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Electron G-Factor Anomaly and the Charge Thickness

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Abstract

The electron g-factor relates the magnetic moment to the spin angular momentum. It was originally theoretically calculated to have a value of exactly 2. Experiments yielded a value of 2 plus a very small fraction, referred to as the g-factor anomaly. This anomaly has been calculated theoretically as a power series of the fine structure constant. This document shows that the anomaly is the result of the electron charge thickness. If the thickness were to be zero, g = 2 exactly, and there would be no anomaly. As the thickness increases, the anomaly increases. An equation relating the g-factor and the surface charge thickness is presented. The thickness is calculated to be 0.23% of the electron radius. The cause of the anomaly is very clear, but why is the charge thickness greater than zero? Using the model of the interior structure of the electron previously proposed by the author, it is shown that the non-zero thickness, and thus the g-factor anomaly, are due to the proposed positive charge at the electron center and compressibility of the electron material. The author's previous publication proposes a theory for splitting the electron into three equal charges when subjected to a strong external magnetic field. That theory is revised in this document, and the result is an error reduced to 0.4% in the polar angle where the splits occur and a reduced magnetic field required to cause the splits.

Keywords

Electron G-Factor Anomaly, Electron Charge Thickness, Electron Positive Charge, Electron Mass Thickness, Electron Fractionalization, Splitting the Electron, Electron Compressibility Factor

1. Introduction

As Einstein famously said towards the end of his life, "You know, it would be sufficient to really understand the electron." [1]

The electron g-factor g is calculated from the charge and magnetic moment. The result shows that $\frac{B}{E}=\frac{g}{2\alpha}$, where B is the magnetic field at the center of the electron, E is the electric field at the electron's surface due to its charge q, and α is the fine structure constant. The magnetic field B is modeled to be created by many spinning subshells of charge, with the outermost subshell having a radius R, the classical electron radius, and the innermost subshell having a radius of $R_{qi}<R$. An equation is derived that relates g to the charge shell outer and inner radii ratio $\frac{R_{qi}}{R}$. The thickness of the outer shell mass was derived in [2]. An improvement to that calculation is presented, and the result is a somewhat smaller inner radius R_i of the outer shell mass.

The charge shell is assumed to be embedded within the mass at the outer surface of the outer shell mass. A model is presented wherein the charge shell is stretched by the positive charge of the central core, thereby creating the non-zero thickness of the shell and thus the g-factor anomaly. From the anomaly, the compressibility factor of the electron's outer shell mass is calculated.

The splitting of the electron into three equal charges was modeled in [2]. That model is modified in this document, with the result of lower error in the predicted angles at which the splits occur. All of the components of pressure on the electron's outer surface are calculated. The splits are defined to occur at the polar angles Θ and $180-\Theta$, where the total pressure on the surface is outward. The minimum magnetic field for splitting to occur is calculated.

Table 1 contains some constants associated with the electron [3]. Unless otherwise specified, all units are CGS.

Table 1. Table of Constants.

constant	symbol	value [CGS]
fine structure constant	α	$7.2973525693\times10^{-3}$
Planck's constant	h	6.62607015×10^{-27}
speed of light	c	2.99792458×10^{10}
electron mass	m	$9.1093837015\times10^{-28}$ $=0.51099895000$ MeV
classical electron radius	R	$2.817940325\times10^{-13}$
electron charge	q	$-4.803204713\times10^{-10}$
outer shell charge	$q^{-}=\frac{3}{2\alpha}q$	$-9.873179349\times10^{-8}$
central core charge	$q^{+}=\left(1-\frac{3}{2\alpha}\right)q$	9.825147301×10^{-8}
electron g-factor	g	2.00231930436256

2. G-Factor Derivation

The electron g-factor g can be derived from the electric field E at the surface of the electron and the magnetic field B at its center [4]. The radius used is the classical electron radius R [5], which is also derived below. M is the electron magnetic dipole moment.

$$E = \frac{q}{R^2}, \quad B = \frac{2M}{R^3}, \quad R = \frac{q^2}{mc^2} \tag{1}$$

