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Calculation of Particle Decay Times in the Standard Model

Jan Helm

Department of Electrical Engineering, Technical University, Berlin, Germany
Email: jan.helm@alumni.tu-berlin.de

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Abstract

We present here a two-step method of classification and calculation for decay rates in the Standard Model. The first step is a phenomenological classification method, which is an extended and improved schematic experimental formula for decay width originally introduced by Chang. This schematic formula separates decays into seven classes. Furthermore, from it is derived a process-specific interaction energy m_X. The second step is a numerical calculation method, which calculates this interaction energy m_X numerically by minimization of action from the Lagrangian of the process, from which follows the decay width via the phenomenological formula. The Lagrangian is based on an extension of the Standard Model, the extended SU(4)-preon-model. A comparison of numerically calculated and observed decay widths for a large selection of decays shows a good agreement.

Keywords

Particle Decay, Decay Width, Interaction Energy, Minimization of Action, Extension of Standard Model

1. Introduction

The particle decays in the Standard Model are characterized by their decay width Γ (or equivalently decay rate $\Gamma_d = \Gamma/\hbar$), and are described by the famous Fermi's golden rule, i.e. an integral with parameters.

A closed expression for Γ can be found in only a few cases, otherwise there are empirical formulas, or simply data tables.

We present here a two-step calculation method for calculation of general decay rates in the Standard Model.

The first step is a **phenomenological classification method**, which is an improved and generalized schematic formula for decay width originally introduced by Chang [1]. It is a general parameterized approximation formula with some special cases, which is in good agreement with measurements.

It introduces seven classes of particle decays, where the interaction constant is roughly class-specific. In other words, it allows to extrapolate and make assessments for decays, for which there is no analytic formula. Furthermore, it supports the notion of decay-mediating virtual particle with interaction energy m_X.

The second step is a **numerical Lagrangian calculation method** for interaction energy m_X, which calculates the interaction energy of the process numerically by minimization of action from the Lagrangian of the process. From the interaction energy follows the decay width using the phenomenological formula. A comparison of numerically calculated and observed decay widths for a large selection of decays shows a good agreement.

The starting point for decay rate $\Gamma_d = \Gamma/\hbar$, or equivalently, its decay width Γ, of a n-body process

$$\Gamma\left(P_0(k,m) \to P(p_1,m_1)P(p_2,m_2)P(p_3,m_3)\cdots P(p_n,m_n)\right)$$

is the Fermi's golden rule

$$\mathrm{d}\Gamma = m\frac{\left|M\left(k,p_1,p_2,p_3,\cdots,p_n\right)\right|^2}{2}\frac{\mathrm{d}^3 p_1}{(2\pi)^3 2m^2E_1}\frac{\mathrm{d}^3 p_2}{(2\pi)^3 2m^2E_2}\frac{\mathrm{d}^3 p_3}{(2\pi)^3 2m^2E_3}\cdots$$
$$\frac{\mathrm{d}^3 p_n}{(2\pi)^3 2m^2E_n}(2\pi)^4 m^4\delta^4\left(k-\left(p_1+p_2+p_3+\cdots+p_n\right)\right)$$

We demonstrate in chap.2 at selected examples, how to derive Γ from Fermi's golden rule in a closed form, which in general has to be done numerically.

For 2-body decays and 3-body decays we can (approximately) split-off the kinematic factor I_Γ of Γ, taking the transition matrix out of the integral.

The phenomenological classification method is described in chap.3 and chap. 5, and the calculated Γ_{calc} are compared with measured Γ_{obs} in chap.6.

The phenomenological formula for the decay width is [1].

$\Gamma = \tilde{G}^2 m_i^k \left|P_l^m(x)\right|^2 = \frac{G^2}{C_1} m_i^k \left|P_l^m(x)\right|^2$, where $P_l^m(x)$ Legendre polynomial $m = l$ or $m = l + 1$, l = isospin I, $x = \frac{m_f}{m_i}$ mass ratio, $\tilde{G} = \frac{G}{\sqrt{C_1}}$ with G = interaction constant, m_i is the initial mass, k is the mass-power-coefficient.

We introduce and derive the interaction energy m_X in the form

$$\Gamma = \left|M\right|^2 I_\Gamma m_i = \left(\frac{m_i^2}{8m_X^2}\right)^2 I_\Gamma m_i .$$

The numerical calculation method is based on an extended version of the Standard Model (SM) introduced by Helm [2], called the extended SU(4)-preon-model (SU4PM). In SU4PM, the Pauli SU(2)-weak interaction is extended to SU(4)-hypercolor (hc) interaction with four charges, 15 hc-boson fields and two subparticles called preons, and SU(2)-weak interaction becomes a Yukawa-approximation via massive (W, Z)-bosons.

The SU4PM model allows to calculate the masses of the SM remarkably well, reducing 29 parameters of the SM to 7.

In chap. 7, we calculate the interaction energy m_X numerically by minimization of action of the SU4PM Lagrangian of the particles in the decay process, and we obtain a good agreement between the calculated values m_{Xcal} and observed values m_{Xexp}.

A remark about units: in particle physics it is customary to use the convention $\hbar = c = 1$, and we adopt it here as well, except in places, where quantities have to be distinguished, e.g. decay width Γ is an energy and is measured in MeV: $[\Gamma] = \text{MeV}$, whereas decay rate $\Gamma_d = \Gamma/\hbar$ is measured in s^{-1}: $[\Gamma_d] = s^{-1}$.

Other quantities are transformed from each other by $\hbar$ and c, e.g. mass $m = E/c^2$, time $t = \hbar/E$, length $x = \hbar c/E$, angular momentum $p = E/c$.

The contents of the paper is as follows.

In chap. 2, first some important decays are discussed, and in 1.8 and 1.9 the general decay width formula for 3-body and 2-body decays.

In chap. 3 the phenomenological formula for the decays is presented, and is discussed for some important decays.

Chap. 4 shows the data of the most important particles.

In chap. 5, the phenomenological formula values and the observed values for the decay width, together with the decay interaction energy m_X are discussed.

In chap. 6 the phenomenological decay width, the observed decay width, and the interaction energy are shown in a table and in a plot, and generally characterized.

In chap. 7, we present a calculation method and a reaction model using electromagnetic, color SU(3), and extended weak SU(4) interaction based on SU4PM model.

Here, the theoretical background and the calculation software is discussed, and the calculated results for m_{Xcal} are compared to the observed values m_{Xexp} from chap. 5, and shown in a table and in plots.

2. Selected Particle Decays with Theoretical Background

In this chapter we discuss some well-understood particle decays, with decay width described by an analytical formula [3] [4] [5] [6] [7].

2.1. Neutron

The free neutron decays into a proton, electron, and antineutrino [8] is shown in Figure 1.

$$n \to p + e^- + \bar{\nu}_e$$

The rest energy $(m_n - m_p - m_e)c^2 = 782\ \text{keV}$ is carried away by e and ν

The transition matrix of the decay is [8] [10]

$$M = \left(G_V \bar{p}\gamma^\mu n - G_A \bar{p}\gamma^\mu \gamma_5 n\right)\left(\bar{e}\gamma_\mu (1-\gamma_5)\nu\right)\delta\left(E_n - E_p - E_e - E_\nu\right)$$

from the interaction Hamiltonian [10]

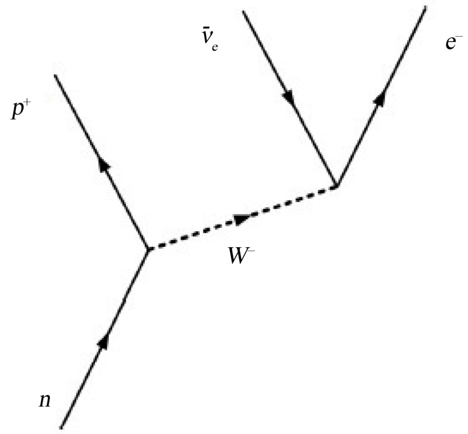


Figure 1. Neutron decay [9].

$$H_{\text{int}} = G_F V_{ud}\left(\bar{p}\gamma^{\mu}\left(1-\frac{G_A}{G_V}\gamma_5\right)n\right)\left(\bar{e}\gamma_{\mu}\left(1-\gamma_5\right)\nu\right)$$

with $G_A/G_V = 1.255 \pm 0.005$

$$E\left(G_V\right) = \frac{1}{\sqrt{G_V}} = 296.7\ \text{GeV}$$

$G_F = 1.1663787(6)\times 10^{-5}\ \text{GeV}^{-2}$ is the Fermi weak coupling constant, $V_{ud} = 0.97417(21)$ (V is the CKM-matrix),
and the weak V-constant is

$G_V = G_F V_{ud} = 1.135\times 10^{-5}\ \text{GeV}^{-2}$, $G_A = G_F V_{ud}\lambda$ and λ is the hadronic strong interaction correction.

We compute the neutron decay probability per unit time using Fermi's golden rule [11]:

$\Gamma_{i\to f} = \frac{2\pi}{\hbar}\left|\langle f\,|\,H\,|\,i\rangle\right|^2 \rho\left(E_f\right)$, where $\rho\left(E_f\right)$ = final state energy density

or in differential form [11]

$$\mathrm{d}\Gamma = \frac{\left|M\left(k_1,k_2,k_3,k_4\right)\right|^2}{2m_1}\frac{\mathrm{d}^3k_2}{(2\pi)^3 2E_2}\frac{\mathrm{d}^3k_3}{(2\pi)^3 2E_3}\frac{\mathrm{d}^3k_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^4\left(k_1-k_2-k_3-k_4\right) \tag{1}$$

where $k_1 = p_n$, $k_2 = p_p$, $k_3 = p_e$, $k_4 = p_\nu$, $m_1 = m_n$ with the (dimensionless) transition matrix $M\left(k_1,k_2,k_3,k_4\right) = \langle f|H_{\text{int}}|i\rangle\delta\left(E_f - E_i\right)$ of the interaction Hamiltonian H_{int}.

Here E_e, p_e, E_n, and p_n are the electron and antineutrino total energy and momentum Δ is the neutron-proton mass difference $\Delta = 1.29333205(51)\,\text{MeV}$.

Integration over the antineutrino and electron momenta gives the beta electron energy spectrum

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E} = \frac{G_V^2 + 3G_A^2}{2\pi^3}E_e p_e\left(\Delta - E_e\right)^2$$

Additional integration over electron energy yields

$$\Gamma = \left(G_V^2 + 3G_A^2\right)\frac{m_e^5}{2\pi^3} f_R$$

Here f_R is the phase-space term, i.e. the value of the integral over the Fermi energy spectrum, including Coulomb, recoil order, and radiative corrections.

The decay width of the decay becomes [8]

$$\Gamma\left(n \to p e \nu_e\right) = \left(G_V^2 + 3G_A^2\right)\frac{m_e^5}{2\pi^3} f_R = G_F^2 V_{ud}^2\left(1+3\lambda^2\right)\frac{m_e^5}{2\pi^3} f_R \quad (2)$$

where $G_V = G_F V_{ud}$, $G_A = G_F V_{ud}\lambda$, $\lambda = 1.255$, $V_{ud} = 0.974$, $G_F = 1.166\times 10^{-5}\ \text{GeV}^{-2}$

with the phase-space term [9]

$$f_R\left(\xi\right) = \frac{1}{60}\left(2\xi^4 - 9\xi^2 - 8\right)\left(\xi^2 - 1\right)^{1/2} + \frac{1}{4}\xi \ln\left(\xi + \left(\xi^2 - 1\right)^{1/2}\right),\quad \xi = \frac{m_n - m_p}{m_e},$$

$f_R = 1.6332$

here the transition probability per unit time is $W = \Gamma\left(n \to p e \nu_e\right)/\hbar$.

The neutron lifetime τ_n becomes

$$\tau_n = \hbar/\Gamma\left(n \to p e \nu_e\right) = \frac{2\pi^3}{\left(G_V^2 + 3G_A^2\right) m_e^5 f_R} = 881.5\ \text{s}$$

2.2. Muon

The muon decays into an electron, an electron-antineutrino and a muon-neutrino is shown in **Figure 2**.

$$\mu \to e^- + \bar{\nu}_e + \nu_\mu$$

For the muon decay we derive the formula for the decay width Γ [6] [12].

The interaction Hamiltonian is the current-current interaction

$$H_{\text{int}} = \frac{g^2}{2M_W^2}\bar{u}_2\left(k_2, s_2\right)\left(\gamma^\mu \frac{1-\gamma^5}{2}\right)u_1\left(k_1, s_1\right)\bar{u}_4\left(k_4, s_4\right)\left(\gamma_\mu \frac{1-\gamma^5}{2}\right)u_3\left(k_3, s_3\right)$$

From the transition matrix element

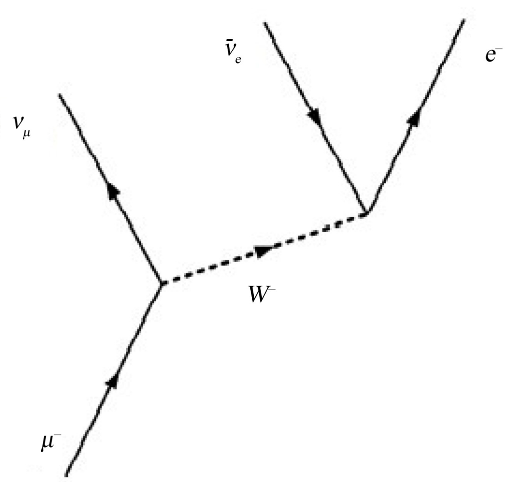


Figure 2. Muon decay [9].

$$\mathcal{M} = \frac{g^2}{2M_W{}^2}\bar{u}_2(k_2,s_2)\left(\gamma^\mu\frac{1-\gamma^5}{2}\right)u_1(k_1,s_1)\bar{u}_4(k_4,s_4)\left(\gamma_\mu\frac{1-\gamma^5}{2}\right)u_3(k_3,s_3)\delta(E_1-E_2-E_3-E_4)$$

with $\frac{G}{\sqrt{2}}=\frac{g^2}{8M_W^2}$ and $G = G_F$, g is the weak dimensionless interaction constant, we obtain after some γ-algebra averaging over the spins and trace-manipulation.

$\left|M(k_1,k_2,k_3,k_4)\right|^2 = 64G^2(k_1\cdot k_3)(k_2\cdot k_4)$ and for the decay rate we have Fermi's golden rule

$$\mathrm{d}\Gamma = \frac{\left|M(k_1,k_2,k_3,k_4)\right|^2}{2m}\frac{\mathrm{d}^3k_2}{(2\pi)^3 2E_2}\frac{\mathrm{d}^3k_3}{(2\pi)^3 2E_3}\frac{\mathrm{d}^3k_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^4(k_1-k_2-k_3-k_4)$$

$$\mathrm{d}\Gamma = \frac{64G^2}{2m}\left((k_1\cdot k_3)(k_2\cdot k_4)\right)\frac{\mathrm{d}^3k_2}{(2\pi)^3 2E_{k_2}}\frac{\mathrm{d}^3k_3}{(2\pi)^3 2E_{k_3}}\frac{\mathrm{d}^3k_4}{(2\pi)^3 2E_{k_4}}(2\pi)^4\delta^4(k_1-k_2-k_3-k_4)$$

In the muon rest frame $k_1=(m,0,0,0)$ and $(k_1\cdot k_3)=mE_3$ and with $k_1=k_2+k_3+k_4$

$$\mathrm{d}\Gamma = \frac{G^2}{8m\pi^5}\left((k_2\cdot k_4)mE_3\right)\frac{\mathrm{d}^3k_2}{|\vec{k}_2|}\frac{\mathrm{d}^3k_3}{|\vec{k}_3|}\frac{\mathrm{d}^3k_4}{|\vec{k}_4|}\delta\left(m-|\vec{k}_2|-|\vec{k}_3|-|\vec{k}_4|\right)\delta^3\left(\vec{k}_2+\vec{k}_3+\vec{k}_4\right)$$

in spherical coordinates

$$\mathrm{d}\Gamma = \frac{mG^2|\vec{k}_3|^2}{8\pi^4}\left(m-2|\vec{k}_3|\right)\frac{\sin\theta\,\mathrm{d}\theta\,\mathrm{d}|\vec{k}_3|\,\mathrm{d}^3k_4}{\left(|\vec{k}_3|^2+|\vec{k}_4|^2+2|\vec{k}_3||\vec{k}_4|\cos\theta\right)|\vec{k}_4|}\delta\left(m-|\vec{k}_3+\vec{k}_4|-|\vec{k}_3|-|\vec{k}_4|\right)$$

with variable $u^2=|\vec{k}_3|^2+|\vec{k}_4|^2+2|\vec{k}_3||\vec{k}_4|\cos\theta$

$$\mathrm{d}\Gamma = \frac{mG^2|\vec{k}_3|}{8\pi^4}\left(m-2|\vec{k}_3|\right)\frac{\mathrm{d}|\vec{k}_3|\,\mathrm{d}^3k_4}{|\vec{k}_4|^2}\int\mathrm{d}u\,\delta\left(m-u^2-|\vec{k}_3|-|\vec{k}_4|\right),$$

and with $E = k_4$

we obtain

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E} = \frac{mG^2}{2\pi^3}E^2\left(\frac{m}{2}-\frac{2E}{3}\right)$$

$$\Gamma = \frac{m^2G^2}{4\pi^3}\int_0^{m/2}E^2\left(1-\frac{4E}{3m}\right)\mathrm{d}E\,,\quad \Gamma=\frac{m^5G^2}{192\pi^3} \tag{3}$$

and the decay time $\tau=\hbar\Gamma^{-1}$,

where $m = 0.1056584\ \text{GeV}$

$$\tau = \hbar\frac{192\pi^3}{(0.1056584\ \text{GeV})^5\left(1.17\times10^{-5}\ \text{GeV}^{-2}\right)^2} = \hbar 3.30\times10^{18}\ \text{GeV}\,,$$

or in seconds, multiplied by $\hbar = 6.58\times10^{-25}\ \text{s}\cdot\text{GeV}$,

we obtain the lifetime $\tau = 2.17\ \mu\text{s}$.

2.3. Tauon

The decay modes of the tauon are

$$\tau \to \mu + \bar{\nu}_\mu + \nu_\tau$$

$$\tau \to e + \bar{\nu}_e + \nu_\tau$$

$$\tau \to d + \bar{u} + \nu_\tau$$

$$\tau \to s + \bar{u} + \nu_\tau$$

The leptonic modes give a factor $f_l = 2$, the hadronic modes a factor $f_h = 3|V_{ud}|^2 + 3|V_{us}|^2 = 2.99$,
and $m_\tau = 16.82 m_\mu$

$$t_{life}(\tau) = \frac{t_{life}(\mu)}{(f_l + f_h)(m_\tau/m_\mu)^2} = 3.23 \times 10^{-13}\ \text{s}$$

2.4. Pions

The particle data for the pions are shown in Table 1.

Charged pion decays

The diagram of charged pion decays is shown in Figure 3.

The $\pi^\pm$ mesons have a mass of 139.6 MeV/c^2 and a mean lifetime of 2.6033 × 10^{-8} s. They decay due to the weak interaction. The primary decay mode of a pion, with a branching fraction of 0.999877, is a leptonic decay into a muon and a muon neutrino:

$$\pi^+ \to \mu^+ + \nu_\mu$$

$$\pi^- \to \mu^- + \bar{\nu}_\mu$$

The pion-muon decay width is [14]

$$\Gamma = f_\pi^2 \frac{G^2}{8\pi} m_\pi^3 m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$

Table 1. Parameters of the pions.

particle	symbol	antipart.	composition	mass (MeV/c^2)	I^G	J^{PC}	S	C	B	lifetime(s)	main decay
ch-pion	π^+	π^-	$u\bar{d}$	139.57018 ± 0.00035	1^-	0^-	0	0	0	2.6033 ± 0.0005 × 10^{-8}	$\mu^+ + \nu_\mu$
n-pion	π^0		$(u\bar{u} - d\bar{d})/\sqrt{2}$	13.9766 ± 0.0006	1^-	$0^\pm$	0	0	0	8.4 ± 0.6 × 10^{-17}	$\gamma + \gamma$

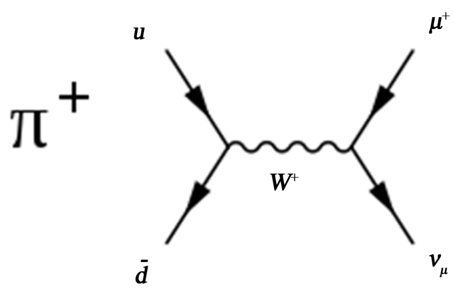


Figure 3. Feynman diagram of the dominant leptonic pion decay [13].

where f_π is the (dimensionless) pion decay factor, $f_\pi = \frac{130.41\,\text{MeV}}{m_\pi} = 0.934$.

