$$Q_{t+1}^j \left((1-\delta) K_{t+1}^j \right) = (1-\delta)^2 K_t^j + (1-\delta) I_t^j - V_{t+1}^j, \; j = 1, \cdots, J, \; V_{t+1}^j \geq 0, \tag{34}$$

where $V_{t+1}^j, j = 1, \cdots, J$ denotes the discount on the equity price of firm *j* at time $t+1$ due to the non-shiftability of firm *j*'s capital stock.

Maximization of the net present value in the second line of Equation (33) implies the following first-order conditions:

$$\alpha M \left[(1-\delta) K_t^j + I_t^j \right]^{\alpha-1} \left(a_{t+1} N_{t+1}^j \right)^{1-\alpha} = \delta + i_{t+1}, \tag{35}$$

$$M \left[(1-\delta) K_t^j + I_t^j \right]^{\alpha} \left(a_{t+1} N_{t+1}^j \right)^{-\alpha} a_{t+1} = w_{t+1}. \tag{36}$$

Since all firms have the same production function (see Equation (32)) and the capital depreciation rate is the same with all firms, the optimal capital labor ratio will be the same for all firms: $\frac{K_t^j}{a_t N_t^j} = \frac{K_t^{j'}}{a_t N_t^{j'}} = \frac{K_t}{a_t N_t}, \; j \neq j' = 1, \cdots, J$. Moreover, since the number of employed workers is $N_t \equiv \sum_{j=1}^{J} N_t^j = L(1-u_t)$, we can rewrite the profit maximization conditions (35) and (36) as follows:

$$\alpha M \left[K_{t+1} / \left(a_{t+1} L (1-u_{t+1}) \right) \right]^{\alpha-1} = \delta + i_{t+1}, \tag{37}$$

$$(1-\alpha) M \left[K_{t+1} / \left(a_{t+1} L (1-u_{t+1}) \right) \right]^{\alpha} a_{t+1} = w_{t+1}. \tag{38}$$

Finally, the GDP function can be rewritten as follows:

$$Y_t \equiv \sum_{j=1}^{J} Y_t^j = M \left(a_t L (1-u_t) \right)^{1-\alpha} (K_t)^\alpha. \tag{39}$$

As in Diamond (1965), the government does not optimize, but is subject to the following budget constraint period by period:

$$L_t u_t \varsigma_t = \tau_t (1-u_t) w_t L_t, \tag{40}$$

where, for the sake of simplicity, it is assumed that the government does not have any other expenditures than the unemployment benefits and that there is no government debt.

As Magnani (2015: pp. 13-14) rightly states, aggregate investment in Solow's

(1956) neoclassical growth model is not micro-, but macro-founded since it is determined by aggregate savings. The same holds true in Diamond's (1965) OLG model of neoclassical growth where perfectly flexible aggregate investment is also determined by aggregate savings of households. As already mentioned in the literature review above, and as the first-order conditions for optimal investment of younger shareholders (35) and (36) show, optimal investment is indeterminate and thus also perfectly flexible in the stock market model of Magill and Quinzii (2003). This is most easily seen if we rewrite Equations (37) and (38) as follows:

$$\alpha M\left(\frac{K_{t+1}}{a_{t+1}L_{t+1}}\right)^{\alpha-1}\left(1-u_{t+1}\right)^{1-\alpha}=\delta+i_{t+1}, \tag{41}$$

$$\left(1-\alpha\right)M\left(\frac{K_{t+1}}{a_{t+1}L_{t+1}}\right)^{\alpha}\left(1-u_{t+1}\right)^{-\alpha}=w_{t+1}. \tag{42}$$

Equations (41) and (42) do not allow for determination of the optimal firm investment quantity.

Morishima (1977), and more recently Magnani (2015), both deviate from neoclassical growth models in maintaining that an independent investment function is needed to determine the level of investment in intertemporal equilibrium models of involuntary unemployment. The big question, however, is where does this function come from in a general equilibrium model with an active stock market and an explicit firm maximization calculus to determine investment quantities?

In order to provide an answer to this question we recall the no-arbitrage condition between shares and corporation bonds (29) $\left(D_{t+1}^{j}+Q_{t+1}^{j}\right)/Q_{t}^{j}=1+i_{t+1},\ j=1,\cdots,J$, with $D_{t+1}^{j}=M\left(K_{t+1}^{j}\right)^{\alpha}\left(a_{t+1}N_{t+1}^{j}\right)^{1-\alpha}-w_{t+1}N_{t+1}^{j}-\left(1+i_{t+1}\right)I_{t}^{j},\ j=1,\cdots,J$. Respecting the first-order conditions for net present value maximization (35) and (36), and assuming that affine equity price expectations are rational, i.e. equation (34) holds, then we can show, following Magill and Quinzii (2003: p. 247), that $\frac{D_{t+1}^{j}+Q_{t+1}^{j}}{Q_{t}^{j}}=\frac{K_{t+1}^{j}\left(\delta+i_{t+1}\right)-\left(1+i_{t+1}\right)I_{t}^{j}+\left(1-\delta\right)K_{t+1}^{j}-V_{t+1}^{j}}{\left(1-\delta\right)K_{t}^{j}-V_{t}^{j}}$, if and only if

$$V_{t+1}^{j}=\left(1+i_{t+1}\right)V_{t}^{j},\ j=1,\cdots,J,\forall t\geq 0. \tag{43}$$

