Journal of Power and Energy Engineering, 2022, 10, 54-74
https://www.scirp.org/journal/jpee
ISSN Online: 2327-5901
ISSN Print: 2327-588X

Comparison of Economic Dispatch, OPF and Security Constrained-OPF in Power System Studies $\mathbb{R}^N$

Etete Obio[1], Siddique Ali[2], Idowu Oyebanjo[2], Daniel Abara[2], Farouk Suleiman[3], Peter Ohiero[1], Daniel Oku[4], Vincent Ogar[5]

[1]Cross River University of Technology, Calabar, Nigeria
[2]The University of Manchester, Manchester, United Kingdom
[3]University of Salford, Salford, United Kingdom
[4]University of Glasgow, Glasgow, United Kingdom
[5]University of Strathclyde, Glasgow, United Kingdom
Email: mrtownjnr@gmail.com

How to cite this paper: Obio, E., Ali, S., Oyebanjo, I., Abara, D., Suleiman, F., Ohiero, P., Oku, D. and Ogar, V. (2022) Comparison of Economic Dispatch, OPF and Security Constrained-OPF in Power System Studies $\mathbb{R}^N$. *Journal of Power and Energy Engineering*, **10**, 54-74.
https://doi.org/10.4236/jpee.2022.108005

Received: June 26, 2022
Accepted: August 26, 2022
Published: August 29, 2022

Abstract

Electricity network is a very complex entity that comprises several components like generators, transmission lines, loads among others. As technologies continue to evolve, the complexity of the electricity network has also increased as more devices are being connected to the network. To understand the physical laws governing the operation of the network, techniques such as optimal power flow (OPF), Economic dispatch (ED) and Security constrained optimal power flow (SCOPF) were developed. These techniques have been used extensively in network operation, planning and so on. However, an in-depth presentation showcasing the merits and demerits of these techniques is still lacking in the literature. Hence, this paper intends to fill this gap. In this paper, Economic dispatch, optimal power flow and security-constrained optimal power flow are applied to a 3-bus test system using a linear programming approach. The results of the ED, OPF and SC-OPF are compared and presented.

Keywords

Economic Dispatch, Linear Programming, Network Operation, Optimal Power Flow, Security Constrained Power Flow

1. Introduction

The aim of the electric power utilities is to supply reliable and affordable electricity to meet the demand of the consumers at the lowest possible operating cost subject to the constraints imposed by the generating units. Economic dispatch (ED) is defined as the process of allocating the generation level among the generating units in such a way that the total system cost is minimized [1]. As far back as the 1920's ED problems have been solved with several methods such as the baseload method where the units are loaded to their maximum capability in increasing order starting with the most efficient units [2]. In the early 1930s, the incremental cost method later known as the equal increment method was recognized to yield the most economic results, this method is based on the principle that the next increment in load should be supplied by the unit with the lowest incremental cost [2].

Normally, the input to output characteristics of the generating units are non-linear, non-smooth and discrete due to prohibited operating zones, ramp rate limits and multi-fuel effects. Thus, the resultant ED becomes a challenging non-convex optimization problem that has been solved by several methods such as newton methods, gradient method and quadratic programming, among others. The major application of economic dispatch is a unit commitment problem, which involves periodic online and offline scheduling of units at the minimum cost subject to units' constraints.

In general, ED is cost-effective as it focuses only on the merit order generation without considering the network (line) constraints [3]. Therefore, for an effective operation and planning of the electricity network, optimal power flow (OPF) calculations are critically important. According to [4], OPF is a static nonlinear optimization problem that calculates the optimal setpoints of all pre-defined electrical variables in an electrical network, for a particular load setting and system parameters. Carpentier *et al.* [5] presented the first form of OPF solution, since then various forms of optimal power flow techniques have been developed with a continuous search for more efficient methods to improve the reliability and accuracy of the OPF solution [5]. A comprehensive review on the state of the art in OPF solution techniques so far applied to power system is presented in [4] [5].

The application of OPF in today's power system is becoming indispensable, especially in the area of network security, operation and control, planning among others [6] [7]. In power system planning, it is required that the network should be robust against all possible uncertainties while maximizing the social welfare of all the participants in the system. In mathematical modelling of power system planning problems, the OPF is a sub-function of the planning model, thus efficient OPF techniques are required to guarantee the best possible solution of the planning process. Nevertheless, some discussion on the application of OPF in power system planning and operation can be traced back to 1974, typically in a work presented by Sasson and Merril *et al.* [8]. Power system planning is a very complex problem that requires a huge amount of work and cannot be solved as a single problem. Hence it is necessary to separate it into various stages and segments such as load forecasting, reliability assessment, stability evaluation and generation/transmission expansion planning. Mathematically, OPF is usually formulated as an optimization problem with an objective function subject to a set of constraints that are dependent on the nature of the problem under investigation.

The main drawback of OPF in power system analysis is that it fails to consider the robustness of the system with respect to credible line contingencies. In a power system, security applies to the ability of the network to continuously operate satisfactorily after an outage of any of the system components such as line and generator. In other words, OPF focuses on a single system state or configuration at a particular time, hence leaving the system operator with no idea of the cost needed to meet the system operating constraints in case of credible contingency [9]. Since the utilities are saddled with the responsibility to produce and continuously supply adequate and secured electrical energy to the consumer at the lowest cost, a better approach than OPF thus becomes necessary. In line with this requirement, the Security constrained optimal power flow (SCOPF) was developed to address network security issues in both intact and contingency conditions of the network [10] [11]. In general, SCOPF is an extended form of optimal power flow that aims at minimizing the generation dispatch subject to the system power balance constraints and components operational limits for the intact network case as well as the contingency case [12]. Practically speaking, network security can be enhanced by using either the preventive or the corrective approach [13] [14].

1) Preventive security approach

In this approach, the system is not provided with the flexibility for re-scheduling the control variables in the post-contingency state and it is sometimes considered as a conservative approach.

2) Corrective security approach

This approach provides the system with the flexibility to make some adjustments to the control variable in the occurrence of a contingency. From an economic perspective, the corrective approach has been proven to be more viable than the preventive approach as it leads to a lower system cost [9]. For simplicity, a preventive Security constrained optimal power flow (PSCOPF) is demonstrated in this report and the results are compared with the initial results obtained in the ED and OPF case studies.

Paper Organization

The rest of the paper is organized as follows. Section II describes the mathematical formulations of the ED, OPF and SCOPF. The models are implemented on a 3-bus network and the results are discussed in Section III. Finally, the main observations and conclusions are summarized in Section IV.

