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Exploring the Implications of the Deformation Parameter and Minimal Length in the Generalized Uncertainty Principle

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Abstract

The breakdown of the Heisenberg Uncertainty Principle occurs when energies approach the Planck scale, and the corresponding Schwarzschild radius becomes similar to the Compton wavelength. Both of these quantities are approximately equal to the Planck length. In this context, we have introduced a model that utilizes a combination of Schwarzschild's radius and Compton length to quantify the gravitational length of an object. This model has provided a novel perspective in generalizing the uncertainty principle. Furthermore, it has elucidated the significance of the deforming linear parameter β and its range of variation from unity to its maximum value.

Keywords

Generalized Uncertainty Principle, Deformed Heisenberg Algebra, Minimal Length

1. Introduction

In the realm of quantum gravity, which is expected to be applicable at extremely small length scales or high energies, the fundamental units of measurement are often considered to be the Planck length (l_p) and Planck mass m_p. These quantities play a significant role in understanding the behavior of gravity at such scales. The Planck length is often associated with a minimum measurable length scale. It is speculated that beyond this scale, the traditional concept of space-time may no longer hold. This minimum length scale is thought to be connected to the granularity or quantization of space-time at very small scales. It is important to note that the existence and precise nature of this minimum measurable length scale are still subjects of ongoing research and conjecture.

When it comes to black holes, general relativity predicts the presence of an event horizon, also known as the Schwarzschild radius. This radius represents the point beyond which nothing, not even light, can escape the gravitational pull of the black hole. The Schwarzschild radius R_s is determined by the mass of the black hole and can be calculated using the formula $R_s = \frac{2GM}{c^2}$, where G is the gravitational constant and c is the speed of light and M is the mass.

In quantum effects, such as quantum fluctuations, there is a concept known as the fluctuation-dissipation theorem. According to this theorem, quantum systems inherently experience fluctuations due to the uncertainty principle. These fluctuations can manifest as variations in various physical quantities, including mass [1].

The interplay between quantum effects and the minimum measurable length scale associated with quantum gravity has significant implications for the lower boundary of mass. According to the uncertainty principle, there is always inherent uncertainty in the measurement of physical quantities, including mass. This uncertainty is directly proportional to the reduced Planck constant ($\hbar$), which is a fundamental constant that characterizes the quantum nature of a system [2].

In contrast, within the context of black holes, general relativity imposes an upper limit on mass. The maximum mass that a black hole can contain is determined by its Schwarzschild radius. If the mass exceeds this limit, the black hole would collapse into a singularity.

Hence, the combination of quantum effects and general relativity establishes boundaries on the mass of objects. The lower boundary is influenced by quantum effects and the minimum measurable length, which is proportional to the Planck length. On the other hand, the upper boundary is determined by general relativity and the maximum mass that can be accommodated within a given region, as indicated by the Schwarzschild radius [3]-[9].

It is important to acknowledge that the regime of quantum gravity, where the Planck quantities become relevant, remains a challenging area to explore experimentally. The study of black holes and their behavior within the framework of quantum gravity continues to be an active field of research, and further investigations are necessary to fully comprehend the implications of these fundamental scales in the context of mass and gravity.

A simple comparison can be made between the generalized uncertainty principle (GUP) and the Heisenberg uncertainty principle. The generalized uncertainty principle (GUP) is a modification of the Heisenberg uncertainty principle, which is a fundamental concept in quantum mechanics. The GUP arises from attempts to reconcile quantum mechanics with gravitational theories, particularly general relativity. It proposes the existence of a minimum measurable length scale due to the effects of quantum gravity.

The GUP modifies the commutation relation between position and momentum operators, which has several implications. In the standard Heisenberg uncertainty principle, the commutation relation is given by $[\hat{x}, \hat{p}] = i\hbar$, where $\hat{x}$

represents the position operator, $\hat{p}$ represents the momentum operator, and ħ is the reduced Planck's constant. However, in the GUP, this commutation relation is modified to include a correction term that depends on a deformation parameter, typically denoted by β : $[\hat{x}, \hat{p}] = i\hbar(1+\beta p^2)$.

The presence of the deformation parameter β in the commutation relation leads to various physical predictions and effects. It is important to note, however, that the GUP is still a theoretical proposal and has not yet been experimentally confirmed. Nonetheless, it has generated significant interest among researchers investigating quantum gravity and related fields.