In line with Farmer (2013), and in addition to Magill and Quinzii's (2003) affine equity price expectations (34), we may now suggest that shareholders form beliefs with respect to the expected equity price divided by the real wage at period $t+1$ over time. Thus: $\Phi_{t}^{J}=Q_{t+1}^{j}/w_{t+1},\ j=1,\cdots,J$, whereby Φ_{t}^{j} depicts the beliefs of investors with respect to the rationally expected equity price of corporation $j\left(=1,\cdots,J\right)$ in period $t+1$ relative to the common wage rate (Farmer, 2013: p. 328). The beliefs are determined by the function $\Phi_{t}^{j}=\left(\Phi_{t-1}^{j}\right)^{\chi}\left(Q_{t}^{j}/w_{t}\right)^{1-\chi}\exp\left(s_{t}^{b}\right),\ s_{t}^{b}\sim N\left(0,\sigma_{b}^{2}\right)$. Unfortunately, however, and contrary to initial thinking, this specification of the belief function is not compatible with the intertemporal equilibrium equations presented thus far. This led us to attempt an adaptation of Plotnikov's (2019) proposal for closure (in his indeterminate intertemporal equilibrium model) to our model above. Since the net value maximizing investment quantity of firm *j* is indeterminate, it appears reasonable to relate firm investment to permanent production (similar to consumption being proportional to permanent income as in Plotnikov (2019)). Thus, $I_t^j = \Phi^j Y_t^{P,j}$, $\Phi^j > 0$ with $Y_t^{P,j}$ denoting permanent production of firm *j* in period t and the belief function $Y_t^{P,j}/w_t = \left(Y_{t-1}^{P,j}/w_{t-1}\right)^{\chi}\left(Y_t^j/w_t\right)^{1-\chi}\exp\left(s_t^b\right)$, $s_t^b \sim N\left(0,\sigma_b^2\right)$. However, even this specification turned out to be incompatible with the intertemporal equilibrium equations presented above. As a consequence, the only specification of a belief function which turned out to be consistent with the other intertemporal equilibrium equations was that of Farmer and Kuplen (2018: p. 9) which stated that firm *j's* investment quantity is equal to an exogenous, time-stationary constant Φ^j, $j = 1,\cdots,J$, and reflects "Keynesian investors' animal spirits" (Magnani, 2015: p. 14):

$$I_t^J = \Phi^j,\ j = 1,\cdots,J. \tag{44}$$

In addition to the restrictions imposed by household and firm optimizations and by the government budget constraint, markets for labor, firm bonds, and equity, ought to clear in all periods (the market for the output of production is cleared by means of Walras' law[2]).

$$L_t\left(1-u_t\right) = \sum_{j=1}^{J} N_t^j = N_t, \forall t. \tag{45}$$

$$L\left(1-u_t\right)b_{t+1}^E + Lu_t b_{t+1}^U = \sum_{j=1}^{J} b_{t+1}^j,\ \forall t. \tag{46}$$

The demand of the younger employed and the unemployed households for firm bonds (left-hand side of Equation (46)) balances with their supply (right-hand side of Equation (46)). Firms finance their investments by the sales of bonds:

$$\sum_{j=1}^{J} I_t^j = \sum_{j=1}^{J} b_{t+1}^j,\ \forall t. \tag{47}$$

The shares of employed and unemployed younger households sum to unity:

$$L\left(1-u_t\right)\theta_t^{j,E} + Lu_t\theta_t^{j,U} = 1,\ j = 1,\cdots,J,\ \forall t. \tag{48}$$

The sales of equity shares by employed and unemployed older households are equal to the share purchases of employed and unemployed younger households:

$$L_{t-1}\left(1-u_{t-1}\right)\theta_{t-1}^{j,E} = L_t\left(1-u_t\right)\theta_t^{j,E},\ j = 1,\cdots,J,\ \forall t, \tag{49}$$

$$L_{t-1}\left(1-u_{t-1}\right)\theta_{t-1}^{j,U} = L_t\left(1-u_t\right)\theta_t^{j,U},\ j = 1,\cdots,J,\ \forall t. \tag{50}$$

Using the definition of savings for younger employed households in (27) and younger unemployed households in (31), together with the bond market clearing condition (46), the investment financing constraint (47) and condition (48), leads us to the following aggregate savings/investment equality:

[2]The proof of Walras' law can be obtained upon request from the author.

$$L_t(1-u_t)s_t^E + L_t u_t s_t^U = \sum_{j=1}^{J} I_t^j + \sum_{j=1}^{J} Q_t^j. \tag{51}$$

