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Tilt Angle Optimality Criteria for Stand Alone PV Systems

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Abstract

The conventional approach to optimizing tilt angles for fixed solar panels aims to maximize energy generation over the entire year. However, in the context of a supply controlled electric grid, where solar energy availability varies, this criterion may not be optimal. This study explores two alternative optimization criteria focused on maximizing baseload supply potential and minimizing required storage capacity to address seasonality in energy generation. The optimal tilt angles determined for these criteria differed significantly from the standard approach. This research highlights additional factors crucial for designing solar power systems beyond gross energy generation, essential for the global transition towards a fully renewable energy-based electric grid in the future.

Keywords

Electric Grid, Fixed Solar Panels, Optimal PV Tilt Angle, Seasonal Solar Variability, Renewable Energy, Supply-Demand Balance

1. Introduction

The surge in demand for clean energy has led to a substantial increase in the installation of solar panels in recent years [1] [2]. This demand is projected to continue growing due to several factors, including the rising prices of fossil fuels, a strong commitment to curbing carbon dioxide emissions, and the declining costs of solar systems, making them increasingly competitive in terms of financial viability against other energy sources.

As solar panels become a significant component of the electric grid, their integration poses numerous technical challenges [3] [4] [5]. One particularly crucial problem revolves around maintaining a balance between power generation and consumption to ensure optimal operation of the grid. Previously, this challenge was easily managed by grid operators, given their ability to control fossil fuel power plants. When electricity demand increased, more fuel could be burned in power plants, and vice versa. This paradigm provided convenience to both suppliers and consumers. However, with solar power becoming the primary energy source feeding the grid, this equilibrium is disrupted. Solar energy generation is uncontrollable, making it increasingly challenging to achieve a balance between supply and demand. In certain instances, excess power must be curtailed, resulting in revenue losses. In other cases, demand may exceed supply, leading to shortages and inconveniences for consumers. As the share of solar energy in the grid continues to grow, maintaining the supply-demand balance becomes even more challenging. Particularly, if the ultimate goal is to replace all fossil fuel power plants with renewable energy, achieving a 100% share of generated power, this problem will manifest itself to the greatest extent, necessitating a practical solution.

One essential solution to this problem lies in the utilization of energy storage systems integrated with the electric grid. These systems can store surplus energy generated during periods of abundant sunshine and distribute it during times of minimal or absent sunlight to meet demand [6] [7] [8]. However, energy storage systems have remained expensive thus far, inhibiting widespread adoption of this solution to address the intermittent nature of solar energy. This is where our work becomes crucial. The objective of this paper is to minimize energy storage requirements through strategic installation of solar panels.

Solar panels capture solar irradiance and convert it into electricity, with maximum energy capture occurring when the panels directly face the sun's rays. There are two technical systems employed: solar panels with sun-tracking mechanisms and fixed solar panels [9]. This paper focuses on the latter.

While solar panels with tracking systems generate more electricity, their higher costs associated with the tracking mechanism limit their adoption compared to fixed panels. Fixed panel systems have lower costs and maintenance requirements, making them the preferred choice for most solar installations worldwide [10] [11].

In fixed panel systems, the installation angle is typically chosen to maximize energy generation throughout the year [12] [13] [14]. From an economic standpoint, this seems reasonable for vendors. However, from the perspective of the supply-demand balance discussed earlier, this may not be optimal. Consequently, the integration of solar panels into the electric grid and the overall share of solar energy in the energy mix are constrained. Our work aims to enhance the supply-demand balance by generating electricity with reduced seasonal variability, ensuring a more even distribution throughout the year and minimizing energy storage requirements.

Another factor in determining the optimal tilt angle of solar panels is minimizing dust accumulation on the panels [15] [16]. While horizontal panels experience higher dust accumulation rates, vertical panels have the lowest rates. Dust accumulation incurs costs associated with the cleaning process required to restore the panels' original efficiency [17]. Thus, an optimal angle could be selected to minimize these expenses.

In this paper, we propose a novel criterion for selecting the optimal tilt angle, focusing on minimizing energy storage requirements by reducing the seasonal variability of solar panel power generation over a year's time frame. The significance of this criterion can be summarized by the following three points:

1. Facilitating higher penetration ratios of solar energy into the electric grid by replacing conventional fossil fuel power plants. This accelerates the increase of clean energy in the energy mix to meet the goals established by governments and various entities.

2. Reducing system costs by minimizing the storage requirements needed to store excess energy when supply exceeds demand. This stored energy can then be utilized when supply alone is insufficient to meet demand.

3. Mitigating inter-seasonal variations in energy generation by supplying a more evenly distributed power throughout the year. This has the additional benefit of reducing curtailed energy during summer and increasing revenue. Furthermore, an increase in energy generation during winter provides an advantage.

Overall, our work aims to address the challenges of seasonality posed by integrating solar energy into the electric grid, optimize the tilt angle of fixed solar panels, minimize energy storage size, and enhance the supply-demand balance, leading to a more efficient and cost-effective utilization of solar power.

2. Method

2.1. Description of Sun Movement

The apparent movement of the sun across the sky is a phenomenon attributed to the Earth's rotation rather than the sun's motion. This motion is a result of two primary rotations of the Earth in space: (1) the yearly orbit around the sun, leading to the apparent north-south movement of the sun, and (2) the daily rotation of the Earth about its axis, causing the apparent east-west movement of the sun across the celestial sphere. These apparent changes in the sun's position allow for the definition of several key angles:

1. Sun Declination (δ): This angle represents the sun's declination at solar noon relative to the equator, with a range of $-23.45^\circ \le \delta \le 23.45^\circ$. As illustrated in **Figure 1**, δ completes one cycle over the course of a year, varying from -23.45° on December 21 to 0° on March 21, then to 23.45° on June 21, and back to 0° on September 21. The day-to-day change in δ is minimal. It can be estimated using the formula [18]:

$$\delta = 23.45\sin\left(360\frac{284+n}{365}\right).$$

where n represents the day of the year ($1 \le n \le 365$).