The generalized Uncertainty Principle (GUP) brings about modifications to the uncertainty relations between position and momentum. This signifies that there is a fundamental limit to the precision at which certain pairs of observables can be simultaneously measured. Unlike the standard Heisenberg uncertainty principle, the GUP suggests that this limit is not an exact equality but is instead altered by the deformation parameter β.

Furthermore, the GUP predicts the existence of a minimum measurable length scale. The inclusion of the deformation parameter β introduces a correction term in the uncertainty of position, indicating that there is a fundamental limit to the accuracy with which distances can be measured. This minimum length scale is often associated with quantum gravitational effects and is speculated to be approximately equal to the Planck length [10].

Additionally, the Generalized Uncertainty Principle (GUP) can lead to modifications in the energy-momentum relations of particles. Specifically, it can introduce corrections to the dispersion relation, which establishes the relationship between a particle's energy and momentum. These modifications have potential implications in the field of high-energy physics, particularly in understanding the behavior of particles at extremely short distances or high energies [11].

When it comes to experimental tests and limitations on the value of the deformation parameter, it is important to acknowledge the inherent challenges in directly probing the GUP. The effects of the GUP are expected to be significant at very high energies or extremely small length scales, where quantum gravitational effects become prominent. However, current experimental capabilities have not yet reached these scales [12].

Nowadays, there have been no direct experimental tests that definitively confirm or rule out the GUP. Consequently, the value of the deformation parameter β remains largely unconstrained by empirical data. Nevertheless, various theoretical frameworks within the realm of quantum gravity, such as string theory and loop quantum gravity, offer some rationale for the existence of a minimum length scale and modifications to the uncertainty principle.

It is worth noting that ongoing and future experiments, including those conducted at high-energy colliders or gravitational wave observatories, may indirectly provide evidence or constraints on the GUP. Nonetheless, this remains an active area of research, and further advancements are necessary to establish experimental boundaries on the value of the deformation parameter.

In this paper, we will give a simple derivation of the generalized uncertainty principle (GUP) and the physical meaning of the linear deformation parameter.

2. Generalized Uncertainty Principle

The expansion of fundamental physical principles emerged as a result of the necessity to elucidate certain phenomena that were beyond the scope of non-expanded physical laws. Wu, X., Wu, B., Li, H., and Wu, Q [13], introduced the concept of the generalized Hamilton principle, which encompasses the description of both heat exchange systems and non-conservative force systems.

In the realm of microscopic measurements, the mass of the particle (m) is less than Planck mass (m_p), ($m < m_p$), it is worth noting that the Compton length λ_c surpasses the Schwarzschild's radius R_s . Consequently, the term $R_s = \frac{2Gm}{c^2}$ is disregarded, and the measuring length is equivalent to the Compton length $\lambda_c = \frac{\hbar}{mc}$. Conversely, in macroscopic measurements ($m > m_p$), the term $\frac{\hbar}{mc}$ is omitted. As a result, quantum effects do not manifest themselves on a macroscopic scale. Therefore, it can be explicitly stated that the mass determines the equation that can be utilized for measurement, whether it be Schwarzschild's radius or the Compton length. This signifies a transformation between the macroscopic and microscopic scales [14].

Several examples are provided to illustrate how the mass of an object determines the equations employed for measurement as in the framework of general relativity, the gravitational field surrounding a massive object is described by the Schwarzschild metric. The Schwarzschild radius (R_s) represents a characteristic length scale associated with the object's mass (m). For macroscopic objects with substantial masses, such as planets or stars, the Schwarzschild radius becomes significant, necessitating the consideration of its effects on the surrounding space-time. Conversely, for microscopic objects with small masses, the Schwarzschild radius is negligible compared to other length scales, and its effects can be disregarded.

In quantum mechanics, the Compton wavelength (λ_c) is linked to the mass of a particle. For macroscopic objects with large masses, the Compton wavelength becomes exceedingly small in comparison to the object's size, rendering quantum effects insignificant. However, for microscopic particles like electrons or photons, the Compton wavelength is significant and determines the characteristic scale at which quantum phenomena become prominent.

Particle collisions play a crucial role in high-energy particle physics experiments, as they provide valuable insights into the fundamental properties of matter and the underlying physical laws. The energy scales at which these interactions occur are determined by the mass of the particles involved. In experiments conducted at particle accelerators such as the Large Hadron Collider (LHC), protons are accelerated to high energies and made to collide with each other. The energy of these collisions is directly linked to the mass of the particles being collided.