2. Problem Formulation and Methodology

2.1. Economic Dispath (ED)

The economic dispatch problem is an optimization problem with an objective function of minimizing the total generation cost C_T, subject to the power balance constraint and the generator upper and lower operating limits. The power balance constraint ensures that the sum of the total power generated is equal to the load demand plus the network losses. In a power system such as the one shown in **Figure 1**, which consist of N_g generating units with generation output P_{gi} connected to a single bus-bar, to supply a load D_k at a cost C_i, the ED problem can be mathematically represented as follows [9]:

$$C_T = C_1 P_{g1} + C_2 P_{g2} + \cdots + C_n P_{gn} = \sum_{i=1}^{n} C_i \left(P_{gi}\right), \tag{1}$$

$$P_{gT} = P_{g1} + P_{g2} + \cdots + P_{gn} = \sum_{i=1}^{n} P_{gi}, \tag{2}$$

$$\min \sum_{i=1}^{n} C_i \left(P_{gi}\right) = \min \sum_{i=1}^{n} C_i \left(a_i + b_i P_{gi} + c_i P_{gi}^2\right), \tag{3}$$

subject to:

$$\sum_{i=1}^{n} P_{gi} = P_{loss} + D, \tag{4}$$

$$P_{gi} - P_{gi}^{\max} \leq 0,\ i = 1, \cdots, n, \tag{5}$$

$$P_{gi}^{\min} - P_{gi} \leq 0,\ i = 1, \cdots, n \tag{6}$$

where P_{loss} represents Network losses, D is the system load demand, $P_{gi}^{\max}$ is the upper bounds of generator, $P_{gi}^{\min}$ is the lower bounds of generator, C_i is the cost function of generator *i*, P_{gi} is power output from the i^{th} generator, N_g is the number of generators and a_i, b_i, c_i are the cost coefficients of generator *i*. Equation (1) represents the total generation cost while (2) sums the power outputs of all the generating units in the system. The objective function for the ED problem is described in Equation (3) which minimizes the total generation cost in the system. Equation (4) enforces the nodal power balance constraint while constraints (5) and (6) ensure that the generators operating limits are respected.

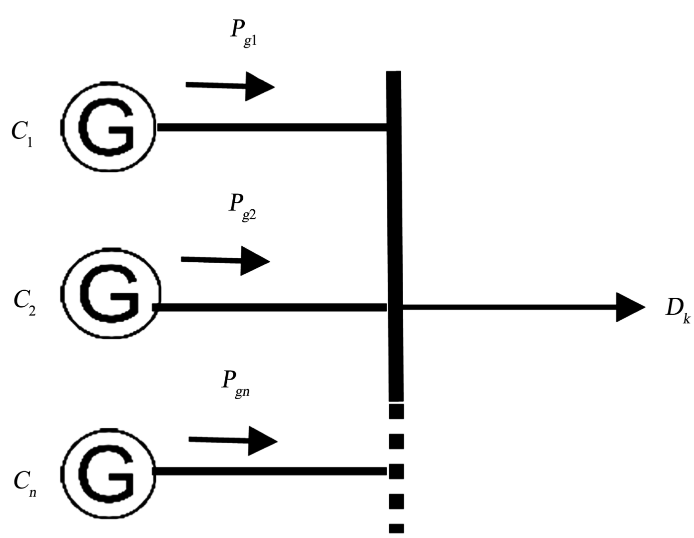


Figure 1. Generating units connected to a single busbar to supply a load.

2.2. Lagrange Multipliers and Complimentary Slackness Condition

For a simple system with linear constraints, the ED problem can easily be solved by the substitution method. But as the size of the system increases with nonlinear constraints, solution by substitution becomes inefficient in handling these constraints. Also, the solution obtained by substitution methods does not offer much insight into the economics of the optimization problem. Hence a more preferable approach to solving the ED problem is to consider the Lagrange multiplier in the solution method. In theoretical economics, the Lagrange multiplier is interpreted as the equilibrium price or the shadow cost of an optimization problem. Since ED is an optimization problem, the Lagrange multiplier obtained from the solution corresponds to the cost of supplying additional power from generating units at the optimal solution.

ED with only equality constraints: Considering the ED problem of the system in Figure 1, with the same objective function (1) and equality constraint (4), and ignoring the network losses, the problem can be formulated with the Lagrange multiplier method as follows [9]:

$$\ell\left(P_{gi},\lambda\right)=\sum_{i=1}^{n}C_i\left(P_{gi}\right)+\lambda_m\left(D-\sum_{i=1}^{n}P_{gi}\right). \tag{7}$$

For optimality, the necessary conditions as given by the gradient of the partial derivative of the Lagrangian are;

$$\frac{\partial\ell\left(P_{gi},\lambda\right)}{\partial P_{gi}}\equiv\frac{\partial C_i\left(P_{gi}\right)}{\partial P_{gi}}-\lambda=0 \tag{8}$$

$i=1,\cdots,n,$

$$\frac{\partial\ell\left(P_{gi},\lambda\right)}{\partial\lambda}\equiv D-\sum_{i=1}^{n}P_{gi}=0. \tag{9}$$

Hence, the incremental cost for the system is given as

$$\frac{\partial C_i\left(P_{gi}\right)}{\partial P_{gi}}=\frac{\partial C_n\left(P_{gn}\right)}{\partial P_{gn}}=\lambda \tag{10}$$

where λ represents the cost of producing one additional megawatt-hour with any of the generators at the optimal solution of the ED. The Lagrange multiplier is also known as the equal incremental cost or shadow price of the electrical energy. It is important to note that any perturbation in the output of the scheduled generating units will affect the optimal generation cost in the objective function (1).

2.3. ED with Inequality Constraints

To solve the ED problem with inequality constraints such as the generator operating limits the Karush-Kuhn-Tucker (KKT) condition is applied. The KKT condition or complementary slackness condition at optimum can be concisely summarised in (11) and the inequality (12). In the KKT condition, two possibilities are likely to exist for each inequality that exists in any particular system [11].

$$\mu_n g_n\left(P_{gn}\right)=0, \tag{11}$$

$$\mu_n \geq 0. \tag{12}$$

From the Equation (11) and inequality (12), the KKT condition or complementary slackness are described as follows:

2.3.1. Condition 1

For binding constraint, $\mu_n > 0$ and $g_n\left(P_{gn}\right)=0$.

2.3.2. Condition 2

For non-binding constraint,

$$\mu_n = 0 \text{ and } g_n\left(P_{gn}\right)<0;$$

where μ_n is the KKT multiplier for the n^{th} inequality constraints in a system.