2. Hour Angle (ω): This angle denotes the angular displacement between the sun and the local meridian, with each hour corresponding to a 15° increment or decrement. In the morning hours, ω is negative, while in the afternoon, it is positive, with solar noon having an angle of 0°. Energy generation occurs only

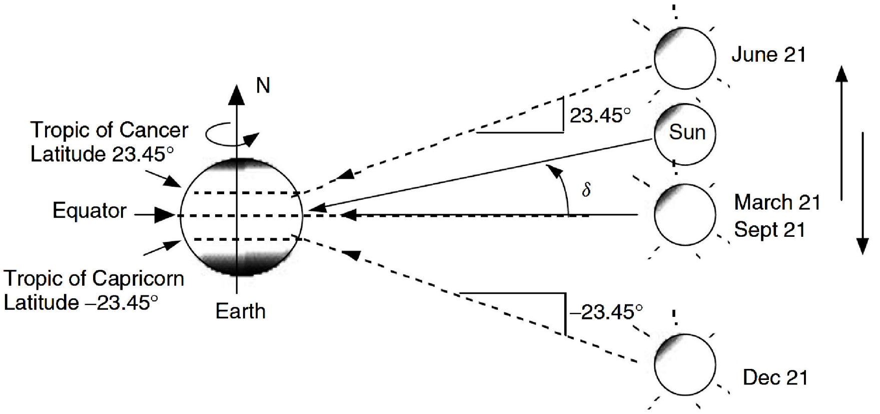

Figure 1. Seasonal sun movement and the angle δ [19].

between sunrise and sunset.

The sunrise and sunset angles are calculated using [18]:

$$\omega_{sr} = -\cos^{-1}\left[-\tan\phi\tan\delta\right], \tag{2}$$

and

$$\omega_{ss} = \cos^{-1}\left[-\tan\phi\tan\delta\right], \tag{3}$$

respectively, where ϕ represents the latitude of the location ($-90^\circ \le \phi \le 90^\circ$), with north latitudes being positive and south latitudes negative.

The range of ω for the daytime period is bounded by ω_{sr} and ω_{ss}:

$$\omega_{sr} \le \omega \le \omega_{ss} \tag{4}$$

However, energy generation occurs only when sunlight directly strikes the front face of solar panels, not when striking the rear face. Consequently, the duration of energy generation may be shorter than the time between sunrise and sunset. The calculation of sunrise and sunset times on the solar panel itself will be elucidated following further definitions in subsequent sections.

2.2. Description of PV Panel Orientation

Figure 2 illustrates the positioning of solar panels along with the definition of various pertinent angles, as summarized below:

- β: Represents the tilt angle of the solar panels relative to a horizontal surface, with a range of $0^\circ \le \beta \le 180^\circ$. Panels facing downward have $\beta > 90^\circ$.
- γ: Denotes the surface azimuth angle, covering $-180^\circ \le \gamma \le 180^\circ$, where zero corresponds to due south, positive azimuth values indicate west-facing panels, and negative azimuth values represent east-facing panels.
- θ: Known as the angle of incidence, it signifies the angle between the incident solar radiation beam and the normal to the panel's surface.

For a given solar system installed at a specific location, the angles β and γ remain fixed, while ω and θ exhibit significant variation throughout the daily cycle of Earth's rotation. Meanwhile, δ remains relatively constant within each day, but its gradual change becomes evident over the course of the year.

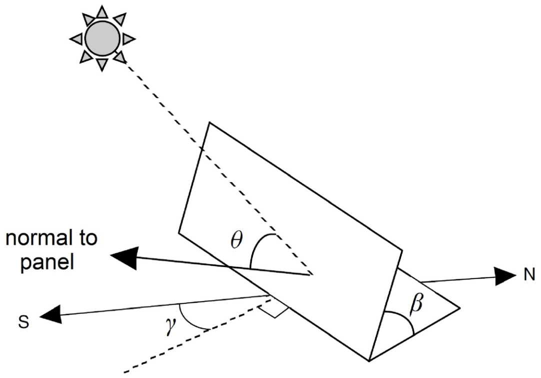

Figure 2. Panel orientation and sun incident angle θ [19].

2.3. Incident Solar Irradiance

Given the nearly constant value of solar irradiance at approximately 1367 W/m², the incident solar irradiance on a panel is governed by the angle of incidence, θ, as determined by the following equation:

$$I = 1367 \times \cos\theta. \tag{5}$$

The angle θ is influenced by the movement of the sun across the sky, which encompasses both daily and seasonal components. It can be calculated using the following equation [18]:

$$\begin{aligned}\theta = \cos^{-1}\big[&\sin\delta\sin\phi\cos\beta - \sin\delta\cos\phi\sin\beta\cos\gamma \\ &+ \cos\delta\cos\phi\cos\beta\cos\omega + \cos\delta\sin\phi\sin\beta\cos\gamma\cos\omega \\ &+ \cos\delta\sin\beta\sin\gamma\sin\omega\big].\end{aligned} \tag{6}$$

It is imperative that the solar incident angle on the panel remains $\theta < 90^\circ$ for energy generation to occur. Conversely, if $\theta > 90^\circ$, the sun will strike the rear face of the solar panel, resulting in no energy generation. To determine the effective duration of sunshine on the panel for energy generation purposes, we utilize Equation (6) with $\theta = 90^\circ$ on the left-hand side and solve for ω (assuming all other variables are known for a specific installation geometry and location). This yields two values for ω: one corresponding to the hour angle when the sun rises on the panel, denoted as ω_{srp}, and the other corresponding to the hour angle when the sun sets on the panel, denoted as ω_{ssp}.