The second most common decay mode of a pion, with a branching fraction of 0.000123, is also a leptonic decay into an electron and the corresponding electron antineutrino. This "electronic mode" was discovered at CERN in 1958

$$\pi^+ \to e^+ + \nu_e$$

$$\pi^- \to e^- + \bar{\nu}_e$$

The suppression of the electronic decay mode with respect to the muonic one is given approximately (up to a few percent effect of the radiative corrections) by the ratio of the half-widths of the pion-electron and the pion-muon decay reactions:

$$R_\pi = \left(m_e/m_\mu\right)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 = 1.283 \times 10^{-4}$$

and is a spin effect known as helicity suppression.

Also observed, for charged pions only, is the very rare "pion beta decay" (with branching fraction of about 10^{-8}) into a neutral pion, an electron and an electron antineutrino (or for positive pions, a neutral pion, a positron, and electron neutrino).

$$\pi^+ \to \pi^0 + e^+ + \nu_e$$

$$\pi^- \to \pi^0 + e^- + \bar{\nu}_e$$

The rate at which pions decay is a prominent quantity in many sub-fields of particle physics, such as chiral perturbation theory. This rate is parametrized by the pion decay constant (f_π), related to the wave function overlap of the quark and antiquark, which is about 130 MeV.

Neutral pion decays

The π^0 meson has a mass of 135.0 MeV/c^2 and a mean lifetime of 8.4×10^{-17} s. It decays via the electromagnetic force, which explains why its mean lifetime is much smaller than that of the charged pion (which can only decay via the weak force).

The neutral pion decay is shown in **Figure 4**.

The dominant π^0 decay mode (anomaly-induced neutral pion decay), with a branching ratio of BR = 0.98823, is into two photons:

$$\pi^0 \to \gamma + \gamma$$

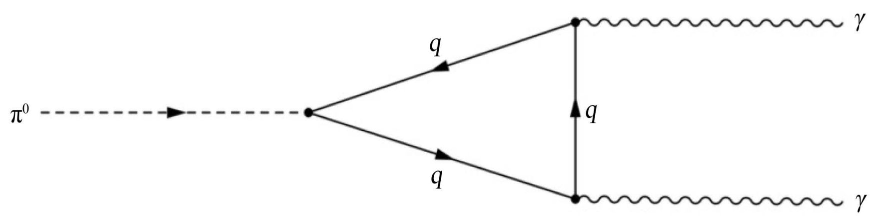


Figure 4. Anomaly-induced neutral pion decay [13].

The second largest π^0 decay mode (BR = 0.01174) is the Dalitz decay (named after Richard Dalitz), which is a two-photon decay with an internal photon conversion resulting a photon and an electron-positron pair in the final state:

$$\pi^0 \to \gamma + e^- + e^+$$

The third largest established decay mode (BR = 3.34 × 10^{-5}) is the double Dalitz decay, with both photons undergoing internal conversion which leads to further suppression of the rate:

$$\pi^0 \to e^- + e^+ + e^- + e^+$$

The fourth largest established decay mode is the loop-induced and therefore suppressed (and additionally helicity-suppressed) leptonic decay mode (BR = 6.46 × 10^{-8}):

$$\pi^0 \to e^- + e^+$$

2.5. Pion-Nucleon Interaction and Decays

The Lagrangian is [6] [15]

$$L = ig\bar{\psi}(x)\gamma_5\vec{\sigma}\cdot\phi(x)\psi(x)$$

with pion $\phi(x)=(\phi_1,\phi_2,\phi_3)$, nucleon $\psi(x)=(\psi_p,\psi_n)$, and Pauli-matrix-vector $\vec{\sigma}$, explicitly

$$L = ig(\bar{\psi}_p,\bar{\psi}_n)\gamma_5\begin{pmatrix}\phi_3 & \phi_1 - i\phi_2 \\ \phi_1 + i\phi_2 & \phi_3\end{pmatrix}(\psi_p,\psi_n) \quad (\phi_1,\phi_2,\phi_3)$$

with Feynman diagrams shown in **Figure 5**.
and with the corresponding hadronic transformations

$$p \to^{\pi^0} p,\ n \to^{\pi^0} n,\ n \to^{\pi^-} p,\ p \to^{\pi^+} n$$

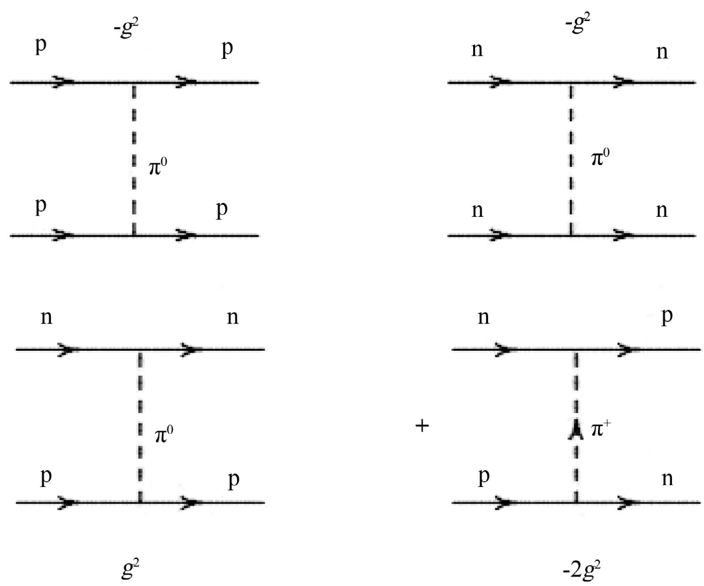


Figure 5. Nucleon interaction via pions.

2.6. Kaons

Kaons exist in charged form ch-kaon = $\left(K^+, K^-\right)$ and neutral form n-kaon = $\left(K^0, \bar{K}^0\right)$, where n-kaon appears in nature as a symmetric and antisymmetric mixture of d and s: the long-lived kaon-L and the short-lived kaon-S.

The particle data for the kaons are shown in Table 2, their decay reactions in Table 3, and their quark decay processes in Figure 6.

Table 2. Parameters of the kaons.

particle	symbol	antipart.	composition	mass (MeV/c²)	I^G	J^{PC}	S	C	B	lifetime(s)	main decay
ch-kaon	K^+	K^-	$u\bar{s}$	439.677 ± 0.016	1/2	0^-	1	0	0	$1.2380 \pm 0.0021 \times 10^{-8}$	$\mu^+ + \nu_\mu$ $\pi^+ + \pi^0$ $\pi^+ + \pi^+ + \pi^-$ $\pi^0 + e^+ + \nu_e$
n-kaon	K^0	$\bar{K}^0$	$d\bar{s}$	497.611 ± 0.013	1/2	0^-	1	0	0		
kaon-S	K_S^0	K_S^0	$\left(d\bar{s} - s\bar{d}\right)/\sqrt{2}$	497.611 ± 0.013	1/2	0^-		0	0	$8.954 \pm 0.004 \times 10^{-11}$	$\pi^+ + \pi^-$ $\pi^0 + \pi^0$
kaon-L	K_L^0	K_L^0	$\left(d\bar{s} + s\bar{d}\right)/\sqrt{2}$	497.611 ± 0.013	1/2	0^-		0	0	$5.116 \pm 0.021 \times 10^{-8}$	$\pi^\pm + e^\mp + \nu_e$ $\pi^\pm + \mu^\mp + \nu_\mu$ $\pi^0 + \pi^0 + \pi^0$ $\pi^+ + \pi^0 + \pi^-$

Table 3. Main decay modes for K^+ [16].

Reaction	Mode	Branching ratio
$\mu^+ + \nu_\mu$	leptonic	63.55% ± 0.11%
$\pi^+ + \pi^0$	hadronic	20.66% ± 0.08%
$\pi^+ + \pi^+ + \pi^-$	hadronic	5.59% ± 0.04%
$\pi^+ + \pi^0 + \pi^0$	hadronic	1.761% ± 0.022%
$\pi^0 + e^+ + \nu_e$	semileptonic	5.07% ± 0.04%
$\pi^0 + \mu^+ + \nu_\mu$	semileptonic	3.353% ± 0.034%

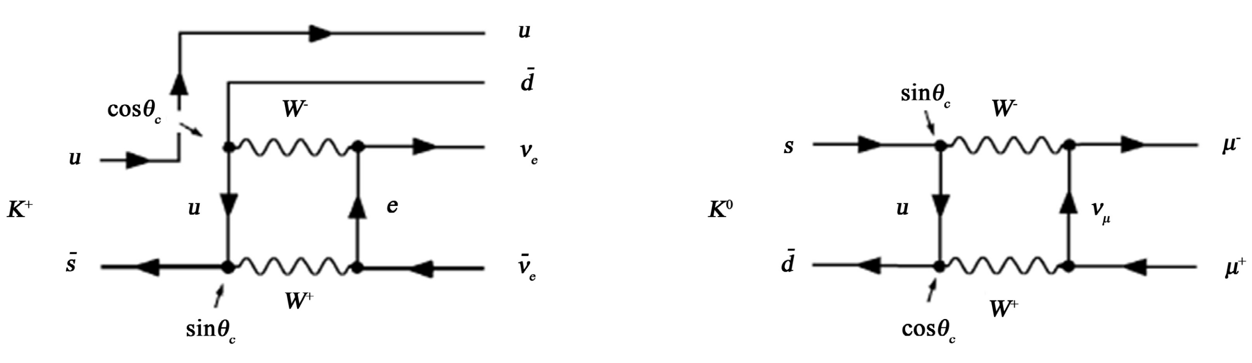


Figure 6. Quark diagrams for K^+ and K^0 decays involving strangeness changing neutral currents [10].

K0 decay and CP-violation

The full kaon-L and kaon-S contain a CP-violating term:

$$K_L = \frac{1}{\sqrt{1+\varepsilon^2}}\left(\frac{K^0+\bar{K}^0}{\sqrt{2}} + \varepsilon\frac{K^0-\bar{K}^0}{\sqrt{2}}\right)$$

$$K_S = \frac{1}{\sqrt{1+\varepsilon^2}}\left(\frac{K^0-\bar{K}^0}{\sqrt{2}} + \varepsilon\frac{K^0+\bar{K}^0}{\sqrt{2}}\right)$$

$\varepsilon = 2.25\times 10^{-3}$ CP violation factor.

2.7. The Kaon-Pion Decay Detailed Theory

In [17] a semi-empirical formula for the transition matrix element A_{++} in $\Gamma\left(K^+(k)\to\pi^+(p_1)\pi^+(p_2)\pi^-(p_3)\right)$ is derived.

First, the kinematic momentum variables s_0, s_1, s_2, s_3 are introduced

$$s_1=(k-p_1)^2,\quad s_2=(k-p_2)^2,\quad s_3=(k-p_3)^2,\quad s_0=\left(m^2+m_1^2+m_2^2+m_3^2\right)/3$$

then, the Dalitz plot variables $x=\frac{s_2-s_1}{\left(m_1^2+m_2^2\right)/2}$, $y=\frac{s_3-s_0}{m_3^2}$ are defined.

We obtain for A_{++} (x,y) the expression

$$A_{++-}(x,y) = (-2\alpha_1+\alpha_3) + \left(-\beta_1+\frac{1}{2}\beta_3-\sqrt{3}\gamma_3\right)y$$
$$-(2\zeta_1+2\zeta_3)\left(y^2+\frac{1}{3}x^2\right)+(\xi_1+\xi_3-\xi_3')\left(y^2-\frac{1}{3}x^2\right)$$

with the constants:

$\alpha_1=(91.71\pm 0.32)\times 10^{-8}$, $\alpha_2=(-7.36\pm 0.47)\times 10^{-8}$,

$\beta_1=(-25.68\pm 0.27)\times 10^{-8}$, $\beta_2=(-2.43\pm 0.41)\times 10^{-8}$, $\gamma_3=(2.26\pm 0.23)\times 10^{-8}$

$\zeta_1=(-0.47\pm 0.15)\times 10^{-8}$, $\zeta_3=(-0.21\pm 0.08)\times 10^{-8}$,

$\xi_1=(-1.51\pm 0.30)\times 10^{-8}$, $\xi_3=(-0.12\pm 0.17)\times 10^{-8}$, $\xi_3'=(-0.21\pm 0.51)\times 10^{-8}$

and $m_2=m_3=m_1$, $s_3=(p_1+p_2)^2=(E_1+E_2)^2-(\vec{p}_1+\vec{p}_2)^2$.

We obtain the following expression for the differential transition width from Fermi's golden rule:

$$\mathrm{d}\Gamma=\frac{\left|M(k,p_1,p_2,p_3)\right|^2}{2m}\frac{\mathrm{d}^3p_1}{(2\pi)^3 2E_1}\frac{\mathrm{d}^3p_2}{(2\pi)^3 2E_2}\frac{\mathrm{d}^3p_3}{(2\pi)^3 2E_3}(2\pi)^4\delta^4(k-p_1-p_2-p_3)$$

or, with Dalitz variables

$$\mathrm{d}\Gamma=\frac{\left|M(k,p_1,p_2,p_3)\right|^2}{2m}\frac{\mathrm{d}^3p_1}{(2\pi)^3 2\sqrt{m_1^2+p_1^2}}\frac{\mathrm{d}^3p_2}{(2\pi)^3 2\sqrt{m_2^2+p_2^2}}\frac{\mathrm{d}^3p_3}{(2\pi)^3 2\sqrt{m_3^2+p_3^2}}(2\pi)^4\delta^4(k-p_1-p_2-p_3)$$

we choose $\vec{k}=0$, $k^0=m$, *i.e.* $\vec{p}_3=-(\vec{p}_1+\vec{p}_2)$

$$s_1=(k-p_1)^2=(m-E_1)^2-\vec{p}_1^2=(m-E_1)^2-\left(E_1^2-m_1^2\right)=m^2+m_1^2-2mE_1$$

$$s_2=(m-E_2)^2-\vec{p}_2^2=m^2+m_1^2-2mE_2$$

$$\vec{p}_1^2=E_1^2-m_1^2,\quad \vec{p}_2^2=E_2^2-m_1^2$$

$$\vec{p}_2^2=\vec{p}_1^2+\vec{p}_2^2+2\vec{p}_1\vec{p}_2\cos\theta_{12}=E_1^2+E_2^2-2m_1^2+2\sqrt{E_1^2-m_1^2}\sqrt{E_2^2-m_1^2}\cos\theta_{12},$$

$$m=E_1+E_2+E_3$$

$$s_3 = (p_1 + p_2)^2 = (m - (E_1 + E_2))^2 - (\vec{p}_1 + \vec{p}_2)^2$$
$$= (m - (E_1 + E_2))^2 - \left(E_1^2 + E_2^2 - 2m_1^2 + 2\sqrt{E_1^2 - m_1^2}\sqrt{E_2^2 - m_1^2}\cos\theta_{12}\right)$$

$$s_3 = (p_1 + p_2)^2$$
$$= 2E_1E_2 - 2m(E_1 + E_2) + m^2 + 2m_1^2 - 2\sqrt{E_1^2 - m_1^2}\sqrt{E_2^2 - m_1^2}\cos\theta_{12}$$

$$E_1 = \sqrt{m_1^2 + \vec{p}_1^2}\ ,\quad E_2 = \sqrt{m_1^2 + \vec{p}_2^2}\ ,$$

$$E_3 = \sqrt{m_1^2 + (\vec{p}_1 + \vec{p}_2)^2} = \left(E_1^2 + E_2^2 - m_1^2 + 2\sqrt{E_1^2 - m_1^2}\sqrt{E_2^2 - m_1^2}\cos\theta_{12}\right)^{1/2}$$

$$x = \frac{s_2 - s_1}{m_1^2} = \frac{2m(E_1 - E_2)}{m_1^2}$$

$$y = \frac{s_3 - s_0}{m_1^2} = \frac{2E_1E_2 - 2m(E_1 + E_2) + 2m^2/3 + m_1^2 - 2\sqrt{E_1^2 - m_1^2}\sqrt{E_2^2 - m_1^2}\cos\theta_{12}}{m_1^2}$$

We insert $|M(k, p_1, p_2, p_3)|^2 = \frac{1}{8\pi}|A_{++-}(x, y)|^2$

and calculate Γ as an integral over $|p_1| = |\vec{p}_1|$, $|p_2| = |\vec{p}_2|$ θ_{12}, integration $\mathrm{d}^3p_3\delta^3(\vec{p}_3 + \vec{p}_1 + \vec{p}_2)$ cancels out

$$\mathrm{d}\Gamma = \frac{|A_{++-}(x,y)|^2}{2m}\frac{4\pi(E_1^2 - m_1^2)\mathrm{d}|p_1|}{(2\pi)^3 2E_1}\frac{2\pi(E_2^2 - m_1^2)\mathrm{d}|p_2|}{(2\pi)^3 2E_2}\frac{1}{(2\pi)^3 2E_3}(2\pi)^4\delta(m - E_1 - E_2 - E_3)$$

and changing to E_1, E_2, θ_{12}: $\mathrm{d}|p_1| = \frac{E_1\mathrm{d}E_1}{\sqrt{E_1^2 - m_1^2}}$, $\mathrm{d}|p_2| = \frac{E_2\mathrm{d}E_2}{\sqrt{E_2^2 - m_1^2}}$,

$|p_1| = \sqrt{E_1^2 - m_1^2}$, $|p_2| = \sqrt{E_2^2 - m_1^2}$

$$\mathrm{d}\Gamma = \frac{|A_{++-}(x,y)|^2}{8m(2\pi)^3}\sqrt{E_1^2 - m_1^2}\,\mathrm{d}E_1\sqrt{E_2^2 - m_1^2}\,\mathrm{d}E_2$$
$$\times\frac{\sin\theta_{12}\mathrm{d}\theta_{12}}{\left(E_1^2 + E_2^2 - 2m_1^2 + 2\sqrt{E_1^2 - m_1^2}\sqrt{E_2^2 - m_1^2}\cos\theta_{12}\right)^{1/2}}$$
$$\times\delta\left(m - E_1 - E_2 - \left(E_1^2 + E_2^2 - 2m_1^2 + 2\sqrt{E_1^2 - m_1^2}\sqrt{E_2^2 - m_1^2}\cos\theta_{12}\right)^{1/2}\right)$$

we solve $\mathrm{d}\Gamma$ for E_2 [2]:

$$E_2 = \frac{-2E_1^2m - m^3 - mm_1^2 + E_1(3m^2 + m_1^2) + \cos(\theta_{12})\left((E_1^2 - m_1^2)\begin{pmatrix}(m^2 - m_1^2)(4E_1^2 - 4E_1m + m^2 - m_1^2)\\ +4m_1^2(E_1^2 - m_1^2)\cos(\theta_{12})^2\end{pmatrix}\right)^{1/2}}{-2(E_1 - m)^2 + 2(E_1^2 - m_1^2)\cos(\theta_{12})^2}$$

and $m - E_1 - E_2 = \sqrt{E_1^2 + E_2^2 - m_1^2 + 2\sqrt{E_1^2 - m_1^2}\sqrt{E_2^2 - m_1^2}\cos\theta_{12}}$

$$\frac{(m - (E_1 + E_2))^2 - E_1^2 - E_2^2 + m_1^2}{2\sqrt{E_1^2 - m_1^2}\cos\theta_{12}} = \sqrt{E_2^2 - m_1^2}$$

now we carry out the integration over E_2 with the delta-function:

$$\Gamma = \int\frac{|A_{++-}(x,y)|^2}{8m(2\pi)^3}\mathrm{d}E_1\frac{(m - (E_1 + E_2))^2 - E_1^2 - E_2^2 + m_1^2}{2\cos\theta_{12}}\frac{\sin\theta_{12}\mathrm{d}\theta_{12}}{m - (E_1 + E_2)}$$

and after simplification

$$\Gamma = \int \frac{\left|A_{++-}(x,y)\right|^2}{8m(2\pi)^3} \frac{\sqrt{E_2^2 - m_1^2}\sqrt{E_1^2 - m_1^2}}{m - (E_1 + E_2)} \sin\theta_{12}\, d\theta_{12} dE_1 \tag{4}$$

The integration boundary in E_1 is $m_1 \le E_1 \le \left(1 + f_{elb1}(m, m_1, \theta_{12})\right) m_1$, in θ_{12} $0 \le \theta_{12} \le \pi$

where $f_{elb1}(m, m_1, \theta_{12}) = \frac{|p_1|}{m_1}$ is the relative momentum, at which E_2 becomes complex

$$f_{elb1}(m, m_1, \theta_{12}) = \frac{2m^3 m_1 - 4m^2 m_1^2 - 2m m_1^3 + 4m_1^4 \sin(\theta_{12})^2 - \sqrt{2}\begin{pmatrix} m^2 m_1^4 \left(m^2 \left(1 - \cos(2\theta_{12})\right) + 2m_1^2 + 6m_1^4 \cos(2\theta_{12})\right) \\ + m_1^8 \left(\cos(4\theta_{12}) - \cos(2\theta_{12})\right) \end{pmatrix}^{1/2}}{4\left(m^2 m_1^2 - m_1^4 \sin(\theta_{12})^2\right)}$$

Numerical integration yields for $m_1 = m(\pi^+) = 0.139\ \text{GeV}$, $m = m(K^+) = 0.493\ \text{GeV}$ [2].