On respecting Equation (44) and firm-specific accumulation equation

$$K_{t+1}^j = (1-\delta)K_t^j + I_t^j,\ j = 1,\cdots,J, \tag{52}$$

the following equilibrium equation results:

$$I_t^j = \Phi^j = K_{t+1}^j - (1-\delta)K_t^j,\ j = 1,\cdots,J,\ \forall t. \tag{53}$$

Equation (53) does not appear in Magill and Quinzii's (2003) stock market model, since they assume full employment of the labor force, which is equivalent to $u_t = 0, \forall t$ in our model. For $u_t > 0$ and u_t being endogenous, equation (53) features as the equilibrium condition which makes the whole set of intertemporal equilibrium equations determinate. In contrast to Morishima (1977: pp. 117-119) and Magnani (2015: p. 14), inflexible firm-specific and aggregate investment is not simply assumed to be macro-founded but turns out to be consistent with an indeterminate, market-value maximizing investment quantity of firm *j*. In this restricted sense, we are entitled to claim that inflexible investment is micro-founded in our modified stock market model of involuntary unemployment.

4. Intertemporal Equilibrium

To start with, assume in line with Magill and Quinzii (2003: p. 249) a balanced-growth intertemporal equilibrium in which firms exhibit at all times the same relative sizes and stock market values. Then, consider initial conditions $(K_0^j, V_0^j) = \nu_j (K_0, V_0)$ with $\nu_j > 0$ and $\sum_{j=1}^{J} \nu_j = 1$. If, for the sequence of (real) wage and interest rates $(w_t, i_{t+1})_{t\ge 0}$, aggregate discounts $(V_t) \ge 0$ and employment-investment decisions $(N_t, I_t)_{t\ge 0}$ satisfy the Equations (34)-(36), (43), (52) and (53), then $(V_t^j, N_t^j, I_t^j) = \nu_j (V_t, N_t, I_t)$ also satisfy Equations (34)-(36), (52) and (53), such that for each firm (N_t^j, I_t^j) is market-value maximizing, its market value is larger than zero, and the return on equity equals i_{t+1}. Hence, the optimal choices of individual firms can be depicted by the market-value maximizing choice of aggregate employment and capital.

Acknowledging the linear-homogeneity of firm production functions (32) and the underemployment equilibrium condition (45), we can switch to aggregate capital per efficient labor $k_t \equiv K_t/(a_t L_t)$ quantities, and rewrite the first-order conditions (41) and (42) as follows:

$$\alpha M (k_{t+1})^{\alpha-1} (1-u_{t+1})^{1-\alpha} = \delta + i_{t+1}, \tag{54}$$

$$(1-\alpha) M a_{t+1} (k_{t+1})^{\alpha} (1-u_{t+1})^{-\alpha} = w_{t+1}. \tag{55}$$

As a next step, the aggregate version of equation (52) is solved for I_t and inserted into the savings/investment equality (51). Assuming that affine equity price expectations (34) also prevailed in period *t*, the savings/investment equality can be rewritten as follows:

$$L_t\left(1-u_t\right)s_t^E + L_t u_t s_t^U = K_{t+1} - \left(1-\delta\right)K_t + \left(1-\delta\right)K_t - V_t = K_{t+1} - V_t. \quad (56)$$

Next, insert into Equation (56) the optimal savings functions (27) and (31) and the government balanced budget condition (40):

$$\begin{aligned} & L_t\left(1-u_t\right)\sigma w_t\left(1-\tau_t\right) + L_t u_t \sigma \varsigma_t \\ & = L_t\left(1-u_t\right)\sigma w_t\left(1-\tau_t\right) + L_t\left(1-u_t\right)\sigma w_t \tau_t \\ & = L_t\left(1-u_t\right)\sigma w_t = K_{t+1} - V_t,\ \sigma \equiv \beta/\left(\varepsilon+\beta\right). \end{aligned} \quad (57)$$

Inserting into Equation (57) the first-order condition (55) for *t*, and dividing the resulting equation on both sides by $a_t L_t$, we obtain:

$$\begin{aligned} & \left(L_t\left(1-u_t\right)\sigma a_t\left(1-\alpha\right)M\left(k_t\right)^{\alpha}\left(1-u_t\right)^{-\alpha} = K_{t+1} - V_t\right)\frac{1}{a_t L_t} \\ & \Leftrightarrow \left(1-\alpha\right)\sigma M\left(k_t\right)^{\alpha}\left(1-u_t\right)^{1-\alpha} = \frac{K_{t+1}}{a_{t+1}L_{t+1}}\frac{a_{t+1}L_{t+1}}{a_t L_t} - \frac{V_t}{a_t L_t} \\ & = k_{t+1}G^n - v_t,\ G^n \equiv \frac{a_{t+1}L_{t+1}}{a_t L_t},\ v_t \equiv \frac{V_t}{a_t L_t}. \end{aligned} \quad (58)$$