For the Network shown in 1, the application of the KKT condition is presented as follows:

$$\min: C_T\left(p_{g1}, p_{gn}\right)=C_1\left(P_1\right)+C_n\left(P_n\right) \tag{13}$$

subject to:

$$y\left(P_{gi}, P_{gn}\right) \equiv D-P_{gi}-P_{gn}=0, \tag{14}$$

$$g_1\left(P_{gi}, P_{gn}\right) \equiv P_{gi}-P_{gi}^{\max} \leq 0, \tag{15}$$

$$g_2\left(P_{gi}, P_{gn}\right) \equiv P_{gi}^{\min}-P_{gi} \leq 0, \tag{16}$$

$$g_3\left(P_{gi}, P_{gn}\right) \equiv P_{gn}-P_{gn}^{\max} \leq 0, \tag{17}$$

$$g_4\left(P_{gi}, P_{gn}\right) \equiv P_{gn}^{\min}-P_{gn} \leq 0. \tag{18}$$

Hence applying the Lagrange method, the function becomes

$$\begin{aligned}
&\ell\left(P_{g1}, P_{gn}, \lambda, \mu_1, \mu_2, \mu_3, \mu_4\right) \\
&=C_1\left(P_{g1}\right)+C_n\left(P_{gn}\right)+\lambda\left(D-P_{g1}-P_{gn}\right)+\mu_1\left(P_{g1}-P_{g1}^{\max}\right) \\
&\quad+\mu_2\left(P_{g1}^{\min}-P_{g1}\right)+\mu_3\left(P_{gn}-P_{gn}^{\max}\right)+\mu_4\left(P_{gn}^{\min}-P_{gn}\right).
\end{aligned} \tag{19}$$

The necessary conditions for optimality of the constrained ED considering the KKT condition are

$$\frac{\partial \ell}{\partial P_{g1}} \equiv \frac{\mathrm{d} C_1\left(P_{g1}\right)}{\mathrm{d} P_{g1}}-\lambda+\mu_1-\mu_2=0, \tag{20}$$

$$\frac{\partial \ell}{\partial P_{gn}} \equiv \frac{\mathrm{d} C_n\left(P_{gn}\right)}{\mathrm{d} P_{gn}}-\lambda+\mu_3-\mu_4=0, \tag{21}$$

$$\frac{\partial \ell}{\partial \lambda} \equiv D-P_{g1}-P_{gn}=0, \tag{22}$$

Applying KKT or complementary conditions,

$$\frac{\partial \ell}{\partial \mu_1} \equiv P_{g1} - P_{g1}^{\max} \leq 0 \text{ and } \mu_1\left(P_{g1} - P_{g1}^{\max}\right) = 0; \mu_1 \geq 0, \tag{23}$$

$$\frac{\partial \ell}{\partial \mu_2} \equiv P_{g1}^{\min} - P_{g1} \leq 0 \text{ and } \mu_2\left(P_{g1}^{\min} - P_{g1}\right) = 0; \mu_2 \geq 0, \tag{24}$$

$$\frac{\partial \ell}{\partial \mu_3} \equiv P_{gn} - P_{gn}^{\max} \leq 0 \text{ and } \mu_3\left(P_{gn} - P_{gn}^{\max}\right) = 0; \mu_3 \geq 0, \tag{25}$$

$$\frac{\partial \ell}{\partial \mu_4} \equiv P_{gn}^{\min} - P_{gn} \leq 0 \text{ and } \mu_4\left(P_{gn}^{\min} - P_{gn}\right) = 0; \mu_4 \geq 0. \tag{26}$$

Following the KKT condition in the ED problem, three possible outcomes corresponding to the generating units in the system are expected to occur.

Case A: In this case, it is assumed that none of the generating units is at the upper or lower limits. Hence this implies that none of the inequality constraints is binding and that the KKT multipliers ($\mu_1, \mu_2, \mu_3, \mu_4$) associated with the inequality constraints (23) - (26) are all equal to zero.

$$\frac{\partial \ell}{\partial P_{g1}} \equiv \frac{\mathrm{d}C_1\left(P_{g1}\right)}{\mathrm{d}P_{g1}} - \lambda = 0, \tag{27}$$

$$\frac{\partial \ell}{\partial P_{gn}} \equiv \frac{\mathrm{d}C_n\left(P_{gn}\right)}{\mathrm{d}P_{gn}} - \lambda = 0, \tag{28}$$

$$\frac{\partial C_1\left(P_{g1}\right)}{\partial P_{g1}} = \frac{\partial C_n\left(P_{gn}\right)}{\mathrm{d}P_{gn}} = \lambda. \tag{29}$$

This solution implies that all the generating units in the system are operating at the same incremental cost.

Case B: Since the generating units can operate at lower or upper limits, in this case, it is assumed that only generator 1 is operating with an output P_{g1} at the upper limit, which implies that the inequality (23) is binding with a corresponding non-zero KKT multiplier μ_1. On the other hand, the inequalities (24) - (26) are not binding and hence their KKT multipliers μ_2, μ_3 and μ_4 are all equal to zero.

$$\frac{\partial \ell}{\partial P_{g1}} \equiv \frac{\mathrm{d}C_1\left(P_{g1}\right)}{\mathrm{d}P_{g1}} - \lambda + \mu_1 = 0, \tag{30}$$

$$\frac{\mathrm{d}C_1\left(P_{g1}\right)}{\mathrm{d}P_{g1}} - \lambda + \mu_1 \leq 0, \tag{31}$$

$$\frac{\partial \ell}{\partial P_{gn}} \equiv \frac{\mathrm{d}C_n\left(P_{gn}\right)}{\mathrm{d}P_{gn}} - \lambda = 0, \tag{32}$$

$$\frac{\mathrm{d}C_n\left(P_{gn}\right)}{\mathrm{d}P_{gn}} = \lambda. \tag{33}$$

Inequality (31) and Equation (33) show that all the generators are not operating at the same incremental cost because the incremental cost of generator 1 is less than that of other generators in the system.

Case C: Similarly, in this case, generator 1 is assumed to be at the lower operating limit. Hence, the inequality (24) is binding with a corresponding none zero KKT multiplier μ_2, while μ_1, μ_3 and μ_4 are all equal to zero.

$$\frac{\partial \ell}{\partial P_{g1}} \equiv \frac{\mathrm{d}C_1\left(P_{g1}\right)}{\mathrm{d}P_{g1}} - \lambda + \mu_2 = 0, \tag{34}$$

$$\frac{\partial \ell}{\partial P_{g1}} \equiv \frac{\mathrm{d}C_1\left(P_{g1}\right)}{\mathrm{d}P_{g1}} - \lambda + \mu_2 \geq 0, \tag{35}$$

$$\frac{\partial \ell}{\partial P_{gn}} \equiv \frac{\mathrm{d}C_n\left(P_{gn}\right)}{\mathrm{d}P_{gn}} - \lambda = 0, \tag{36}$$

$$\frac{\mathrm{d}C_n\left(P_{gn}\right)}{\mathrm{d}P_{gn}} = \lambda. \tag{37}$$

Equation (35) indicates that the marginal cost of generator 1 is higher than the marginal cost of all the generating units in the system and as such to supply the next MW load, any other generating unit with marginal cost in Equation (37) will have to be dispatched.

2.4. OPF Objective Function

In OPF calculation, the full AC power flow model offers the best solution since all the quantities are fully represented using the complete electrical network equations without any approximation. The objective function of OPF can either be a single decision variable or a sum of different decision variables. Depending on the nature of the problem, the optimization formulation can take the form of a convex or a non-convex problem which in turn influences the choice of algorithm for solving the problem. Some possible objective functions of an OPF may include; [15] [16] [17] [18] [19]

1) Minimization of losses in the system,
2) Minimization of CO_2 emission,
3) Maximization of profits,
4) Minimization of interruption cost,
5) Minimization of network investment cost,
6) Maximization of transmission line capacity, etc.