Energy generation occurs only during the interval:

$$\max\left\{\omega_{sr}, \omega_{srp}\right\} \le \omega \le \min\left\{\omega_{ss}, \omega_{ssp}\right\} \tag{7}$$

The mathematical formulations delineated in Sections 2.1 and 2.2 are grounded on an idealized solar irradiance scenario, forming the basis for all simulations conducted in this study.

2.4. Optimal Tilt Angle Choices

In the subsequent sections, we propose four distinct system configurations labeled as SYSTEM A, SYSTEM B, SYSTEM C, and SYSTEM D. Each of these configurations shares the common objective of providing a consistent baseload, defined as the daily energy demand that remains constant throughout the entire year. It's pertinent to clarify that our focus lies on the aggregated daily demand over a twenty-four-hour period. Thus, we deliberately overlook intra-daily power mismatches between supply and demand to streamline the analysis. This approach is adopted partly due to our belief that the inter-seasonal energy variations in solar energy are primarily responsible for impeding higher levels of photovoltaic (PV) energy generation in the electric grid.

For all the systems delineated below, we employ a standardized PV system size of 1 kW as a basis. Subsequently, we optimize the tilt angle, compute the daily supplied baseload energy, and ascertain the requisite energy storage system size accordingly.

2.4.1. SYSTEM A

In SYSTEM A, the PV system is supplemented with a sufficiently large storage system. The primary objective of the energy storage system is to ensure that the constant baseload is reliably supplied by storing surplus energy generated during summer months and utilizing it when generation alone cannot meet the baseload demand during winter. The tilt angle of the solar panels in SYSTEM A is optimized to minimize the required energy storage system size. Mathematically, the optimization for the tilt angle is expressed as:

$$\hat{\beta}_A = \arg\min_{\beta} \{\text{storage system size}\}. \tag{8}$$

This equation signifies the determination of the optimal tilt angle ($\hat{\beta}_A$) that minimizes the size of the energy storage system for SYSTEM A.

2.4.2. SYSTEM B

In SYSTEM B, no energy storage system is utilized, and the PV system alone is tasked with supplying the baseload power consistently on a daily basis throughout the entire year. This objective can only be achieved if the minimum energy generation by the PV system over the course of the year equals or exceeds the baseload energy requirement. The tilt angle of the solar panels for SYSTEM B is optimized to maximize the energy generated by the PV system. This tilt angle typically reflects the choice made by most installers to maximize generated energy and consequently their profit margins. Mathematically, the optimization for the tilt angle is articulated as:

$$\hat{\beta}_B = \arg\max_{\beta} \{\text{generated energy}\}. \tag{9}$$

Here, $\hat{\beta}_B$ represents the optimal tilt angle that maximizes the generated energy for SYSTEM B.

2.4.3. SYSTEM C

In SYSTEM C, we adopt the same tilt angle optimization criterion as SYSTEM B, aiming to maximize the generated energy by the PV system. However, we enhance the system by integrating a sufficiently large storage system to ensure the continuous supply of the constant baseload throughout the entire year. Mathematically, the tilt angle optimization is expressed as:

$$\hat{\beta}_C = \arg\max_{\beta} \{\text{generated energy}\}. \tag{10}$$

Here, $\hat{\beta}_C$ represents the optimal tilt angle that maximizes the generated energy for SYSTEM C.

2.4.4. SYSTEM D

In SYSTEM D, no energy storage system is employed, and the PV system alone is tasked with supplying the baseload power consistently on a daily basis throughout the entire year. This can be achieved only if the minimum energy generation by the PV system over the entire year equals or exceeds the baseload energy requirement. Therefore, the tilt angle for SYSTEM D is optimized to maximize the minimum year-round generated energy by the PV system. Mathematically, the optimization for the tilt angle is expressed as:

$$\hat{\beta}_D = \arg\max_{\beta} \{\min\{\text{generated energy}\}\}. \tag{11}$$

Here, $\hat{\beta}_D$ represents the optimal tilt angle that maximizes the minimum year-round generated energy for SYSTEM D.

For all systems, the panels typically face the south direction ($\gamma = 0^\circ$) in the northern hemisphere and the north direction ($\gamma = 180^\circ$) in the southern hemisphere. The tilt angle (β) solely depends on the latitude.

Table 1 below summarizes the conditions of the four systems.

2.5. Storage Size Estimation

To compute the energy storage system size for SYSTEM A and SYSTEM C, we first define the storage state as:

$$\text{storage state} = \int_0^t \left(P(\tau) - \bar{P}\right) \mathrm{d}\tau = \int_0^t P(\tau)\,\mathrm{d}\tau - \bar{P}t. \tag{12}$$

Here, $P(t)$ represents the time function of the generated power from the panel, and $\bar{P}$ denotes the average power generation, which, in this analysis, equals the baseload power. Consequently, the required storage size is calculated as:

$$\text{storage size} = \max\{\text{storage state}\} - \min\{\text{storage state}\} \tag{13}$$

2.6. Economic Comparison of the Systems

In this section, we undertake an economic comparison of SYSTEM A, SYSTEM B, SYSTEM C, and SYSTEM D. The Levelized Cost of Electricity (LCOE) is utilized as the metric for comparison, defined as [20]:

Table 1. The four stand alone PV systems.