By using the capital-output ratio $\kappa_t \equiv K_t/Y_t = K_t\Big/\left[Ma_t L_t\left(1-u_t\right)^{1-\alpha}\left(k_t\right)^{\alpha}\right] = \left(k_t\right)^{1-\alpha}\Big/\left[M\left(1-u_t\right)^{1-\alpha}\right]$ or $k_t = M^{1/(1-\alpha)}\left(\kappa_t\right)^{1/(1-\alpha)}\left(1-u_t\right)$, Equation (58) can be transformed into Equation (59):

$$\begin{aligned} & \left(1-\alpha\right)\sigma M^{1/(1-\alpha)}\left(\kappa_t\right)^{\alpha/(1-\alpha)}\omega_t \\ & = G^n M^{1/(1-\alpha)}\left(\kappa_{t+1}\right)^{1/(1-\alpha)}\omega_{t+1} - v_t,\ \omega_t \equiv 1-u_t,\ \forall t. \end{aligned} \quad (59)$$

Equation (59) represents the first difference equation of the intertemporal equilibrium in our stock-market model of involuntary unemployment.

The second dynamic equation results from summing Equation (53) over all firms and dividing the resulting equation on both sides by $a_t L_t$:

$$\begin{aligned} & \frac{\sum_{j=1}^{J}\Phi^j}{a_t L_t} \equiv \phi = G^n k_{t+1} - \left(1-\delta\right)k_t \\ & = G^n M^{1/(1-\alpha)}\left(\kappa_{t+1}\right)^{1/(1-\alpha)}\omega_{t+1} - \left(1-\delta\right)M^{1/(1-\alpha)}\left(\kappa_t\right)^{1/(1-\alpha)}\omega_t. \end{aligned} \quad (60)$$

The third equilibrium-dynamics equation pops up when Equation (43) is divided on both sides by $a_t L_t$, and when the definition of v_t and the first-order condition (54) are used:

$$\begin{aligned} & G^n v_{t+1} = \left[1-\delta+\alpha M\left(k_t\right)^{\alpha-1}\left(1-u_t\right)^{1-\alpha}\right]v_t \\ & = \left(1-\delta+\alpha/\kappa_t\right)v_t,\ \text{with } 0 \le v_{t+1} \le \left(1-\delta\right)^2 k_t. \end{aligned} \quad (61)$$

The three-dimensional dynamic system (59)-(61) can be reduced to two dimensions by solving equation (59) for $G^n M^{1/(1-\alpha)}\left(\kappa_{t+1}\right)^{1/(1-\alpha)}\omega_{t+1}$, inserting the result into Equation (60) and solving the resulting equation for ω_t:

$$\omega_t = 1 - u_t = \frac{\phi - v_t}{M^{1/(1-\alpha)}\left[(1-\alpha)\sigma(\kappa_t)^{\alpha/(1-\alpha)} - (1-\delta)(\kappa_t)^{1/(1-\alpha)}\right]}. \tag{62}$$

Reinserting (62) for t and $t + 1$ into Equation (60), generates the following two-dimensional dynamic system:

$$\begin{aligned} &\frac{G^n(\kappa_{t+1})^{1/(1-\alpha)}(\phi - v_{t+1})}{(1-\alpha)\sigma(\kappa_{t+1})^{\alpha/(1-\alpha)} - (1-\delta)(\kappa_{t+1})^{1/(1-\alpha)}} \\ &= \phi + \frac{(1-\delta)(\kappa_t)^{1/(1-\alpha)}(\phi - v_t)}{(1-\alpha)\sigma(\kappa_t)^{\alpha/(1-\alpha)} - (1-\delta)(\kappa_t)^{1/(1-\alpha)}}, \end{aligned} \tag{63}$$

$$G^n v_{t+1} = (1-\delta+\alpha/\kappa_t)v_t, \text{ with } 0 \le v_{t+1} \le (1-\delta)^2 M^{1/(1-\alpha)}(\kappa_t)^{1/(1-\alpha)}(1-u_t), \tag{64}$$

whereby $\kappa_0 = \underline{\kappa} > 0$ and $v_0 = \underline{v} > 0$, $\underline{\kappa}$ and $\underline{v}$ exogenously given.

5. Existence of Steady States

The steady states of the equilibrium dynamics depicted by the difference Equations (63) and (64) are defined as $\lim_{t\to\infty} v_t = v$ and $\lim_{t\to\infty} \kappa_t = \kappa$. Due to the relative simplicity of the dynamic system (63) and (64) explicit steady-state solutions are possible. As in Magill and Quinzii (2003), there are two different steady-state solutions of the equilibrium dynamics (63) and (65): 1) The zero-discount, or so-called Diamond-solution $\kappa_D > 0$ and $v_t = v = 0, \forall t$, and 2), the positive-discount steady state $\kappa > 0$ and $v_0 = \underline{v} > 0$, $v_t > 0, \forall t$, and $v > 0$. Here we focus on solution (2). This leads us to the following proposition 1:

Proposition 1. Suppose that $G^n/\left[G^n - (1-\delta)\right] > (1-\alpha)\sigma/\alpha$ and $\phi < M^{\frac{1}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}\left[G^n - (1-\delta)\right]^{\frac{-\alpha}{1-\alpha}}$. Then, the following steady solution for $(\kappa, v) > 0$ and $0 < u < 1$ exists:

$$\kappa = \frac{\alpha}{G^n - (1-\delta)}, \tag{65}$$

$$v = \phi\left\{\frac{G^n}{G^n - (1-\delta)} - \frac{(1-\alpha)\sigma}{\alpha}\right\}, \tag{66}$$

$$u = 1 - \frac{\phi}{M^{1/(1-\alpha)}\kappa^{1/(1-\alpha)}\left[G^n - (1-\delta)\right]}. \tag{67}$$

Remark: Rearranging the steady-state solution (65) brings forth: $1 - \delta + \alpha/\kappa = 1 + i = G^n$. This means that the positive-discount steady state features the so-called Golden Rule intertemporal consumption allocation which is long-run efficient.

6. Dynamic Stability of the Positive-Discount Steady State

The next step is to investigate the local dynamic stability of the unique, positive-discount, steady-state solution. To this end, the intertemporal equilibrium Equations (63) and (64) are totally differentiated with respect to $\kappa_{t+1}, v_{t+1}, \kappa_t, v_t$.

Then, the Jacobian matrix $J(\kappa, v)$ of all partial differentials with respect to κ_t and v_t is formed as follows:

$$J(\kappa, v) \equiv \begin{bmatrix} \frac{\partial \kappa_{t+1}}{\partial \kappa_t}(\kappa, v) & \frac{\partial \kappa_{t+1}}{\partial v_t}(\kappa, v) \\ \frac{\partial v_{t+1}}{\partial \kappa_t}(\kappa, v) & \frac{\partial w_{t+1}}{\partial v_t}(\kappa, v) \end{bmatrix}, \tag{68}$$

with

$$\frac{\partial \kappa_{t+1}}{\partial \kappa_t} \equiv j_{11} = 1 - \frac{\alpha}{(1-\alpha)\sigma},$$

$$\frac{\partial \kappa_{t+1}}{\partial v_t} \equiv j_{12} = \frac{\alpha^2}{(1-\alpha)\sigma G^n \phi},$$

$$\frac{\partial v_{t+1}}{\partial \kappa_t} \equiv j_{21} = \frac{\left[G^n - (1-\delta)\right]\phi\left\{\sigma\left[G^n - (1-\delta)\right] - \alpha\left\{G^n + \sigma\left[G^n - (1-\delta)\right]\right\}\right\}}{\alpha^2 G^n},$$

$$\frac{\partial v_{t+1}}{\partial v_t} = 1.$$

Due to the simplicity of the elements of the Jacobian (68), its eigenvalues φ_1 and φ_2 can be directly calculated as follows:

$$\varphi_1 = \frac{1-\delta}{G^n}, \tag{69}$$

$$\varphi_2 = 1 + \frac{G^n - (1-\delta)}{G^n} - \frac{\alpha}{(1-\alpha)\sigma}. \tag{70}$$

Proposition 2. Suppose the assumptions of Proposition 1 and additionally $\frac{(1-\alpha)\sigma}{\alpha} > \frac{G^n}{2G^n - (1-\delta)}$ hold. Then, the eigenvalues φ_1 and φ_2 of the Jacobian (68) at the steady-state solution (65)-(67) are strictly larger than zero and smaller than unity ($0 < \varphi_1 < 1$, $0 < \varphi_2 < 1$) which implies that the equilibrium dynamics in the neighborhood of the steady state is (locally) asymptotically stable.

Proof. Since $0 < \delta < 1$ and $G^n > 1$, $0 < \varphi_1 < 1$ is obvious.

$\phi_2 > 0 \Leftrightarrow \frac{2G^n - (1-\delta)}{G^n} > \frac{\alpha}{(1-\alpha)\sigma} \Leftrightarrow \frac{(1-\alpha)\sigma}{\alpha} > \frac{G^n}{2G^n - (1-\delta)}$ on account of the additional assumption in Proposition 2.

$\phi_2 < 1 \Leftrightarrow \frac{G^n - (1-\delta)}{G^n} - \frac{\alpha}{(1-\alpha)\sigma} < 0 \Leftrightarrow \frac{(1-\alpha)\sigma}{\alpha} < \frac{G^n}{G^n - (1-\delta)}$ on account of the assumption in Proposition 1. Q.E.D.

In other words: the dynamics with initial values $\kappa_0 = \underline{\kappa} > 0$ and $v_0 = \underline{v} > 0$ in the neighborhood of the positive-discount, steady-state solution in our stock market model with involuntary unemployment is non-oscillating and converges towards the steady state as time approaches infinity.

7. Comparative Dynamics of the Steady-State Solution and the Intertemporal Equilibrium Dynamics

Before concluding it is apt to investigate firstly the comparative dynamics of the positive-discount steady state. The effects of infinitesimal, isolated parameter changes on the positive-discount steady-state solution (65)-(67) are summarized in the following Proposition 3.