The gyromagnetic ratio of the electron can be derived from the following equation [6], where $S_z = \pm\frac{1}{2}\frac{h}{2\pi}$.

$$\frac{M}{S_z} = \frac{q}{2mc}g \tag{2}$$

The electron charge q can be expressed in terms of the fine structure constant, Planck's constant, and the speed of light [3]:

$$q = -\sqrt{\alpha\left(\frac{h}{2\pi}\right)c} \tag{3}$$

$$\frac{B}{E} = \frac{2M}{R^3}\frac{R^2}{q} = \frac{2}{Rq}\frac{hRc}{8\pi q}g = \frac{hc}{4\pi}\frac{2\pi}{hc\alpha}g = \frac{g}{2\alpha} \tag{4}$$

The following variables are defined for use in the equations below: The radius r is that of a charge subshell; $\mathrm{d}r$ is the thickness and spacing of the subshells. The radius of the outermost charge subshell is R, the classical electron radius. The radius of the innermost charge subshell is R_{qi}. The inner radius of the outer shell mass is R_i.

For an electron surface charge shell having non-zero thickness, B is considered to be created by a spinning nest of concentric subshells, each having a charge $\mathrm{d}q^-$. The charge shell has a total charge of $q^- = \frac{3}{2\alpha}q$ [3].

$$\mathrm{d}q^- = 4\pi r^2 \sigma \mathrm{d}r \tag{5}$$

The charge volume density σ of the charge shellis [2]:

$$\sigma = \frac{3q^-}{4\pi R^3\left(1-\left(\frac{R_{qi}}{R}\right)^3\right)} \tag{6}$$

The period and frequency of rotation of the charge shell are:

$$T = \frac{2\pi R}{c}; \quad \omega = \frac{2\pi}{T} = 2\pi\frac{c}{2\pi R} = \frac{c}{R} \tag{7}$$

The magnetic moment $\mathrm{d}M$ of each charge subshell is [7]:

$$\mathrm{d}M = \frac{1}{3}dq^-\frac{\omega}{c}r^2 = \frac{1}{3}\left(4\pi r^2\sigma\mathrm{d}r\right)\frac{r^2}{R} = \frac{4\pi}{3R}\sigma r^4\mathrm{d}r \tag{8}$$

The ratio of the magnetic field B at the electron center created by the spinning charge shell to the electric field E at the electron surface is:

$$\frac{B}{E}=\int_{R_{qi}}^{R}\frac{2\mathrm{d}M}{r^3E}=\frac{4\pi}{3R}\frac{\sigma}{E}R^2\left[1-\left(\frac{R_{qi}}{R}\right)^2\right]=\frac{3}{2\alpha}\frac{1-\left(\frac{R_{qi}}{R}\right)^2}{1-\left(\frac{R_{qi}}{R}\right)^3}=\frac{g}{2\alpha}\tag{9}$$

$$g=\frac{3}{1-\left(\frac{R_{qi}}{R}\right)^3}\left[1-\left(\frac{R_{qi}}{R}\right)^2\right]\tag{10}$$

For a charge shell of zero thickness, $R_{qi}=R$ and $g = 2$ exactly. Factor g has been calculated theoretically as a power series of the fine structure constant [3], so the charge thickness can also be calculated theoretically from Equation (10). Conversely, given the charge shell thickness, the g-factor can be calculated. Therefore,

$$R_{qi}=0.9976825R\tag{11}$$

and the charge shell thickness is $0.0023175R$.

3. Derivation of the Classical Electron Radius

Many radii have been proposed for the free electron, and have a wide range of values. This document uses classical physics for deriving some of the electron's attributes. It seems appropriate, therefore, to use the classical electron radius. One of the author's previous papers [3] shows that it, along with other attributes, are simple functions of the fine structure constant, a useful feature in some of the derivations in this document. The electron radius is also defined by the National Institute of Standards and Technology as the classical radius, which also seems to be the commonly accepted radius. It can be derived very simply using classical physics.

