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The Contribution of the Scholar and Biblical Commentator Abraham ibn Ezra in the Transfer of Advances in Mathematics from Islamic Spain to Christian Europe

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Abstract

In this paper we will present some of the mathematical achievements of Abraham ibn Ezra, to which he was not given proper credit or credit of being the first to introduce these mathematical concepts to Europe. As a result, we will correct to a degree the history of mathematics. It is very possible that Abraham ibn Ezra did not achieve general recognition for his work as his books were written in Hebrew and never translated into Latin. However, he still deserves recognition for this.

Keywords

Sefer ha-Mispar, Abraham ibn Ezra, Abraham bar Hiyya ha-Nasi, Liber Abbaci, The Transfer of Mathematical Learning from Moslem Spain to Christian Europe

1. Introduction

At the beginning of the eighth century the Umayyad Moslems conquered the Iberian Peninsula (Spain, Portugal, Andora and Gibraltar) and called it Al-Andalus (hereafter "Spain"). Their rule continued until the 15th century.

One of the effects of the Moslem rule was the introduction of Islamic culture into the European continent.

The Persian scholar Muhammad ibn Musa al-Khwarizmi, whose name gave rise to the term's algebra and algorithm, was active mainly in Bagdad and wrote several scientific books on various subjects. The Islamic rulers saw an importance in preserving and spreading scientific advances and knowledge in the

Arabic language. Al-Ma'mun the ruler of the Moslem Empire at the beginning of the 9th century encouraged and financed the translation of existing scientific works into Arabic. He built the House of Wisdom the world's largest library at the time and supported groups of scholars to translate and publish in Arabic.

At the beginning of the twelfth century there were two Jewish scholars by the name of Abraham that mastered the Islamic scientific progress in Spain and served as a bridge to transmit this knowledge from the Islamic countries to Christian Europe (hereafter "Europe").

The first one, Abraham bar Hiyya ha-Nasi, occupied a high position in the court of Alfonso the first in Barcelona. He was also known as Abraham Savasorda which is a Latinisation of the Arabic Sahib al-Shurta which means chief of the police. He was an expert in astronomy, mathematics and philosophy and was a pioneer in the writing of scientific works in the Hebrew language.

One of his principal works was Hibbur ha-Meshihah ve-ha-Tishboret (Treatise on measurement and calculation) a treatise on the calculation of areas and mathematical proofs that preceded European scholars. The book became famous and was translated in 1145 to Latin by Plato Tiburtinus and titled in Latin Liber ambadorum. In this textbook there appears for the first time in Christian Europe the mathematical proof of Quadratic equations. As a result this year is known as the "year of birth of European Algebra".
(https://www.encyclopedia.com/science/encyclopedias-almanacs-transcripts-and-maps/robert-chester; Smith, 1951)

In the same year Abraham ibn Ezra published a book on mathematics in Hebrew called Sefer ha-Mispar (The book of numbering). This was the first book published in Europe which included the Hindu-Arabic numeral system—a positional decimal numeral system (a system in which the weight of each digit depends on its distance from the decimal point).

2. The Travels of Abraham ibn Ezra from Spain (Called by the Moslems Al-Andalus) to Europe

Abraham ibn Ezra was born in Spain at the end of the 11th century. In the fifth decade of his life he immigrated to North Africa and later to Italy, France and England. He arrived in Europe around the year 1140 when he was approximately fifty years old.

During the twenty years he lived in Europe he wrote his scientific compositions (24 works) (Sela, 1999) (**Figure 1**).

During his residence in Europe, he wrote books on various topics: Mathematics, Astronomy (astronomical tables), calendars astrology and others.

In this paper we will concentrate on a few original findings from Sefer ha-Mispar that was published in Hebrew in 1145 while Abraham ibn Ezra was in Italy.

This book was published in a Hebrew version with German translation by Moshe Silverberg in 1895.