By carefully studying the outcomes of these collisions, physicists can probe the intricate details of matter and gain a deeper understanding of its fundamental nature. These experiments allow scientists to investigate the behavior of particles at both macroscopic and microscopic scales. It is essential to consider the interplay between quantum mechanics and general relativity to comprehensively describe the physical world. The mass of an object or particle plays a significant role in shaping the equations and phenomena that are relevant for measurement and analysis in these experiments.

Several modified iterations of the conventional Heisenberg argument have been proposed in order to obtain physical solutions, such as what Beckwith [15] did to answer the question, if initial vacuum field corresponds to a configuration of early universe space-time at the start of inflation?.

In one such version, outlined in [16], a comprehensive description is provided. According to this version, a stream of photons possessing an energy E has the potential to theoretically identify an object with an approximate size Δx. This estimation assumes the dispersion relation $E = pc$, P is a momentum.

$$\Delta x = \frac{\hbar c}{2E} \tag{1}$$

As previously mentioned, Heisenberg's thought experiment initially disregards the influence of gravity. However, if we consider the potential creation of micro black holes during high-energy scatterings, with a gravitational radius $R_s = R_s(E)$ that is approximately proportional to the scattering energy E ,as referenced in [8], it becomes evident that the conventional uncertainty relation needs to be modified.

$$\Delta x = \frac{\hbar c}{2E} + \beta R_s(E) \tag{2}$$

$R_s(E)$ represents the gravitational effect and β the proportional constant.

The Heisenberg Uncertainty Principle (HUP), which states that the uncertainty in position (Δx) multiplied by the uncertainty in momentum (Δp) is approximately equal to Planck's constant ($\hbar$), is not applicable when dealing with energies close to the Planck scale. At this scale, the Schwarzschild radius, which represents the size of a black hole, becomes comparable to the Compton wavelength, which characterizes the quantum nature of a particle. Both of these quantities are approximately equal to the Planck length. As the energy increases, the Schwarzschild radius also increases, leading to a modified version of the uncertainty principle: Δx is proportional to the square of the Planck length (l_P), $\Delta x \sim l_P^2 \frac{\Delta p}{\hbar}$ multiplied by Δp divided by Planck's constant ($\hbar$). This observation, supported by thought experiments and rigorous derivations, suggests that the Generalized Uncertainty Principle (GUP) holds true at all scales. The GUP is represented by the equation, $\Delta x_i \Delta p_i \ge \frac{\hbar}{2}\left[1 + \alpha\left((\Delta p)^2 + \langle p \rangle^2\right) + 2\alpha\left(\Delta p_i^2 + \langle p_i \rangle^2\right)\right]$, $i = 1,2,3$ and

$\alpha = \frac{l_P^2}{2\hbar^2}$ as it referenced in [17].

The threshold mass can be defined as the mass at which the transition occurs between the microscopic system and the macroscopic system, it can be defined as the mass that satisfies the equality between Compton length and Schwarzschild radius,

$$\frac{\hbar}{cl} = \frac{c^2 r}{2G} \tag{3}$$

The solution depends on the length, $l = r = l_P$, where the threshold mass becomes Planck's mass m_P.

Now, if an individual attempt to calculate the gravitational radius of an object with an average mass $\overline{m}$ by employing both equations, $r_1 = \frac{2G\overline{m}}{c^2}$ and $r_2 = \frac{\hbar}{\overline{m}c}$, The resulting measurement, denoted as $r = \overline{r} \pm \Delta\overline{r}$, can be expressed as,

$$r = \frac{1}{2}\left(\frac{2G\overline{m}}{c^2} + \frac{\hbar}{\overline{m}c}\right) \pm \frac{1}{2}\left(\frac{2G\overline{m}}{c^2} - \frac{\hbar}{\overline{m}c}\right) \tag{4}$$

When $\overline{m} = m_p$ equation (4) gives Planck length,

$$r = \overline{r} = \frac{1}{2}\left(\frac{2Gm_p}{c^2} + \frac{\hbar}{m_p c}\right) = \frac{2Gm_p}{c^2} = \frac{\hbar}{m_p c} = \sqrt{\frac{2\hbar G}{c^3}} = l_P, \text{ with } \Delta\overline{r} = 0 \tag{5}$$

where l_P is Planck length. But in general, $r_1 \neq r_2$ and $\Delta\overline{r} \neq 0$.