Consider a spherical shell of charge q with a radius r of infinity ∞. Each charge increment of the shell sees the remainder of the charge as a point charge at the center. Energy is required to shrink the sphere radius to R. This energy is converted to mass m. The relationship between radius R and mass m is expressed by:

$$-\int_{\infty}^{R}\left(\frac{q}{r}\right)^2\mathrm{d}r=mc^2=-q^2\left[-\frac{1}{r}\right]_{\infty}^{R}=\frac{q^2}{R},\quad R=\frac{q^2}{mc^2}\tag{12}$$

4. Mass Outer Radius

Equation (12) for R expresses the radius of the charge shell, and not necessarily that of the mass. It seems to be commonly believed that the two radii are the same. However, the mass outer radius can actually be as low as $0.886R$ with the outer shell still having the spin angular momentum predicted by quantum theory. As the mass radius is reduced, the outer shell thickness is also reduced, until it is very thin.

Equations (45)-(47) below, which predict the splitting of the electron, contain terms expressing the contribution to pressure from centrifugal force. The mass outer radius is assumed to be *R*. Upon reducing the mass radius and adjusting $\frac{R_i}{R}$ accordingly to preserve spin angular momentum, it was seen that the error in the splitting polar angle increased. As a result, it is assumed in this document that the mass radius is equal to *R*.

It does not seem to be known whether the charge shell floats like a cloud above the mass surface or is embedded within the mass. Since this document assumes that the charge and mass radii are the same, then it follows that the charge embedded within the mass is also assumed.

5. Outer Shell Mass Thickness

The thickness of the outer shell mass was calculated in [2]. Equations (10) and (11) of that paper are approximations, made to simplify the thickness calculation. A more accurate equation for $\mathrm{d}m^+$ is:

$$\mathrm{d}m^+ = 4\pi\sigma R\left[r\mathrm{d}r - \left(\frac{R_i}{R}\right)^3 \sqrt{\frac{1-x^2}{1-\left(\frac{R_i}{R}\right)^2 x^2}}\,x\mathrm{d}x\right], \tag{13}$$

where σ is the outer shell mass density, R_i is the inner radius of the outer shell and $x = \frac{r}{R_i} = 0 \to 1$.

The mass m^+ of the outer shell then becomes:

$$m^+ = 4\pi\sigma R^3\left[\int_0^1 x\mathrm{d}x - \left(\frac{R_i}{R}\right)^3 \int_0^1 \sqrt{\frac{1-x^2}{1-\left(\frac{R_i}{R}\right)^2 x^2}}\,x\mathrm{d}x\right] \tag{14}$$

The equation to be solved for $\frac{R_i}{R}$ then becomes:

$$\begin{aligned}&\frac{1}{4} - \left(\frac{R_i}{R}\right)^5 \int_0^1 \sqrt{\frac{1-x^2}{1-\left(\frac{R_i}{R}\right)^2 x^2}}\,x^3\mathrm{d}x \\ &= \left(\frac{2\alpha}{3mcR}S = 0.57735027\right)\left[\frac{1}{2} - \left(\frac{R_i}{R}\right)^3 \int_0^1 \sqrt{\frac{1-x^2}{1-\left(\frac{R_i}{R}\right)^2 x^2}}\,x\mathrm{d}x\right]\end{aligned} \tag{15}$$

The solution to Equation (15) is:

$\frac{R_i}{R} = 0.6371$ and the thickness of the outer shell is $0.3629R$.

The mass m^+ of the outer shell is $\frac{3}{2\alpha}m$ [3]. Equation (14) can therefore be written as:

$$
\begin{aligned}
m^{+} &= 4\pi\sigma R^{3}\left(0.40521234\right) = \frac{3}{2\alpha}m \\
&= \frac{3}{2\alpha}\frac{q^{2}}{c^{2}R} = \frac{3}{2\alpha}\frac{1}{c^{2}R}\left(\frac{2\alpha}{3}q^{-}\right)^{2} = \frac{2\alpha\left(q^{-}\right)^{2}}{3c^{2}R}
\end{aligned} \tag{16}
$$

The outer shell mass density can now be calculated as:

$$
\sigma = \frac{m^{+}}{4\pi R^{3}\left(0.40521234\right)} = \frac{\left(q^{-}\right)^{2}}{4\pi c^{2}R^{4}}\frac{2\alpha}{3\left(0.40521234\right)} = \frac{\left(q^{-}\right)^{2}}{4\pi R^{4}c^{2}}\left(0.012005808\right) \tag{17}
$$

6. Charge Shell Thickness

It has been shown above that the anomaly in the g-factor value can be explained by the non-zero thickness of the electron charge shell. But the question remains: Why is the charge shell thickness non-zero? The following proposes that the answer is the presence of the positive charge at the electron center and the elasticity of the electron material.