Constraints: In OPF formulation, the constraints could be modelled as equality, inequality or both, depending on the nature and purpose of solving the OPF problem. The operation of the power network is governed by physical laws which describe the interaction of the electrical variable at every operating point of the system. Kirchhoff's Current and Voltage laws are among the few laws that must be obeyed in the network during the OPF calculation. Since the network consists of individual components which are not infinite, it is equally required that the operational limits of all the components be respected during the OPF calculation. The basic constraints in OPF are equality and inequality constraints as represented in (39) and (40).

$$\min f\left(c_1, c_2, \cdots, c_n\right). \tag{38}$$

$$\begin{cases} \left(c_1, c_2, \cdots, c_n\right) = 0, \\ x_k\left(c_1, c_2, \cdots, c_n\right) = 0. \end{cases} \tag{39}$$

$$\begin{cases} y_1\left(c_1, c_2, \cdots, c_n\right) \le 0, \\ y_m\left(c_1, c_2, \cdots, c_n\right) \le 0. \end{cases} \tag{40}$$

The former refers to the power flow equation which attends attain a nonlinear status for the full AC OPF model and linear for the DC model. While the latter is related to the problem variables which are characterized by a fixed upper and/or lower limit of the OPF variables. Also, the inequality constraints may be expressed as a continuous or discrete function of the variables, depending on the characteristic of the problem. Generally, the OPF variable is made up of control and state variables. The basic state and control variables are as follows:

State variable

1) Voltage magnitude at each bus apart from the generator bus,

2) Voltage angle at each bus excluding the swing bus.

Control variable

1) The active power output of the generating units,

2) The voltage at the generating units,

3) Position of the transformer taps,

4) Status of the switched capacitors and reactors,

5) Amount of load disconnected, etc.

2.5. Security Constrained OPF (SCOPF)

Mathematically, the SCOPF can be formulated as presented in [20]:

$$\min : f\left(x_k, u\right), \tag{41}$$

subject to:

$$y_k\left(x_k, u\right) = 0, k = 0, 1, \cdots, n, \tag{42}$$

$$g_k\left(x_k, u\right) \ge 0, k = 0, 1, \cdots, n, \tag{43}$$

From the expressions, Equation (41) represents the objective function subject to the equality and inequality constraints in (42) and (43) respectively. The SCOPF consist of the intact and contingency network, where $k = 0$ corresponds to the intact state while $k = 1, \cdots, n$ corresponds to the contingency states of the network and n is the number of contingencies considered. Also, the vector of the control variables (such as generation dispatch, transformer taps setting, load interruption, etc.) is represented by x_k while the state variable vector under the k^{th} state is represented by u. It is also important to mention that with SCOPF, the dimension of the optimization problem is greater than that of the OPF problem, this is due to the increase in the number of variables and constraints.

In some cases, there may be no feasible solution to the SCOPF problem or it might be over-constrained, especially when the system is operating under severe conditions or very close to the system allowable operating limits [14]. Hence, to ensure that the SCOPF problem is not over-constrained, the limits of the control variable in the contingency state are sometimes relaxed to increase the feasible region of the solution. The ED, OPF and SCOPF are performed on a 3-bus test system in Figure 2 to compared using the system information given in Table 1, Table 2 and Table 3.

2.6. Unconstrained Economic Dispath (ED)

The ED problem is formulated as a linear programming problem as shown in (44) - (49).

$$\min \sum_{i=1}^{n} C_i\left(P_{Gi}\right) = 8P_{G1} + 10P_{G2} + 15P_{G3} + 20P_{G4}. \tag{44}$$

subject to:

$$P_{G1} + P_{G2} + P_{G3} + P_{G4} = D_1 + D_2 + D_3 = 410, \tag{45}$$

$$0 \le P_{G1} \le 140 \text{ MW}, \tag{46}$$

$$0 \le P_{G2} \le 285 \text{ MW}, \tag{47}$$

$$0 \le P_{G3} \le 200 \text{ MW}, \tag{48}$$

$$0 \le P_{G4} \le 90 \text{ MW}. \tag{49}$$

Manual computation of the Transmission Line flows: Given the transmission line data and generation dispatch, the power balance Equation (KCL) for the network in 2 is written as follows;

$$\text{Bus 1}: P_{G1} + P_{G2} - F_{12} - F_{13} - 50 = 0, \tag{50}$$

$$\text{Bus 2}: P_{G3} - F_{12} - F_{23} - 60 = 0, \tag{51}$$

$$\text{Bus 3}: P_{G4} - F_{13} - F_{23} - 300 = 0. \tag{52}$$

Table 1. Three-bus test system load data.

Bus	Index	Demand
1	D_1	50
2	D_2	60
3	D_3	300

Table 2. Three-bus test system generator data.

Bus	Index	Max. Capacity (MW)	Min. Capacity (MW)	marginal cost (£/MWh)
1	G_1	0	140	8
1	G_2	0	285	10
2	G_3	0	200	15
3	G_4	0	90	20

And for the KVL, the loop equation is given as

$$0.2F_{12} + 0.1F_{23} - 0.2F_{13} = 0. \tag{53}$$

Substituting for the generation dispatch in Equations (50) - (53), the KCL is thus reduced to;

$$\text{Bus 1}: 360 - F_{12} - F_{13} = 0, \tag{54}$$

$$\text{Bus 2}: F_{12} - F_{23} - 60 = 0, \tag{55}$$

$$\text{Bus 3}: F_{13} + F_{23} = 300. \tag{56}$$

From (54),

$$F_{12} = 360 - F_{13}. \tag{57}$$

Substituting F_{12} in (57) into the KVL in (53);

$$-0.4F_{13} + 0.1F_{23} = -72. \tag{58}$$

Multiplying (56) by 0.4,

$$0.4F_{13} + 0.4F_{23} = 120 \text{ MW}. \tag{59}$$

And adding (58) and (59),

$$0.5F_{23} = 48 \text{ MW}, \tag{60}$$

$$F_{23} = 96 \text{ MW}. \tag{61}$$

Substituting the value of F_{23} in (55) and (56), the values of F_{12} and F_{13} can be calculated respectively as;

$$F_{12} = 156 \text{ MW}, F_{13} = 204 \text{ MW}.$$

2.7. Optimal Power Flow (OPF)

For a lossless transmission network, the DC OPF can concisely be formulated with the same objective function in Equation (42) subject to constraints (5) and (6) with additional of the following constraints (62) - (65) as follows:

$$\sum_{i=1}^{N_g} P_{gi} = D_k + \sum_{l=1}^{N_l} f_l, \tag{62}$$

$$f_l - \frac{\theta_m - \theta_k}{x_l} = 0, \tag{63}$$

$$f_l^{\min} \le f_l \le f_l^{\max}, \tag{64}$$

$$\theta_k = 0. \tag{65}$$

In the OPF, the power system physical laws which are Kirchhoff's current and voltage laws (KCL and KVL respectively) are represented by the constraints (62) and (63). Equation (62) enforces the power balance constraint at all the nodes in the network while Equation (63) calculates the line flows. To ensure that the transmission lines transfer limits are not exceeded, the inequality in (64) is used. Equation (64) represents the voltage angle at the reference bus. For the three-bus network in **Figure 2**, the objective functions of the DC OPF along with the constraints are presented as follows.