In this book Abraham ibn Ezra explains the mathematics behind the Hindu–Arabic numeral system which was developed in India in the sixth century,

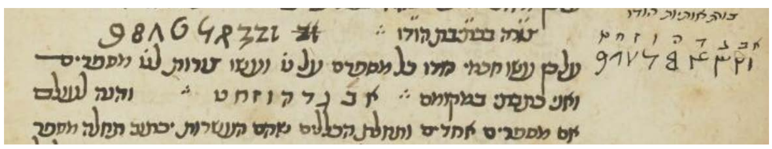
Figure 1. Map of the travels of Abraham ibn Ezra.

and adopted by the Arab world as a result of the work of Muhammad ibn Musa al-Khwarizmi in the 9th century. In his book, Abraham ibn Ezra represents the digits with letters from the Hebrew alphabet.

It was only fifty years later in 1202 that Leonardo Bonacci (known as Fibonacci) published his book on mathematics, Liber abbaci (from the Latin abacus).

It is customary to accredit the introduction of this numerical system into Europe to Fibonacci although Abraham ibn Ezra preceded him. For instance, Eves writes (Eves, 1983):

> "Let us, however, return to work that played an influential role in spreading knowledge and use of the Hindu-Arabic numerical system.
> By far the most influential was a book called the Liber abaci that appeared in Italy in the year 1202. The appearance of this work was a Great Moment in Mathematics…".



A section of a manuscript from Sefer ha-Mispar from the Bibliothèque Nationale de France Ms. Hebr. 1052. (Paris National Library) In this manuscript the author explains the Hindu-Arabic numeral system using Hebrew letters.

3. Examples of the Mathematical Innovations in Sefer ha-Mispar

3.1. The Transfer to Europe of the Digit "Zero" and the Positional Numeral System

In the positional decimal numeral system in which each digit represents it's value times the weight of its position we must invent a new digit to "save the position" in cases that that position has no value. That new digit is zero. It is difficult today to understand but, in that period, it was a revolutionary idea. Abraham ibn Ezra writes in Sefer ha-Mispar:

> "… If there is a unit's number before the general number which is the tens then one should first write the number of units followed by the number of generals (tens). However, if there are no units and there are tens one should place a figure in the shape of a wheel O in the first place in order to signify that there are no units… This symbol, a wheel O is like chaff in the wind, and is there only to save the space (In Latin and Spanish = Cifra)".

One can assume that the circulation of Sefer ha-Mispar was limited because it was written in Hebrew and this new innovation spread slowly until Fibonacci published his book.

3.2. Libra

In this case Abraham ibn Ezra had to invent a new term in Hebrew for this mathematical method and he termed it the Hebrew word for Libra.

Libra is a method of checking the correctness of mathematical calculations.

The method is based on the construction of "small" numbers that represent the original numbers. For this he used the remainder of division (modulo). The method works as follows:

Let us assume that the original calculation is of the following form:

$$\begin{array}{r} \times \begin{array}{r} \text{ABC} \\ \text{DEF} \end{array} \\ \hline \text{GHIJK} \end{array}$$

The test is performed by calculating the modulo of each number. Instead of ABC we use the number ABC(mod x), Instead of DEF we use DEF(mod x) and instead of GHIJK we use GHIJK(mod x).

An example:

$123 \times 456 = 56{,}088$

We choose modulo 7

$123 (\text{mod } 7) = 4$, $456 (\text{mod } 7) = 1$,

And $56{,}088 (\text{mod } 7) = 4$

If the result had not been 4 we could assume that there was a mistake in our calculation. However, if it is correct there is still a probability that there was a mistake equal base (in this case 7. For instance, if we calculated incorrectly and the result was 56,081).

3.2.1. The "Nine" Test

Abraham ibn Ezra uses a special method using modulo base 9.

After the chapter on multiplication the author writes as follows:

And a last test for Libra: For the first number of the calculation add up the digits. Then subtract 9 from the result. Keep doing this until you get a single digit.

Do the same for the second number of the calculation. Now, multiply these two results and then once again subtract 9 from the results until you get a single figure. If one of these two results is 9, it is not necessary to continue the calculations (the final result is also 9). This result should be equal to a similar calculation done on the number of the result. If it is not, a mistake was made in your original calculation.