To calculate its thickness as a function of elasticity, the charge shell is sliced into many nested concentric subshells. A subshell sees the attractive (inward) force of the positive central core, the repulsive (outward) force of other subshells inside of it, and the inward repulsive force of each charge increment of the charge subshells outside of it. As subshell radius increases, the attractive (inward) force is reduced by the negative subshells inside of it, which offset the positive charge. The inward force is also reduced because of fewer subshells outside of it. Since the charge of the negative outer shell is greater than the positive charge of the central core, eventually the subshell radius increases to the point where the net force from all charges becomes repulsive (outward). So inner subshells will see inward forces while outer subshells will see outward forces. Therefore, the positive central core charge creates a force on the negative outer charge shell that tends to pull it apart.

The fact that the charge shell has non-zero thickness suggests that the electron material has non-zero but finite elasticity. It is assumed for this model that it is the elasticity that enables the non-zero thickness of the charge shell. There might be alternative explanations for why the charge is distributed along the radials, such as diffusion. But diffusion is counterintuitive, since the net force on the charge shell is outward, not inward.

Let each subshell have the same charge:

$$
\mathrm{d}q^{-} = \frac{q^{-}}{R - R_{qi}}\mathrm{d}r = \frac{q^{-}}{R\left(1 - \frac{R_{qi}}{R}\right)}\mathrm{d}r \tag{18}
$$

The electric field seen by a subshell at radius r and due to the positive central core and subshells inside of it is expressed as:

$$\left(q^{+}+\frac{r-R_{qi}}{R-R_{qi}}q^{-}\right)\frac{1}{r^{2}}=\left(q^{+}-\frac{\frac{R_{qi}}{R}}{1-\frac{R_{qi}}{R}}q^{-}\right)\frac{1}{r^{2}}+\frac{\frac{1}{R}}{1-\frac{R_{qi}}{R}}q^{-}\frac{1}{r} \tag{19}$$

The electric force on a subshell charge $\mathrm{d}q^{-}$ at radius r is expressed as:

$$\begin{aligned}&\frac{q^{-}}{R\left(1-\frac{R_{qi}}{R}\right)}\left[\left(q^{+}-\frac{\frac{R_{qi}}{R}}{1-\frac{R_{qi}}{R}}q^{-}\right)\frac{1}{r^{2}}+\frac{\frac{1}{R}}{1-\frac{R_{qi}}{R}}q^{-}\frac{1}{r}\right]\mathrm{d}r\\&=\frac{\left(q^{-}\right)^{2}}{R\left(1-\frac{R_{qi}}{R}\right)}\left[\left(\frac{q^{+}}{q^{-}}-\frac{\frac{R_{qi}}{R}}{1-\frac{R_{qi}}{R}}\right)\frac{1}{r^{2}}+\frac{\frac{1}{R}}{1-\frac{R_{qi}}{R}}\frac{1}{r}\right]\mathrm{d}r\end{aligned} \tag{20}$$

Each subshell applies a force via the outer shell material to the outer surface of the charge shell. The total force is:

$$\begin{aligned}&\frac{\left(q^{-}\right)^{2}}{R\left(1-\frac{R_{qi}}{R}\right)}\left[\left(\frac{q^{+}}{q^{-}}-\frac{\frac{R_{qi}}{R}}{1-\frac{R_{qi}}{R}}\right)\int_{R_{qi}}^{R}\frac{\mathrm{d}r}{r^{2}}+\frac{\frac{1}{R}}{1-\frac{R_{qi}}{R}}\int_{R_{qi}}^{R}\frac{\mathrm{d}r}{r}\right]\\&=\frac{\left(q^{-}\right)^{2}}{R^{2}}\left[\frac{q^{+}}{q^{-}}\frac{R}{R_{qi}}-\frac{1}{1-\frac{R_{qi}}{R}}-\frac{1}{\left(1-\frac{R_{qi}}{R}\right)^{2}}\ln\left(\frac{R_{qi}}{R}\right)\right]\end{aligned} \tag{21}$$