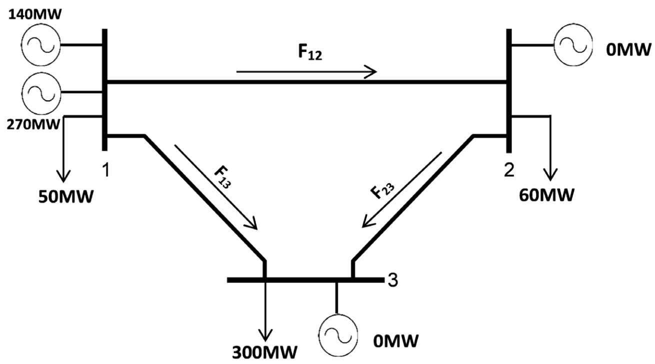


Figure 2. Manual computation of ED on 3-bus network with line flow directions.

$$\min \sum_{i=1}^{n} C_i\left(P_{Gi}\right) = 8P_{G1} + 10P_{G2} + 15P_{G3} + 20P_{G4}, \tag{66}$$

subject to:

The power balance constraints (KCL);

$$\text{Bus 1}: P_{G1} + P_{G2} = D_1 + F_{12} + F_{13}, \tag{67}$$

$$\text{Bus 2}: P_{G3} = D_2 - F_{12} + F_{23}, \tag{68}$$

$$\text{Bus 3}: P_{G4} = D_3 - F_{23} - F_{13}. \tag{69}$$

The KVL;

$$\text{Line 1-2}: F_{12} - \frac{\theta_1 - \theta_2}{x_{12}} = 0, \tag{70}$$

$$\text{Line 1-3}: F_{13} - \frac{\theta_1 - \theta_3}{x_{13}} = 0, \tag{71}$$

$$\text{Line 2-3}: F_{23} - \frac{\theta_2 - \theta_3}{x_{23}} = 0, \tag{72}$$

$$\text{Slack}: \theta_1 = 0. \tag{73}$$

The transmission lines transfer capability limit;

$$\text{Line 1-2}: -120 \le F_{12} \le 120, \tag{74}$$

$$\text{Line 1-3}: -250 \le F_{13} \le 250, \tag{75}$$

$$\text{Line 3-2}: -130 \le F_{23} \le 130. \tag{76}$$

The generator limits;

$$\text{G1}: 0 \le P_{G1} \le 140, \tag{77}$$

$$\text{G2}: 0 \le P_{G2} \le 285, \tag{78}$$

$$\text{G3}: 0 \le P_{G3} \le 200, \tag{79}$$

$$\text{G4}: 0 \le P_{G4} \le 90. \tag{80}$$

2.8. Security Constrained Optimal Power Flow (SCOPF)

Using the three-bus test system in Figure 2, the preventive security-constrained optimal power flow is implemented. The demonstration is based on the N − 1 security criterion which requires the network to continuously operate within the thermal limits after the outage of a single component in the system. In this paper, only the outage of the transmission line is considered in a deterministic way, which means that the probability of the outage occurrence is not considered. Also, it is assumed that the power outputs of the generating units are the same for both intact and contingency networks. Hence the SCOPF for the three-bus test system is formulated with the same objective function in (66) as follows:

$$\min \sum_{i=1}^{n} C_i\left(P_{Gi}\right) = 8P_{G1} = 10P_{G2} + 15P_{G3} + 20P_{G4}, \tag{81}$$

subject to:

Intact network constraints:

The power balance constraints (KCL);

$$\text{Bus 1}: P_{G1} + P_{G2} = D_1 + F_{12} + F_{13}, \tag{82}$$

$$\text{Bus 2}: P_{G3} = D_2 - F_{12} + F_{23}, \tag{83}$$

$$\text{Bus 3}: P_{G4} = D_3 - F_{23} - F_{13}, \tag{84}$$

The KVL is represented as shown in (70) to (73);

Contingency network constraints:

The outage of Transmission Line ($L_{1\text{-}2}$):

The power balance constraints (KCL);

$$\text{Bus 1}: P_{G1} + P_{G2} = D_1 + F_{12}^{outage12} + F_{13}^{outage12}, \tag{85}$$

$$\text{Bus 2}: P_{G3} = D_2 - F_{12}^{outage12} + F_{23}^{outage12}, \tag{86}$$

$$\text{Bus 3}: P_{G4} = D_3 - F_{13}^{outage12} - F_{23}^{outage12}, \tag{87}$$

The KVL;

$$\text{Line 1-2 outage}: F_{12}^{outage12} = 0, \tag{88}$$

$$\text{Line 1-3}: F_{13}^{outage12} - \frac{\theta_1^{outage12} - \theta_3^{outage12}}{x_{12}} = 0, \tag{89}$$

$$\text{Line 2-3}: F_{23}^{outage12} - \frac{\theta_2^{outage12} - \theta_3^{outage12}}{x_{23}} = 0, \tag{90}$$

$$\text{Slack}: \theta_1^{outage12} = 0. \tag{91}$$

The outage of Transmission Line ($L_{1\text{-}3}$):

The power balance constraints (KCL);

$$\text{Bus 1}: P_{G1} + P_{G2} = D_1 + F_{12}^{outage13} + F_{13}^{outage13}, \tag{92}$$

$$\text{Bus 2}: P_{G3} = D_2 - F_{12}^{outage13} + F_{23}^{outage13}, \tag{93}$$

$$\text{Bus 3}: P_{G4} = D_3 - F_{13}^{outage13} - F_{23}^{outage13},. \tag{94}$$

The KVL;

$$\text{Line 1-2}: F_{12}^{outage13} - \frac{\theta_1^{outage13} - \theta_2^{outage12}}{x_{12}} = 0, \tag{95}$$

$$\text{Line 1-3 outage}: F_{13}^{outage13} = 0, \tag{96}$$

$$\text{Line 2-3}: F_{23}^{outage13} - \frac{\theta_2^{outage13} - \theta_3^{outage13}}{x_{23}} = 0, \tag{97}$$

$$\text{Slack}: \theta_1^{outage13} = 0. \tag{98}$$

The outage of Transmission Line ($L_{2\text{-}3}$):

The power balance constraints (KCL);

$$\text{Bus 1}: P_{G1} + P_{G2} = D_1 + F_{12}^{outage23} + F_{13}^{outage23}, \tag{99}$$

$$\text{Bus 2}: P_{G3} = D_2 - F_{12}^{outage23} + F_{23}^{outage23}, \tag{100}$$

$$\text{Bus 3}: P_{G4} = D_3 - F_{13}^{outage23} - F_{23}^{outage23}, \tag{101}$$