SYSTEM	Storage	Tilt angle optimality criterion	Baseload supplied
A	√	minimize storage size	√
B	×	maximize generation	√
C	√	maximize generation	√
D	×	maximize the minimum generation	√

$$\text{LCOE} = \frac{\text{system cost}}{\text{baseload energy supplied}} = \frac{\text{PV system cost} + \text{storage system cost}}{\text{baseload energy supplied}} \tag{14}$$

Here, the PV system cost and the storage system cost are given by:

$$\text{PV system cost} = \text{PV system size} \times \text{PV price per kW} = 1 \times \text{PV price per kW} \tag{15}$$

$$\text{Storage system cost} = \text{storage system size} \times \text{storage price per kWhr} \tag{16}$$

Let's abbreviate the following:

$$\text{PV}_{\text{price}} = \text{PV price per kW} \tag{17}$$

$$\text{storage}_{\text{price}} = \text{storage price per kWhr} \tag{18}$$

$$\text{Storage}_{\text{A}} = \text{storage system size for SYSTEM A} \tag{19}$$

$$\text{Storage}_{\text{C}} = \text{storage system size for SYSTEM C} \tag{20}$$

For SYSTEM A, (15) and (16) become

$$\text{PV system cost} = \text{PV}_{\text{price}} \tag{21}$$

$$\text{Storage system cost} = \text{storage}_{\text{A}} \times \text{storage}_{\text{price}} \tag{22}$$

For SYSTEM B, (15) becomes

$$\text{PV system cost} = \text{PV}_{\text{price}} \tag{23}$$

For SYSTEM C, (15) and (16) become

$$\text{PV system cost} = \text{PV}_{\text{price}} \tag{24}$$

$$\text{Storage system cost} = \text{storage}_{\text{C}} \times \text{storage}_{\text{price}} \tag{25}$$

For SYSTEM D, (15) becomes

$$\text{PV system cost} = \text{PV}_{\text{price}} \tag{26}$$

Additionally, let's define the following baseload energy ratios:

$$\text{Energy}_{\text{X/Y}} = \frac{\text{baseload energy supplied by SYSTEM X}}{\text{baseload energy supplied by SYSTEM Y}} \tag{27}$$

where X and Y could be A, B, C, or D.

2.6.1. SYSTEM A vs B

To ensure that SYSTEM A is more economical than SYSTEM B, i.e., $\text{LCOE}_{\text{A}} < \text{LCOE}_{\text{B}}$, the following inequality must hold:

$$\frac{\text{PV}_{\text{price}} + \text{storage}_{\text{A}} \times \text{storage}_{\text{price}}}{\text{baseload energy supplied by SYSTEM A}} < \frac{\text{PV}_{\text{price}}}{\text{baseload energy supplied by SYSTEM B}} \tag{28}$$

This inequality can be further simplified as:

$$\text{PV}_{\text{price}} + \text{storage}_{\text{A}} \times \text{storage}_{\text{price}} < \text{PV}_{\text{price}} \times \text{Energy}_{\text{A/B}} \tag{29}$$

$$\text{storage}_{\text{A}} \times \text{storage}_{\text{price}} < \text{PV}_{\text{price}} \left(\text{Energy}_{\text{A/B}} - 1\right) \tag{30}$$

$$\frac{\text{storage}_{\text{price}}}{\text{PV}_{\text{price}}} < \frac{\text{Energy}_{\text{A/B}} - 1}{\text{storage}_{\text{A}}} \tag{31}$$

These equations provide an upper bound on the ratio of storage price per kWhr to PV price per kW for SYSTEM A to be more economically feasible than SYSTEM B.

2.6.2. SYSTEM A vs C

To establish SYSTEM A as more economical than SYSTEM C, i.e., $\mathrm{LCOE_A} < \mathrm{LCOE_C}$, the following inequality must hold:

$$\begin{aligned}&\frac{\mathrm{PV_{price}} + \mathrm{storage_A} \times \mathrm{storage_{price}}}{\text{baseload energy supplied by SYSTEM A}}\\&< \frac{\mathrm{PV_{price}} + \mathrm{storage_C} \times \mathrm{storage_{price}}}{\text{baseload energy supplied by SYSTEM C}}\end{aligned} \tag{32}$$

This inequality can be further simplified as:

$$\mathrm{PV_{price}} + \mathrm{storage_A} \times \mathrm{storage_{price}} < \left(\mathrm{PV_{price}} + \mathrm{storage_C} \times \mathrm{storage_{price}}\right) \times \mathrm{Energy_{A/C}} \tag{33}$$

$$\mathrm{PV_{price}}\left(1 - \mathrm{Energy_{A/C}}\right) < \mathrm{storage_{price}}\left(\mathrm{storage_C} \times \mathrm{Energy_{A/C}} - \mathrm{storage_A}\right) \tag{34}$$

$$\frac{\mathrm{storage_{price}}}{\mathrm{PV_{price}}} > \frac{1 - \mathrm{Energy_{A/C}}}{\mathrm{storage_C} \times \mathrm{Energy_{A/C}} - \mathrm{storage_A}} \tag{35}$$

These equations provide a lower bound on the ratio of storage price per kWhr to PV price per kW for SYSTEM A to be more economically feasible than SYSTEM C.