$\Gamma(m_1, m) = 0.033 \times 10^{-16}\ \text{GeV}$, the measured decay width is 0.0297 × 10^{-16} GeV (see below).

2.8. The General 3-Body Decay

We use the momentum notations $\Gamma\left(P_0(k,m) \to P(p_1,m_1) P(p_2,m_2) P(p_3,m_3)\right)$ [14] [18].

We start again with Fermi's golden rule:

$$d\Gamma = \frac{\left|M(k,p_1,p_2,p_3)\right|^2}{2m} \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{d^3 p_3}{(2\pi)^3 2E_3} (2\pi)^4 \delta^4 (k - p_1 - p_2 - p_3) \tag{5}$$

we choose $\vec{k} = 0$, $k^0 = m$, *i.e.* $\vec{p}_3 = -(\vec{p}_1 + \vec{p}_2)$

$\vec{p}_1^2 = E_1^2 - m_1^2$, $\vec{p}_2^2 = E_2^2 - m_2^2$

$\vec{p}_3^2 = \vec{p}_1^2 + \vec{p}_2^2 + 2\vec{p}_1\vec{p}_2 \cos\theta_{12} = E_1^2 + E_2^2 - m_1^2 - m_2^2 + 2\sqrt{E_1^2 - m_1^2}\sqrt{E_2^2 - m_2^2} \cos\theta_{12}$,

$m = E_1 + E_2 + E_3$

$E_1 = \sqrt{m_1^2 + \vec{p}_1^2}$, $E_2 = \sqrt{m_2^2 + \vec{p}_2^2}$,

$$E_3 = \sqrt{m_3^2 + (\vec{p}_1 + \vec{p}_2)^2} = \left(E_1^2 + E_2^2 + m_3^2 - m_1^2 - m_2^2 + 2\sqrt{E_1^2 - m_1^2}\sqrt{E_2^2 - m_2^2} \cos\theta_{12}\right)^{1/2}$$

now we calculate Γ as an integral over $|p_1| = |\vec{p}_1|$, $|p_2| = |\vec{p}_2|$ θ_{12}, integration $d^3 p_3 \delta^3 (\vec{p}_3 + \vec{p}_1 + \vec{p}_2)$ cancels out

$$d\Gamma = \frac{\left|M(k,p_1,p_2,p_3)\right|^2}{2m} \frac{4\pi\left(E_1^2 - m_1^2\right) d|p_1|}{(2\pi)^3 2E_1} \frac{2\pi\left(E_2^2 - m_2^2\right) d|p_2|}{(2\pi)^3 2E_2} \frac{1}{(2\pi)^3 2E_3} (2\pi)^4 \delta(m - E_1 - E_2 - E_3)$$

and changing to E_1, E_2, θ_{12}: $d|p_1| = \frac{E_1 dE_1}{\sqrt{E_1^2 - m_1^2}}$, $d|p_2| = \frac{E_2 dE_2}{\sqrt{E_2^2 - m_2^2}}$,

$|p_1| = \sqrt{E_1^2 - m_1^2}$, $|p_2| = \sqrt{E_2^2 - m_2^2}$

$$\mathrm{d}\Gamma=\frac{\left|M\left(k,p_1,p_2,p_3\right)\right|^2}{8m\left(2\pi\right)^3}\sqrt{E_1^2-m_1^2}\,\mathrm{d}E_1\sqrt{E_2^2-m_2^2}\,\mathrm{d}E_2$$
$$\times\frac{\sin\theta_{12}\mathrm{d}\theta_{12}}{\left(E_1^2+E_2^2+m_3^2-m_1^2-m_2^2+2\sqrt{E_1^2-m_1^2}\sqrt{E_2^2-m_2^2}\cos\theta_{12}\right)^{1/2}}$$
$$\times\delta\left(m-E_1-E_2-\left(E_1^2+E_2^2+m_3^2-m_1^2-m_2^2+2\sqrt{E_1^2-m_1^2}\sqrt{E_2^2-m_2^2}\cos\theta_{12}\right)^{1/2}\right)$$

we solve for E_2 [2]:

$$E_2=\Big(m\left(-2E_1^2-m^2-m_1^2+m_3^2\right)+E_1\left(3m^2+m_1^2+m_2^2-m_3^2\right)+\cos\left(\theta_{12}\right)\Big(m^4+2m^2m_1^2$$
$$+m_1^4-2m^2m_2^2+m_2^4-2m^2m_3^2-2\left(m_1^2+m_2^2\right)m_3^2+m_3^4+2E_1^2\left(2m^2-m_2^2\right)$$
$$-4E_1m\left(m^2+m_1^2-m_2^2-m_3^2\right)+2m_2^2\left(E_1^2-m_1^2\right)\cos\left(2\theta_{12}\right)\Big)^{1/2}\Big)$$
$$/\left(-2\left(E_1-m\right)^2+2\left(E_1^2-m_1^2\right)\cos\left(\theta_{12}\right)^2\right)$$

and $m-E_1-E_2=\left(E_1^2+E_2^2+m_3^2-m_1^2-m_2^2+2\sqrt{E_1^2-m_1^2}\sqrt{E_2^2-m_2^2}\cos\theta_{12}\right)^{1/2}$

$$\frac{\left(m-\left(E_1+E_2\right)\right)^2-E_1^2-E_2^2-m_3^2+m_1^2+m_2^2}{2\sqrt{E_1^2-m_1^2}\cos\theta_{12}}=\sqrt{E_2^2-m_2^2}$$

now we carry out the integration over E_2 with the delta-function:

$$\Gamma=\int\frac{\left|M\left(k,p_1,p_2,p_3\right)\right|^2}{8m\left(2\pi\right)^3}\mathrm{d}E_1\frac{\left(m-\left(E_1+E_2\right)\right)^2-E_1^2-E_2^2-m_3^2+m_1^2+m_2^2}{2\cos\theta_{12}}\frac{\sin\theta_{12}\mathrm{d}\theta_{12}}{m-\left(E_1+E_2\right)}$$

and after simplification

$$\Gamma=\int\frac{\left|M\left(k,p_1,p_2,p_3\right)\right|^2}{8m\left(2\pi\right)^3}\frac{\sqrt{E_2^2-m_1^2}\sqrt{E_1^2-m_2^2}}{m-\left(E_1+E_2\right)}\sin\theta_{12}\,\mathrm{d}\theta_{12}\mathrm{d}E_1$$

setting $\left|M\left(k,p_1,p_2,p_3\right)\right|=1$ we obtain the *partial kinematic factor* for 3-body decay $I_{\Gamma3}\left(\frac{m_1}{m},\frac{m_2}{m},\frac{m_3}{m}\right)$

$$\Gamma=\int\frac{1}{8m\left(2\pi\right)^3}\frac{\sqrt{E_2^2-m_1^2}\sqrt{E_1^2-m_2^2}}{m-\left(E_1+E_2\right)}\sin\theta_{12}\,\mathrm{d}\theta_{12}\mathrm{d}E_1=m\,I_{\Gamma3}\left(m,m_1,m_2,m_3\right)\quad(6)$$

The integration boundary in E_1 is $m_1\le E_1\le\left(1+f_{e1b1}\left(m,m_1,m_2,m_3,\theta_{12}\right)\right)m_1$, in θ_{12} $0\le\theta_{12}\le\pi$

where $f_{e1b1}\left(m,m_1,m_2,m_3,\theta_{12}\right)=\frac{\left|p_1\right|}{m_1}$ is the relative momentum, at which E_2 becomes complex

$$f_{e1b1}\left(m,m_1,m_2,m_3,\theta_{12}\right)$$
$$=\Big(2\left(m^3m_1-2m^2m_1^2+mm_1^3-mm_1\left(m_2^2+m_3^2\right)+m_1^2m_2^2-m_1^2m_2^2\cos\left(2\theta_{12}\right)\right)$$
$$-\Big(-\Big(m^2\left(m^2-4mm_1+6m_1^2-2m_2^2-2m_3^2\right)-4mm_1^3+m_1^4+4mm_1\left(m_2^2+m_3^2\right)$$

$$-2m_1^2\left(m_2^2+m_3^2\right)+m_2^4+m_3^4-2{m_2}^2{m_3}^2\Big)\Big(4m^2{m_1}^2-2m_1^2m_2^2+2m_1^2{m_2}^2\cos\left(2\theta_{12}\right)\Big)$$
$$+4\Big(-m^3m_1+2m^2m_1^2-mm_1^3+mm_1\left(m_2^2+m_3^2\right)+m_1^2m_2^2\left(\cos\left(2\theta_{12}\right)-1\right)\Big)^2\Big)^{1/2}\Big)$$
$$/\Big(2\Big(2m^2m_1^2-m_1^2m_2^2+m_1^2m_2^2\cos\left(2\theta_{12}\right)\Big)\Big) \tag{6a}$$

The kinematic factor $I_{\Gamma 3}\left(m,m_1,m_2,m_3\right)$ can be calculated numerically.

The *total kinematic factor* results from $I_{\Gamma 3}\left(m,m_1,m_2,m_3\right)$ by symmetrization over all 6 index permutations

$$I_{\Gamma 3s}\left(m,m_1,m_2,m_3\right)=\left(I_{\Gamma 3}\left(m,m_1,m_2,m_3\right)+I_{\Gamma 3}\left(m,m_1,m_3,m_2\right)+\cdots\right)/6$$

Here is the plot of $I_{\Gamma 3s}\left(1,m_1,0.1,0.1\right)10^6$ shown in **Figure 7**.

Example: $\Gamma\left(\mu\to e\bar{\nu}_e\nu_\mu\right)$ with kinematic factor:
$I_{\Gamma 3s}\left(m(\mu),m(e),0,0\right)=0.3835$.

2.9. The General 2-Body Decay

We use the momentum notations $\Gamma\left(P_0(k,m)\to P\left(p_1,m_1\right)P\left(p_2,m_2\right)\right)$ [14] [18].

We start again with Fermi's golden rule for 2-body decay [11]:

$$\mathrm{d}\Gamma=m\frac{\left|M\left(k,p_1,p_2\right)\right|^2}{2}\frac{\mathrm{d}^3p_1}{(2\pi)^3\,2E_1}\frac{\mathrm{d}^3p_2}{(2\pi)^3\,2E_2}(2\pi)^4\,\delta^4\left(k-p_1-p_2\right) \tag{7}$$

we choose $\vec{k}=0$, $k^0=m$, *i.e.* $\vec{p}_2=-\vec{p}_1$

$$\vec{p}_1^{\,2}=E_1^2-m_1^2,\quad \vec{p}_2^{\,2}=E_2^2-m_2^2$$
$$m=E_1+E_2$$
$$E_1=\sqrt{m_1^2+\vec{p}_1^{\,2}},\quad E_2=\sqrt{m_2^2+\vec{p}_1^{\,2}}=\sqrt{E_1^2+m_2^2-m_1^2}=m-E_1,$$
$$\delta^4\left(k-p_1-p_2\right)=\delta\left(m-E_1-E_2\right)\delta^3\left(\vec{p}_1+\vec{p}_2\right)$$

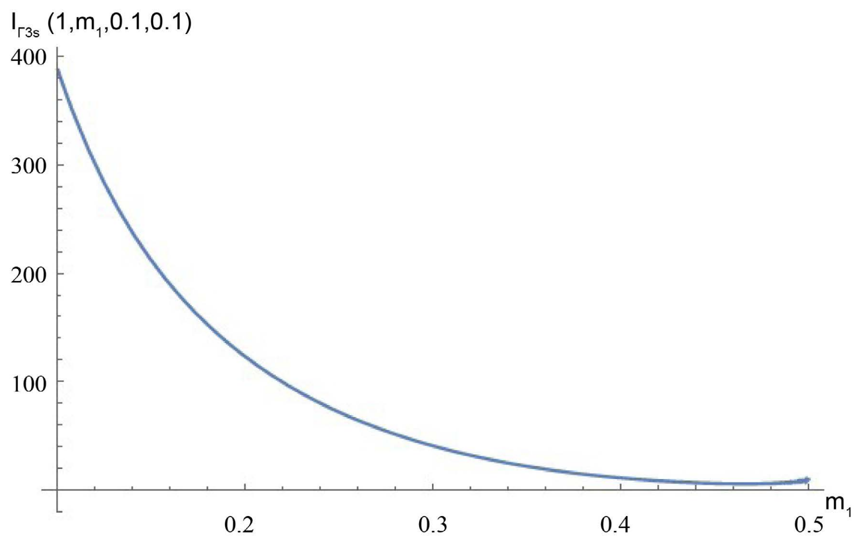


Figure 7. Plot of $I_{\Gamma 3s}\left(1,m_1,0.1,0.1\right)10^6$.

now we calculate Γ as an integral over $|p_1| = |\vec{p}_1|$, integration $\mathrm{d}^3 p_2 \delta^3(\vec{p}_1 + \vec{p}_2)$ cancels out

and changing to E_1: $\mathrm{d}|p_1| = \frac{E_1 \mathrm{d}E_1}{\sqrt{E_1^2 - m_1^2}}$, $|p_1| = \sqrt{E_1^2 - m_1^2}$

$$\mathrm{d}\Gamma = m \frac{|M(k, p_1, p_2)|^2}{8(2\pi)^2} \frac{4\pi\sqrt{E_1^2 - m_1^2}\,\mathrm{d}E_1}{\sqrt{E_1^2 + m_2^2 - m_1^2}} \delta(m - E_1 - E_2)$$

from $\sqrt{E_1^2 + m_2^2 - m_1^2} = m - E_1$ we obtain the solution [2]

$$E_{10} = \frac{m^2 + m_1^2 - m_2^2}{2m},$$

the integration $\mathrm{d}E_1 \delta(m - E_1 - E_2)$ cancels out and we obtain, setting $|M(k, p_1, p_2, p_3)| = 1$

$$\begin{aligned} \Gamma &= \frac{m}{4(2\pi)} \frac{\sqrt{E_{10}^2 - m_1^2}}{\sqrt{E_{10}^2 + m_2^2 - m_1^2}} \\ &= \frac{1}{8m\pi} \frac{\left(m^4 + m_1^4 + m_2^4 - 2m^2 m_1^2 - 2m^2 m_2^2 - 2m_1^2 m_2^2\right)^{1/2}}{\left(m^4 + m_1^4 + m_2^4 - 2m^2 m_1^2 + 2m^2 m_2^2 - 2m_1^2 m_2^2\right)^{1/2}} \end{aligned} \quad (8)$$

we obtain the *kinematic factor* for 2-body decay $I_{\Gamma 2}(m, m_1)$

$$I_{\Gamma 2}(m, m_1, m_2) = \frac{\Gamma}{m} = \frac{1}{8\pi} \frac{\left(m^4 + m_1^4 + m_2^4 - 2m^2 m_1^2 - 2m^2 m_2^2 - 2m_1^2 m_2^2\right)^{1/2}}{\left(m^4 + m_1^4 + m_2^4 - 2m^2 m_1^2 + 2m^2 m_2^2 - 2m_1^2 m_2^2\right)^{1/2}} \quad (8a)$$

The *total kinematic factor* for 2-body decay results from the symmetrized $I_{\Gamma 2}(m, m_1)$

$$I_{\Gamma 2s}(m, m_1, m_2) = \frac{I_{\Gamma 2}(m, m_1) + I_{\Gamma 2}(m, m_2)}{2}$$

As an example, here is the plot $I_{\Gamma 2s}(1, m_1, 0.5)$ shown in **Figure 8**.

Example: $\Gamma(\pi \to \mu \nu)$, with kinematic factor: $I_{\Gamma 2s}(m(\pi), m(\mu), 0) = 0.0251$.

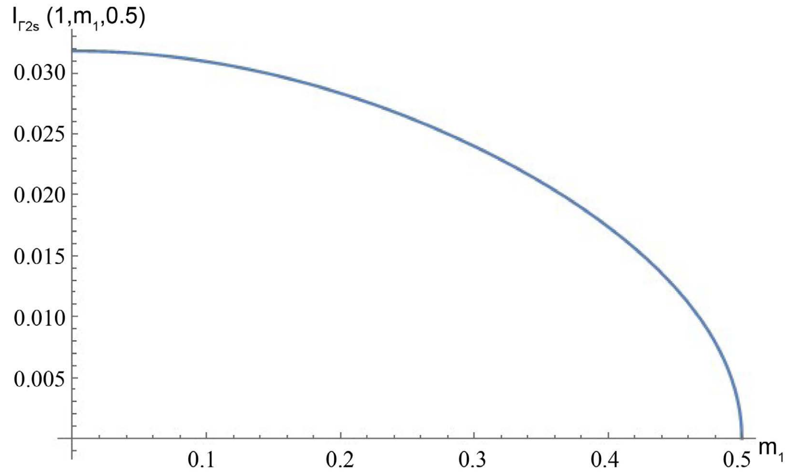
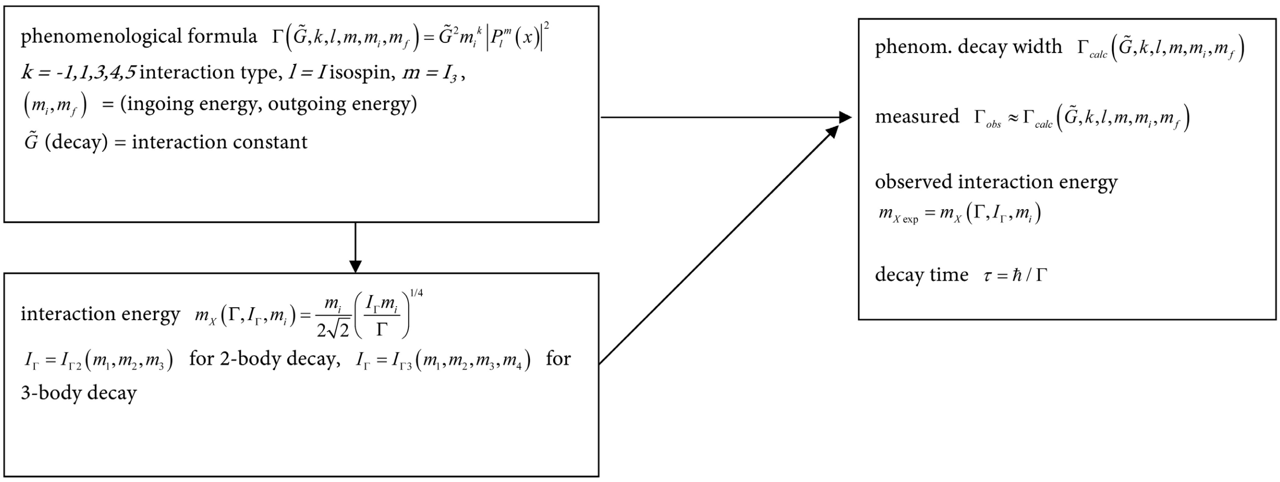


Figure 8. Plot of $I_{\Gamma 2s}(1, m_1, 0.5)$.

3. The Theoretical Background and the Phenomenological Decay Formula

Schematics: decay width phenomenological formula, interaction energy



In this chapter, we follow the following scheme underlying the phenomenological formula by Chang [1] for decay width, the derived interaction energy, and resulting decay time.

The phenomenological formula is a semi-empirical scheme for the calculation of the decay width Γ of a decay: $\Gamma_{calc}(\tilde{G},k,l,m,m_i,m_f)=\tilde{G}^2 m_i^k |P_l^m(x)|^2$, depending on (m_i, m_f) = (ingoing energy, outgoing energy), interaction constant (decay dependent) $\tilde{G}$, interaction type $k = -1$, 1, 3, 4, 5, and extended isospin I with the traditional notations $l = I$, $m = I_3$.

From the decay width, the decay time follows immediately $\tau = \hbar/\Gamma$.

The agreement between the phenomenological Γ_{calc} and the observed values Γ_{obs} is remarkably good (see chap. 6).

The interaction energy $m_X(\Gamma, I_\Gamma, m_i)$ between the incoming and the outgoing state in the process (corresponding to the energy of the mediating particle in the Feynman diagram, e.g. W^- boson in the neutron decay), can be calculated from Γ using kinematics factors $I_\Gamma(m_1, m_2, \cdots)$.

The kinematics factors describe the statistics of the process and depend on the involved particle masses.

In 1.8 and 1.9 we calculated the kinematic factor and obtained for decay width the general formula $\Gamma = |M|^2 I_\Gamma m_i$.

The formula for interaction energy $m_X(\Gamma, I_\Gamma, m_i)=\frac{m_i}{2\sqrt{2}}\left(\frac{I_\Gamma m_i}{\Gamma}\right)^{1/4}$ is derived below in chap. 3.1.