The KVL;

$$\text{Line 1-2}: F_{12}^{outage23} - \frac{\theta_1^{outage23} - \theta_2^{outage23}}{x_{12}} = 0, \tag{102}$$

$$\text{Line 1-3}: F_{13}^{outage23} \frac{\theta_1^{outage23} - \theta_3^{outage23}}{x_{13}} = 0, \tag{103}$$

$$\text{Line 2-3 outage}: F_{23}^{outage23} = 0, \tag{104}$$

$$\text{Slack}: \theta_1^{outage23} = 0. \tag{105}$$

The trans. lines transfer capability limit for the intact network:

$$\text{Line 1-2}: -120 \le F_{12} \le 120, \tag{106}$$

$$\text{Line 1-3}: -250 \le F_{13} \le 250, \tag{107}$$

$$\text{Line 2-3}: -130 \le F_{23} \le 130. \tag{108}$$

Trans. lines transfer capability limit for contingency network:

$$\text{Line 1-3}: -250 \le F_{13}^{outage12} \le 250, \tag{109}$$

$$\text{Line 2-3}: -130 \le F_{23}^{outage12} \le 130, \tag{110}$$

$$\text{Line 1-2}: -120 \le F_{12}^{outage13} \le 120, \tag{111}$$

$$\text{Line 2-3}: -130 \le F_{23}^{outage13} \le 130, \tag{112}$$

$$\text{Line 1-2}: -120 \le F_{12}^{outage23} \le 120, \tag{113}$$

$$\text{Line 1-3}: -250 \le F_{13}^{outage23} \le 250, \tag{114}$$

The generator limits;

$$\text{G1}: 0 \le P_{G1} \le 140, \tag{115}$$

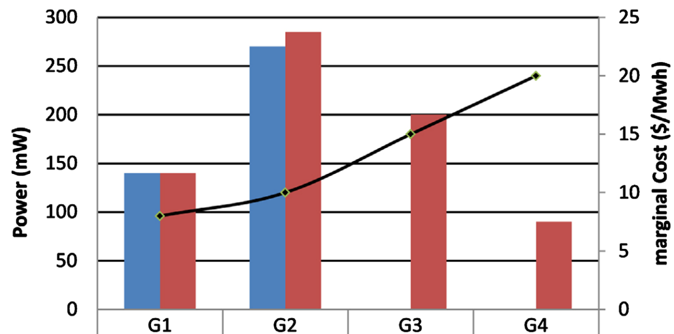
$$\text{G2}: 0 \le P_{G2} \le 285, \tag{116}$$

$$\text{G3}: 0 \le P_{G3} \le 200, \tag{117}$$

$$\text{G4}: 0 \le P_{G4} \le 90. \tag{118}$$

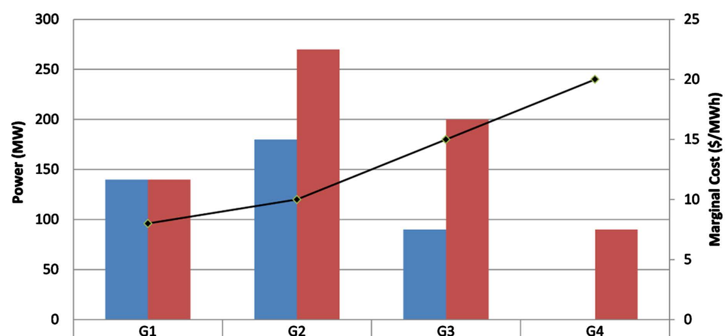
3. Results and Discussion

In this section, the three models are tested on a 3-bus network depicted in Figure 2. In the system under investigation, the generator G_1 is the least expensive then followed by G_2 hence, only the two generators are dispatched to satisfy the demand as demonstrated in **Figure 3**. Since generators G_3 and G_4 are not dispatched, the marginal generator G_2 in the system is hence making the marginal cost of the system equal \$10/MWh which is the cheapest while ignoring the line constraints. As already discussed in previous sections, the system marginal cost represents the Lagrange multiplier or the shadow cost of the system. Using the branch data given in **Table 3**, the resulting line flows for the ED are manually computed by applying Kirchhoff's voltage and current laws. Correspondingly, it can be observed that the resulting flow F_{12} on line $L_{1\text{-}2}$ shown in **Figure 10** is above the maximum transmission line capacity. This condition makes the unconstrained economic dispatch solution unsuitable when considering the security constraints of the network. With the network constraints considered in the ED problem, **Figure 4** shows the generation dispatched together with the marginal cost of the generators. It can be observed from the graph that generator G_2 is operating below its maximum capacity even though being less expensive than generator G_3. This out of merit order generation is a result of the limitation in the transmission line capacity, thus constraining the cheaper generator G_2 from operating at full capacity.



	G1	G2	G3	G4
Gen Output (MW)	140	270	0	0
Gen Capacity (MW)	140	285	200	90
Gen Marginal cost ($/Mwh)	8	10	15	20

Figure 3. Economic dispatch for the three-bus test system.



	G1	G2	G3	G4
Gen Output (MW)	140	180	90	0
Gen Capacity (MW)	140	270	200	90
Gen Marginal cost ($/Mwh)	8	10	15	20

Figure 4. Generators output for the optimal power flow (OPF).

Table 3. Three-bus test system branch data.

Line	Bus Origin	Bus destination	Capacity (MW)	x (p.u)
$L_{1\text{-}2}$	1	2	120	0.2
$L_{1\text{-}3}$	1	3	250	0.2
$L_{2\text{-}3}$	2	3	130	0.1

On the other hand, the out-of-merit order generation is to ensure that the network security is not violated even though being more expensive than the merit-order generation in the ED. However, the total system operating cost is represented in **Figure 5**, with the OPF resulting in an extra cost of $450. The extra cost incurred in the OPF case is considered as the cost of network security. Also, the power flows on each of the lines are shown in **Figure 6**. From the graph, it can be seen that Line $L_{1\text{-}2}$ is operating at 100% of its capacity while $L_{1\text{-}3}$ and $L_{2\text{-}3}$ are operating at 60% and 46.2% of their capacities respectively.

From the simulation results, **Figure 7** shows a comparison of the power outputs of the generators in the ED, OPF and PSCOPF scenarios. From the graph, it can be seen that generator G_1 operates at full capacity in all scenarios, mainly due to its low incremental cost, while generator G_2 output is reduced by 150 MW in the SCOPF scenario. Correspondingly, the output of generators G_3 and G_4 are increased by 110 MW and 40 MW respectively, despite being more expensive than generator G_2. This increase in the outputs of expensive generators is to continuously supply the heavy demand at bus 3 due to the line contingencies. Considering **Figure 8**, the total system cost for the three case studies is compared to illustrate the trade-off between system network reliability and the cost of system operation.

From the comparison, SCOPF shows an increase in the total system cost by $1400 of the initial ED system cost and is also higher than OPF system cost by $950. This shows that the total system cost increases when operating the system with network security constraints. The line flows for the intact and contingency network is shown in **Figure 9**. By comparing the flows with the basic OPF in **Figure 6**, it can be observed that the line flows are lower due to the power demand at each bus being supplied by the generators closer to them or connected to the same bus.