2.6.3. SYSTEM A vs D

Applying a procedure akin to the one employed in Section 2.6.1, we determine that for SYSTEM A to be more economical than SYSTEM D, i.e., $\mathrm{LCOE_A} < \mathrm{LCOE_D}$, the following inequality holds:

$$\frac{\mathrm{storage_{price}}}{\mathrm{PV_{price}}} < \frac{\mathrm{Energy_{A/D}} - 1}{\mathrm{storage_A}} \tag{36}$$

This equation delineates a criterion based on the ratio of storage price per kWhr to PV price per kW for SYSTEM A to exhibit greater economic viability than SYSTEM D.

2.6.4. SYSTEM C vs B

Employing a procedure akin to the one utilized in Section 2.6.1, we establish that for SYSTEM C to surpass SYSTEM B in economic feasibility, i.e., $\mathrm{LCOE_C} < \mathrm{LCOE_B}$, the following inequality holds:

$$\frac{\mathrm{storage_{price}}}{\mathrm{PV_{price}}} < \frac{\mathrm{Energy_{C/B}} - 1}{\mathrm{storage_C}} \tag{37}$$

This equation delineates a criterion based on the ratio of storage price per kWhr to PV price per kW for SYSTEM C to exhibit greater economic viability than SYSTEM B.

2.6.5. SYSTEM C vs D

Applying a similar procedure as the one employed in Section 2.6.1, we determine that for SYSTEM C to be more economical than SYSTEM D, i.e., $\text{LCOE}_\text{C} < \text{LCOE}_\text{D}$, the following inequality holds:

$$\frac{\text{storage}_{\text{price}}}{\text{PV}_{\text{price}}} < \frac{\text{Energy}_{\text{C/D}} - 1}{\text{storage}_{\text{C}}} \tag{38}$$

This equation provides a criterion based on the ratio of storage price per kWhr to PV price per kW for SYSTEM C to demonstrate greater economic viability than SYSTEM D.

2.6.6. SYSTEM B vs D

LCOE_D is always less than LCOE_B.

Equations (31), (35), (36), (37), and (38) will serve as the foundation for the economic comparison of the four systems, as elaborated upon in the results section. The relative economic advantage of one system over another is contingent upon these inequalities.

3. Results

3.1. Optimizing the Tilt Angles

Figure 3 illustrates the relationship between tilt angle and latitude across the four system configurations. The corresponding tilt angles are detailed in Table 2, alongside additional performance metrics such as baseload supplied, required storage, and average curtailment.

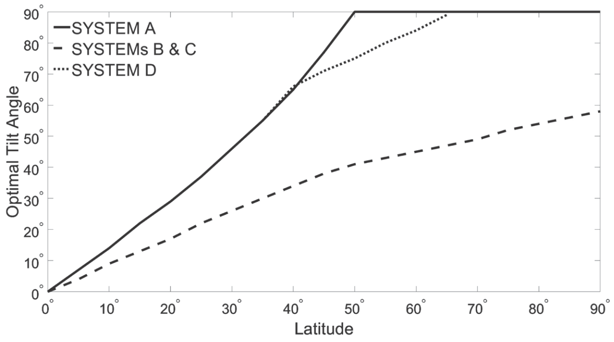

Figure 3. Optimal tilt angle as a function of latitude.

Table 2. Comparison of the four system configurations.

	SYSTEM A			SYSTEM B			SYSTEM C			SYSTEM D		
Latitude “°”	**Tilt angle “°”**	**Baseload supplied “kWhr/day”**	**Required storage “kWhr”**	**Tilt angle “°”**	**Baseload supplied “kWhr/day”**	**Avg. curtailment “kWhr/day”**	**Tilt angle “°”**	**Baseload supplied “kWhr/day”**	**Required storage “kWhr”**	**Tilt angle “°”**	**Baseload supplied “kWhr/day”**	**Avg. curtailment “kWhr/day”**
0	0	7.00	23.45	0	6.60	0.40	0	7.00	23.45	0	6.60	0.40
5	7	6.98	24.41	4	6.38	0.61	4	6.99	42.21	7	6.56	0.42
10	14	6.92	26.36	9	6.19	0.77	9	6.95	56.72	14	6.48	0.45
15	22	6.81	28.13	13	5.90	0.99	13	6.89	79.57	22	6.35	0.47
20	29	6.67	29.87	17	5.57	1.23	17	6.81	104.4	29	6.16	0.51

Continued

25	37	6.46	32.87	22	5.26	1.44	22	6.69	124.8	37	5.89	0.57
30	46	6.16	36.36	26	4.82	1.72	26	6.54	154.6	46	5.54	0.62
35	55	5.78	41.10	30	4.32	2.04	30	6.36	187.7	55	5.05	0.73
40	65	5.27	48.69	34	3.72	2.40	34	6.12	225.1	66	4.41	0.80
45	77	4.55	54.33	38	3.02	2.81	38	5.83	268.0	71	3.60	1.30
50	90	3.65	62.41	41	2.17	3.31	41	5.48	322.4	75	2.63	1.94
55	90	3.53	143.7	43	1.21	3.84	43	5.05	386.5	80	1.51	2.58
60	90	3.29	238.9	45	0.33	4.22	45	4.55	450.9	84	0.42	3.18
65	90	3.03	319.3	47	0	4.06	47	4.06	498.7	89	0	3.07
70	90	2.81	367.3	Not applicable			49	3.64	519.1	Not applicable		
75	90	2.59	395.0	Not applicable			52	3.24	520.0	Not applicable		
80	90	2.34	404.8	Not applicable			54	2.83	505.8	Not applicable		
85	90	2.07	396.3	Not applicable			56	2.43	475.5	Not applicable		
90	90	1.75	368.1	Not applicable			58	2.02	428.4	Not applicable		

