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Reinterpreting the Sharpe Ratio as a Measure of Investment Return from Alpha

Michele Anelli

Department of Business and Law, School of Economics and Management, University of Siena, Siena, Italy
Email: anelli2@student.unisi.it

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Abstract

This paper examines the fundamental building blocks of the Sharpe ratio to debate over the economic interpretation of this well-known tool used to measure the risk-adjusted performance of various financial portfolios and funds. It focuses on the risk-adjusted expected return of an investment versus a benchmark portfolio (or index) return. By leveraging on a set of statements and assumptions, I isolate the information content of the ratio as expression of the investment return from alpha. I finally derive that, under the efficient market hypothesis (*EMH*) or perfectly diversified portfolios, the Sharpe ratio is zero.

Keywords

Sharpe Ratio, Alpha Return

1. Introduction and Literature Review

Traditionally Sharpe, Treynor and Jensen's ratios are used to measure the performance of various financial portfolios (Mistry & Shah, 2013). Measures of risk adjustment and performance evaluation enable portfolio managers and traders to compare the ex-ante or ex-post returns associated with different levels of risk. The main and widely used measure of historic or potential risk-adjusted performance is the Sharpe Ratio (Sharpe, 1966). It is built on Markowitz' mean-variance paradigm, which assumes that the mean and standard deviation of the distribution of one-period return is sufficient statistics for evaluating the prospects of an investment portfolio. The Sharpe ratio is indeed the ratio of the excess realized or expected return of an investment versus a benchmark portfolio or a risk-free rate (numerator) to its return standard deviation over the same period of time (denominator). The denominator represents, according to the financial community, a quantitative measure of volatility (i.e., risk). The traditional Sharpe ratio based on realized returns and volatility is an indicator of a fund's historic risk-adjusted performance over a defined time horizon, while the ratio based on the expected returns and volatility is an indicator of a fund's potential risk-adjusted performance over a future time interval. A higher Sharpe ratio indicates a good investment performance, given the associated risk. Many researchers, however, focused on the limitations of this ratio, particularly on the estimation of potential risk-adjusted fund's performance without questioning the information content of the fundamental building blocks of the ratio. Lo (2002) states that the accuracy of Sharpe ratio estimators hinges on the statistical properties of the expected returns and volatilities. Since they are unknown variables, they must be estimated statistically and, therefore, are subject to estimation error. These statistical properties, moreover, can vary considerably among portfolios, strategies, and over time. The author highlights that the timeseries properties of investment strategies (e.g., mean reversion, momentum, and other forms of serial correlation) can have a nontrivial impact on the Sharpe ratio estimator itself. And of the same opinion is Kourtis (2016), according to which investors often adopt mean-variance efficient portfolios for achieving superior risk-adjusted returns, but such portfolios are sensitive to estimation errors, which affect portfolio performance. Dowd (2000) derives an operational decision rule that enables a manager to correctly assess alternative investment opportunities or past investment decisions using an improved Sharpe ratio. If returns are normal, the standard Sharpe ratio gives the correct result if the investments being considered are independent of the rest of our portfolio but cannot be relied upon otherwise (it can be altered by investments that do not have normally distributed returns). In the latter case, the author suggests the so-called "generalized Sharpe ratio" because it is valid regardless of the correlations of the investments being considered with the rest of the portfolio. Goetzmann et al. (2002) stress the weakness points of the Sharpe ratio and other related reward-to-risk measures that may be manipulated with option-like strategies, widely spread in the hedge fund industry. The evident limitations of the ratio are acknowledged by Sharpe himself (Sharpe, 1994). He indeed not only recognizes the limitations of his own measure (especially as it relates to multi-period investments), but offers some variations to adjust for these shortcomings, but these adjustments may not have been adequate (Muralidhar, 2015).