Modelling Software

In this paper, the advanced interactive multidimensional modelling system (AIMMS) optimization package is used. A comparison of various modelling languages used for optimization in power system study is presented in [21]. Among the modelling language compared include, Lingo, GAMs, Aimms and Yalmip. As part of the comparison, the author categorized the features of the modelling language based on the following attributes:

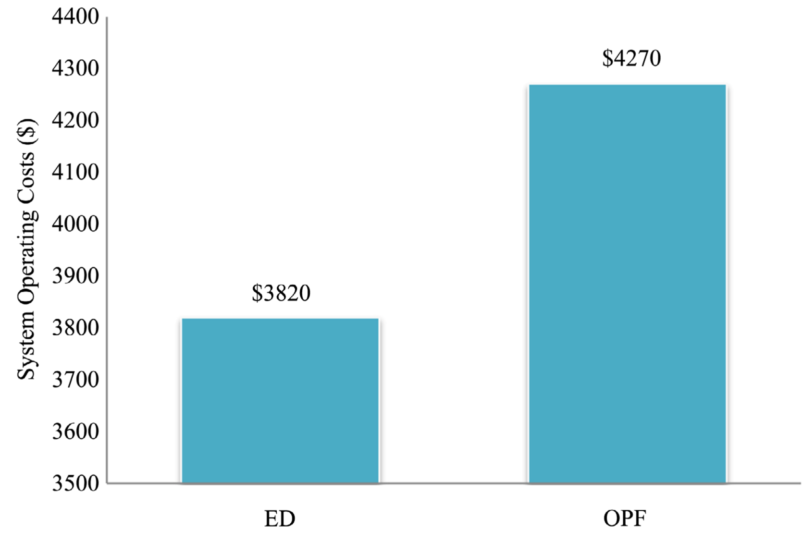


Figure 5. Total system operating cost for ED and OPF.

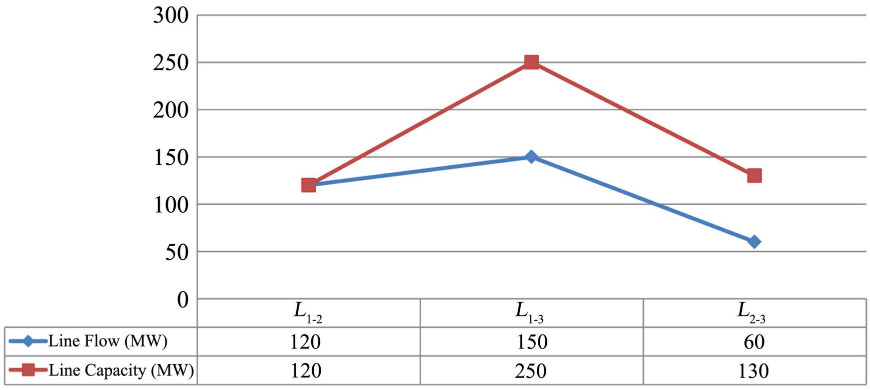


Figure 6. Comparison of transmission line flows and line capacities for OPF.

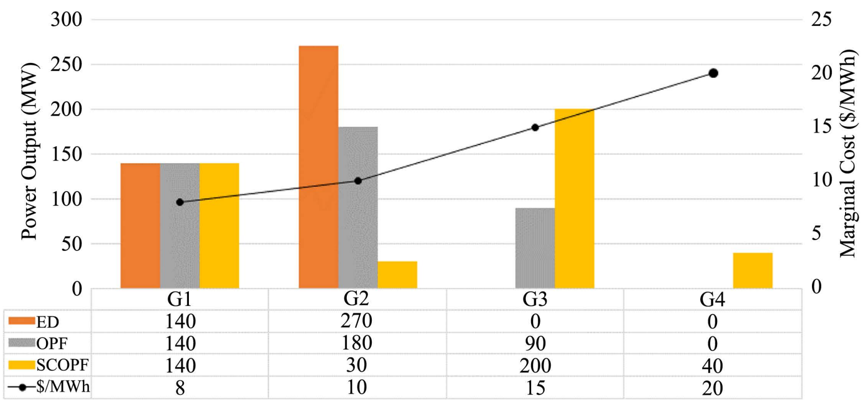


Figure 7. Comparison of generator outputs for ED, OPF and PSCOPF.

Flexibility with MS Excel Spreadsheets: AIMMS offers the flexibility of importing and exporting data from a spreadsheet using the Ms excel function library. Also, the Ms Excel add-in (AIMMS interface setup) allows for easy loading of data directly from the spreadsheet into the required location in AIMMS.

Figure 8. Total system operating cost for ED, OPF and PSCOPF.

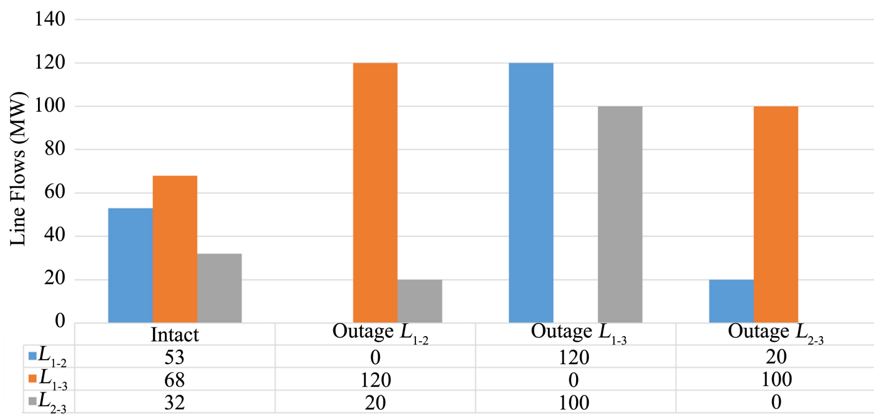


Figure 9. Transmission line flows for the PSCOPF.

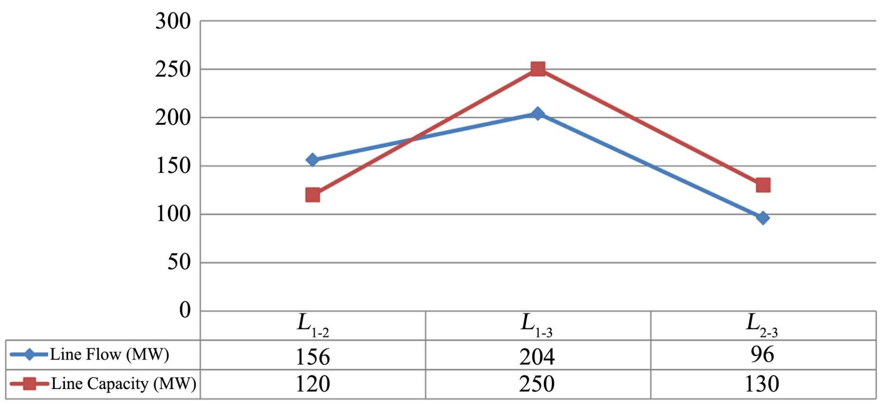


Figure 10. Transmission line flows for unconstrained economic dispatch.