3.2. Modeling of Optimal Tilt Angles

In this section, we will derive mathematical models describing the optimal tilt angles for the four discussed systems. These models consist of piecewise linear functions that closely approximate the optimal tilt angles.

$$\hat{\beta}_A = \begin{cases} 1.5\phi, & 0^\circ \le \phi \le 30^\circ \\ 2.2(\phi - 30) + 45, & 30^\circ \le \phi \le 50^\circ \\ 90^\circ & 50^\circ \le \phi \le 90^\circ \end{cases} \tag{39}$$

$$\hat{\beta}_B = \begin{cases} 0.85\phi, & 0^\circ \le \phi \le 45^\circ \\ 0.44(\phi - 45) + 38.25, & 45^\circ \le \phi \le 66.6^\circ \\ \text{Not applicable} & 66.6^\circ \le \phi \le 90^\circ \end{cases} \tag{40}$$

$$\hat{\beta}_C = \begin{cases} 0.85\phi, & 0^\circ \le \phi \le 45^\circ \\ 0.44(\phi - 45) + 38.25, & 45^\circ \le \phi \le 90^\circ \end{cases} \tag{41}$$

$$\hat{\beta}_D = \begin{cases} 1.56\phi, & 0^\circ \le \phi \le 45^\circ \\ 0.92(\phi - 45) + 70.1, & 45^\circ \le \phi \le 66.6^\circ \\ \text{Not applicable} & 66.6^\circ \le \phi \le 90^\circ \end{cases} \tag{42}$$

3.3. Economic Analysis

The findings presented in this section provide insights into the economic viability of implementing the four systems discussed, based on the relative pricing of energy storage compared to photovoltaic (PV) systems.

Both SYSTEMs A and C integrate energy storage, albeit with differing criteria for selecting tilt angles. While SYSTEM C prioritizes maximizing generated energy, SYSTEM A selects tilt angles to minimize the required energy storage. On the other hand, SYSTEM D aims to fulfill baseload energy requirements without relying on energy storage, which necessitates selecting tilt angles maximizing winter generation but may result in substantial curtailment during summer due to the absence of storage facilities.

SYSTEM A represents an optimal configuration benefiting from two key features: efficient energy storage utilization and prudent tilt angle selection to minimize storage size requirements.

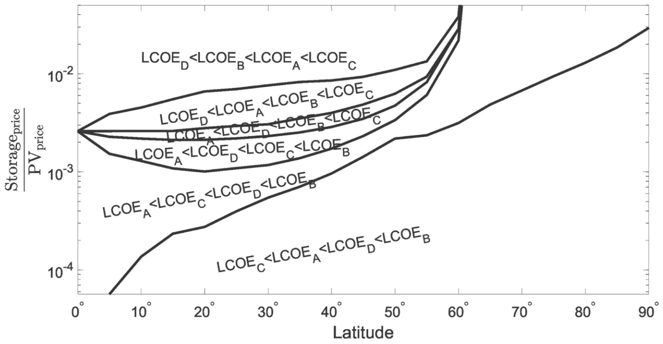

Figure 4. Economical comparison of SYSTEMs A, B, C, and D.

In the economic comparison of the four systems, as of 2024, the prevailing scenario sees PV costs per kilowatt relatively low while energy storage costs per kilowatt-hour remain high. Consequently, as depicted in **Figure 4**, SYSTEM D emerges as the most economically advantageous option (upper region in **Figure 4**), followed by SYSTEM A, with SYSTEM C being the least favorable. Thus, currently, aligning PV tilt angles with those of SYSTEM D appears optimal for transitioning energy usage away from fossil fuels towards solar-based alternatives.

However, prevailing industry practice often favors SYSTEM B criteria, maximizing immediate energy generation and associated profits. Future shifts in electricity pricing, particularly favoring winter consumption over summer, could prompt installers to prioritize profit maximization over energy generation maximization when selecting tilt angles.

Adopting an electricity pricing scheme favoring winter consumption could accelerate the transition to a fully renewable energy-based grid by incentivizing tilt angles that enhance winter generation. Additionally, rapid declines in storage system prices may alter the economic landscape, potentially favoring SYSTEM A as storage costs decrease.

Ultimately, if storage costs significantly undercut PV costs, a scenario where SYSTEM C becomes economically favorable (lower region in **Figure 4**) may arise, although this remains speculative given projected future PV and storage prices.

3.4. LCOE and Forecasted System Prices

In this section, we compute the LCOE based on projected prices of PV and storage systems sourced from the NREL website [21]. Forecasted PV prices are depicted in **Figure 5**, while battery storage prices are illustrated in **Figure 6**. The LCOE is assessed for all systems, as depicted in **Figure 7**, considering a location at 30˚ N latitude and assuming a 30-year system lifetime.

Consistent with expectations, the lowest electricity prices are achieved using SYSTEM D, followed by SYSTEM B, SYSTEM A, and finally, SYSTEM C, representing the most expensive option. This trend is anticipated to persist throughout the entire forecasted period until 2050.