By utilizing the aforementioned principles, we can establish a correlation between measurement in the microscopic realm and measurement in the macroscopic domain through the application of the subsequent equation,

$$\frac{\hbar c}{m} \to 2GM; M \to \frac{\alpha}{m}, \alpha = \frac{\hbar c}{2G} \tag{6}$$

The symbol "m" represents the microscopic mass, which is smaller than the Planck mass (m_p), $m < m_p$, while the symbol "M" represents the macroscopic mass, which is larger than the Planck mass $M > m_p$.

A connection has been established between the macroscopic and microscopic realms, with the Planck mass serving as the dividing line between them. When the mass surpasses the Planck mass, a shift occurs between the two systems. This shift can be interpreted as a duality in mass [18]. Consequently, in Equation (4), the transition signs become equivalent to the Planck mass. This observation inspired Arbab [19] to base his work on the characteristics of Planck's constant $\hbar_c$, which is contingent upon the system's size.

In the context of T-duality, there is a relation between the length scales in the two dual theories. This relation gives rise to a modified uncertainty principle known as the T-duality uncertainty principle. According to this principle, the spatial resolution (Δl) is bounded not only by the reciprocal of the momentum spread (Δp), as in the standard Heisenberg uncertainty principle, but also by the string scale L_s [20].

The string scale L_s is a fundamental length scale in string theory, and it is related to the tension of the string. It represents the characteristic size of strings in the theory. It is given by $L_s = \sqrt{\acute{\beta}}$, where $\acute{\beta}$ is the Regge slope parameter, which is related to the string tension.

In the T-duality uncertainty principle, the spatial resolution (Δl) is constrained by the product of the momentum spread (Δp) and the string scale L_s, such that $\Delta l \Delta p > 1/L_s$. This implies that there is a fundamental limit to the precision with which one can simultaneously measure position and momentum in the context of T-duality.

3. The Deformation Parameter

Return to the measurement, the uncertainty in measuring the length l is Δl. The standard deviation (The standard deviation is essentially the width of the range over which the function f is distributed around its mean value, $\langle f \rangle$) is defined by

$$\Delta l = \sqrt{\langle l^2 \rangle - \langle l \rangle^2} \tag{7}$$

By substituting $l = l_P^2 \frac{p}{2\hbar} + \frac{\hbar}{2p}$, noting that, the first term is proportioning to the momentum P which representing the macroscopic scale. The second one is proportioning to the reciprocal of the momentum and represents the microscopic scale. Now we can estimate the uncertainty in measuring l by the following inequality,

$$\Delta l \geq l_P^2 \frac{\Delta p}{2\hbar} \sqrt{1 + \frac{\hbar^4}{l_P^4} \frac{\left\langle \frac{1}{p^2} \right\rangle - \left\langle \frac{1}{p} \right\rangle^2}{\langle p^2 \rangle - \langle p \rangle^2} + \frac{2\hbar^2}{l_P^2 (\Delta p)^2} \left(1 - \langle p \rangle \left\langle \frac{1}{p} \right\rangle \right)} \tag{8}$$

where p is the momentum, $\frac{G\bar{m}}{c^2} = l_P^2 \frac{p}{2\hbar}$. The standard deviation in microscopic scale is measured by $\Delta\left(\frac{1}{p}\right)$, $\Delta\left(\frac{1}{p}\right) = \sqrt{\left\langle \frac{1}{p^2} \right\rangle - \left\langle \frac{1}{p} \right\rangle^2}$. And the standard deviation in macroscopic scale is measured by ΔP, $\Delta P = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$.

If the term $\frac{\hbar^4}{l_P^4} \frac{\left\langle \frac{1}{p^2} \right\rangle - \left\langle \frac{1}{p} \right\rangle^2}{\langle p^2 \rangle - \langle p \rangle^2} + \frac{2\hbar^2}{l_P^2 (\Delta p)^2} \left(1 - \langle p \rangle \left\langle \frac{1}{p} \right\rangle \right) \ll 1$,