Proposition 3. Suppose that the assumptions of Propositions 1 and 2 hold. Then, the effects of infinitesimal, isolated changes of main model parameters on the positive-discount steady-state solution (65)-(67) read as follows:

$$\frac{\partial\kappa}{\partial\alpha}=\frac{1}{G^n-(1-\delta)}>0,\ \frac{\partial\kappa}{\partial G^n}=-\frac{\alpha}{\left[G^n-(1-\delta)\right]^2}<0,\quad \frac{\partial\kappa}{\partial\delta}=-\frac{\alpha}{\left[G^n-(1-\delta)\right]^2}<0, \tag{71}$$

$$\frac{\partial v}{\partial\phi}=\left\{\frac{G^n}{G^n-(1-\delta)}-\frac{(1-\alpha)\sigma}{\alpha}\right\}>0,\ \frac{\partial v}{\partial G^n}=\frac{-(1-\delta)\phi}{\left[G^n-(1-\delta)\right]^2}<0,\quad \frac{\partial v}{\partial\delta}=\frac{-G^n\phi}{\left[G^n-(1-\delta)\right]^2}<0,\ \frac{\partial v}{\partial\alpha}=\frac{\phi\sigma}{\alpha^2}>0,\ \frac{\partial v}{\partial\sigma}=\frac{-(1-\alpha)\phi}{\alpha}<0. \tag{72}$$

$$\begin{aligned}&\frac{\partial u}{\partial\phi}=\frac{-1}{M^{1/(1-\alpha)}\kappa^{1/(1-\alpha)}\left[G^n-(1-\delta)\right]}<0,\\&\frac{\partial u}{\partial G^n}=\frac{-\alpha\kappa^{1/(\alpha-1)}M^{1/(\alpha-1)}\phi}{(1-\alpha)\left[G^n-(1-\delta)\right]^2}<0,\\&\frac{\partial u}{\partial\delta}=\frac{-\alpha\kappa^{1/(\alpha-1)}M^{1/(\alpha-1)}\phi}{(1-\alpha)\left[G^n-(1-\delta)\right]^2}<0,\\&\frac{\partial u}{\partial\alpha}=\frac{\kappa^{(2-\alpha)/(\alpha-1)}\phi M^{-1/(1-\alpha)}\left[1-\alpha+\alpha(\log\kappa+\log M)\right]}{(1-\alpha)^2\left[G^n-(1-\delta)\right]^2}>0.\end{aligned} \tag{73}$$

Considering the results of the comparative-dynamics experiment in (71)-(73) one encounters well-known and not so familiar findings. It is well-known from the theory of exogenous growth that a higher capital income share ($d\alpha>0$), a lower natural growth rate $d\left(G^n-1\right)<0$ and a lower capital depreciation rate ($d\delta<0$) increase the capital-output ratio ($d\kappa>0$). Moreover, marginal changes of the saving rate (σ) do not impact the steady-state capital-output ratio. New are the findings with respect to the effects of all parameters on the steady-state discount (see the partial derivatives in (72)). More investors' optimism ($d\phi>0$) and a higher capital income share ($d\alpha>0$) increase the steady-state discount ($dv>0$), while a higher natural growth rate $d\left(G^n-1\right)>0$, a larger capital depreciation rate ($d\delta>0$) and a higher saving rate ($d\sigma>0$) decrease the discount ($dv<0$). Also new and most important for the topic of this paper are the effects of marginal parameter changes on the unemployment rate. Here, the partial derivatives in (73) show that only a larger capital income share

($d\alpha > 0$) increases the unemployment rate ($du > 0$), while more investor's optimism ($d\phi > 0$), a larger natural growth rate ($d\left(G^n - 1\right) > 0$) and a larger depreciation rate ($d\delta > 0$) decrease the steady-state unemployment rate. Notice this typical "Keynesian" result in our neo-classical growth model: more optimistic investors reduce the steady-state unemployment rate. Notice also that an altered saving rate does not change the steady-state unemployment rate.

Thus, it remains to see whether and if yes how the saving rate impacts the unemployment rate along the intertemporal equilibrium path towards the new steady state. In order to be able to answer these questions we switch to a numerical specification of our stock market model of involuntary unemployment. The main model parameters are chosen such that the assumptions of Propositions 1 and 2 hold. Moreover, we choose the following "typical"[3] parameter set which accords rather well with medium-term stylized facts regarding the growth rate of gross domestic product, the interest rate, the savings ratio, the investment ratio and the unemployment rate of the global economy averaged over the time period between 1990 and 2020 (see IMF, 2008; 2014; 2020): $G^n = 2.1$, $\beta = 0.5$, $\delta = 0.7$, $\varepsilon = 0.9$, $M = 10$, $\phi = 2.622$. Inserting into the steady-state equations (65)-(67) these parameter values, these equations generate the following steady-state solution: $\kappa = 0.1389$, $v = 0.2497$, $u = 0.06$.