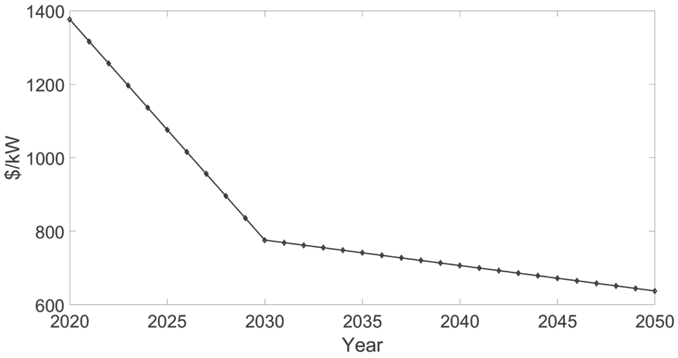

Figure 5. Forecasted PV prices.

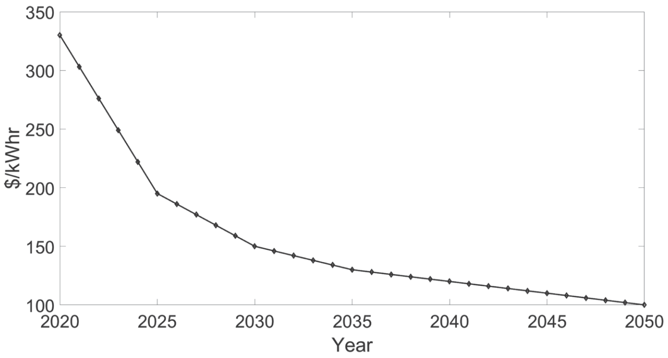

Figure 6. Forecasted battery storage prices.

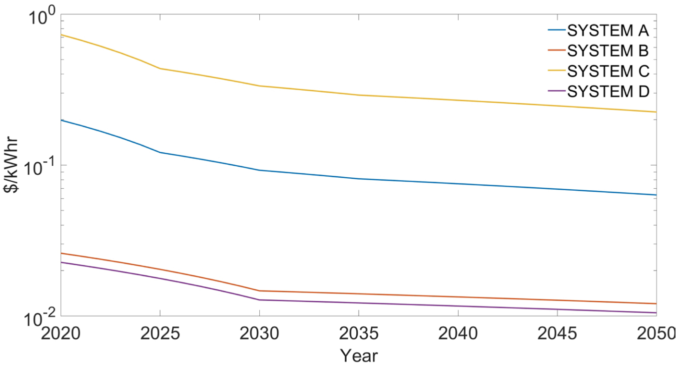

Figure 7. LCOE for all systems.

4. Discussion

4.1. Generated Energy Variation with Latitude

The energy generated varies with latitude primarily due to changes in the duration of daylight and nighttime throughout the year. Near the equator, both daytime and nighttime are roughly equal, with each lasting around twelve hours. Consequently, these latitudes experience the highest energy generation on Earth. However, as you move towards higher latitudes, there is a greater variation in the duration of daylight and nighttime between the summer and winter seasons.

Although a fixed solar panel's surface could potentially receive a maximum of twelve hours of sunlight within a twenty-four hour period, during winter, the

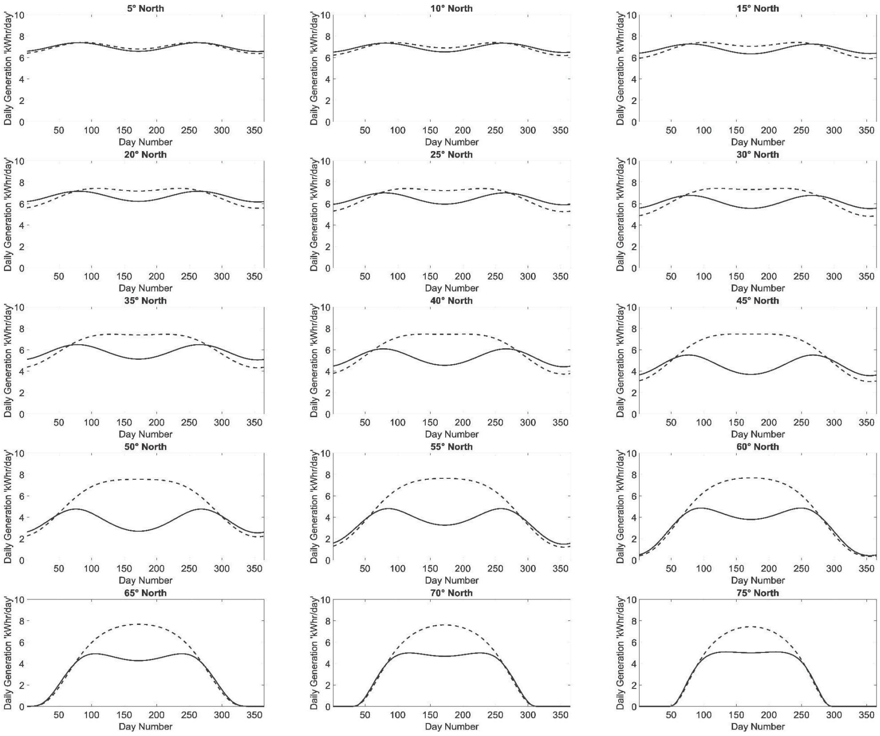

Figure 8. Daily power generation for SYSTEM A (solid) and SYSTEM B (dashed).

duration of daytime is less than twelve hours, resulting in reduced energy generation, particularly in winter. This decrease is not compensated for during the summer because fixed solar panels still receive a maximum of twelve hours of sunlight, even though the daytime exceeds twelve hours. As a result, the peak energy generation during summer remains constant across all latitudes. On the other hand, energy generation in winter decreases as you move towards higher latitudes as can been seen in **Figure 8**. In extreme polar latitudes, there are consecutive days during winter when the sun does not rise at all, causing energy generation to drop to zero. This overall trend leads to a decrease in generated energy throughout the year as you move towards higher latitudes. This trend is apparent in **Table 2** for all systems.