3.1. The Phenomenological Decay Formula and Interaction Energy

The phenomenological decay formula

The phenomenological formula for the decay width is [1]

$$\Gamma = \tilde{G}^2 m_i^k \left|P_l^m(x)\right|^2 = \frac{G^2}{C_1} m_i^k \left|P_l^m(x)\right|^2 \quad (9)$$

where $P_l^m(x)$ Legendre polynomial $m = l$ or $m = l+1$, $l =$ isospin I, $x = \frac{m_f}{m_i}$ mass ratio, $\tilde{G} = \frac{G}{\sqrt{C_1}}$ with $G =$ *interaction constant*, m_i is the initial mass.

The constant C_1 is process-dependent, standard value $C_1 = 4\pi$, with exceptions for muon: $C_1 = 192\pi^3$, and for neutron: $C_1 = 2\pi^3$.

The G constants are: for kaons $G^2 = g_1^2 = 2.06\times10^{-14}$, for pions $G^2 = g_0^2 = 2.18\times10^{-14}$, for leptonic decays $A \to A' e\,\bar{\nu}_e\,(\Delta S = 0)$

$G = G_l = 1.02\times10^{-5}\left(\frac{1}{m_p^2}\right) = 1.16\times10^{-5}\ \text{GeV}^{-2}$, for hyperons

$G = g_{h1} = 5.81\times10^{-7}\ \text{GeV}$ or $G = g_{h2} = 1.4\times10^{-8}\ \text{GeV}^{-3/2}$.

The interaction constants for hyperons in [1] were given by

$G = g_h = 6.2\times10^{-7}\left(\frac{m_p}{m_i}\right)$ or $G = g_{h'} = 1.28\times10^{-8}\left(\frac{m_i}{m_p}\right)^{3/2}$, they have been corrected, since G must not depend on initial mass m_i, also the power coefficient k and the data tables were corrected accordingly.

The power coefficient is

$k = 1$ for a dimensionless $\tilde{G}$, like in pion decay

$$\Gamma(\pi \to \mu\nu_\mu) = \tilde{G}^2 m_i x^2 \left(1-x^2\right)^2 = \frac{G^2 m_i}{4\pi} x^2 \left(1-x^2\right)^2, \quad G^2 = g_0^2 = 2.18\times10^{-14}$$

$k = 5$ for a dimensional $\tilde{G}$, $[\tilde{G}] = \text{GeV}^{-2}$, like in muon decay

$$\Gamma(\mu \to e\nu_e\nu_\mu) = \tilde{G}^2 m_i^5 \left(1-x^2\right)^4 = \frac{G_F^2 m_i^5}{192\pi^3}\left(1-x^2\right)^4$$

$k = 3$ for a dimensional $\tilde{G}$, $[\tilde{G}] = \text{GeV}^{-1}$, like in π^0 decay

$$\Gamma(\pi^0 \to \gamma\gamma) = \tilde{G}^2 m_i^3 \left(1-x^2\right)^3$$

$k = -1$ for a dimensional $\tilde{G}$, $[\tilde{G}] = \text{GeV}$, for kaon decays with $\tilde{G} \cong g_{h1}$

$k = 4$ for a dimensional $\tilde{G}$, $[\tilde{G}] = \text{GeV}^{-3/2}$, for non-kaon hyperon decays with $\tilde{G} \cong g_{h2}$

so in all cases $[\Gamma] = \text{m}$, i.e. the dimension is energy, as it must be.

The *extended isospin* I [5] [19] includes higher generation quarks, $I(s) = I(c) = 1/2$ and $I(l) = 1$ for leptons l as well as $I(\gamma) = 1$ for the photon.

The extended isospin has the following values shown in **Table 4**.

The angular momentum in decay width: $l = |\Delta I| = \left|I_i \pm I_f\right|$ is the difference or sum of the initial and final isospin.

The interaction energy

The *interaction energy* m_X is the (excitation) energy of the mediating virtual exchange boson (for pure weak decays: W or Z-boson).

We can deduce the interaction energy from the phenomenological formula in

Table 4. Extended isospin.

$I(\Lambda)=1/2$
$I(\Sigma)=1/2$
$I(\Xi)=1/2$
$I(K)=1$
$I(\gamma)=1$
$I(\pi)=1$
$I(l)=1$ for lepton l
$I(p)=1/2$, $I(n)=1/2$
but $I(ud)=I(dd)=I(uu)=1$
$I(\eta)=1$

the following way.

We consider the phenomenological formula for $k = 5$ (e.g. muon decay) in the form $\Gamma=\tilde{G}^2 m_i^5\left(1-x^2\right)^4$, where $x=\frac{m_f}{m_i}$.

We separate the kinematic factor as in chap. 2.8 and 2.9 in the form $\Gamma=|M|^2 I_\Gamma m_i$.

So we conclude $I_\Gamma=\left(1-x^2\right)^4$, $M^2=\tilde{G}^2 m_i^4$.

In analogy to the weak interaction mediated by the W-boson (as in muon decay chap.3.3) $\tilde{G}=\frac{G_F}{\sqrt{2}}=\frac{g^2}{8M_W^2}$ (g weak interaction constant), we make the ansatz for the matrix element M and the interaction energy m_X

$$M=\frac{m_i^2}{8m_X^2}$$

The decay width with the matrix element M and the kinematic I_Γ factor becomes then

$\Gamma=|M|^2 I_\Gamma m_i=\left(\frac{m_i^2}{8m_X^2}\right)^2 I_\Gamma m_i$ or in general $\frac{m_X}{m_i}=f_I\left(\frac{m_i}{\Gamma}\right)^{1/4}$, where $f_I=\frac{I_\Gamma^{1/4}}{2\sqrt{2}}$.

From this formula we can derive a **general semi-empirical formula** for the interaction energy m_X, where the kinematic factor is dimensionless $\left[I_\Gamma\right]=1$,

$$m_X=\frac{m_i}{2\sqrt{2}}\left(\frac{I_\Gamma m_i}{\Gamma}\right)^{1/4} \tag{10}$$

where $I_\Gamma=I_{\Gamma 2}\left(m_1,m_2,m_3\right)$ for 2-body decay,

$I_\Gamma=I_{\Gamma 3}\left(m_1,m_2,m_3,m_4\right)$ for 3-body decay (see 1.8, 1.9)

For $k = 1$ $\frac{m_i^4}{m_X^4}=\frac{64\tilde{G}^2}{I_\Gamma}\left|P_l^m(x)\right|^2$ with the phenomenological formula

$$\Gamma=\tilde{G}^2 m_i\left|P_l^m(x)\right|^2 \tag{10a}$$

For $k = 5$ $\frac{1}{m_X^4} = \frac{64\tilde{G}^2}{I_\Gamma}\left|P_l^m(x)\right|^2$ with $\Gamma = \tilde{G}^2 m_i^5 \left(1-x^2\right)^4 = \frac{G_F^2 m_i^5}{192\pi^3}\left(1-x^2\right)^4$

(10b)

For $k = 3$ $\frac{1}{m_X^2} = \frac{64\tilde{G}^2}{I_\Gamma}\left|P_l^m(x)\right|^2$ with $\Gamma = \tilde{G}^2 m_i^3 \left(1-x^2\right)^3$ (10c)

3.2. Derivation of Angular Momentum Dependence in the Phenomenological Formula

Laplace operators in spherical coordinates reads [1]

$$\Delta\psi = \frac{1}{r^2}\left(\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial p}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2}\right)$$

quantum kinetic energy is $E_{kin}(\psi) = -\frac{\hbar^2\Delta\psi}{2m}$

with the ansatz $\psi = R(r)\Theta(\theta,\varphi)$

for rigid rotator r=*const* and we obtain the angular momentum spectrum

$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2} = l(l+1)$ with kinetic energy

$E_{kin} = \frac{L^2}{2I} = \frac{h^2 l(l+1)}{8\pi^2 I}$

where l is the angular momentum quantum number

with eigenfunctions

$$\psi_{lm} = N_{lm}P_l^{|m|}(\cos\theta)\exp(im\varphi),\quad \left|\psi_{lm}\right|^2 = \frac{1}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}(2l+1)P_l^{|m|}(\cos\theta)^2$$

and decay width $\Gamma = A\left|\psi_{lm}\right|^2 = \frac{A}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}(2l+1)P_l^{|m|}(\cos\theta)^2$

with $x = \cos\theta = \frac{p_z}{|\vec{p}|}$ and $x = \frac{\sum m_f}{m_i}$, $m = l$ or $m = l-1$

Legendre functions $P_l^m(x) = \frac{1}{\Gamma(1-m)}\left(\frac{1+x}{1-x}\right)^{m/2} {}_2F_1\left(-l, l+1; 1-m; \frac{1-x}{2}\right)$

with hypergeometric function ${}_2F_1$

associated Legendre polynomials

$P_l^m(x) = (-1)^m 2^l \left(1-x^2\right)^{m/2}\sum_{k=m}^{l}\frac{k!}{(k-m)!}x^{k-m}\binom{l}{k}\binom{(l+k-1)/2}{l}$

$P_{l+1}^l(x) = x(2l+1)P_l^l(x)$, $P_l^l(x) = (-1)^l(2l-1)!!\left(1-x^2\right)^{1/2}$,

$P_{l+1}^{l+1}(x) = -(2l+1)\sqrt{1-x^2}P_l^l(x)$

$P_0^0(x) = 1$, $P_1^0(x) = x$, $P_1^{-1}(x) = \frac{1}{2}\left(1-x^2\right)^{1/2}$, $P_1^1(x) = -\left(1-x^2\right)^{1/2}$,

$P_2^0(x) = \frac{1}{2}\left(3x^2-1\right)$

and the decay width becomes $\Gamma_{l,\alpha}(x) = Cx^{2(\alpha-1)}\left(1-x^2\right)^l$, $\alpha = 1,2$

In the following, the angular momentum L is replaced by the isospin I.

3.3. Muon Decay Theory

The analytical formula from the Feynman diagram is

$$\Gamma\left(\mu \to e\nu_e \nu_\mu\right) = \frac{G_F^2 m_\mu^5}{192\pi^3} \quad [6],\ G_F \text{ Fermi-constant} \tag{11}$$

exact formula with corrections [10]

$$\Gamma\left(\mu \to e\bar{\nu}_e \nu_\mu\right) = \frac{G_F^2 m_\mu^5}{192\pi^3} f\left(\frac{m_e^2}{m_\mu^2}\right)\left(1 + f_{rc}\left(\frac{\alpha}{\pi}\right)\right)\left(1 + \frac{3}{5}\frac{m_\mu^2}{m_W^2} + \cdots\right) \tag{11a}$$

$$f_{rc}\left(x\right) = \frac{x}{2}\left(\frac{25}{4} - \pi^2\right)\left(1 + x\left(\frac{2}{3}\log\frac{m_\mu}{m_e} - 3.7\right) + x^2\left(\frac{4}{9}\log^2\frac{m_\mu}{m_e} - 2\log\frac{m_\mu}{m_e} + C\right) + \cdots\right) \tag{11b}$$

and the phenomenological formula

$$\Gamma\left(\mu \to e\nu_e \nu_\mu\right) = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1 - \frac{m_e^2}{m_\mu^2}\right)^4 = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1 - x^2\right)^4 \quad [1],\ l = 4 \tag{11c}$$

This is the general formula for a *leptonic weak* 3-*body decay*, setting initial mass $m_i = m_\mu$.

The (charged) weak interaction in the Feynman-Gell-Mann form reads

$H_{weak} = \frac{G_F}{\sqrt{2}} J^\mu J_\mu^\dagger$ [6] (9.1), where J_μ is the charged leptonic-hadronic current

$J_\mu = L_\mu + H_\mu$ and $L_\mu\left(x\right) = 2\bar{e}_L\left(x\right)\gamma_\mu \nu_e\left(x\right) + \cdots$ is the leptonic current,

$H_\mu\left(x\right) = 2\bar{u}_L\left(x\right)\gamma_\mu d_L\left(x\right) + \cdots$ is the analogous hadronic current.

In the standard model, the (charged) weak interaction is mediated by the massive W-boson W_μ with mass M_W for the charged current, with the Lagrangian

$$L_{weak} = -\frac{g}{2\sqrt{2}}\left(J^\mu \bar{W}_\mu + W^\mu \bar{J}_\mu\right) \tag{12}$$

where the effective interaction constant is $G = \frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$.

We can use the total current and use an *excited intermediate W-boson*, which includes the hadronic part, with the total mass $m_X > M_W$ and calculate it from the effective measured coupling constant $G = \frac{G_F}{\sqrt{2}}$, setting $g = 1$:

$$G^2 = \frac{1}{64m_X^4}$$

The isospin numbers are $l = \Delta I = I_f + I_i = 3 + 1 = 4$ and $m = l = 4$.

3.4. Pion Decay Theory

The analytical formula from the Feynman diagram is

$$\Gamma\left(\pi \to \mu\nu_\mu\right) = \frac{G_F^2 m_\pi}{8\pi} f_\pi^2 V_{ud}^2 m_\mu^2\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 \approx \frac{G_F^2 m_\pi^5}{8\pi}\frac{m_\mu^2}{m_\pi^2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 \quad [[11], (13.26)] \tag{13}$$

G_F Fermi-constant, and the phenomenological formula

$$\Gamma\left(\pi \to \mu \nu_\mu\right) = \frac{G^2 m_\pi}{4\pi}\frac{m_\mu^2}{m_\pi^2}\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2 = \frac{G^2 m_\pi}{4\pi}x^2\left(1-x^2\right)^2 \quad [1] \tag{13a}$$

The (charged) weak interaction has the form

$H_{weak} = \frac{G_F}{\sqrt{2}} J^\lambda\left(\mu,\nu_\mu\right) J_\lambda\left(u,d\right)$ with the leptonic current

$J^\lambda\left(\mu,\nu_\mu\right) = \bar{\mu}\gamma^\lambda\left(1-\gamma_5\right)\nu$ and the hadronic current $J^\lambda\left(u,d\right) = \bar{u}\gamma^\lambda d$ for $\pi^- = \bar{u}d$ ([6], (13.6)).

Using the same procedure as above with the excited intermediate W-boson, we calculate M_X from the above two formulas:

$$\frac{G_F^2}{8\pi}\left(\frac{m_\pi}{m_X}\right)^4 = \frac{G^2}{4\pi} \quad \text{and} \quad \frac{G_F}{\sqrt{2}} = \frac{g_F^2}{8M_X^2}$$

and setting $g_F = 1$ and initial mass $m_i = m_\pi$

we obtain $\left(\frac{m_i}{m_X}\right)^4 = 64G^2$.

The isospin numbers are $l = \Delta I = I_f + I_i = 2+1 = 3$ and $m = l-1 = 2$.

3.5. Kaon Pion Decay Theory

The kaon-pion decay is shown in **Figure 9**.

The generalized and isospin-adapted 3-body semi-leptonic formula (from the muon) decay is

$$\Gamma\left(K^\pm \to \pi^\pm \pi^+ \pi^-\right) = \frac{g^4 m_i}{32\times 192\pi^3}\left(\frac{m_i^4}{m_X^4}\right)\left(1-\frac{m_f^2}{m_i^2}\right)^4 \tag{14}$$

and the phenomenological formula

$$\Gamma\left(K^\pm \to \pi^\pm \pi^+ \pi^-\right) = \frac{G^2 m_{K+}}{4\pi}\left(1-\frac{m_\pi^2}{m_{K+}^2}\right)^2 \quad [1], \text{ where } G = 2g_1\sqrt{\alpha} \tag{14a}$$

From these two formulas setting $g = 1$ we obtain for m_x:

$$\frac{m_i^4}{m_X^4} = 8\times 192\pi^2 G^2 = 32\times 192\pi^2 g_1^2 \alpha$$

The interaction is mediated by W-boson and a gluon: it is a weak-hadronic transformation.

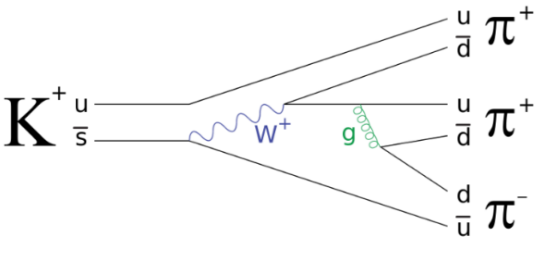


Figure 9. Kaon decay.

The isospin numbers are $l = \Delta I = I_f - I_i = 3 - 1 = 2$ and $m = l = 2$.

3.6. Neutron Decay Theory

The quark process of the neutron decay is shown in Figure 10.

The analytical formula from the Feynman diagram is [8]

$$\Gamma\left(n \to pe\nu_e\right) = \left(G_V^2 + 3G_A^2\right)\frac{m_e^5}{2\pi^3} f_R = G_F^2 V_{ud}^2 \left(1 + 3\lambda^2\right)\frac{m_e^5}{2\pi^3} f_R \tag{15}$$

$G_V = G_F V_{ud}$, $G_A = G_F V_{ud}\lambda$, $\lambda = 1.255$, $V_{ud} = 0.974$, $G_F = 1.166 \times 10^{-5}\,\text{GeV}^{-1}$

with the phase-space term [9]

$$f_R = \frac{1}{60}\left(2\xi^4 - 9\xi^2 - 8\right)\sqrt{\xi^2 - 1} + \frac{1}{4}\xi \log\left(\xi + \sqrt{\xi^2 - 1}\right) \quad \text{with} \quad \xi \equiv \frac{m_n - m_p}{m_e} = 2.53,$$

so $f_R = 1.6332$

and the phenomenological formula for decay width is

$$\Gamma\left(n \to pe\nu_e\right) = G^2 m_i^5 \left(1 - \frac{m_f^2}{m_i^2}\right)^4 \text{ [1]}, \quad l = 4 \tag{15a}$$

with initial mass $m_i = m_n$ and final mass $m_f = m_p + m_e$ and obtain with the same ansatz as for $\Gamma\left(\mu \to e\nu_e\nu_\mu\right)$:

$$G^2 = \frac{1}{192\pi^3\, 32 M_X^4}$$

The neutron decay involves in fact only 2 quarks

$\Gamma\left(n \to pe\nu_e\right) = \Gamma\left(dd \to ud\, e\nu_e\right)$

so the isospin numbers are $l = \Delta I = I_f + I_i = 3 + 1 = 4$ and $m = l = 4$ with

$I_f = I(ud) + I(e) + I(\nu) = 1 + 1 + 1 = 3$ and $I_i = I(dd) = 1$.

3.7. Theory of 3-Body Eta-Pion Decay

The eta-pion decay is shown in Figure 11.

The generalized and isospin-adapted 3-body semi-leptonic formula (from the muon) decay is

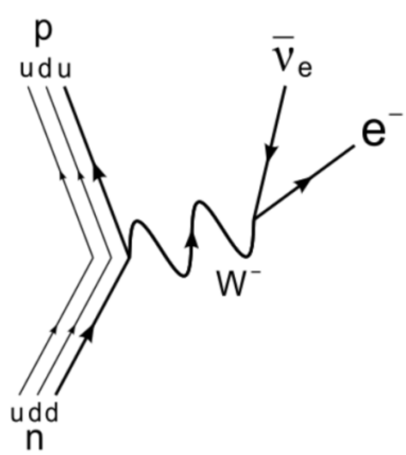


Figure 10. Neutron decay.

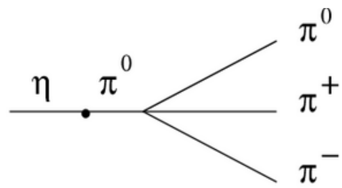


Figure 11. Eta-pion decay.

$$\Gamma\left(\eta \to \pi^0 \pi^+ \pi^-\right) = \frac{g^4 m_i}{32 \times 192\pi^3}\left(\frac{m_i^4}{m_X^4}\right)\left(1 - \frac{m_f^2}{m_i^2}\right)^4 \tag{16}$$

and the phenomenological formula [1]

$$\Gamma\left(\eta \to \pi^0 \pi^+ \pi^-\right) = \frac{G^2}{4\pi} m_i \left(1 - \frac{m_f^2}{m_i^2}\right)^4 \tag{16a}$$

From this setting $g = 1$ we obtain for m_x:

$$\frac{m_i^4}{m_X^4} = 8 \times 192\pi^2 G^2$$

The decay is mainly hadronic, but the kinematics is one of a 3-body decay, so we can use the generalized 3-body semi-leptonic formula from above.

The intermediate boson here is π^0, so $m_X \propto m\left(\pi^0\right)$ and the isospin numbers are $l = \Delta I = I_f + I_i = 3 + 1 = 4$ and $m = l = 4$.