We can use the binomial expansion formula, which gives the expansion of $(1 + x)^n$ where "n" is a rational number. This expansion has an infinite number of terms,

$$(1+x)^n = 1 + nx + [n(n-1)/2!]x^2 + [n(n-1)(n-2)/3!]x^3 + \cdots$$

Equation (8) simplified to

$$\Delta l \geq l_P^2 \frac{\Delta p}{2\hbar} + \frac{\hbar^3 \left(\Delta\left(\frac{1}{p}\right)\right)^2}{4 l_P^2 \Delta p} + \frac{\hbar}{2\Delta p} \left(1 - \langle p \rangle \left\langle \frac{1}{p} \right\rangle \right) \tag{9}$$

But the term $\Delta\left(\frac{1}{p}\right)$ acts in microscopic scale in the regime of Compton length, where,

$$\Delta\left(\frac{1}{p}\right)=\frac{\Delta l}{\hbar} \tag{10}$$

Therefore, the generalized uncertainty takes the form

$$\Delta l\Delta P\geq\frac{\hbar}{2}\left\{1+\left(\frac{\Delta p l_P}{\hbar}\right)^2+\frac{1}{2}\left(\frac{\Delta l}{l_P}\right)^2-\langle p\rangle\left\langle\frac{1}{p}\right\rangle\right\} \tag{11}$$

We note that the right side consists of four terms. Where the first term represents the known value of uncertainty as it is known. We will explain the following three terms.

The first term $\left(\frac{\Delta p l_P}{\hbar}\right)^2$, accounts for the contribution of the macroscopic system to uncertainty.

The second term $\left(\frac{\Delta l}{l_P}\right)^2$, accounts for the contribution of the microscopic system to uncertainty.

The last term $\langle p\rangle\left\langle\frac{1}{p}\right\rangle$, accounts for the contribution of the interaction between macroscopic and microscopic systems to uncertainty.

Moreover, the commutation relation for position and momentum is given by

$$[X,P]=i\hbar\left(1+\left(\frac{\Delta p l_P}{\hbar}\right)^2+\frac{1}{2}\left(\frac{\Delta l}{l_P}\right)^2-\langle p\rangle\left\langle\frac{1}{p}\right\rangle\right) \tag{12}$$

By comparing (11) with the deformed Heisenberg uncertainty [21] [22] [23]

$$\Delta x\Delta p\geq\frac{\hbar}{2}\left(1+\sigma(\Delta p)^2+\sigma\langle P\rangle^2\right) \tag{13}$$

Upon comparing (11) with the deformed Heisenberg uncertainty relation, where σ represents the deformation parameter, it becomes evident

$$\sigma=\frac{l_P^2}{\hbar^2} \tag{14}$$

Moreover, the minimum measuring length is

$$\Delta x_{\min}(\langle P\rangle)=\hbar\sqrt{\sigma}\sqrt{1+\sigma\langle P\rangle^2} \tag{15}$$

The smallest measuring length according to (14) and (15), when $\langle P\rangle=0$ [24] [25] is

$$\Delta x_{\min}=\hbar\sqrt{\sigma}=l_P \tag{16}$$

According to the model Equation (9) gives the smallest measuring length $\Delta l=l_P$ in Plank's scale which implies

$$\Delta\left(\frac{1}{P}\right)=\frac{l_P^2}{\hbar^2}\sqrt{2}\Delta P=\sqrt{2}\sigma\Delta P,\quad l_{\min}\Delta p=\hbar,\ \text{and}\ \langle p\rangle\left\langle\frac{1}{p}\right\rangle=1 \tag{17}$$

The above result consents with the result in (5). The measurement $r=l_P$ when $m=m_p$.

By rewriting Equation (9) in linear combination

$$\Delta l \Delta P \geq \frac{\hbar}{2}\left[1-\langle p\rangle\left\langle\frac{1}{p}\right\rangle+a\left(\Delta\left(\frac{1}{p}\right)\right)^2+\frac{1}{2}a^{-1}(\Delta p)^2\right] \tag{18}$$

With the linear parameter $a=\frac{\hbar^2}{2l_p^2}$.

The general form of the linear parameter *a*, is β. It can be formed and related to Plank's length as

$$\beta_n=\frac{1}{(n+1)!}\left(\frac{\hbar}{l_p}\right)^{2n} \tag{19}$$

β_n is a new deformation parameter, which demands and conserves the dimension of the inequality (18).

$n=0\pm1,\pm2$ represents the state of the scale e.g. $n=0$ is the Plank's scale where $m=m_p$ .we find the coupling of macroscopic and microscopic state, relation (18) shows this state in terms of ($\langle p\rangle\left\langle\frac{1}{p}\right\rangle$). also the state $n=+1$ represents microscopic scale where $m<m_p$. The state $n=-1$ represents the macroscopic scale with $m>m_p$, the β parameter index indicates scale order.