Consider now a small positive and unexpected shock on ε from 0.9 towards 0.91 implying a small decrease of the saving rate. Then, the following **Table 1** exhibits the intertemporal equilibrium path of main endogenous variables towards the new steady state: $\kappa = 0.1389$, $v = 0.2688$, $u = 0.06$.

A glance on **Table 1** reveals that a small reduction of the saving rate temporarily reduces the capital-output ratio and the unemployment rate, while the equity price discount increases. After theoretically infinite periods (practically after 80 periods) the capital-output ratio and the unemployment rate return towards the pre-shock values, while the equity price increases. That the unemployment rate temporarily (in the short-term) decreases with a lower saving rate sounds again "Keynesian" in our neo-classical growth model with involuntary unemplyoment.

Starting again from the same steady-state solution as before the saving-rate shock, we increase now the "animal spirits" parameter from $\phi = 2.622$ towards

Table 1. Intertemporal equilibrium path of $\left(\kappa_t, v_t, u_t\right)_{t>1}$ after a small negative saving-rate shock.

t	0	1	2	3	4	5	6	…	40
κ_t	0.1389	0.1380	0.1379	0.1380	0.1381	0.1382	0.1382	…	0.1389
v_t	0.2497	0.2497	0.2510	0.2524	0.2538	0.2545	0.2561	…	0.2688
u_t	0.06	0.0520	0.0516	0.0523	0.0529	0.0534	0.0539	…	0.06

Source: Author's own calculation.

[3]Why the chosen parameter set might be called „typical" is discussed more extensively in Farmer (2022).

$\phi = 2.65$: all other parameters remain on their pre-saving-rate-shock values. The effects of this small, positive investment shock on the capital-output ratio, the equity price discount and on the unemployment rate along the intertemporal equilibrium path are depicted in Table 2.

As Table 2 reveals, the positive shock on investment temporarily decreases the capital-output ratio and (rather starkly) the unemployment rate, while the equity price discount increases in the short- and long-term. While the unemployment rate increases again along the intertemporal equilibrium path, it turns out to be lower in the new steady state: A Keynes-like result even in the long run.

Our last shock experiment concerns the natural growth rate (the qualitative impacts of a higher depreciation rate are similar). Starting once more from the steady state before the saving-rate shock and the parameters implying it, we increase the natural growth factor from $G^n = 2.1$ to $G^n = 2.15$. The impacts on the capital-output ratio, the equity price discount and on the unemployment rate along the intertemporal equilibrium path are depicted in Table 3.

A marginally higher natural growth rate decreases temporarily and permanently the capital-output ratio, the equity price discount and the unemployment rate. A similar effect results from a higher depreciation rate.

8. Conclusion

This paper aimed at investigating micro-foundations for inflexible aggregate investment in Diamond-type OLG models with involuntary unemployment. After reviewing some recent attempts to micro-found aggregate investment and involuntary unemployment in intertemporal equilibrium models, it turned out that a

Table 2. Intertemporal equilibrium path of $(\kappa_t, v_t, u_t)_{t>1}$ after a small positive "animal-spirits" shock.

t	0	1	2	3	4	5	6	…	40
κ_t	0.1389	0.1388	0.1387	0.1387	0.1387	0.1388	0.1388	…	0.1389
v_t	0.2497	0.2497	0.2498	0.2501	0.2502	0.2504	0.2506	…	0.2524
u_t	0.06	0.0488	0.0488	0.0489	0.0490	0.0491	0.0491	…	0.0499

Source: Author's own calculation.

Table 3. Intertemporal equilibrium path of $(\kappa_t, v_t, u_t)_{t>1}$ after a small positive natural-growth shock.

t	0	1	2	3	4	5	6	…	40
κ_t	0.1389	0.1358	0.1354	0.1353	0.1353	0.1353	0.1353	…	0.1351
v_t	0.2497	0.2439	0.2428	0.2423	0.2419	0.2416	0.2414	…	0.2379
u_t	0.06	0.0546	0.0536	0.0534	0.0532	0.0531	0.0529	…	0.0514

Source: Author's own calculation.

modification of Magill and Quinzii's (2003) stock market OLG model with full employment seems to be best suited to attribute involuntary unemployment to inflexible aggregate investment. While at first sight promising, Farmer's (2012, 2013) approach to micro-found the empirical correlation between stock market values and the unemployment rate through investor's beliefs about future equity price proved to be inconsistent with the intertemporal equilibrium in Magill and Quinzii's (2003) stock market model. Also, Plotnikov's (2019) use of Friedman's (1957) permanent income hypothesis to close his indeterminate intertemporal equilibrium model could not be applied to investment in the modified Magill and Quinzii (2003) stock-market model. And finally, Miyashita's adjustment-cost micro-foundation of aggregate investment must not be applied to a widespread unemployment situation since convex adjustment costs presume full employment (Ebel, 1978). Thus, the only specification of inflexible investment which is consistent with the intertemporal equilibrium in Magill and Quinzii's (2003) stock market model is that of Farmer and Kuplen (2018) who assume that aggregate investment per efficiency capita is determined by exogenously given "animal spirits". The corresponding model parameter can be seen as a degenerate belief function à la Farmer (2020). Moreover, this belief's determined investment quantity is consistent with optimally indeterminate firm-level investment. In that sense, inflexible aggregate investment is micro-founded in our stock market model with involuntary unemployment.