3.8. Theory of 2-Photon Meson Decay

The formula for the radiative 2-photon meson decay is:

$$\Gamma\left(\pi^0 \to \gamma\gamma\right) = \frac{\alpha^2 m_i^3}{64\pi^3 m_X^2} \tag{17}$$

where $m_i = m\left(\pi^0\right)$ and $m_X = F_\pi$, $F_\pi = \frac{2m_u}{\sqrt{Z_\pi}}$ pseudoscalar weak decay constant [20].

the phenomenological formula is

$$\Gamma\left(\pi^0 \to \gamma\gamma\right) = \frac{G^2 m_i^3}{4\pi}\left(1 - \frac{m_f^2}{m_i^2}\right)^3 \tag{17a}$$

From this we obtain for m_x: $m_X = \frac{4\pi\alpha}{G}$.

The intermediate boson here is the strongly excited π^0, so $m_X \propto 10m\left(\pi^0\right)$ and the isospin numbers are $l = \Delta I = I_f + I_i = 2 + 1 = 3$ and $m = l = 3$.

3.9. Theory of 1-Photon Hyperon Decay

The photon hyperon decay is shown in Figure 12.

The interaction becomes for the transition $s \to d\gamma$

$H_{sd\gamma} = \bar{d}\sigma_{\mu\nu}\left(a + b\gamma_5\right) s q^\mu A^\nu \alpha$, where A^ν, q^μ are the photon and its momentum.

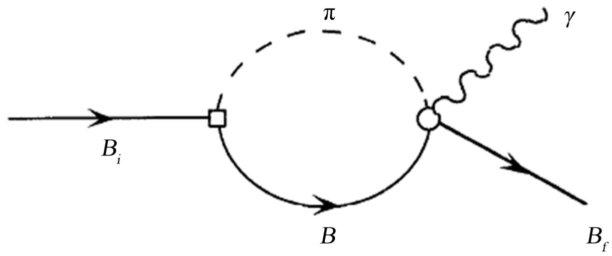


Figure 12. Photon hyperon decay.

For the analytical formula we can use the extended isospin-adapted expression from the pion decay (here $\Gamma\left(\Sigma^+ \to p\gamma\right)$)

$$\frac{G_F^2 m_\pi^5}{8\pi}\frac{m_\mu^2}{m_\pi^2}\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2$$

$$\Gamma\left(\Sigma^+ \to p\gamma\right) = f_\pi \frac{G_F^2 \alpha^2 m_i^5}{8\pi}\left(1-\frac{m_f^2}{m_i^2}\right)^2 \tag{18}$$

where $m_i = m\left(\Sigma^+\right)$ and $m_f = m(p)$, f_π is the hadronic correction factor the phenomenological formula is

$$\Gamma\left(\Sigma^+ \to p\gamma\right) = \frac{G^2 m_i^5}{4\pi}\left(1-\frac{m_f^2}{m_i^2}\right)^2 \tag{18a}$$

From this we obtain for m_x: $m_X^4 = \frac{\alpha^2}{64G^2}$ or $m_X = \sqrt{\frac{\alpha}{8G}}$

and the isospin numbers are $l = \Delta I = I_f + I_i = (1/2+1) + 1/2 = 2$ and $m = l = 2$.

3.10. The Generalized Weak Decay Formula

We have seen in 2.3 for the muon decay that the decay interaction has the Feynman-Gell-Mann form

$$H_{weak} = \frac{G_F}{\sqrt{2}}\bar{J}^\mu J_\mu \quad \text{where} \quad G_F/\sqrt{2} = g^2/\left(8M_W^2\right)$$

or in generalized form (in natural units)

$$H_{\text{int}} = \frac{g^2}{8m_X^2}\bar{J}_1^\mu \left(J_2\right)_\mu \tag{19}$$

where g is the (dimensionless) interaction constant, m_X is the interaction energy (excitation energy of the intermediate boson), J_1 and J_2 are the currents involved, e.g. the lepton current $\left(J_2\right)_\mu = \bar{e}(x)\gamma_\mu\left(1-\gamma_5\right)\nu_e(x)$.

The current has dimension *length*$^{-3}$, so the formula in cgs units reads

$H_{\text{int}} = (\hbar c)^3 \frac{g^2}{8m_X^2}\bar{J}_1^\mu\left(J_2\right)_\mu$, so H_{int} has dimension *energy/length*^{3}, *i.e.* energy density, as it should be.

The decay width (energy) becomes then

$$\Gamma = \int H_{\text{int}}(x)\,\mathrm{d}^3x \tag{19a}$$

4. Particle Data

In the following Table 5 we present the data for the particles involved in the decays [6] [19] [21].

Table 5. Particle data.

name	Mass [GeV]	e-charge	color-charge	chirality	spin	isospin
e	0.000511	−1	0	0	1/2	1
nue	3×10^{-13}	0	0	1	1/2	1
u	0.0023	2/3	3	0	1/2	1/2
d	0.0048	−1/3	3	0	1/2	1/2
mu	0.106	−1	0	0	1/2	1
numu	1.1×10^{-11}	0	0	1	1/2	1
c	1.34	2/3	3	0	1/2	1/2
s	0.106	−1/3	3	0	1/2	1/2
tau	1.78	−1	0	0	1/2	1
nutau	9.8×10^{-11}	0	0	1	1/2	1
t	171	2/3	3	0	1/2	1/2
b	4.2	−(1/3)	3	0	1/2	1/2
W^-	80.4	−1	0	1	1	1
Z	91.2	0	0	0	1	1
gamma	0	0	0	0	1	1
g	0	0	8	0	1	0
H	125.1	0	0	0	0	0
p	0.93827	1	3	0	1/2	1/2
n	0.93956	0	3	0	1/2	1/2
Lambda	1.1157	0	3	0	1/2	1/2
Sigma+	1.1894	1	3	0	1/2	1
Sigma0	1.1926	0	3	0	1/2	1
Sigma−	1.19745	−1	3	0	1/2	1
Xi0	1.31486	0	3	0	1/2	1/2
Xi−	1.3217	−1	3	0	1/2	1/2
rho+	0.7751	1	3	0	1	1
rho0	0.77526	0	3	0	1	1
omega	0.78265	0	3	0	1	0
phi	1.01946	0	3	0	1	0
K*+	0.89166	1	3	0	1	1
K*0	0.89581	0	3	0	1	1
pi+	0.13957	1	3	0	0	1
pi0	0.134977	0	3	0	0	1
eta	0.54786	0	3	0	0	1
eta'	0.95778	0	3	0	0	1
K+	0.49368	1	3	0	0	1
K0	0.49761	0	3	0	0	1
KS0	0.49761	0	3	0	0	1
KL0	0.49761	0	3	0	0	1

5. Decay Width and Interaction Energy for Different Types of Decays

In this chapter, we compare the observed decay bandwidths with the ones calculated from the semi-empirical formula. As we shall see, there is in general a satisfactory agreement between the observed and the calculated values [1] [22].

Here, m_X is calculated according to the formula in 2.1 from the observed decay width Γ_{obs}

$$m_X = \frac{m_i}{2\sqrt{2}}\left(\frac{I_\Gamma m_i}{\Gamma_{obs}}\right)^{1/4} \tag{20}$$

5.1. Strange Hyperon Decays with Pions

Here we have [23]

$$\Gamma = C\left|\psi_{1,1}\right|^2 = \frac{G^2}{4\pi} m_i^{-1}\left(1-x^2\right),\ \left|\Delta S\right| = 1,\ l = \Delta I = 1,\ m = 1,\ x = \frac{m_f}{m_i},\ k = -1$$

with $g_{h1} = 5.81\times10^{-7}$ GeV

$\Lambda \to p\pi^-$, $\Lambda \to n\pi^0$, $G = g_{h1}$ resp. $G = g_{h1}/\sqrt{2}$

$\Sigma^+ \to p\pi^0$, $\Sigma^+ \to n\pi^+$, $\Sigma^- \to n\pi^-$, $G = g_{h1}$

$\Xi^0 \to \Lambda\pi^0$, $\Xi^- \to \Lambda\pi^-$, $G = g_{h1}$ resp. $G = g_{h1}/\sqrt{2}$

The data for the strange hyperon decays with pions are shown in Table 6.

The decays can be roughly ordered according to the interaction energy.

Lambda into nucleon pion $m_X \approx 400$ GeV.

Sigma into nucleon pion $m_X \approx 400$ GeV.

5.2. Two-Body Non-Strange Decays of Mesons

$$\Gamma = C\left|\psi_{3,2}\right|^2 = \frac{G^2}{4\pi} m_i x^2\left(1-x^2\right),\ \Delta S = 0,\ l = \Delta I = 3,\ m = 2,\ x = \frac{m_f}{m_i},\ k = 1$$

$\pi^\pm \to l\nu$, $G = g_0$

$K^\pm \to l\nu$, $G = g_1$

$K_S^0 \to \pi^+\pi^-$, $G = 2g_1\alpha 443$, $K_S^0 \to \pi^0\pi^0$, $G = 2g_1\alpha 443/\sqrt{2}$

$K^\pm \to \pi^\pm\pi^0$, $G = 2g_1\alpha\sqrt{443}$

$K_L^0 \to \pi^+\pi^-$, $G = 2g_1\alpha$, $K_L^0 \to \pi^0\pi^0$, $G = 2g_1\alpha/\sqrt{2}$

Table 6. Strange hyperon decays with pions.

decay	Γcalc [GeV]	Γobs [GeV]	rel. Δwidth	I_i	I_f	$l=\Delta I$	G [GeV]	x	m_X	G formula
$\Lambda \to p\pi^-$	1.61032×10^{-15}	1.599×10^{-15}	0.00813008	1/2	3/2	1	6.2×10^{-7}	0.966066	352.232	$G = g_{h1}$
$\Lambda \to n\pi^0$	8.74088 × 10−16	8.96 × 10−16	0.0145089	1/2	3/2	1	4.38×10^{-7}	0.963106	423.206	$G = g_{h1}/\sqrt{2}$
$\Sigma^+ \to p\pi^0$	4.20623×10^{-15}	4.233×10^{-15}	0.00590598	1/2	3/2	1	6.2×10^{-7}	0.902343	426.379	$G = g_{h1}$
$\Sigma^+ \to n\pi^+$	4.00357×10^{-15}	3.966×10^{-15}	0.00630358	1/2	3/2	1	6.2×10^{-7}	0.907289	424.288	$G = g_{h1}$
$\Sigma^- \to n\pi^-$	4.22472×10^{-15}	4.444×10^{-15}	0.00562556	1/2	3/2	1	6.2×10^{-7}	0.90119	430.76	$G = g_{h1}$
$\Xi^0 \to \Lambda\pi^0$	1.95069×10^{-15}	2.259×10^{-15}	0.0110668	1/2	3/2	1	6.2×10^{-7}	0.951186	471.39	$G = g_{h1}$
$\Xi^- \to \Lambda\pi^-$	3.99332×10^{-15}	4.011×10^{-15}	0.00623286	1/2	3/2	1	4.38×10^{-7}	0.949739	399.863	$G = g_{h1}/\sqrt{2}$

The data for the non-strange two-body meson decays are shown in Table 7.

The decays can be roughly ordered according to the interaction energy

Pion-lepton $m_X \approx 100\ \text{GeV}$,

Kaon-lepton: $m_X \approx 400\ \text{GeV}$,

Kaon-pion: $m_X \approx 600\ \text{GeV}$,

Short-lived K_{s0}-pion: $m_X \approx 150\ \text{GeV}$,

Long-lived K_{L0}-pion: $m_X \approx 3200\ \text{GeV}$.

5.3. Three-Four-Body Decays of Strange Mesons

$$\Gamma = C\left|\psi_{l,m}\right|^2 = \frac{G^2}{4\pi} m_i \left(1-x^2\right)^l, \quad |\Delta S| = 1, \quad l = \Delta I = 2, \quad m = 2, \quad x = \frac{m_f}{m_i}, \quad k = 1$$

except $K^\pm \to \pi^\pm \pi^+ \pi^- \gamma$, where $\Delta I = 1$, $m = 1$

$K^\pm \to \pi^0 l\nu$, $G = g_1\sqrt{\frac{\alpha}{2}}$

$K_L^0 \to \pi^\pm l\nu$, $G = g_1\sqrt{\alpha}$

$K^\pm \to \pi^\pm \pi^+ \pi^-$, $G = 2.53 g_1\sqrt{\alpha}$

$K^\pm \to \pi^\pm \pi^0 \pi^0$, $G = 2.53 g_1\sqrt{\frac{\alpha}{2}}$

$K_L^0 \to \pi^0 \pi^0 \pi^0$, $G = 2 g_1\sqrt{\alpha}$

$K_L^0 \to \pi^+ \pi^- \pi^0$, $G = 2 g_1\sqrt{\alpha}\big/\sqrt{3/2}$

$K_L^0 \to \pi^0 \pi^+ e^- \nu_e$, $G = g_1\frac{\alpha}{\pi}$

$K^\pm \to \pi^\pm \pi^\mp l^\pm \nu$, $K^\pm \to \pi^0 \pi^0 l^\pm \nu$, $G = g_1\frac{\alpha}{\pi}$

$K^\pm \to \pi^0 \pi^\pm \gamma$, $G = g_1\left(\frac{\alpha}{\sqrt{2}}\right)$

$K^\pm \to \pi^\pm \pi^+ \pi^- \gamma$, $G = g_1 \alpha\sqrt{2}$

The data for the three-four-body decays of strange mesons are shown in Table 8.

The decays can be roughly ordered according to the interaction energy

Table 7. Two-body non-strange decays of mesons.

decay	width_calc[GeV]	width_obs[GeV]	rel. Δwidth	I_i	I_f	$l = \Delta I$	G	x	m_X	G formula
$\pi^+ \to \mu^+ \nu_\mu$	2.50122×10^{-17}	2.528×10^{-17}	0.000791139	1	2	3	1.47648×10^{-7}	0.759476	96.3675	$G = g_0$
$\pi^+ \to e^+ \nu_e$	3.24552×10^{-21}	3.11×10^{-21}	0.0186495	1	2	3	1.47648×10^{-7}	0.00366125	107.988	$G = g_0$
$K^+ \to \mu^+ \nu_\mu$	3.3949×10^{-17}	3.372×10^{-17}	0.00266904	1	2	3	1.43527×10^{-7}	0.214714	383.074	$G = g_1$
$K^+ \to e^+ \nu_e$	8.67067×10^{-22}	8.238×10^{-22}	0.0581452	1	2	3	1.43527×10^{-7}	0.00103508	387.415	$G = g_1$
$K_S^0 \to \pi^+ \pi^-$	5.0423×10^{-15}	5.084×10^{-15}	0.00354052	1	2	3	9.28211×10^{-7}	0.560961	146.47	$G = 2 g_1 \alpha 443$
$K_S^0 \to \pi^0 \pi^0$ 0	2.5002×10^{-15}	2.255×10^{-15}	0.00798226	1	2	3	6.56344×10^{-7}	0.542501	174.823	$G = 2 g_1 \alpha 443\big/\sqrt{2}$
$K^+ \to \pi^+ \pi^0$	1.12741×10^{-17}	1.112×10^{-17}	0.00719424	1	2	3	4.41006×10^{-8}	0.556123	667.317	$G = 2 g_1 \alpha\sqrt{443}$
$K_L^0 \to \pi^+ \pi^-$	2.56934×10^{-20}	2.543×10^{-20}	0.0247739	1	2	3	2.09528×10^{-9}	0.560961	3082.85	$G = 2 g_1 \alpha$
$K_L^0 \to \pi^0 \pi^0$	1.27399×10^{-20}	1.119×10^{-20}	0.204647	1	2	3	1.48159×10^{-9}	0.542501	3679.59	$G = 2 g_1 \alpha\big/\sqrt{2}$

Table 8. Three-four-body decays of strange mesons.

decay	width_calc [GeV]	width_obs [GeV]	rel. Δwidth	I_i	I_f	$l=\Delta I$	G	x	m_X	G formula
$K^+ \to \pi^0 e^+ \nu_e$	2.52543×10^{-18}	2.565×10^{-18}	0.0105263	1	3	2	8.67078×10^{-9}	0.274445	2168.89	$G = g_1\sqrt{\alpha}/\sqrt{2}$
$K^+ \to \pi^0 \mu^+ \nu_\mu$	1.7138×10^{-18}	1.764×10^{-18}	0.0272109	1	3	2	8.67078×10^{-9}	0.488124	1899.19	$G = g_1\sqrt{\alpha}/\sqrt{2}$
$K_L^0 \to \pi^+ e^- \bar{\nu}_e$	5.04792×10^{-18}	5.217×10^{-18}	0.0120759	1	3	2	1.22623×10^{-8}	0.281508	1830.36	$G = g_1\sqrt{\alpha}$
$K_L^0 \to \pi^+ \mu^- \bar{\nu}_\mu$	3.40719×10^{-18}	3.478×10^{-18}	0.00920069	1	3	2	1.22623×10^{-8}	0.493499	1602.52	$G = g_1\sqrt{\alpha}$
$K^+ \to \pi^+ \pi^+ \pi^-$	2.97836×10^{-18}	2.971×10^{-18}	0.00538539	1	3	2	3.10237×10^{-8}	0.84814	127.367	$G = 2.53 g_1\sqrt{\alpha}$
$K^+ \to \pi^+ \pi^0 \pi^0$	9.19439×10^{-19}	9.34×10^{-19}	0.0289079	1	3	2	1.55119×10^{-8}	0.829533	191.076	$G = 2.53 g_1\sqrt{\alpha}/2$
$K_L^0 \to \pi^0 \pi^0 \pi^0$	2.71785×10^{-18}	2.518×10^{-18}	0.0353455	1	3	2	2.45247×10^{-8}	0.813752	159.959	$G = 2 g_1\sqrt{\alpha}$
$K_L^0 \to \pi^+ \pi^- \pi^0$	1.50061×10^{-18}	1.617×10^{-18}	0.0185529	1	3	2	2.00243×10^{-8}	0.832212	167.813	$G = 2 g_1\sqrt{\alpha}/\sqrt{3/2}$
$K^+ \to \pi^+ \pi^- e^+ \nu_e$	2.01491×10^{-21}	2.174×10^{-21}	0.0367985	1	3	2	3.33475×10^{-10}	0.566462	8453.24	$G = g_1\alpha/\pi$
$K^+ \to \pi^+ \pi^- \mu^+ \nu_\mu$	6.69205×10^{-22}	7.44×10^{-22}	0.642473	1	3	2	3.33475×10^{-10}	0.780141	6435.05	$G = g_1\alpha/\pi$
$K^+ \to \pi^0 \pi^0 e^+ \nu_e$	1.06991×10^{-21}	1.169×10^{-21}	0.182207	1	3	2	2.35802×10^{-10}	0.547855	10293.3	$G = g_1\alpha/(\pi\sqrt{2})$
$K^+ \to \pi^0 \pi^0 \mu^+ \nu_\mu$	3.8545×10^{-22}	4.2×10^{-22}	0.5	1	3	2	2.35802×10^{-10}	0.761534	7981.24	$G = g_1\alpha/(\pi\sqrt{2})$
$K_L^0 \to \pi^0 \pi^+ e^- \nu_e$	2.12372×10^{-21}	2.764×10^{-21}	0.0209841	1	3	2	3.33475×10^{-10}	0.552758	8671.14	$G = g_1\alpha/\pi$
$K_L^0 \to \pi^0 \pi^+ \mu^- \nu_\mu$	7.58984×10^{-22}	$8. \times 10^{-22}$	0.0725	1	3	2	3.33475×10^{-10}	0.76475	6715.43	$G = g_1\alpha/\pi$
$K^+ \to \pi^0 \pi^+ \gamma$	1.37146×10^{-20}	1.462×10^{-20}	0.0581395	1	1	2	8.55396×10^{-10}	0.556123	5707.21	$G = g_1\alpha/\sqrt{3/2}$
$K^\pm \to \pi^\pm \pi^+ \pi^- \gamma$	6.05074×10^{-21}	5.53×10^{-21}	0.385172	1	0	1	7.40795×10^{-10}	0.84814	3574.06	$G = g_1\alpha\sqrt{2}$

K^+, K_L^0 into pi 2lepton $m_X \approx 1.8$ TeV
K^+, K_L^0 into 3 pi $m_X \approx 150$ GeV
K^+, K_L^0 into 2 pi 2lepton $m_X \approx 7...10$ TeV
K^+ into 2pi photon $m_X \approx 5.7$ TeV
K^+ into 3 pion photon $m_X \approx 3.5$ TeV

5.4. Three-Body Decays of Strange Hyperons

$$\Gamma = C\left|\psi_{l,m}\right|^2 = \frac{G^2}{4\pi} m_i^4 \left(1-x^2\right)^l, \quad |\Delta S| = 1, \quad l = \Delta I = 2, \quad m = 2, \quad x = \frac{m_f}{m_i}, \quad k = 4$$

with $g_{h2} = 1.4 \times 10^{-8}\ \text{GeV}^{-3/2}$

$\Lambda \to p l \nu$, $G = g_{h2}\sqrt{3}$

$\Sigma^- \to n l \nu$, $G = g_{h2}\, 2$

The data for the three-body decays of strange hyperons are shown in Table 9.
The decays can be roughly ordered according to the interaction energy
Λ into pi 2lepton $m_X \approx 1.2$ TeV
Σ into pi 2lepton $m_X \approx 1.7$ TeV

5.5. Non-Strange Leptonic Three-Body Decays

$A' \to A e \nu \left(\Delta S = 0\right)$

$$\Gamma = C\left|\psi_{4,4}\right|^2 = G^2 m_i^5 \left(1-x^2\right)^4, \quad l = \Delta I = 4, \quad m = 4, \quad k = 5$$

$$\pi^\pm \to \pi^0 e^\pm \nu\,,\quad G = G_0 \Big/ \sqrt{192\times 50\pi^3}$$
$$n \to p e^- \bar{\nu}_e\,,\quad G = G_0 \Big/ \sqrt{192\times 175\pi^3}$$
$$\Sigma^\pm \to \Lambda e^\pm \nu\,,\quad G = G_0 \Big/ \sqrt{192\times 65\pi^3}$$
$$\mu^- \to e^- \bar{\nu}_e \nu_\mu\,,\quad \tau^- \to e^- \bar{\nu}_e \nu_\tau\,,\quad \tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau\,,\quad G = G_0 \Big/ \sqrt{192\pi^3}$$

The data for non-strange leptonic three-body decays are shown in Table 10.