Finally, the general form of the uncertainty and the commutation relation for position and momentum according to relation (18) and Equation (19) is given by

$$\Delta l \Delta P \geq \frac{\hbar}{2}\left[1-\langle p\rangle\left\langle\frac{1}{p}\right\rangle+\frac{\alpha_n}{(n+1)!}\cdot\left(\frac{\hbar}{l_p}\right)^{2n}\right] \tag{20}$$

$$[X,P]=i\hbar\left[1-\langle p\rangle\left\langle\frac{1}{p}\right\rangle+\frac{\alpha_n}{(n+1)!}\cdot\left(\frac{\hbar}{l_p}\right)^{2n}\right] \tag{21}$$

where α_n is the relation of the standard deviation of momentum and the fundamental length of the scale, which ensures that the dimension of the term $\Delta l \Delta P$ is conserved.

n	β_n	α_n	$\Delta l \Delta p$	$[X, P]$
0	1	$\langle p\rangle\left\langle\frac{1}{p}\right\rangle$	$\frac{\hbar}{2}$	$i\hbar$
1	$\frac{1}{2}\left(\frac{\hbar}{l_p}\right)^2$	$\left(\frac{\Delta l}{\hbar}\right)^2$	$\frac{\hbar}{2}\left[1-\langle p\rangle\left\langle\frac{1}{p}\right\rangle+\frac{1}{2}\left(\frac{\Delta l}{l_p}\right)^2\right]$	$i\hbar\left[1-\langle p\rangle\left\langle\frac{1}{p}\right\rangle+\frac{1}{2}\left(\frac{\Delta l}{l_p}\right)^2\right]$
−1	$\left(\frac{\hbar}{l_p}\right)^{-2}$	$(\Delta p)^2$	$\frac{\hbar}{2}\left[1-\langle p\rangle\left\langle\frac{1}{p}\right\rangle+\left(\frac{\hbar}{l_p}\right)^{-2}(\Delta p)^2\right]$	$i\hbar\left[1-\langle p\rangle\left\langle\frac{1}{p}\right\rangle+\left(\frac{\hbar}{l_p}\right)^{-2}(\Delta p)^2\right]$
2	$\frac{1}{6}\left(\frac{\hbar}{l_p}\right)^4$	$\left(\frac{\Delta l}{\hbar}\right)^4$	$\frac{\hbar}{2}\left[1-\langle p\rangle\left\langle\frac{1}{p}\right\rangle+\frac{1}{6}\left(\frac{\Delta l}{l_p}\right)^4\right]$	$i\hbar\left[1-\langle p\rangle\left\langle\frac{1}{p}\right\rangle+\frac{1}{6}\left(\frac{\Delta l}{l_p}\right)^4\right]$
−2	0	$(\Delta p)^4$	$\frac{\hbar}{2}\left(1-\langle p\rangle\left\langle\frac{1}{p}\right\rangle\right)$	$i\hbar\left(1-\langle p\rangle\left\langle\frac{1}{p}\right\rangle\right)$

The reason for the value of β_n to be equal to zero is the term $\frac{1}{(n+1)!}$, which leads to zero for negative values of $n+1$, when $n<-1$.

4. Minimal Length

Since the negative values of n represent the macroscopic system, the model gives a constant relationship to uncertainty for all values achieved $n=-2,-3,\cdots$. Thus, we find that uncertainty leads to

$$\Delta l \Delta P = \frac{\hbar}{2}\left(1-\langle p\rangle\left\langle\frac{1}{p}\right\rangle\right) \tag{22}$$

$$[X,P] = i\hbar\left(1-\langle p\rangle\left\langle\frac{1}{p}\right\rangle\right) \tag{23}$$

If a particle is confined within a specific volume, such as a box, the particle's average momentum $\langle p\rangle$ will be zero. As a result, the Heisenberg uncertainty relations, described by Equations (22) and (23), emerge due to the uncertainty and commutation relation. However, in free space, the particle possesses an average momentum $\langle p\rangle$ that is greater than zero. Consequently, when measuring the average momentum of macroscopic bodies in the microscopic system, a very small value is obtained, leading to the term $\langle p\rangle\left\langle\frac{1}{p}\right\rangle$ approaching zero. Once again, the well-known Heisenberg uncertainty relations are derived from relations (22) and (23). This process ensures that relations (22) and (23) do not yield a negative value.