The electrical pressure on the charge shell at radius R is:

$$\frac{\left(q^{-}\right)^{2}}{4\pi R^{4}}\left[\frac{q^{+}}{q^{-}}\frac{R}{R_{qi}}-\frac{1}{1-\frac{R_{qi}}{R}}-\frac{1}{\left(1-\frac{R_{qi}}{R}\right)^{2}}\ln\left(\frac{R_{qi}}{R}\right)\right] \tag{22}$$

The inward electric force on a subshell at radius r created by all of the subshells outside of it is calculated next. The subshell appears as a point at the center to all of the subshells outside of it. Of course, the net vector sum of all of the forces is zero, since the electric field inside of the outer subshells is zero, but the absolute value of these forces on the subshell surface is not zero.

$$\mathrm{d}q^{-}(r)\int_{r}^{R}\mathrm{d}q^{-}(x)\frac{1}{x^{2}}=\left[\frac{q^{-}}{R\left(1-\frac{R_{qi}}{R}\right)}\right]^{2}\left(\frac{1}{r}-\frac{1}{R}\right)\mathrm{d}r \tag{23}$$

The negative (repulsive) force on all subshells due to all subshells outside of each subshell is:

$$\left[\frac{q^-}{R\left(1-\frac{R_{qi}}{R}\right)}\right]^2 \int_{R_{qi}}^{R}\left(\frac{1}{r}-\frac{1}{R}\right)\mathrm{d}r = -\left[\frac{q^-}{R\left(1-\frac{R_{qi}}{R}\right)}\right]^2 \left[\left(1-\frac{R_{qi}}{R}\right)+\ln\left(\frac{R_{qi}}{R}\right)\right] \quad (24)$$

The negative (repulsive) pressure at the charge shell surface from all subshells due to all subshells outside of each subshell is:

$$-\frac{\left(q^-\right)^2}{4\pi R^4}\left[\frac{1}{\left(1-\frac{R_{qi}}{R}\right)}+\frac{1}{\left(1-\frac{R_{qi}}{R}\right)^2}\ln\left(\frac{R_{qi}}{R}\right)\right] \quad (25)$$

The total outward pressure on the charge shell due to the central core charge and all of the charge subshells is:

$$\frac{\left(q^-\right)^2}{4\pi R^4}\left[\frac{q^+}{q^-}\frac{R}{R_{qi}}-\frac{2}{1-\frac{R_{qi}}{R}}-\frac{2}{\left(1-\frac{R_{qi}}{R}\right)^2}\ln\left(\frac{R_{qi}}{R}\right)\right]=\frac{\left(q^-\right)^2}{4\pi R^4}(0.004101107) \quad (26)$$

The electrical pressure on an infinitely thin ($R_{qi}=R$) charge shell is:

$$\frac{\left(q^-\right)^2}{4\pi R^4}\left(\frac{q^+}{q^-}+1\right)=\frac{\left(q^-\right)^2}{4\pi R^4}(0.004865) \quad (27)$$

The outward pressure on the charge outer surface is lower for non-zero charge thickness than for zero thickness. The outer shell material is stretched and the charge shell is along with it. The pressure differential $\mathrm{d}P$ across the charge shell which causes the stretch is:

$$\mathrm{d}P=\frac{\left(q^-\right)^2}{4\pi R^4}(0.004865-0.004101107)=\frac{\left(q^-\right)^2}{4\pi R^4}(0.000763893) \quad (28)$$

Without the positive central core, there would be no inward force on the charge subshells, and they would all collapse to radius R due to their repulsion.