Here pure-leptonic transitions are bi-quark transitions

$n \to p e \bar{\nu}$ becomes $du \to uu\, e\bar{\nu}$

$\Sigma^+ \to \Lambda \bar{e} \nu$ becomes $uu \to ud\, \bar{e}\nu$

The decays can be roughly ordered according to the interaction energy

lepton into lepton 2 neutrino $m_X \approx 700$ GeV

pi into pi 2 lepton $m_X \approx 300$ GeV

neutron decay n ->p e νe $m_X \approx 100$ GeV

Σ into Λ 2lepton $m_X \approx 800$ GeV

5.6. Three-Body Decays Eta-Pions

The decay width is [24]

$$\Gamma = C\left|\psi_{4,4}\right|^2 = \frac{G^2 m_i}{4\pi}\left(1-x^2\right)^4,\quad l = \Delta I = 4\,,\quad m = 4\,,\quad k = 1$$
$$\eta \to \pi^0\pi^0\pi^0\,,\quad \eta \to \pi^+\pi^-\pi^0\,,\quad G = 0.0145$$
$$\eta \to \pi^+\pi^-\gamma\,,\quad G = 0.00213$$

The data for three-body eta-pion decays are shown in Table 11.

The decays can be roughly ordered according to the interaction energy

Table 9. Three-body decays of strange hyperons.

decay	width_calc [GeV]	width_obs [GeV]	rel. Δwidth	I_i	I_f	$l = \Delta I$	G[GeV$^{-3/2}$]	x	m_X	G formula
$\Lambda \to p e^- \bar{\nu}_e$	2.21509×10^{-18}	2.081×10^{-18}	0.0168188	1/2	5/2	2	2.42×10^{-8}	0.841428	1504.53	$G = g_{h2}\sqrt{3}$
$\Lambda \to p \mu^- \bar{\nu}_\mu$	3.99111×10^{-19}	3.93×10^{-19}	0.223919	1/2	5/2	2	2.42×10^{-8}	0.935977	1037.67	$G = g_{h2}\sqrt{3}$
$\Sigma^- \to n e^- \bar{\nu}_e$	4.42673×10^{-18}	4.526×10^{-18}	0.0393283	1/2	5/2	2	2.8×10^{-8}	0.785061	1879.71	$G = g_{h2}\, 2$
$\Sigma^- \to n \mu^- \bar{\nu}_\mu$	1.69761×10^{-18}	2.003×10^{-18}	0.0888667	1/2	5/2	2	2.8×10^{-8}	0.873155	1546.45	$G = g_{h2}\, 2$

Table 10. Non-strange leptonic three-body decays.

decay	width_calc [GeV]	width_obs [GeV]	rel. Δwidth	I_i	I_f	$l = \Delta I$	G	x	m_X	G formula
$\mu^- \to e^- \bar{\nu}_e \nu_\mu$	3.01736×10^{-19}	2.954×10^{-19}	0.014218	1	3	4	1.50165×10^{-7}	0.00482075	717.027	$G = G_0 / \sqrt{192\pi^3}$
$\tau^- \to e^- \bar{\nu}_e \nu_\tau$	4.02938×10^{-13}	4.041×10^{-13}	0.00296956	1	3	4	1.50165×10^{-7}	0.000287079	717.105	$G = G_0 / \sqrt{192\pi^3}$
$\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$	3.97253×10^{-13}	3.932×10^{-13}	0.00305188	1	3	4	1.50165×10^{-7}	0.0595506	695.878	$G = G_0 / \sqrt{192\pi^3}$
$\pi^+ \to \pi^0 e^+ \nu_e$	2.63622×10^{-25}	2.619×10^{-25}	0.067583	1	3	4	2.12366×10^{-8}	0.970753	468.38	$G = G_0 / \sqrt{192\times 50\pi^3}$
$n \to p e^- \bar{\nu}_e$	7.12154×10^{-28}	7.239×10^{-28}	0.000897914	1	3	4	1.13514×10^{-8}	0.999171	204.69	$G = G_0 / \sqrt{192\times 175\pi^3}$
$\Sigma^+ \to \Lambda e^+ \nu_e$	1.67174×10^{-19}	1.642×10^{-19}	0.249695	1	3	4	1.86257×10^{-8}	0.938466	703.24	$G = G_0 / \sqrt{192\times 65\pi^3}$
$\Sigma^- \to \Lambda e^- \bar{\nu}_e$	2.5218×10^{-19}	2.55×10^{-19}	0.0470588	1	3	4	1.86257×10^{-8}	0.932157	740.33	$G = G_0 / \sqrt{192\times 65\pi^3}$

Table 11. Three-body decays eta-pions.

decay	width_calc [GeV]	width_obs [GeV]	rel. Δwidth	I_i	I_f	$l=\Delta I$	G	x	m_X	G formula
$\eta\to\pi^0\pi^0\pi^0$	3.88428×10^{-7}	4.226×10^{-7}	0.00851869	1	3	4	0.0145	0.739114	0.26643	$G=0.0145$
$\eta\to\pi^+\pi^-\pi^0$ 0	3.09444×10^{-7}	2.951×10^{-7}	0.0176211	1	3	4	0.0145	0.755881	0.25729	$G=0.0145$
$\eta\to\pi^+\pi^-\gamma$	5.94407×10^{-8}	6.097×10^{-8}	0.021978	1	3	4	0.00213	0.50951	7.4388	$G=0.00213$

eta into 3 pion $m_X\approx0.3$ GeV

eta into 2 pion photon $m_X\approx1$ GeV

5.7. Photon-Radiative Decays

The decay width is [24] [25] [26]

$$\Gamma=C\left|\psi_{3,3}\right|^2=\frac{G^2m_i^3}{4\pi}\left(1-x^2\right)^3,\quad l=\Delta I=3,\quad m=3,\quad k=3$$

with $\alpha=\frac{e^2}{4\pi}$, $g_{ph}'=0.138$

pseudoscalar mesons $P\to\gamma\gamma$, $P=\pi^0,\eta,\eta'$ theory $\Gamma\left(P\to\gamma\gamma\right)=\frac{e^4g_{ph}^2}{64\pi}m_P^3$,

$x=0$, $G=2\pi\alpha C_1g_{ph}'$, $C_1=1,\sqrt{5/4},\sqrt{5/3}$

$$\Gamma=C\left|\psi_{2,2}\right|^2=\frac{G^2m_i^5}{4\pi}\left(1-x^2\right)^2,\quad l=\Delta I=2,\quad m=2$$

$$g_{ph}=9.769\times10^{-9},\quad G=C_1g_{ph}$$

hyperons $\Lambda\to n\gamma$, $\Sigma^+\to p\gamma$, $\Xi^0\to\Lambda\gamma$, $\Xi^0\to\Sigma^0\gamma$, $\Xi^-\to\Sigma^-\gamma$,
$C_1=\sqrt{7/2},2,1,\sqrt{8},1/\sqrt{2}$

The data for photon-radiative decays are shown in Table 12.

The decays can be roughly ordered according to the interaction energy

pi, eta into 2 photon $m_X\approx20$ GeV

Λ, Σ into nucleon photon $m_X\approx130$ GeV

Xi into Λ photon $m_X\approx180$ GeV

Xi into Σ photon $m_X\approx100,\ldots,200$ GeV

6. Characterization and Calculation of Different Types of Decays Based on Interaction Energy

6.1. Table of Decays Based on Interaction Energy

In the following Table 13, are shown the collected decay data from chap.5.

In the above table, the decays are grouped according to type and interaction energy m_X.

Consider the general decay

$$P_{in}\to P_1+P_2+P_3+P_4$$

In the table above, the column P_{in} contains the structure of the original particle, the column P_i contains the structures of the outgoing particles, separated by

Table 12. Photon-radiative decays.

decay	width_calc [GeV]	width_obs [GeV]	rel. Δwidth	I_i	I_f	$I = \Delta I$	G	x	m_X	G formula
$\pi^0 \to \gamma\gamma$	7.86×10^{-9}	7.84×10^{-9}	0.0687023	1	2	3	0.00632905	0.	23.8527	$G = 2\pi\alpha g_{ph}'$
$\eta \to \gamma\gamma$	6.4×10^{-7}	6.55×10^{-7}	0.21875	1	2	3	0.00707609	0.	21.334	$G = 2\pi\alpha g_{ph}'\sqrt{5/4}$
$\eta' \to \gamma\gamma$	4.57×10^{-6}	4.67×10^{-6}	0.0547046	1	2	3	0.0081707	0.	18.476	$G = 2\pi\alpha g_{ph}'\sqrt{5/3}$
$\Lambda \to n\gamma$	3.88647×10^{-18}	3.96×10^{-18}	0.0368098	1/2	3/2	2	1.82761×10^{-8}	0.842126	164.32	$G = g_{ph}\sqrt{7/2}$
$\Sigma^+ \to p\gamma$	1.03154×10^{-17}	9.81×10^{-18}	0.0487805	1/2	3/2	2	1.9538×10^{-8}	0.78886	161.01	$G = 2g_{ph}$
$\Xi^0 \to \Lambda\gamma$	2.33984×10^{-18}	2.34×10^{-18}	0.150943	1/2	3/2	2	9.769×10^{-9}	0.848531	224.40	$G = g_{ph}$
$\Xi^0 \to \Sigma^0\gamma$	7.50752×10^{-18}	7.87×10^{-18}	0.120787	1/2	3/2	2	2.76309×10^{-8}	0.907017	131.51	$G = g_{ph}\sqrt{8}$
$\Xi^- \to \Sigma^-\gamma$	4.91693×10^{-19}	4.99×10^{-19}	0.179688	1/2	3/2	2	6.90773×10^{-9}	0.905992	263.09	$G = g_{ph}/\sqrt{2}$

slash, the rows $m_1, \cdots, m_4$ and m_X contain the respective mass.

The configuration is described either by quarks (like Λ = uds) or by *l* (lepton) or by Z, W.

The scheme in the last column describes the QHCD/QCD model of the interaction energy with number of active hc-bosons, e.g. sd’(2h) → Z → π^0(2h) for the decay Ξ → Λ *π*.

E.g. the generic decay Λ/Σ → n *π* has the incoming configuration P_{in} = uds and the outgoing generic configuration P_{12} = (n = udd)/(π^0 = (uu’-dd’)), with the interaction energy $m_X \approx$ 400 GeV, and the decay scheme sd’(2h) → Z → π^0, where the significant incoming current is $s\bar{d}$ interacting via 2 hc-bosons, the intermediate boson is the Z-boson, and the outgoing current is $\pi^0 = \left(u\bar{u} - d\bar{d}\right)/\sqrt{2}$. The number of active hc-bosons (or active gluons, in the pion-mediated decays) determines roughly the energy level.

Discussion of the results

The table reveals a simple principle for the scheme:

$q_1\bar{q}_2 \to b \to p$ or $\bar{q}_1 q_2 \to b \to p$, where q_1, q_2 are quarks in the incoming quark-current, b is the mediating boson $b = W, Z, \pi^0$, p are the outgoing particles, $p = \pi^0, \pi, W, \gamma$, where p can be represented as one or more quark-currents except for the photon γ, which is itself the electromagnetic current.

The resulting interaction energy m_X in the table above is not distributed uniformly, but accumulates around certain values, the energy classes.

$E_{h1} \approx 150$ GeV for 1 hc-boson

$E_{h2} \approx 400$ GeV for 2 hc-bosons

$E_{h4} \approx 700$ GeV for 4 hc-bosons

$E_{h6} \approx 1500$ GeV for 6 hc-bosons

$E_{h12} \approx 3500$ GeV for non-diagonal 12 hc-bosons outgoing W (1hcb)

$E_{h12,3h} \approx 5700$ GeV for non-diagonal 12 hc-bosons outgoing W (3hcb)

$E_{h15} \approx 7500$ GeV for all 15 hc-bosons outgoing W (3hcb)

$E_{h15,3h} \approx 9000$ GeV for all 15 hc-bosons outgoing W (6hcb)

$E_{c1} \approx 0.3$ GeV for 3 gluons (color interaction, factor 1000 weaker than hc-interaction)

Table 13. Decays based on interaction energy.

decay	P_{in}	P_i	m_1[GeV]	m_2[GeV]	m_3[GeV]	m_4[GeV]	m_{Xexp}[GeV]	scheme
Λ → n π	uds	udd/(uu'-dd')	1.1157	0.93956	0.134977	0.	387.719	sd'(2h) → Z → π0(2h)
Σ→ n π	uds	udd/(uu'-dd')	1.1894	0.93956	0.134977	0.	427.142	sd'(2h) → Z → π0(2h)
Ξ→Λ π	uss	uds/(uu'-dd')	1.31486	1.1157	0.134977	0.	435.627	sd'(2h) → Z → π0(2h)
π+ → l ν	ud'	2rL−	0.13957	0.106	1.1×10^{-11}	0.	102.178	ud'(1h) → W → W
K+ → l ν	us'	2rL−	0.49368	0.106	1.1×10^{-11}	0.	385.244	us'(2h) → W → W
K+ → π+ π0	us'	ud'/(uu'-dd')	0.49368	0.13957	0.134977	0.	667.317	sd'(4h) → Z → 2π0(3h)
KS0 → π π	(ds'+sd')	2(uu'-dd')	0.49761	0.13957	0.13957	0.	160.647	ds'(1h) → Z → 2π0
K+ → π+ π π	us'	ds'/2(uu'-dd')	0.49368	0.13957	0.134977	0.134977	159.222	us'(1h) → W → π+ 2π0
KL0 → π0 π π	ds'	(uu'-dd')/2(uu'-dd')	0.49761	0.134977	0.134977	0.134977	163.886	ds'(1h) → Z → π0 2π0
KL0 → π π	(ds'-sd')	2(uu'-dd')	0.49761	0.13957	0.13957	0.	3381.22	ds'(12h) → Z → 2π0(4h)
K+ → π0 l ν	us'	(uu'-dd')/W	0.49368	0.134977	0.106	1.1×10^{-11}	2034.04	us'(6h) → W → π0 W(6h)
KL0 → π+ l ν	ds'	ud'/W	0.49761	0.13957	0.106	1.1×10^{-11}	1716.44	ds'(6h) → Z → π+ W(2h)
K+ → π+ π− l ν	us'	2(uu'-dd')/W	0.49368	0.134977	0.134977	0.106	7444.14	us'(15h) → W → 2π0 W(6h)
K+ → π0 π0 l ν	us'	2(uu'-dd')/W	0.49368	0.134977	0.134977	0.106	9137.27	us'(15h) → W → 2π0 W(15h)
KL0 → π+ π0 l ν	us'	ud'/(uu'-dd')/W−	0.49761	0.13957	0.134977	0.106	7693.29	us'(15h) → W → 2π0 W(6h)
K+ → π+ π+ π− γ	us'	ud'/(ud'+u'd)	0.49368	0.13957	0.27914	0.	3574.06	us'(12h) → W → π+π-π+ γ(6h)
K+ → π0 π+ γ	us'	(uu'-dd')/ud'	0.49368	0.134977	0.13957	0.	5707.21	us'(12h) → W → π0π+γ(12h)
Λ → p *l* ν	uds	uud/W	1.1157	0.93827	0.106	1.1×10^{-11}	1271.1	su'(6h) → W → W(1h)
Σ- → n *l* ν	dds	udd/W	1.1197	0.93956	0.106	1.1×10^{-11}	1713.08	su'(6h) → W → W(2h)
μ/τ → e νe ν	*l*	*l*	1.78	0.000511	$3. \times 10^{-13}$	1.1×10^{-11}	717.06	*l* ν'(4h) → W → W
τ → μ νμ νμ	*l*	*l*	1.78	0.106	1.1×10^{-11}	9.8×10^{-11}	695.878	*l* ν'(4h) → W → W
π+ -> π0 *l* ν	ud'	(uu'-dd')/W	0.13957	0.134977	0.106	1.1×10^{-11}	468.38	d'u(2h) → W → W
n → p e νe	udd	uud/W	0.93956	0.93827	0.000511	$3. \times 10^{-13}$	204.69	du'(1h) → W → W
Σ+ → Λ *l* ν	uus	uds/W	1.1894	1.1157	0.106	1.1×10^{-11}	721.78	ud'(4h) → W → W
η → π0 π0 π0	(uu'+dd'-2ss')	3(uu'-dd')	0.54786	0.134977	0.134977	0.134977	0.26186	sd'(3g) → π0 → 3π0
η → π0 π0 γ	(uu'+dd'-2ss')	2(uu'-dd')	0.54786	0.134977	0.134977	0.	7.4388	sd'(6g) → π0 → 2π0 γ
π0/η → γ γ	(uu'-dd')		0.134977	0.	0.	0.	21.221	uu'(8g) → π0 → 2γ
Λ/Σ → n γ	uds	udd	1.1157	0.93956	0.	0.	162.669	sd'(1h) → Z → Z γ
Σ0 → Λ γ	uss	uds	1.31486	1.1157	0.	0.	224.404	sd'(2h) → Z → Z γ
Ξ0 → Σ0 γ	uss	uds	131486	11926	0.	0.	131.511	sd'(1h) → Z → Z γ
Ξ- → Σ- γ	dss	dds	1.3217	1.19745	0.	0.	263.089	sd'(2h) → Z → Z γ (2h)

$E_{c6} \approx 7\ \text{GeV}$ for 6 non-diagonal gluons; $E_{c8} \approx 20\ \text{GeV}$ for all 8 gluons

For weak decays the energy span in m_X is roughly: $\frac{E_{h15}}{E_{h1}} = \frac{9000\ \text{GeV}}{150\ \text{GeV}} = 60$, so the energy span scales like $\frac{E_{h15}}{E_{h1}} = 60 \approx (n_h)^{3/2}$

6.2. The Interaction Energy and the Decay Width

In 2.1 a general relationship between the interaction energy m_X and the decay width Γ was derived:

$$\frac{m_X}{m_i} = f_I \left(\frac{m_i}{\Gamma}\right)^{1/4} \tag{21}$$

where $f_I = \frac{I_\Gamma^{1/4}}{2\sqrt{2}}$

The following plot in Figure 13 depicts this relationship for all 54 decays of the quarks u, d, s and all leptons, dealt with in this chapter [2].

The x-axis is $x = \left(\frac{m_i}{\Gamma}\right)^{1/4}$, the y-axis is $y = \frac{m_X}{m_i}$, the labels consist of the first 3 characters of the name of the corresponding decay, followed by the number in the total decay table, e.g. pi0 ->γ γ has the number 47, and the label "pi047".

One sees immediately, that the decays separate in two large groups: those with $x > 1000$ are weak, *i.e.* hypercolor decays, those with $x < 60$ are strong (pure color) decays.

If there are 1 or 2 photons on the right side, then the electromagnetic Lagrangian component is used in the calculation in chap.7.