In the previous example we do these calculations.

123(mod 9) ≡ 1 + 2 + 3 = 6

456(mod 9) ≡ 4 + 5 + 6 = 6

56088(mod 9) ≡ 5 + 6 + 8 + 8 = 0

(the calculation is modulo 9, therefore 5 + 6 + 8 + 8 is equal to 0)

And thus

6 × 6 = 36 ≡ 3 + 6 = 0

The author teaches us that the sum of the digits of the result is equal to the Libra test with modulo 9.

3.2.2. Digital Roots: The Identification of Mistakes Made in Arithmetic Calculations

Using the digital roots, also called repeated digital sum, we ignore the position of the digits and just add up all the digits as if they are all units.

We use the result from the previous iteration to compute a digit sum. The process continues until a single-digit number is reached. The result is identical to modulo 9.

$$-9*10^k = -(10-1)10^k = -10*10^k + 1*10^k = -1*10^{k+1} + 1*10^k$$

where $k = 0, 1, 2\ldots$

Explanation: subtracting 9 from a number is identical to subtracting 10 and adding 1 (that is the moving of 1 from the 10's position to the unit's position). Similarly, subtracting 90 is identical to subtracting 100 and adding 10. That is to say the result of digital roots is identical to modulo 9 of that number.

The digital root $dr(n)$ of an integer n can be computed without actually performing the iteration using the simple congruence formula

$$\begin{aligned} dr(n) &= \begin{cases} n(\bmod 9), & n \neq 0(\bmod 9) \\ 9, & n \equiv 0(\bmod 9) \end{cases} \\ &= 1 + [n - 1(\bmod 9)] \end{aligned}$$

From this we see that the digital root function is equal to modulo 9, the addition of digits is a much simpler calculation than division of the number.

4. Increasing a Number by Its Square

The author presents an interesting method in which he shows that it is simpler to square a number if it is first divided by three.

We must differentiate between three different situations: a number which is divisible by 3 with no remainder, remainder 1, remainder 2.

4.1. A Number Divisible by 3 with No Remainder (mod 3 = 0)

Divide the number by 3, square the result and then subtract from this the square of a third of the original number. The result is the square of the original number.

In mathematical notation:

$$10\left(\frac{x}{3}\right)^2-\left(\frac{x}{3}\right)^2=9\left(\frac{x}{3}\right)^2=\frac{9x^2}{9}=x^2$$

4.2. A number Divisible by 3 with Remainder 1 *x*(mod 3 = 1)

Subtract 1 from the original number. Continue with this new number as in the previous calculation then add the original number doubled minus 1.

In mathematical notation:

$$10\left(\frac{x-1}{3}\right)^2-\left(\frac{x-1}{3}\right)^2+x-1+x=9\left(\frac{x-1}{3}\right)^2+2x-1=\left(\frac{9(x-1)}{9}\right)^2+2x-1$$
$$=(x-1)^2+2x-1=x^2+1-2x+2x-1=x^2$$

4.3. A Number Divisible by 3 with Remainder 2 *x*(mod 3 = 2)

Add 1 to the original number. Continue with this new number as in the first calculation then add the original number doubled plus 1.

In mathematical notation:

$$10\left(\frac{x+1}{3}\right)^2-\left(\frac{x+1}{3}\right)^2-(x+1+x)=9\left(\frac{x+1}{3}\right)^2-2x-1=\left(\frac{9(x+1)}{9}\right)^2-2x-1$$
$$=(x+1)^2-2x-1=x^2+1+2x-2x-1=x^2$$

5. The Sum of a Mathematical Series

After bringing known methods that he copied from other sources, Abraham ibn Ezra brings a method that he developed himself.

We are interested in calculating 1 + 2 + … x

Square x add to this x and then divide the sum by 2.

$$1+2+\cdots x=\frac{x^2+x}{2}$$

6. Conclusion

- We have presented several mathematical examples that illustrate Abraham ibn Ezra's profound understanding of number theory.
- In modern mathematical notations these may seem obvious but for the middle of the 12th century they were a great innovation.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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