7. Compressibility Factor

When the outer shell material between radii R and R_{qi} is stretched, the material between R_{qi} and R_i is compressed. The compressibility factor k is defined by:

$$k=\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}P}, \quad (29)$$

where V is the volume of outer shell material between R_{qi} and R_i, $\mathrm{d}V$ is the change in V caused by the stretching, and $\mathrm{d}P$ is the pressure differential across $\mathrm{d}V$.

$$\mathrm{d}V=\frac{4}{3}\pi\left(R^3-R_{qi}^3\right),\quad V=\frac{4}{3}\pi\left(R_{qi}^3-R_i^3\right) \quad (30)$$

$$\mathrm{d}P = \frac{\left(q^-\right)^2}{4\pi R^4}\left(0.000763893\right) = \frac{1}{k}\frac{R^3 - R_{qi}^3}{R_{qi}^3 - R_i^3} = \frac{1}{k}\frac{1-\left(\frac{R_{qi}}{R}\right)^3}{\left(\frac{R_{qi}}{R}\right)^3 - \left(\frac{R_i}{R}\right)^3} \tag{31}$$

The compressibility factor is:

$$k = \frac{9.444\times10^{-3}}{7.63893\times10^{-4}}\frac{4\pi R^4}{\left(q^-\right)^2} = 12.363\frac{4\pi R^4}{\left(q^-\right)^2} = 1.005\times10^{-34} \tag{32}$$

The electron material is compressible, but the compressibility is extremely low.

8. Splitting the Electron

In Reference [2], a model was proposed which predicts the splitting of the electron into three equal charges when a very strong external magnet field is applied. In the following, the model is modified, and the result is a lower error in the polar angle at which the splits occur and a lower magnetic field required to cause the splitting. Instead of only considering the pressure on the electron surface for specific polar angles, the contributions to the total pressure for all angles are included. Also, the electrical pressure is reduced slightly as the result of the non-zero thickness of the charge shell.

The individual components of the pressure at the electron outer surface are described next.

8.1. Magnetic Surface Tension

The pressure P_M created by the magnetic surface tension was derived in [2] and expressed by Equation (35) of that article:

$$P_M = -\frac{\left(q^-\right)^2}{12\pi^2 R^4} = \frac{\left(q^-\right)^2}{4\pi R^4}\left(\frac{1}{3\pi}\right) = \frac{\left(q^-\right)^2}{4\pi R^4}\left(0.1061032954\right) \tag{33}$$

The magnetic surface tension pressure is expected to be nearly the same for both zero and non-zero charge shell thicknesses. The pressure of each subshell is the pressure of the zero-thickness surface tension divided by the number of subshells. But then the total pressure on the charge shell is the pressure of each subshell multiplied by the number of subshells. One might expect the total pressure to be reduced slightly by pulling the charge shell apart slightly. If this were to be the case, the error in the calculated polar angle of the split would be reduced.

The magnetic attraction of subshells to each other has been estimated to be very low, and therefore has been neglected. It would be compressive, counteracting the electrical expansive pressure. Obviously, the magnetic differential pressure must be less than the electrical differential pressure, or else the charge shell thickness would be zero.

8.2. Electrical Pressure

From Equation (26) above, the total outward pressure on the charge shell due to the central core charge and all of the charge subshells is:

$$P_E = \frac{(q^-)^2}{4\pi R^4}(0.004101107) \tag{34}$$

8.3. Centrifugal Pressure

From [2], Equation (27), the centrifugal pressure at polar angle θ is:

$$P_C(\theta) = \sigma c^2 \frac{\sin\theta + \cos\theta}{\sin^3\theta} \int_{\frac{R_i \sin\theta}{R}}^{\sin\theta} \frac{x^3}{\sqrt{1-x^2}} \mathrm{d}x \tag{35}$$

The centrifugal force on the surface ring at polar angle θ is:

$$F_C(\theta) = 2\pi R \sin\theta (R d\theta) \sigma c^2 \frac{\sin\theta + \cos\theta}{\sin^3\theta} \int_{\frac{R_i}{R}\sin\theta}^{\sin\theta} \frac{x^3}{\sqrt{1-x^2}} \mathrm{d}x \tag{36}$$