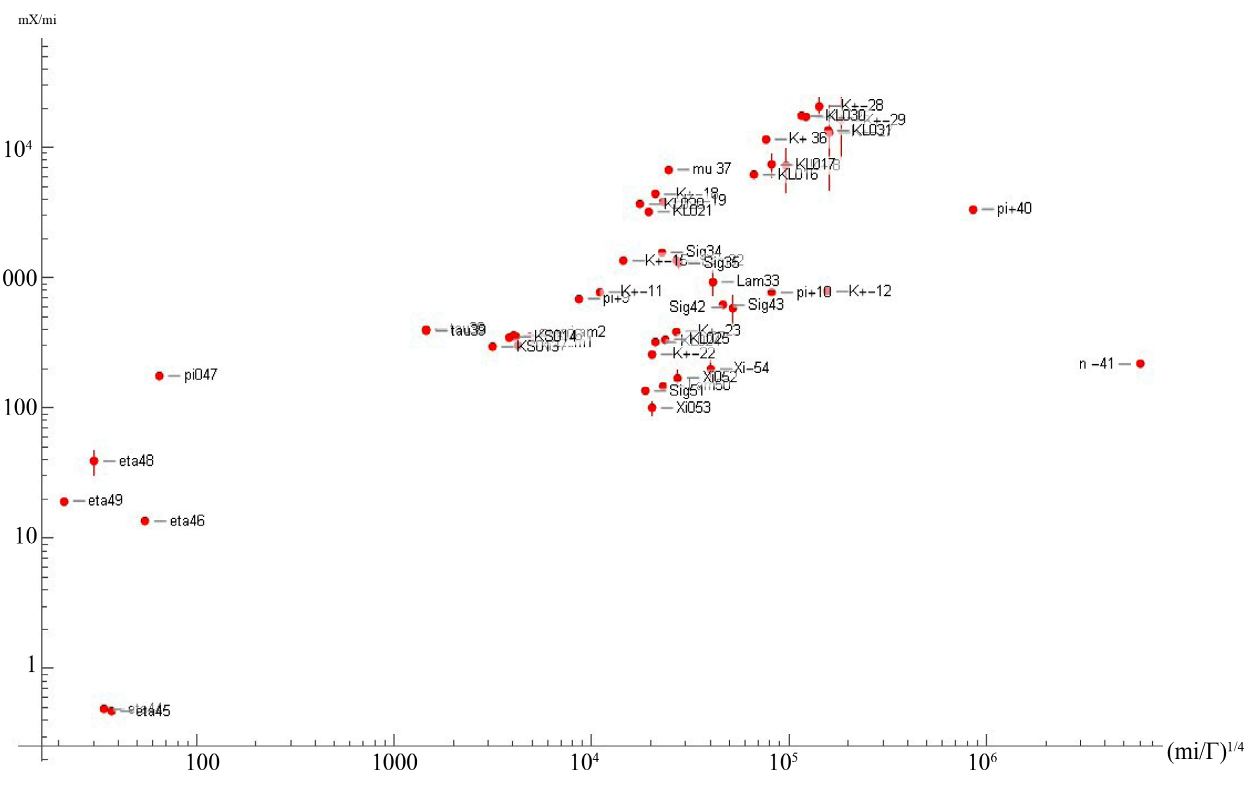


Figure 13. Interaction energy m_X in dependence of initial mass-energy m_i and decay width Γ.

In the pure-color decays only the color SU(3)-Lagrangian is used, in the weak decays both the SU(3) and the hypercolor SU(4)-Lagrangian is used.

7. Numerical Calculation: Method and Results

Schematics: calculated, observed interaction energy

phenomenological formula $\Gamma\left(\tilde{G},k,l,m,m_i,m_f\right)=\tilde{G}^2 m_i^{\,k}\left|P_l^m\left(x\right)\right|^2$
$k = -1,1,3,4,5$ interaction type, $l = I$ isospin, $m = I_3$,
$\left(m_i,m_f\right)$ = (ingoing energy, outgoing energy)
$\tilde{G}$ (decay) = interaction constant

observed interaction energy $m_{Xexp}=m_X\left(\Gamma,I_\Gamma,m_i\right)$

calculated interaction energy
$m_{Xcal}=\sum_i E_{in}\left(u_i\right)+\sum_j E_{in}\left(A_j\right)$
$=\sum_i E_{out}\left(u_i\right)+\sum_j E_{out}\left(A_j\right)$

action $S=\int L\left(u_i\left(x^\mu\right),A_i\left(x^\mu\right)\right)d^4x$ = min, condition $E_{in} = E_{out}$
$L=L_{in}+L_{out}+L_{int}$ Lagrangian
for SU(3)-QCD and SU(4)-weak-QFT(= QHCD)
Dirac part $L_D=\bar{u}\left(i\hbar D_\mu\gamma^\mu-mc\right)u$, $D_\mu=\partial_\mu-igA^a{}_\mu\lambda_a$, with hc-field $A^a{}_\mu$, Lie structure constants f^{abc}
field part $L_f=-\frac{1}{4}F^{a\mu\nu}F_{a\mu\nu}$, field tensor $F^a{}_{\mu\nu}=\partial_\mu A^a{}_\nu-\partial_\nu A^a{}_\mu+gf^{abc}A^b{}_\mu\tilde{A}^c_\nu$
for SU(3)-QCD $\Psi=\left(u_1,u_2,u_3\right)$, e.g. proton $p=\left(u,u,d\right)$
for SU(4)-weak- QHCD $\Psi=\left(u_{L-},u_{L+},u_{R-},u_{R+}\right)$, e.g. electron $e^-=\left(r_{L-},0,r_{R-},0\right)$
generations $f_1\simeq 1A_g$ $f_2\simeq 4A_g$ $f_3\simeq 15A_g$
for QED $D_\mu=\partial_\mu-igA_\mu$, $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$
$L_{QHCD}=L_{hc,D}\left(u_i,A_{g,j}\right)+L_{hc,f}\left(A_{g,j}\right)$, $\lambda_a=15$ 4x4 Gell-Mann matrices
$L_{QCD}=L_{c,D}\left(u_i,A_{c,j}\right)+L_{c,f}\left(A_{c,j}\right)$, $\lambda_a=8$ 3x3 Gell-Mann matrices
$L_{QED}=L_{e,D}\left(u_i,A_e\right)+L_{e,f}\left(A_e\right)$
$L_{JJ}(J(d\bar{u}),J(W))=\frac{\left(u_1^{\,+}\gamma^\mu u_2\right)\left(u_3^{\,+}\gamma_\mu u_4\right)}{m_X^{\,2}}$
$S = min$ yields solution $u_i,A_{g,j},A_{c,j},A_{e,j}$ with energy $E\left(u_i\right),E\left(A_{g,j}\right),E\left(A_{c,j}\right),E\left(A_{e,j}\right)$, total energy $m_{Xcal}=E_{in}=E_{out}$

Introduction of extended SU(4)-preon-model SU4PM

In this chapter, we follow a theoretical scheme, different from the phenomenological ansatz from chap. 3.

We calculate the interaction energy directly from the minimization of the action, based on the Lagrangian of the gauge-field theory of the underlying interaction, SU(1)-QED for electromagnetic interaction with photons, SU(3)-QCD for color interaction with 8 gluon-fields, SU(4)-QHCD for extended weak (hypercolor hc) interaction with 15 hc-boson-fields [5] [10] [27]-[32].

The SU(4)-QHCD model of extended weak (hypercolor) interaction introduced in [2], treats the Pauli SU(2)-weak interaction as a Yukawa-approximation via massive (W, Z)-bosons and extends it to SU(4)-hypercolor interaction with four charges, 15 hc-boson fields and two subparticles called preons.

With the weak interaction extended to SU(4)-QHCD, Standard Model (SM) becomes the extended SU(4)-preon-model (SU4PM).

The SU4PM model allows to calculate the masses of the SM remarkably well, reducing 29 parameters of the SM to 7 [2] [29] [33].

In the following chap. 7.1 we present the basics of the SU4PM model, which the numerical calculation of decays is based on.

In SU4PM, the action $S=\int L\left(u_i\left(x^\mu\right),A_i\left(x^\mu\right)\right)\mathrm{d}^4x$ depends on the total Lagrangian $L=L_{in}+L_{out}+L_{int}$, where each Lagrangian contains the determining fields in the process hyper-color L_{QHCD}, color L_{QCD}, electromagnetic L_{QED}, e.g. for the neutron decay $L=L_{QHCD}+L_{QED}$.

The minimization of action $S=\int L\left(u_i\left(x^\mu\right),A_i\left(x^\mu\right)\right)\mathrm{d}^4x=\min$ with condition $E_{in}=E_{out}$, yields a solution in preons and fields $u_i,A_{g,j},A_{c,j},A_{e,j}$ with energies $E(u_i),E\left(A_{g,j}\right),E\left(A_{c,j}\right),E\left(A_{e,j}\right)$, and interaction energy m_{Xcal}, where $m_{Xcal}=\sum_i E_{in}(u_i)+\sum_j E_{in}\left(A_j\right)=\sum_i E_{out}(u_i)+\sum_j E_{out}\left(A_j\right)$.

The calculated total energy m_{Xcal} is compared to the observed value m_{Xexp} derived from the observed decay width Γ_{obs}.

The agreement is quite good (see chap. 7.3).

7.1. The Configuration of the Standard Model in the Extended SU(4)-Preon-Model

Every basic particle of the SM is assigned a preon and a hc-boson configuration [2] [29] [32].

The preon configuration of a fermion (leptons and quarks) occupies two of the 4 positions in a hc-quadruplet by a Dirac-bispinor, e.g. for electron with index pair (*1*,*3*) we have $\begin{pmatrix} rL- \\ 0 \end{pmatrix}$ in position 1 and $\begin{pmatrix} rR- \\ 0 \end{pmatrix}$ in position 3, according to the hc-charge. The hc-quadruplet has the hc-charges (*L*–, *L*+, *R*–, *R*+).

There are 3 possible hc-boson configurations for an index-pair (*i*,*j*), which are consistent with the SU(4)-symmetry: 1 hc-boson *Aij* corresponding to first generation of flavor = 1, 4 hc-bosons $Aij+\bar{A}ij+Akl+\bar{A}kl$ corresponding to flavor = 2 (the bar specifies the conjugate coupler, and (*k*,*l*) is the complementary index pair, e.g. for electron it is (2,4)), and finally all 15 hc-bosons corresponding to flavor = 3.

The fermions (leptons and quarks) have two independent preon-components *u*1 and *u*2, they form a bispinor with spin *S* = 1/2.

The bosons (weak boson *W*, *Z*, *H*) have only one independent preon-component u1, which is a linear combination of two preons, the spins add up to *S* = 1 for *W* and *Z*, or to *S* = 0 for *H*, e.g. for *Z* = *Z*0

$u1=\left((rL-)+(rR-)\right)/\sqrt{2}$ and $Z0=\left(\begin{pmatrix} u1 \\ 0 \end{pmatrix},\begin{pmatrix} 0 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ 0 \end{pmatrix},\begin{pmatrix} 0 \\ u1 \end{pmatrix}\right)\Big/\sqrt{2}$. The weak bosons *W* and *Z*0 are carrier of the residual weak interaction.

In the following, we present the basics of the SU(3)-color, the SU(4)-hypercolor, and its Yukawa weak Pauli force in three tables **Tables 14(a)-(c)** [29].

Table 14. (a) SU(3) strong (color) interaction; (b) SU(4)-hypercolor interaction; (c) SU(2) weak Pauli interaction.

(a)

gauge-group = Lie-group SU(3) 8 generators Gell-Mann-matrices λ_α, corresponding 8 massless spin-1 gauge bosons, 3 charges (colors) *r g b*, short range, pure quantum, asymptotically free (confinement),
energy scale $E_{col} = 220\,\text{MeV}$, with the zero-shift constant in the Callan-Symantzik relation to remove the singularity $c_{GE} = 0.69$
confinement length scale
$r_{col} = \frac{\hbar c}{E_{col}} = 0.89 \times 10^{-15}\ \text{m} \approx r_{pr}$
cut-off energy
$M_{col} = \frac{\hbar c}{r_{col}} = 2.33 \times 10^{10}\ \text{eV} \approx 23\,\text{GeV}$
the pion π with mass 106MeV is the Yukawa field boson of the color interaction
Lagrangian $L_{col} = -\frac{1}{4} G^a{}_{\mu\nu} G^{a\mu\nu}$
with the field tensor $G^a{}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$
covariant derivative $D_\mu \equiv \partial_\mu - ig A^a_\mu \lambda_a / 2$, where g is the coupling constant, is the gluon gauge field, for eight different gluons $\alpha = 1, \cdots, 8$ and where is one of the eight Gell-Mann matrices, $\alpha = 1, \cdots, 8$.
commutation relations with structure constants $[\lambda_a, \lambda_b] = 2i f^{abc} \lambda_c$

(b)

gauge-group = Lie-group SU(4), 15 generators massless bosons spin = 1, short range, pure quantum
4 charges $r_L^- r_L^+ r_R^- r_R^+$ for r-preon and $q_L^- q_L^+ q_R^- q_R^+$ for q-preon resp. with charge + or − and helicity *L* or *R*,
antiparticle $C(r_L^+) = r_R^-$, $C(r_L^-) = r_R^+$
the weak-interaction is the Yukawa-limit of the hypercolor interaction
L-R-symmetry breaking
$SU(4) = SU(2)_L \otimes SU(1)_R \otimes SU(1)_{em}$
covariant derivative $D_\mu \equiv \partial_\mu - ig A^a_\mu \lambda_a / 2$
commutation relations with structure constants
$[\lambda_a, \lambda_b] = 2i f^{abc} \lambda_c$
field tensor $G^a{}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$
Lagrangian $L_{hc} = -\frac{1}{4} G^a{}_{\mu\nu} G^{a\mu\nu}$
The coupling from the Callan-Symanzik equation is

$$g_{hc}(m) = 4\pi \sqrt{\frac{3}{76\sqrt{\left(\log\left(\frac{m}{\Lambda_{hc}}\right)\right)^2 + c_{GE1}^2}}}$$

critical energy $\Lambda_{hc} = 2m(Z_0) = 180\,\text{GeV}$ in analogy to the QCD, and zero-shift constant $c_{GE1} = \frac{1}{\log\left(\frac{m(t)}{m(d)}\right)} = 0.095$

Continued

(c)
gauge-group = Lie-group SU(2) 3 generators Pauli matrices σ_p, corresponding 3 massive gauge bosons Z, W^+, W^-, short range, pure quantum, are Yukawa bosons of hypercolor interaction cut-off energy $M_Z = 91.17$ GeV, length scale $r_{weak} = 10^{-17}$ m energy scale $E_{weak} \approx 10^{-4} E_{em} = 7$ eV Lagrangian, $L_1 = -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ where $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g f^{abc} W^b_\mu W^c_\nu$, $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ where the physical gauge fields Z, W^+, W^- are $Z_\mu = \frac{g W^3_\mu + g' B_\mu}{\sqrt{g^2 + g'^2}} = \cos\theta_W W^3_\mu + \sin\theta_W B_\mu$ $W^\pm_\mu = \frac{1}{\sqrt{2}}\left(W^1_\mu \pm i W^2_\mu\right)$ covariant derivatives left and right with Pauli matrices σ_i $D_\mu R = \left(\partial_\mu + i g' B_\mu\right) R$, $D_\mu L = \left(\partial_\mu + \frac{i}{2} g' B_\mu - \frac{i}{2} g \sigma_i W^i_\mu\right) L$

7.2. The Interaction Model and the Lagrangian in Two Examples

Example 1: neutron decay $n \to p e \bar{\nu}$

The basic idea of the Fermi model of weak 3-body decay in the Feynman picture mediated by the weak boson W is explained at the example of the neutron decay $n \to p e \bar{\nu}$ with the decay scheme $d\bar{u}(1h,3g) \to W \to W(1h)$.

The incoming Lagrangian is

$L(d\bar{u}) = L_{QHCD}\left(x^\mu, \{u_1, u_2\}, \{Ag_4\}\right) + L_{QCD}\left(x^\mu, \{u_1, u_2, 0\}, \{Ac_1, Ac_2, Ac_3\}\right)$ with the quark wavefunctions $u_1 = d = r^- q^+$, $u_2 = \bar{u} = r^- q^-$ in the hypercolor-SU(4)-preon model, and one hc-boson Ag_4 corresponding to the SU(4) generalized Gell-Mann matrix λ_4 and the SU(4) index pair {1,3} and the interaction $r_L^- \leftrightarrow r_R^-$ in the hc-charge-quadruple $\left(r_L^-, r_L^+, r_R^-, r_R^+\right)$ [2]. Furthermore, there are 3 gluons $\{Ac_1, Ac_2, Ac_3\}$, which carry the color interaction.

We recall that both L_{QHCD} and L_{QCD} have the generic form.

Dirac part $L_D = \bar{u}\left(i\hbar D_\mu \gamma^\mu - mc\right) u$, covariant derivative $D_\mu = \partial_\mu - igA^a{}_\mu \lambda_a$, with field $A^a{}_\mu$, field part $L_{gf} = -\frac{1}{4} F^{a\mu\nu} F_{a\mu\nu}$, field tensor $F^a{}_{\mu\nu} = \partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu + g f^{abc} A^b{}_\mu A^c{}_\nu$, where λ_a are the Gell-Mann matrices, with the structure constants f^{abc} of the respective Lie algebra (SU(3) or SU(4)) and λ_a are the generators of the algebra,

From the preon composition of $d\bar{u}$ results the following form of the SU(4) quadruple wavefunction

$$u_{11} = \left(r_L^- + q_L^+\right)/\sqrt{2}, \quad u_{12} = \left(r_R^- + q_R^+\right)/\sqrt{2}, \quad d = \left(\begin{pmatrix} u_{11} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} u_{12} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right)$$

$$u_{21}=\left(r_L^-+q_L^-\right)/\sqrt{2},\quad u_{22}=\left(r_R^-+q_R^-\right)/\sqrt{2},\quad \bar{u}=\left(\begin{pmatrix}u_{21}\\u_{21}\end{pmatrix},\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}u_{21}\\u_{21}\end{pmatrix},\begin{pmatrix}0\\0\end{pmatrix}\right)\Big/\sqrt{2}$$

The outgoing Lagrangian is $L(W)=L_{QHCD}\left(x^\mu,\{u_3,u_3\},\{Ag'_4\}\right)$ with the weak boson W $u_3=r_L^-r_R^-$ and another hc-boson Ag'_4.

$$u_{31}=r_L^-,\quad u_{32}=r_R^-,\quad W=\left(\begin{pmatrix}u_{31}\\0\end{pmatrix},\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}u_{31}\\0\end{pmatrix},\begin{pmatrix}0\\0\end{pmatrix}\right)$$

The interaction Lagrangian is the Fermi current-current interaction with the mediating exchange boson

$L_{JJ}\left(J\left(d\bar{u}\right),J\left(W\right)\right)=\dfrac{\left(u_1^{+}\gamma^\mu u_2\right)\left(u_3^{+}\gamma_\mu u_4\right)}{m_X^2}$, with the notation Dirac-conjugate u_1^{+}

The interaction energy is

$$\begin{aligned}m_X&=E_{out}=E\left(u_3\right)+m_W+E\left(Ag'_4\right)\\&=E_{in}=E\left(u_1\right)+E\left(u_2\right)+m_u+m_d+E\left(Ag_4\right)+\sum_{i=1}^{3}Ac_i\end{aligned}$$

So we have in total two particle configurations, the incoming $n\bar{p}$ and the outgoing $e\bar{\nu}$, each with an interaction Lagrangian, coupled by the Fermi current-current interaction, and mediated by the corresponding W-boson $W=r_L^-r_R^-$.

In the incoming system $d\bar{u}$ we have to take into account the color interaction of the quarks $L_C\left(d\bar{u}\right)=L_{QCD}\left(x^\mu,\{u_1,u_2\},\{Ac_2,Ac_5,Ac_7\}\right)$ in the basic gluon configuration with 3 rgb-gluons.

Feynman diagram of the decay $n\rightarrow pe\bar{\nu}$, with the notation of the antiparticle $\bar{p}=p^c$ (conjugate), is shown in **Figure 14**.

The quark-hc-boson-gluon decay-scheme in the SU4PM model $d\bar{u}\left(1h,3g\right)\rightarrow W\rightarrow W\left(1h\right)$, where the mediating boson $W=r_L^-r_R^-$ acts via the current-current-interaction L_{JJ} is shown in **Figure 15**.

In the decay-scheme the weak (SU(4)) interaction is carried on the left side by 1 h = 1 hypercolor SU(4) boson Ag_4, and the color (SU(3)) interaction by 3 g = 3 (anticoupler) gluons Ac_2 Ac_5 Ac_7.

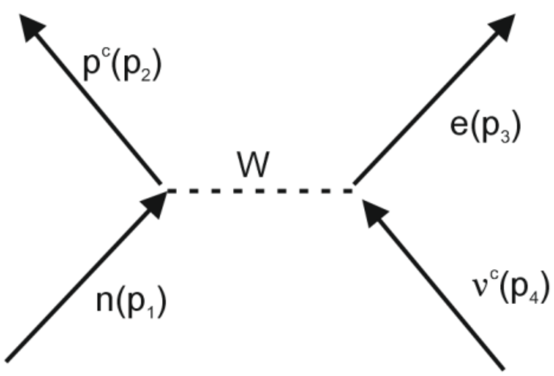


Figure 14. Feynman diagram of the decay $n\rightarrow pe\bar{\nu}$.