Although the classic Sharpe ratio becomes a questionable tool for constructing optimal portfolios if the assumption of normality in return distributions is relaxed, it is a fundamental tool with significant theoretical implications. Differently from the cited literature, I conduct my investigation by analyzing the nature and economic meaning of the ratio components rather than focusing on its statistical property limitations and optimal estimation model to exploit the information content of the ratio. The theoretical idea is coherent for other measures of risk-adjusted investment performance, but the focus is on the Sharpe ratio. What does a Sharpe Ratio really tell us? Muralidhar (2015) asks this question and demonstrates that the Sharpe Ratio effectively only informs the user about the time needed to determine how skillful a manager may be in beating either the risk-free rate or a benchmark. According to us, this is the focal point. Dalio (2011) claims that three basic building blocks of all returns are the risk-free return (i.e., whatever rate best neutralizes risks), returns from betas (i.e., the excess returns of asset classes over the risk-free return) and returns from alphas (i.e., the value added by managers, which is derived from managers deviating from the betas). As a rough approximation, I derive that the ratio is a measure of alpha per unit of risk, provided that the numerator of the ratio is calculated versus a perfectly diversified benchmark portfolio (or index) return. "Alpha" is indeed the most well-known measure of the abnormal return on an investment (Ferson et al., 2014). This abnormal return is the excess return of an investment over the beta return (which is in turn the excess return of an investment over the risk-free return). If we accept and interpolate the statements of Muralidhar (2015), Dalio (2011) and Ferson et al. (2014), we can finally derive that the Sharpe ratio is the value added by managers per unit of risk, which is obtained from managers deviating from the betas. The numerator of the Sharpe ratio is indeed a quantitative measure of alpha. Sharpe (1966) himself states, indeed, that different funds could exhibit different degrees of variability in ex-post return due to the inability of some managers to select incorrectly priced securities and/or to diversify their holdings properly. But is the denominator of the ratio a quantitative measure of risk? Everyone acknowledge it is an approximate quantitative tool. Volatility (i.e., the standard deviation of returns) is not a perfect measure of risk. Risk is the probability of a bad outcome. Volatility is, at best, an indicator of the presence of risk (Marks, 2022). According to Marks (2022), the Sharpe ratio may hint at risk-adjusted performance in the same way that volatility hints at risk, but since volatility isn't risk, the Sharpe ratio is a very imperfect measure. However, risk adjustment is an essential concept and returns should absolutely be evaluated relative to the risk that was taken to achieve them. I agree with the Marks's idea, but I also acknowledge it is a handy quantitative tool available for measuring risk. On the ground of this theoretical concept, I suggest an enticing interpretation of the Sharpe ratio, under the hypothesis of normally distributed returns and independent investments. The theoretical idea supports Zakamulin (2011). The paper takes as its objective to stimulate discussion and raise questions over the economic interpretation and practical implications of the Sharpe ratio rather than to provide answers and an estimation model. The suggested theoretical interpretation raise serious questions concerning the theory of random walks (i.e., on the general behavior of stock-market prices) and emphasize the need to take into account the skill of investment managers in the valuation of securities.

2. Reinterpreting the Sharpe Ratio: Theoretical Implications

Sharpe (1994) defines the ex-ante and ex-post Sharpe Ratio as follows, respectively, in section (2.1) and (2.2).

2.1. The Ex-Ante Sharpe Ratio

Let R_F^e represent the expected return on fund F in the forthcoming period and R_B^e the expected return on a benchmark portfolio. Define d^e the expected differential return, as:

$$d^e = R_F^e - R_B^e \tag{1}$$

Let σ_d^e be the predicted standard deviation of d^e. The ex-ante (or potential) Sharpe Ratio (S^e) is:

$$S^e = \frac{d^e}{\sigma_d^e} \tag{2}$$

The ratio indicates the expected differential return per unit of risk associated with the differential return.

2.2. The Ex-Post Sharpe Ratio

Let R_F represent the realized return on fund F in period *t* and R_B the realized return on a benchmark portfolio in the same period *t*. Define d_t, the realized differential return in period *t*, as:

$$d_t = R_{F_t} - R_{B_t} \tag{3}$$

Let d-bar ($\bar{d}$) be the average value of *d* over the historic period from *t* = 1 through *T*:

$$\bar{d} = \frac{1}{T}\sum_{t=1}^{T}\left(R_{F_t} - R_{B_t}\right) \tag{4}$$

Let σ_d be the realized standard deviation of *d* over the historic period from *t* = 1 through *T*. The ex-post (or historic) Sharpe Ratio (S_h) is:

$$S_h = \frac{\bar{d}}{\sigma_d} \quad \text{with} \quad \sigma_d = \frac{1}{T-1}\sqrt{\sum_{t=1}^{T}\left(d_t - \bar{d}\right)^2} \tag{5}$$

The ratio indicates the historic average differential return per unit of historic variability of the differential return.