The total centrifugal force on one hemisphere is:

$$F_C = \int_0^{\pi/2} F_C(\theta) = 2\pi R^2 \sigma c^2 (0.3678061), \tag{37}$$

and the centrifugal pressure, not including $P_C(\Theta)$, is:

$$P_C = \frac{F_C}{2\pi R^2} = \sigma c^2 (0.3678061) = \frac{(q^-)^2}{4\pi R^4}(0.004415809418), \tag{38}$$

where the centrifugal pressure $P_C(\Theta)$ at polar angle Θ is:

$$\begin{aligned} P_C(\Theta) = \frac{(q^-)^2}{4\pi R^4}(0.012005808) \frac{\sin\Theta + \cos\Theta}{\sin^3\Theta} \Bigg[\frac{1}{3}\sqrt{(1-\sin^2\Theta)^3} \\ - \sqrt{1-\sin^2\Theta} - \frac{1}{3}\sqrt{\left(1-\left(\frac{R_i}{R}\right)^2 \sin^2\Theta\right)^3} + \sqrt{1-\left(\frac{R_i}{R}\right)^2 \sin^2\Theta} \Bigg] \end{aligned} \tag{39}$$

8.4. Lorentz Pressure

As derived in [2], the pressure $P_B(\theta)$ along the radial at polar angle θ due to the external magnetic field B_{ext} is:

$$P_B(\theta) = \frac{F_B}{2\pi R \sin\theta (R d\theta)} = \frac{q^-}{4\pi R^2} B_{ext} \sin\theta (\cos\phi + \sin\phi)(\sin\theta + \cos\theta) \tag{40}$$

where $\phi = 54.736$.

The corresponding force along the ring at polar angle θ is:

$$\begin{aligned} F_B(\theta) &= (2\pi R \sin\theta R \mathrm{d}\theta) P_B(\theta) \\ &= \frac{q^-}{2} B_{ext} (1.393845224)(\sin^3\theta + \sin^2\theta\cos\theta)\mathrm{d}\theta \end{aligned} \tag{41}$$

The total force on a hemisphere, excluding the force created at polar angle Θ, is:

$$\begin{aligned}F_B &= \int_0^{\frac{\pi}{2}} F_B(\theta) \\ &= \frac{q^-}{2} B_{ext}(1.393845224)\left[-\frac{1}{3}\cos\theta\sin^2\theta - \frac{2}{3}\cos\theta + \frac{1}{3}\sin^3\theta\right]_0^{\frac{\pi}{2}} \\ &= \frac{q^-}{2} B_{ext}(1.393845224)\end{aligned} \tag{42}$$

The outward pressure on the hemisphere due to the external magnetic field B_{ext} is:

$$P_B = \frac{1}{2\pi R^2}\frac{q^-}{2} B_{ext}(1.393845224) = \frac{q^-}{4\pi R^2} B_{ext}(1.393845224), \tag{43}$$

where the pressure $P_B(\Theta)$ created by the external magnetic field B_{ext} at polar angle Θ is:

$$P_B(\Theta) = \frac{q^-}{4\pi R^2} B_{ext}(1.393845224)\sin\Theta(\sin\Theta + \cos\Theta). \tag{44}$$

8.5. Polar Angles of the Splitting

The polar angle at which electron splitting can occur is defined as that angle Θ at which the sum of all pressure components at the charge shell surface is zero for a minimum magnetic field B_{ext}. For a greater field than the minimum B_{ext}, the net outward pressure on the charge shell at angle Θ will be greater than zero, and there will be nothing to hold it together, except possibly the tensile strength of the outer shell material.