Declaration of equation, variables and constraints: Declaration in AIMMS is very straightforward as it does not require the user to write any specific programme in the integrated development environment (IDE) instead a set-oriented syntax is used. The AIMMS IDE allows the user to define all the required variables, constraints and parameters for the model with ease.

Easy debugging/clarity of code: Editing of source code is generally not allowed in the AIMMS IDE, however, the user gets an error message, this can easily be identified on the command line or using a tool known as mathematical programme inspector to search for the error in the user-defined algorithm.

Choice of Solvers and licenses and access: The AIMMS modelling package has several solvers as part of the optimization package that can automatically select the appropriate solver for any particular optimization problem. It also has some functional models to support new users of the software as well as a free academic license for easy installation of the software on any computer [22].

The criteria discussed in this section form the basics of choosing the AIMMS modelling language for this research work. However, the use of other programming languages like Java and MATLAB will equally be necessary as AIMMS is an optimization package.

4. Conclusion

In this paper, a thorough review of the basic economic dispatch (ED), optimal power flow (OPF) and security-constrained optimal power flow (SCOPF) as applied to a three-bus test system is presented. The review revealed that by incorporating network constraints into the basic economic dispatch, the violation of the network constraints can be avoided but results in a higher system operating cost than the basic ED. Furthermore, this cost is further increased by considering the network security constraints. The concept and formulation of the Lagrange multiplier and the KKT condition were reviewed in this chapter. Finally, a description of the modelling software used in this research work is also presented.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Chowdhury, E.H. and Tech, V. (1990) A Review of Recent Advances in Economic Dispatch. *IEEE Transactions on Power Systems*, **5**, 1248-1259. https://doi.org/10.1109/59.99376

[2] Happ, H.H. (1977) Optimal Power Dispatch—A Comprehensive Survey. *IEEE Transactions on Power Apparatus and Systems*, **96**, 841-854. https://doi.org/10.1109/T-PAS.1977.32397

[3] FERC (2005) Economic Dispatch: Concepts, Practices and Issues. Palm Springs, 11.

[4] Huneault, M. and Galiana, F.D. (1991) A Survey of the Optimal Power Flow Literature. *IEEE Transactions on Power Systems*, **6**, 762-770. https://doi.org/10.1109/59.76723

[5] Carpentier, J. (1979) Optimal Power Flows. *International Journal of Electrical Power & Energy Systems*, **1**, 3-15. https://doi.org/10.1016/0142-0615(79)90026-7

[6] Cain, M.B., O'neill, R.P. and Castillo, A. (2012) History of Optimal Power Flow and Formulations. Federal Energy Regulatory Commission, Washington DC, 1-36.

[7] Momoh, J.A., Adapa, R. and El-Hawary, M.E. (1999) A Review of Selected Optimal Power Flow Literature to 1993. I. Nonlinear and Quadratic Programming Approaches. *IEEE Transactions on Power Systems*, **14**, 96-104. https://doi.org/10.1109/59.744492

[8] Sasson, A.M. and Merrill, H.M. (1974) Some Applications of Optimization Techniques to Power Systems Problems. *Proceedings of the IEEE*, **62**, 959-972. https://doi.org/10.1109/PROC.1974.9548

[9] Capitanescu, F., Glavic, M., Ernst, D. and Wehenkel, L. (2006) Applications of Security-Constrained Optimal Power Flows. *Proceedings of Modern Electric Power Systems Symposium*, *MEPS*06, Online, 7, http://hdl.handle.net/2268/13538

[10] Capitanescu, F., *et al.* (2011) State-of-the-Art, Challenges, and Future Trends in Security Constrained Optimal Power Flow. *Electric Power Systems Research*, **81**, 1731-1741. https://doi.org/10.1016/j.epsr.2011.04.003

[11] Daniel, G.S. and Kirschen, S. (2004) Fundamentals of Power System Economics. John Wiley & Sons, West Sussex. https://doi.org/10.1002/0470020598

[12] Cañizares, C., Rosehart, W., Berizzi, A. and Bovo, C. (2001) Comparison of Voltage Security Constrained Optimal Power Flow Techniques. 2001 *Power Engineering Society Summer Meeting. Conference Proceedings*, Vol. 3, 1680-1685. https://doi.org/10.1109/PESS.2001.970328

[13] Monticelli, A., Pereira, M.V.F. and Granville, S. (1987) Security-Constrained Optimal Power Flow with Post-Contingency Corrective Rescheduling. *IEEE Transactions on Power Systems*, **2**, 175-180. https://doi.org/10.1109/TPWRS.1987.4335095

[14] Allen, G.B.S., Wood, J. and Wollenberg, B.F. (2003) Power Generation, Operation, and Control. John Wiley & Sons, Hoboken.

[15] Das, C.K., Bass, O., Kothapalli, G., Mahmoud, T.S. and Habibi, D. (2018) Overview of Energy Storage Systems in Distribution Networks: Placement, Sizing, Operation, and Power Quality. *Renewable and Sustainable Energy Reviews*, **91**, 1205-1230. https://doi.org/10.1016/j.rser.2018.03.068

[16] Ahmed, M.Z. and Padhiyar, N. (2020) Multi Objective Optimization of a Tri-Reforming Process with the Maximization of H2 Production and Minimization of CO_2 Emission & Power Loss. *International Journal of Hydrogen Energy*, **45**, 22480-22491. https://doi.org/10.1016/j.ijhydene.2020.06.210

[17] Prajapati, V.K. and Mahajan, V. (2021) Reliability Assessment and Congestion Management of Power System with Energy Storage System and Uncertain Renewable Resources. *Energy*, **215**, Article ID: 119134. https://doi.org/10.1016/j.energy.2020.119134

[18] Nourizadeh, H., Mosallanejad, A. and SetayeshNazar, M. (2022) Optimal Placement of Fixed Series Compensation and Phase Shifting Transformer in the Multi-Year Generation and Transmission Expansion Planning Problem at the Pool-Based Market for Maximizing Social Welfare and Reducing the Investment Costs. *IET Generation*, *Transmission & Distribution*, **16**, 2959-2976. https://doi.org/10.1049/gtd2.12488

[19] Yu, S., Zhou, S. and Qin, J. (2022) Layout Optimization of China's Power Transmission Lines for Renewable Power Integration Considering Flexible Resources and Grid Stability. *International Journal of Electrical Power & Energy Systems*, **135**, Article ID: 107507. https://doi.org/10.1016/j.ijepes.2021.107507

[20] Dong, F., Huang, L., Lam, B.P., Member, S. and Xu, X. (2012) Practical Applications of Preventive Security Constrained Optimal Power Flow. 2012 *IEEE Power and Energy Society General Meeting*, San Diego, 22-26 July 2012, 1-5.

[21] Podhradsky, M. (2010) Modelling Languages for Optimization. Bachelor Thesis, Czech Technical University, Prague.

[22] https://www.aimms.com/support/licensing/