2.3. Reinterpreting the Sharpe Ratio: A Measure of Investment Return from Alpha

Let focus on the ex-ante version of the Sharpe ratio (S^e)[1]. According to my interpretation of statements interpolation of Muralidhar (2015), Dalio (2011) and Ferson et al. (2014), let me define the expected differential return d^e, as:

$$d^e = R_F^e - R_B^e = \alpha^e \tag{6}$$

The expected differential return d^e is the expected return from alpha α^e, that is the expected value added by fund F's managers deviating from the expected beta in the forthcoming period.

[1]Expected returns are arguably the most important input into investment decisions. Many investors determine their expectations for returns on investments in highly subjective ways, based on discretionary views. More objective predictions are anchored on historical experience, financial theories, and observation of prevailing market conditions (Ilmanen, 2012).

Let $\sigma_d^e = \sigma_\alpha^e$ be the predicted standard deviation of α^e. It is the so-called "tracking error". The ex-ante Sharpe Ratio (S^e) is:

$$S^e = \frac{d^e}{\sigma_d^e} = \frac{\alpha^e}{\sigma_\alpha^e} \tag{7}$$

The ratio therefore indicates the expected return from alpha per unit of alpha risk (with the latter measured by the tracking error).

Let assume (for descriptive and theoretical simplicity) one-single security (e.g., fund F), returns are normal, the investments are independent of the rest of our portfolio and consider the one-period dimension. By exploiting the Equation (5), (6) and (7), let derive the fund F's expected return from alpha in the forthcoming period, as:

$$\alpha^e = E[\alpha_T] = d_t\phi(S_h) = \alpha_t\phi(S_h) \quad \text{with} \quad S_h \sim N(0,1), \quad \phi(S_h) = \frac{1}{\sqrt{2\pi\sigma^2}} \mathrm{e}^{-\frac{1}{2}S_h^2} \tag{8}$$

Equation (8), under the specified hypothesis, estimates the expected return from alpha of a fund F by leveraging on the fundamental building blocks of the ex-post (or historic) Sharpe ratio. Since what portfolio managers and traders risk is, in the worst case scenario (e.g., investment default probability is one), their invested capital C (with $C = 1$, since returns are expressed in percentage points) in fund F, α^e represents thus the expected return from alpha per unit of invested capital. Equation (7), therefore, gets (in turn) the ex-ante (or potential) Sharpe ratio, as:

$$S^e = \alpha^e = \alpha_t\phi(S_h) \tag{9}$$

Under the latter (strong) assumption, it is possible to observe that the Sharpe ratio is a pure measure of alpha return from fund F. Equation (9) proves that, if we assume a perfectly diversified fund F, the Sharpe ratio is zero (since there is no alpha return). In other terms, under the efficient market hypothesis (Fama, 1970), there is no alpha from the investment, The Sharpe ratio is zero by definition.

Let specify that, if the numerator of the ratio is calculated versus a risk-free rate rather than a benchmark portfolio (or index) return, the Sharpe ratio is a measure of alpha plus beta returns from fund F per unit of (total) risk.

3. Conclusion

This paper investigates the economic interpretation of the most used tool for measuring the risk-adjusted performance of financial portfolios and funds, i.e. the Sharpe ratio (Sharpe, 1966; Sharpe, 1994). Many studies focus on its statistical property limitations and optimal estimation model rather than stressing the information content of the ratio. The latter aspect is the main goal of this study. Under a specific set of assumptions (normally distributed returns and independent investments) and set of statements (Muralidhar, 2015; Dalio, 2011; Ferson et al., 2014), I suggest a simple and enticing interpretation of the Sharpe ratio as a measure of investment return from alpha, provided that the numerator of the ratio is calculated versus a benchmark portfolio (or index) return. The theoretical idea supports Zakamulin (2011). I finally derive that, under the efficient market hypothesis (Fama, 1970) or perfectly diversified portfolios, the Sharpe ratio is zero. The paper takes as its objective to stimulate discussion and raise questions over the economic interpretation of the Sharpe ratio rather than to provide answers and an estimation model. The suggested theoretical interpretation indeed raises serious questions concerning the theory of random walks (i.e., on the general behavior of stock-market prices) and emphasize the need to take into account the skill of investment managers in the valuation of securities.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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