In contradistinction to Magill and Quinzii's (2003) full employment model, in our model the unemployment rate appears as additional dynamic variable with the consequence that the intertemporal equilibrium dynamics is in principle three- instead of two-dimensional as in Magill and Quinzii (2003). The step-by-step derivation of the intertemporal-equilibrium equations from the first-order conditions for intertemporal utility and market value maxima, the government budget constraint, the degenerate belief function of investors and the market-clearing conditions brings forth that the unemployment rate is not a slowly moving dynamic variable but a sort of a jump variable. Slowly moving or truly dynamic variables are the capital-output ratio and the equity price discount as in Magill and Quinzii (2003), making our intertemporal equilibrium dynamics also two-dimensional. Knowing the intertemporal equilibrium path of these truly dynamic variables, the unemployment rate in each period can in principle be calculated from a combination of the savings/investment and the capital-accumulation equation.

We then investigate the existence of steady-state solutions whereby the capital-output ratio and the equity price discount does not change over time any longer. As in Magill and Quinzii (2003), there are two steady-state solutions: 1) the zero-discount or Diamond steady state and 2) the positive-discount steady state whereby the capital-output ratio accords to the Golden rule of intertemporal consumption allocation: one plus the interest rate equals the natural growth rate. We focus on the second steady state and find in Proposition 1 that a positive-discount steady state exists if the natural growth factor divided by the sum of the natural growth rate plus the depreciation rate is larger than the aggregate saving rate (=wage share times saving rate of younger households) over the capital income share, and the animal-spirits parameter is not too large, made precise in Proposition 1.

In order to be able to perform comparative dynamics of the effects of parameter shocks on main variables we then check the dynamic stability of the equilibrium dynamics in the neighborhood of the positive-discount steady state. We find that local asymptotic stability of the equilibrium dynamics is ensured when the existence condition holds, and the natural growth factor divided by the sum of 2 times the natural growth and the depreciation rate is smaller than the aggregate savings rate over the capital income share. Both eigenvalues are then larger than zero and smaller than unity.

Having proven in Propositions 1 and 2 the existence and dynamic stability of the positive-discount steady state, we are entitled to perform local comparative dynamic experiments whereby we investigate the impacts of infinitesimal changes of the main model parameters on the steady-state capital-output ratio, the equity price discount and on the unemployment rate. We find that a higher capital income share increases the capital-output ratio, while both a higher natural growth rate and a higher depreciation rate decrease the capital-output ratio. In comparison to these well-known responses of the capital-output ratio, the reactions of the equity price discount are more interesting while new: more investor's optimism and a higher capital income share increase the equity price discount, a higher natural growth rate, a larger depreciation rate and a higher saving rate decrease the equity price discount. Most interesting are the responses of the steady-state unemployment rate which increases with a larger capital income share and decreases with higher natural growth, a larger depreciation rate and more investor's optimism. This last result accords well with short-term Keynesian insights, and it turns out to be valid even in the long run.

Completely in accordance with the insights from neo-classical growth theory, variations of the saving rate do neither impact the steady-state capital-output ratio nor the steady-state unemployment rate. Thus, we finally investigate the effects of saving-rate changes on the intertemporal equilibrium path of the capital-output ratio, the equity price discount, and the unemployment rate. Due to the analytical complexity of the algebra of the partial derivatives of these dynamic variables with respect to marginal parameter variations, we resort to a numerical specification of main model parameters which are in line with the assumptions of Propositions 1 and 2 and are representative for "typical" numerical parameter values within this sort of stylized intertemporal equilibrium models. We find that a marginally smaller saving rate temporarily reduces the capital-output ratio and the unemployment rate, while the equity price discount increases. After about 80 periods (theoretically after an infinite number of time periods) the capital-output ratio and the unemployment rate return to their pre-shock values, while the equity price discount permanently increases.

Moreover, we also investigate the intertemporal-equilibrium effects of more investor's optimism and a larger natural growth rate. We find that the former temporarily decreases the capital-output ratio and rather strongly the unemployment rate, while the equity price discount increases in the short and long run. Moreover, the positive investment shock reduces the unemployment rate also in the long run. Finally, a marginally higher natural growth rate decreases temporarily and permanently the capital-output ratio, the equity price discount and the unemployment rate. A similar effect results from a higher depreciation rate.

Obviously, there is ample space for future research. Highest on the agenda in this respect is the search for a non-degenerate belief function which is consistent with intertemporal equilibrium in our modified stock-market model of involuntary unemployment.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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