4.2. Seasonality of Generated Energy with Latitude

The seasonality of generated energy is closely linked to the latitude of a location on Earth. Let's explore how latitude influences the seasonality of generated energy:

1. Low-Latitude Regions: Locations near the equator exhibit relatively minimal seasonality in generated energy. Day length remains relatively consistent throughout the year, with slight variations attributed to changes in sun declination across the four seasons.

2. Mid-Latitude Regions: Locations at intermediate latitudes experience more pronounced seasonal variations in generated energy compared to low-latitude regions for SYSTEM A. These regions show distinct differences in energy gener-

ation between spring and fall, on one hand, and summer and winter, on the other. Spring and fall exhibit two peaks in energy generation, while summer and winter demonstrate two minima. For instance, at latitude of 40 with a panel tilt angle of 65 for SYSTEM A, the angle of incidence at solar noon is close to zero during winter. However, due to the shorter daytime period, energy generation in winter is minimized. In contrast, during spring and fall, when the angle of incidence is approximately 25 and the daytime is the same for both seasons, we can expect similar energy generation. It's important to note that the daytime in spring and fall is around twelve hours, whereas in winter, it's only six hours at latitude of 40. Consequently, even though the angle is optimized for the winter season, energy generation is lower compared to spring and fall due to the shorter day length. Similarly, during summer, with a substantial angle of incidence of 50, the sunshine period on the solar panels cannot exceed twelve hours. This reduces the amount of generated energy compared to spring and fall, resulting in the second minimum in addition to the winter minimum.

3. High-Latitude Regions: Locations near the poles, like the Arctic and Antarctic regions, experience extreme seasonality in generated energy. During summer, when the respective pole is tilted towards the sun, these regions receive 24 hours of daylight. However, the solar panels are exposed to sunlight for only 12 hours, similar to the sun exposure in spring and fall, resulting in a plateau in generated energy observed during these seasons. In contrast, during winter, when the pole is tilted away from the sun, these regions experience polar night, with no sunlight or very limited daylight, leading to low or negligible energy generation.

In summary, the seasonality of generated energy varies based on latitude. Low-latitude regions exhibit minimal variations, mid-latitude regions show more pronounced peaks and minima, and high-latitude regions experience extreme seasonality with distinct plateaus and periods of limited energy generation.

4.3. Energy Storage

The utilization of energy storage poses considerable economic challenges, particularly in the context of renewable energy reliance. This necessity arises due to the inherent intermittency and seasonal variability of renewable energy sources, which necessitate backup solutions for uninterrupted power supply.

In this study, we delved into a novel approach for selecting the tilt angle of solar panels, with the objective of minimizing the energy storage requirements for standalone PV systems. However, given the current high costs associated with energy storage systems, there is limited incentive to augment PV systems with energy storage.

The absence of energy storage leads to a significant increase in curtailed energy as the adoption of PV energy systems, and renewable energy in general, continues to rise. Curtailment represents a financial loss for energy providers, and in the current economic landscape, avoiding energy storage remains the prevailing paradigm. Nevertheless, as storage technology advances and costs decline, there exists the potential for a future scenario where the incorporation of energy storage becomes economically viable. The timing of this transition is uncertain and hinges on the rate at which storage prices decrease.

It is pertinent to note that energy storage solutions are already present in certain sectors of the energy industry, albeit to a limited extent. These systems are typically employed for short-term energy storage to mitigate the intermittent nature of renewable energy sources. However, if the global aim is to transition entirely to renewable for energy needs, addressing the challenge of long-term energy storage becomes imperative, ideally on a seasonal timescale. The longer duration of storage necessitates larger storage capacities, thereby increasing implementation costs—a significant barrier to the widespread adoption of such systems in reality.

5. Conclusions

In summary, the current electricity pricing schemes prioritize tilt angles that maximize energy generation for the sake of profit optimization. While this approach initially accelerates the integration of solar energy into the grid, its long-term implications include an escalation of curtailed energy as we shift away from fossil fuel sources. Moreover, this trend impedes the seamless deployment of PV systems.

It is imperative to reconsider our strategy and adopt a tilt angle for PV systems that minimizes inter-seasonal variability in energy generation. This proactive adjustment aims to reduce curtailment during the summer months and alleviate energy shortages in winter, thereby enhancing the year-round utilization of PV systems. By implementing this approach, we anticipate a significant increase in the overall penetration of solar energy in the electric grid and a rise in the percentage of electricity sourced from renewable means. Additionally, this shift contributes to a reduction in our dependence on finite fossil fuel resources.

Looking forward, the proposed tilt angle selection also positions us favorably for the potential integration of storage systems. By minimizing curtailment and optimizing energy generation, this strategy ensures that the required storage system size remains as small as possible. To incentivize the adoption of this criterion by PV installers, a pricing scheme favoring winter energy generation over summer generation is suggested. Such a scheme not only encourages the widespread adoption of the proposed tilt angle but also prepares the electric grid for the seamless integration of storage systems as their prices become more competitive.

In conclusion, a strategic reevaluation of tilt angle selection not only addresses immediate profitability concerns but also establishes a foundation for a more sustainable and adaptable electric grid. This forward-thinking approach allows for a smoother transition to renewable energy sources and positions us to embrace emerging technologies, such as storage systems, as they become increasingly viable.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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