In the region of $n = 1$, where

$$\Delta l \Delta P = \frac{\hbar}{2}\left[1-\langle p\rangle\left\langle\frac{1}{p}\right\rangle+\frac{1}{2}\left(\frac{\Delta l}{l_p}\right)^2\right] \tag{24}$$

For the system where $\Delta l \gg l_p$, and $1 \geq \langle p\rangle\left\langle\frac{1}{p}\right\rangle$. Relation (24) construes to

$$\Delta P \approx \hbar\frac{\Delta l}{4l_p^2} \tag{25}$$

By comparing the smallest measuring length $l_{\min}\Delta p = \hbar$ in (17) with the relation (25), we find

$$l_{\min} = \frac{4l_p^2}{\Delta l} \tag{26}$$

As the value of minimum measurement can be detected is Planck length, then $l_{\min} \geq l_p$. This leads to verify the domain of Δl as

$$\Delta l \leq 4l_p \tag{27}$$

By following the same previous treatment, the minimum length of the macroscopic system can be calculated when $n = -1$

$$l_{\min} = \frac{l_p^2}{2\Delta l} \tag{28}$$

Then the uncertainty in l and p become

$$\Delta l \leq \frac{l_p}{2} \tag{29}$$

$$\Delta P \geq p_P \tag{30}$$

$p_P = \sqrt{\frac{\hbar c^3}{G}}$ is Planck momentum

The above can be summarized by specifying the minimum length range according to relations (26) and (28) as follows

$$\frac{l_p^2}{2\Delta l} \leq l_{\min} \leq \frac{4 l_p^2}{\Delta l} \tag{31}$$

From above there are numerous values of β_n related to the various scales or energies.

According to equation (18), the hidden physical meaning of the deformation parameter, β_n became clear as the following:

- β_0 is the mass flow rate or the creation-annihilation rate.
- β_2 is the cosmic string energy loss (cosmic string had formed at a phase transition in the early universe, which is responsible for the large-scale structure of the universe). β_2 demands that the energy loss mechanism is sufficient so that the energy density of strings will scale as l^{-4} as is necessary for the consistency of the string scenario [26] [27] [28] [29].

Hence, β_n varies through the range $1 \leq \beta_n \leq l_P^{-4}$. This result is in a good agreement with the values predicted in [30] and references therein.

Moreover, Das S and Vagenas [10], showed that the GUP effect is unobservable with $\beta_0 = 1$, this consists of the arguments given above. That is, in Plank's scale from Equation (17) $\langle p \rangle \left\langle \frac{1}{p} \right\rangle = 1$ and $\Delta\left(\frac{1}{P}\right) = \frac{l_P^2}{\hbar^2}\sqrt{2}\Delta P$. By substituting these in Equation (18) the first term in RHS vanished in Plank's scale and the term $\frac{\hbar^2}{2l_P^2}\left(\Delta\left(\frac{1}{p}\right)\right)^2 + \frac{l_P^2}{\hbar^2}(\Delta p)^2 = 2$. Therefore, Equation (18) construes to the well-known form of uncertainty relation $\Delta l \Delta P \geq \hbar$ and the GUP effect disappeared.

5. Conclusion

Instructively, in this work, we generalized the uncertainty principle by combining microscopic and macroscopic measurements. Equations (20) and (21) are the complete generalizations of uncertainty and commutation relation formulation respectively. The physical meaning of the linear parameters, β_n, can be used to explain mass (creation-annihilation) rate and cosmic string energy loss.

Most quantum theories of gravity indicate that, β_0 in order of unity in the Planck scale. On the other hand, β_n is believed to be an energy scale constant and varies with the scale [26].

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Notation List

represents the position operator	$\hat{x}$
represents the momentum operator	$\hat{p}$
reduced Planck's constant	$\hbar$
Speed of light in the vacuum	c
Planck length	l_P
Planck mass	m_P
gravitational radius	R_s
the relation of the standard deviation of momentum and the fundamental length of the scale, which ensures that the dimension of the term $\Delta l \Delta P$ is conserved.	α_n
Regge slope parameter	$\grave{\beta}$
string scale	L_s
mean value	$\langle f \rangle$
standard deviation	Δf
deformation parameter	β_n