$$P_B + P_B(\Theta) + P_C + P_C(\Theta) + P_E + P_M = 0 \tag{45}$$

The polar angle Θ at which the external magnetic field is at a minimum for splitting is calculated from:

$$P_B + P_B(\Theta) = -P_M - P_E - P_C - P_C(\Theta) \tag{46}$$

$$\begin{aligned}\frac{R^2}{q^-}B_{ext} &= \frac{0.07001234971}{1+\sin\Theta(\sin\Theta+\cos\Theta)} - \frac{0.008613444181}{1+\sin\Theta(\sin\Theta+\cos\Theta)}\frac{(\sin\Theta+\cos\Theta)}{\sin^3\Theta} \\ &* \left[\frac{1}{3}\sqrt{(1-\sin^2\Theta)^3} - \sqrt{1-\sin^2\Theta} - \frac{1}{3}\sqrt{\left(1-\left(\frac{R_i}{R}\right)^2\sin^2\Theta\right)^3} + \sqrt{1-\left(\frac{R_i}{R}\right)^2\sin^2\Theta}\right]\end{aligned} \tag{47}$$

The solution to Equation (47) is $\Theta = 70.25$ degrees and

$$B_{ext} = 0.029943020\frac{q^-}{R^2} = 3.72\times10^{16}\text{ gauss} = 3.72\times10^{12}\text{ Tesla} \tag{48}$$

The splits occur along lines of latitude having polar angles Θ and $180-\Theta$.

The polar angles of the latitudes which separate the surface charge into exactly $\frac{1}{3}q$ portions are 70.52877937 and 109.4712206 degrees [2]. Therefore, the error in the calculated polar angle Θ for splitting the electron into exactly three equal charges is: 0.28 degree or -0.4%. By comparison, the magnetic field at the center of the electron is:

$B = \dfrac{q}{\alpha R^2} = 8.29 \times 10^{17}$ gauss, so the minimum external magnetic field for splitting is about 22 times weaker.

The model for splitting the electron proposed in this document specifically assumes a strong magnetic field and the Lorentz force as the agent for the partitioning. The author recognizes that other models have been proposed which predict different partitions and for different reasons, and the author has no reason to doubt their validity.

9. Summary

The g-factor anomaly, that is, the fractional component of the g-factor value, is shown to be related to the non-zero thickness of the charge on the electron surface. An equation has been calculated which expresses this relationship, and a value for the charge thickness has been derived. The fact that the charge shell has a non-zero thickness is claimed to be due to the presence of a positive charge at center of the electron. That charge was proposed in the author's previous papers to reconcile the discrepancy in the magnetic moment calculated from quantum and classical theories. The fact that charge can also explain the g-factor anomaly adds support to the proposal of such a charge.

The value of the outer shell mass thickness has been revised from the author's previous paper. The compressibility factor of the outer shell material has been derived from the g-factor anomaly.

A revised model of the splitting of the electron in a very strong external magnetic field has been presented, with the error in the polar angles at which the splits would occur reduced to 0.4%. The minimum magnetic field required to cause the splits is estimated to be 4.5% of the strength of the field at the electron center.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Mac Gregor, M.H. (2013) The Enigmatic Electron, Chapters 7, 17. 2nd Edition, El Mac Books, Santa Cruz, Reprint of 1992 Original.

[2] Young, A. (2023) *Journal of Modern Physics*, **14**, 1563-1577. https://doi.org/10.4236/jmp.2023.1412090

[3] Young, A. (2023) *Journal of Modern Physics*, **14**, 553-561. https://doi.org/10.4236/jmp.2023.145031

[4] McDonald, K.T. (2008, August 10) Electromagnetic Fields of a Rotating Shell of Charge. Joseph Henry Laboratories, Princeton University, Princeton, NJ. https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=&ved=2ahUKEwii0fvEyLb9AhXJiO4BHReXBIEQFnoECCUQAQ&url=http%3A%2F%2Fkirkmcd.princeton.edu%2Fexamples%2Frotatingshell.pdf&usg=AOvVaw3nuuv3640EW2oztXV2nNdL

[5] Weisstein, E.W. (2007) Electron Radius. WolframResearch, Champaign, IL. https://scienceworld.wolfram.com/physics/ElectronRadius.html

[6] Yavorsky, B. and Detlaf, A. (1975) Handbook of Physics. Mir Publishers, Moscow, 477.

[7] Young, A. (2022) *Journal of Modern Physics*, **13**, 1117-1127. https://doi.org/10.4236/jmp.2022.137064