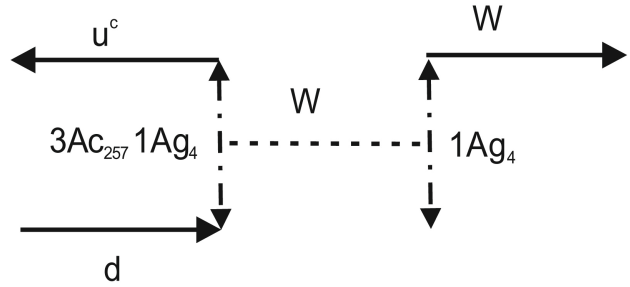


Figure 15. Quark decay-scheme $d\bar{u}(1h,3g)\to W\to W(1h)$ of the decay $n\to pe\bar{\nu}$.

The incoming color Lagrangian is

$L_{QCD}\left(x^{\mu},\{u_1,u_2,0\},\{Ac_1,Ac_2,Ac_3\}\right)$, where the color triple is $\{u_1,u_2,0\}$, on which act the 3x3 color Gell-Mann matrices λ_i.

On the right side, the weak (SU(4)) interaction is carried by 1 h = 1 hypercolor SU(4) boson Ag_4 and there is no color interaction, as the mediating boson W has only a weak charge, no color charge.

Example 2: 4-body kaon-pion photonic decay $K^+\to\pi^+\pi^+\pi^-\gamma$

We illustrate the calculation ansatz in more detail in the more complicated and computationally much more challenging example of the 4-body kaon-pion photonic decay $K^+\to\pi^+\pi^+\pi^-\gamma$ with the quark-hcboson-gluon decay scheme $u\bar{s}(12h,3g)\to W\to\pi^+\pi^+\pi^-(6h,3g,1\gamma)$.

The Feynman diagram of the process is shown in Figure 16.

The corresponding decay-scheme in the SU4PM model is shown in Figure 17.

The incoming Lagrangian is

$L(u\bar{s})=L_{QHCD}\left(x^{\mu},\{u_1,u_2\},\{Ag_n\}\right)+L_{QCD}\left(x^{\mu},\{u_1,u_2,0\},\{Ac_2,Ac_5,Ac_7\}\right)$, with $K^+=u\bar{s}$, with the quark wavefunctions $u=r^+q^+$, $\bar{s}=r^+q^-$,

$$u_{11}=\left(r_L^{\ +}+q_L^{\ +}\right)/\sqrt{2},\ \ u_{12}=\left(r_R^{\ +}+q_R^{\ +}\right)/\sqrt{2},\ \ u=\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}u_{11}\\u_{11}\end{pmatrix},\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}u_{12}\\u_{12}\end{pmatrix}\right)\Big/\sqrt{2}$$

$$u_{21}=\left(r_L^{\ +}+q_L^{\ -}\right)/\sqrt{2},\ \ u_{22}=\left(r_R^{\ +}+q_R^{\ -}\right)/\sqrt{2},\ \ \bar{s}=\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}0\\u_{21}\end{pmatrix},\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}u_{21}\\u_{21}\end{pmatrix}\right)$$

and 12 non-diagonal hc-bosons Ag_n corresponding to the non-diagonal SU(4) generator matrices λ_i and the SU(4) indices $n=\{1,2,4,5,6,7,9,10,11,12,13,14\}$.

It contains also 3 gluons $\{Ac_2,Ac_5,Ac_7\}$ which carry the color interaction.

The outgoing Lagrangian is

$$\begin{aligned}L\left(\pi^+\pi^-\pi^+\right)&=L_{QHCD}\left(x^{\mu},\{u_3,u_4,u_5\},\{Ag'_i\}\right)\\&\quad+L_{QCD}\left(x^{\mu},\{u_3,u_4,u_5\},\{Ac'_2,Ac'_5,Ac'_7\}\right)+L_e\left(\pi^+\pi^-\pi^+\right)\end{aligned}$$, indices

$i=\{1,4,6,9,11,13\}$, with the pions and their corresponding wavefunctions

$$\pi^+\pi^-=u\bar{d}d\bar{u}=r_L^{\ -}+r_R^{\ +}+q_L^{\ -}+q_R^{\ +}$$

$$u_{31}=u_{41}=\left(r_L^{\ -}+q_L^{\ -}\right)/\sqrt{2},\ \ u_{32}=u_{42}=\left(r_R^{\ +}+q_R^{\ +}\right)/\sqrt{2},$$

$$\pi^+\pi^-=\left(\begin{pmatrix}u_{31}-u_{41}\\u_{31}-u_{41}\end{pmatrix},\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}u_{32}-u_{42}\\u_{32}-u_{42}\end{pmatrix}\right)\Big/2$$

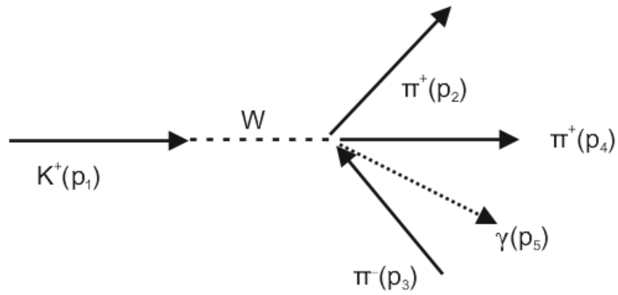


Figure 16. Feynman diagram of the decay $K^+ \to \pi^+ \pi^+ \pi^- \gamma$.

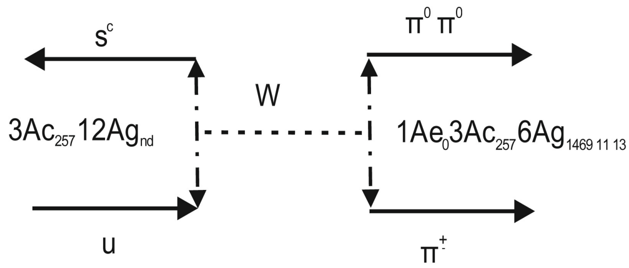


Figure 17. Quark decay-scheme $u\overline{s}(12h,3g) \to W \to \pi^+ \pi^+ \pi^- (6h,3g,1\gamma)$ of the decay $K^+ \to \pi^+ \pi^+ \pi^- \gamma$.

$\pi^+ = u\overline{d} = r_L{}^+ + r_R{}^+ + q_L{}^- + q_R{}^+$, $u_{51} = \left(r_L{}^+ + q_L{}^-\right)/\sqrt{2}$, $u_{52} = \left(r_R{}^+ + q_R{}^-\right)/\sqrt{2}$,

$$\pi^+ = \left(\begin{pmatrix} 0 \\ u_{51} \end{pmatrix}, \begin{pmatrix} 0 \\ u_{51} \end{pmatrix}, \begin{pmatrix} 0 \\ u_{52} \end{pmatrix}, \begin{pmatrix} 0 \\ u_{52} \end{pmatrix}\right) \Big/ \sqrt{2}$$

and the 6 hc-bosons Ag'_i , which are the 6 couplers of SU(4).

It contains also 3 diagonal gluons $\{Ac'_2, Ac'_5, Ac'_7\}$ which carry the color interaction.

The interaction Lagrangian is

$$L_{JJ}\left(J\left(u\overline{s}\right), J\left(\pi^+\pi^-\pi^+\right)\right) = \frac{\left(u_1{}^+\gamma^\mu u_2\right)\left(u_3{}^+\gamma_\mu u_3 + u_4{}^+\gamma_\mu u_4 + u_5{}^+\gamma_\mu u_5\right)}{m_X^2}$$, with the notation Dirac-conjugate $u_1{}^+$.

The interaction energy is

$$\begin{aligned} m_X = E_{out} &= E(u_3) + E(u_4) + E(u_5) + 3m_\pi + \sum_i E(Ag'_i) + \sum_i E(Ac'_i) + Ae_0 \\ &= E_{in} = E(u_1) + E(u_2) + m_u + m_s + + \sum_n E(Ag_n) + \sum_i E(Ac_i) \end{aligned}$$

So we have in total two particle configurations, the incoming $K^+ = u\overline{s}$ and the outgoing $\pi^+\pi^-\pi^+$, each with an interaction Lagrangian, coupled by the Fermi current-current interaction, and mediated by the corresponding W-boson $W = r_L{}^- r_R{}^-$,

In the incoming system $K^+ = u\overline{s}$ and the outgoing $\pi^+\pi^-\pi^+$ we have to take into account the color interaction of the quarks

$L_C\left(u\overline{s}\right) = L_{QCD}\left(x^\mu, \{u_1, u_2, 0\}, \{Ac_2, Ac_5, Ac_7\}\right)$ and

$L_C\left(\pi^+\pi^-\pi^+\right) = L_{QCD}\left(x^\mu, \{u_3, u_4, u_5\}, \{Ac'_2, Ac'_5, Ac'_7\}\right)$ in the basic gluon configuration with 3 rgb-gluons.

The outgoing photon is active in the additional third electromagnetic Lagrangian $L_e\left(\pi^+\pi^-\pi^+\right)=L_e\left(x^\mu,\{u_3,u_4,u_5\},\{Ae_0\}\right)$.

7.3. The Calculation Method

Now we minimize the action $S=\int L\left(x^\mu,u_i,Ag_i\right)r^2\sin\theta\,\mathrm{d}t\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\varphi$ for the total Lagrangian $L\left(x^\mu,u_i,Ag_i\right)=L\left(d\bar{u}\right)+L_{JJ}\left(W\right)+L\left(J\left(d\bar{u}\right),J\left(W\right)\right)+L_C\left(d\bar{u}\right)$ under the constraint of energy conservation $E\left(d\bar{u}\right)=E\left(W\right)$, as required in the Feynman diagram of the process.

We have for the particle wavefunctions $\{u_1,u_2,u_3\}$ the normalization condition $\int\left|u_i\right|^2\mathrm{d}^3x=1$ and for the field bosons we set up a boundary condition for $r=r_0$ $Ag_i\left(r_0\right)=0$ and $Ac_i\left(r_0\right)=0$ and the Lorenz-gauge-condition $\partial_\mu\left(Ag_i\right)^\mu=0$ and $\partial_\mu\left(Ac_i\right)^\mu=0$.

The energy, length, and time are made dimensionless by using the units: E($E_0=\dfrac{\hbar c}{1\text{ am}}=0.196\text{ TeV}$), r(fm), t(am/c) am = 10^{-18} m. We can assume axial symmetry, so we can set $\varphi=0$ and use the spherical coordinates (t,r,θ).

We choose the equidistant lattice for the intervals $(t,r,\theta)\in[0,1]\times[0,1]\times[0,\pi]$ with 21 × 21 × 11 points and, for the minimization n_{sub} in parallel, n_{sub} random sublattices of length l_{sub}, where n_{sub} = 8 or 16, and l_{sub} = 25 or 50 or 100 according to the complexity of the corresponding Lagrangian.

$$l[ix,j]=\left\{\left\{\left(t_{i1},r_{i2},t_{i3}\right)|\left(i1,i2,i3\right)=random\left(lattice,j=1,\cdots,l_{sub}\right)\right\}|\,ix=1,\cdots,n_{sub}\right\}.$$

For the Ritz-Galerkin expansion we use the 12 functions

$$f_k(r,\theta)=\left\{bfunc\left(r,r_0,dr_0\right)r^{k_1},k_1=0,\cdots,n_r\right\}\times\left\{\left(\cos^{k_2}\theta,\cos^{k_2}\theta\sin\theta\right),k_2=0,\cdots,n_\theta\right\}$$

The action $S=\int L\left(x^\mu,u_i,Ag_i\right)r^2\sin\theta\,\mathrm{d}t\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\varphi$ becomes a mean-value on the sublattice $l[ix]$

$$\tilde{S}[ix]=\frac{1}{N\left(l[ix]\right)}\sum_{x\in l[ix]_{sub}}L\left(x,u_i,Ag_i\right)2\pi V_{tr\theta},$$ where $V_{tr\theta}=\pi$ the (t,r,θ)-volume and $l_{sub}=N\left(l[ix]\right)$ is the number of points. We impose the boundary condition for $Ag_i\left(r=r_0\right)=0$ via penalty-function (imposing exact conditions is possible, but slows down the minimization process enormously).

$\tilde{S}$ is minimized n_{sub} x in parallel with the Mathematica-minimization method "simulated annealing".

The proper parameters of the particles u_i and the hc-bosons Ag_i are:

$$par\left(p_i\right)=\{Eu_i,a_i,ru_i,\theta u_i,dru_i\},\quad par\left(Ag_i\right)=\{EA_i,aA_i\},$$
$$par\left(Ac_i\right)=\{EAc_i,aAc_i\}$$

The complexities and execution times (on a 2.7 GHz Xeon E5 work-station) differ greatly for different decays.

For the neutron decay $n\to pe\bar{\nu}$ with the scheme $d\bar{u}(1h)\to W\to W(1h)$ (1hc-boson on both sides) and color interaction $L\left(d\bar{u},3g\right)$ with basic 3 gluons complexity (Lagrangian) = (3.7 + 4.8) × 10^6 terms, minimization time t (minimization) = 111 s.

The mathematical details of the calculation, and the results can be studied in depth in the corresponding Mathematica programs [34].

7.4. Discussion of Calculated Decays

Table 15 of decays with calculation results mX and experimental values mXexp according to the formula in 2.1 from the observed decay width Γobs, is as follows [34].

Table description

The scheme (last) column describes the model of the decay, on which the calculation is based, where the notation *q'* is used for the antiparticle $\bar{q}$.

Here the calculation result (m_{Xcal}) and the value from decay time (m_{Xexp}) are given in GeV.

$m_X(er)$ is the calculated m_X-value with uncertainty *er* in GeV.

E_{col} specifies the calculated color interaction energy in GeV and the number of active gluons on left side of the process, e.g. 250 (3 g), E_{em} is the electromagnetic energy of the involved photons, if any.

<*r*12> and <*dr*12> are the mean radius in *am*-units (1 am = 10^{-18} m) and its quantum "smear-out" in the left-side (incoming) part of the scheme.

The mean boson amplitude (hypercolor, color, electromagnetic) of the incoming and outgoing system *Agi Aci Aei* expressed in units am^{-1} is given in column four.

$m_X = E(B_{med})$, where for weak decays the mediating boson is $B_{med} = W$ or $B_{med} = Z$.

Classification according to m_X: strong decays

There are here 3 strong (color) decays: pion and eta decays, with scales $m_X \approx (0.3, 7.5, 20)\,\text{GeV}$, mediated by a pion

η -> π0 π0 π0, η -> π0 π0 γ, π0/η -> γ γ

Strong decays have an assessed upper limit of interaction energy m_X for strong decays: $E_{c,max} = N_{comp} M_{max}$, where M_{max} is the maximum energy-mass, for 1+2-generation $M_{max} = m_c = 1.3\,\text{GeV}$ for charm-quark, and maximum number of components $N_{comp} = 3 + 15$, where 3 stands for 3 quarks, and 15 stands for 15 hc-bosons, so $E_{c,max} = 18 \times 1.3 = 23.4\,\text{GeV}$.

Classification according to m_X: weak decays

The minimum interaction energy m_X for weak decays is
$E_{w,min} = m_W = 80.4\,\text{GeV}$.

The weak decays considered here can be put into 3 categories.

- low interaction energy 100 - 400 GeV

$n \to p e \bar{\nu}$, $m_X \sim 200\,\text{GeV}$

schematic photonic $\Lambda/\Sigma/\Xi \to \gamma \ldots$, $m_X = 130 \ldots 260\,\text{GeV}$

schematic pionic $\Lambda/\Sigma/\Xi \to \pi \ldots$, $m_X \sim 400\,\text{GeV}$

schematic one-pion $K \to \pi$, $m_X \sim 160\,\text{GeV}$

schematic leptonic $K/\pi \to l\nu$, $m_X = 130 \ldots 180\,\text{GeV}$

-middle interaction energy 700 - 1700 GeV

schematic nucleonic $\Lambda/\Sigma \to (p/n)\nu$, $m_X = 1300 \ldots 1700\,\text{GeV}$

Table 15. Decays with calculation results mX and experimental values mXexp.

decay	P_{in}	P_{out}	Agi Aci Aei	<r12>	<dr12>	Ecol Eem	mX(er)cal	mX exp	scheme
Λ → n π	uds	udd/(uu'-dd')						387.719	sd'(2h) → Z → π0(2h)
Σ → n π	uds	udd/(uu'-dd')						427.142	sd'(2h) → Z → π0(2h)
Ξ → Λ π	uss	uds/(uu'-dd')	{0.271, 1.043} {0.258, 0.348}	0.472	0.461	195(3g)	505(71)	435.627	sd'(2h) → Z → π0(2h)
π+ → l ν	ud'	2rL-	{0.171, 0.323}	0.195	0.559	0	112(16)	102.178	ud'(1h) → W → W
K+ → l ν	us'	2rL-						385.244	us'(2h) → W → W
K+ → π+ π0	us'	ud'/(uu'-dd')	{0.314, 0.219} {0.245, 0.213}	1.13	1.34	194(3g)	705(34)	667.317	us'(4h) → W → π+π0(1h)
KS0 → π π	(ds'+sd')	2(uu'-dd')	{0.804, 0.122} {0.225}	0.300	0.207	64(3g)	159(19)	160.647	ds'(1h) → Z → 2π0
K+ → π+ π π	us'	ds'/2(uu'-dd')						159.222	us'(1h) → W → π+ 2π0
KL0 → π0 π π	ds'	3(uu'-dd')						163.886	ds'(1h) → Z → π0 2π0
KL0 → π π	(ds'-sd')	2(uu'-dd')						3381.22	ds'(12h) → Z → 2π0(4h)
K+ → π0 l ν	us'	(uu'-dd')/W	{0.894, 0.351} {0.249, 0}	0.505	0.535	333(3g)	1940(89)	2034.04	us'(6h) → W → π0 W(6h)
KL0 → π+ l ν	ds'	ud'/W						1716.44	ds'(6h) → Z → π+ W(2h)
K+ → π+ π− l ν	us'	2(uu'-dd')/W						7444.14	us'(15h) → W → 2π0 W(6h)
K+ → π0 π0 l ν	us'	2(uu'-dd')/W	{0.889, 0.365} {0.267, 0.250}	0.449	0.555	2810(8g)	8880(280)	9137.27	us'(15h) → W → 2π0 W(15h)
KL0 → π+ π0 l ν	us'	ud'/(uu'-dd')/W						7693.29	us'(15h) → W → π+π0W(6h)
K+ → π+ π+ π− γ	us'	ud'/(ud'+u'd)	{0.920, 0.284} {0.278, 0.036}	0.987	1.59	930(3g), 640	3470(170)	3574.06	us'(12h) → W → π+π-π+ γ(6h)
K+ → π0 π+ γ	us'	(uu'-dd')/ud'						5707.21	us'(12h) → W → π0π+γ(12h)
Λ → p *l*ν	uds	uud/W	{0.205, 0.135} {0.338, 0}	0.245	0.513	328(3g)	1270(53)	1271.1	su'(6h) → W → W
Σ- → n *l*ν	dds	udd/W						1713.08	su'(6h) → W → W(2h)
μ/τ → e νe ν	*l*	*l*	{0.857, 0.122}	0.461	0.374		768(117)	717.06	*l* *ν*'(4h) → W → W
τ → μ νμ νμ	*l*	*l*						695.878	*l* *ν*'(4h) → W → W
π+ → π0 *l*ν	ud'	(uu'-dd')/W						468.38	ud'(2h) → W → W
n → p e νe	udd	uud/W	{0.275, 0.250} {0.221}	0.341	0.199	87(3g)	197(9.3)	204.69	du'(1h) → W → W
Σ+ → Λ *l*ν	uus	uds/W						721.78	ud'(4h) → W → W
η → π0 π0 π0	(uu'+dd'-2ss')	3(uu'-dd')	{0.200, 0.164}	0.270	0.384	(3g)	0.388(0.109)	0.26186	sd'(3g) → π0 → 3π0
η → π0 π0 γ	(uu'+dd'-2ss')	2(uu'-dd')						7.4388	sd'(6g) → π0 → 2π0 γ
π0/η → γ γ	(uu'-dd')		{0, 0} {0.153, 0.156} 0.065	0.284	0.250	(8g)2.9	23.8(7.2)	21.221	uu'(8g) → π0 → 2γ
Λ/Σ → n γ	uds	udd	{0.294, 0.580} {0.757} 0.667	0.680	2.742	64(3g)9.7	181(41)	162.669	sd'(1h) → Z → Z γ
Σ0 → Λ γ	uss	uds						224.404	sd'(2h) → Z → Z γ
Ξ0 → Σ0 γ	uss	uds						131.511	sd'(1h) → Z → Z γ
Ξ- → Σ-γ	dss	dds						263.089	sd'(2h) → Z → Z γ(2h)

schematic pure leptonic $\mu / \tau \to l\, 2\nu$, $m_X \sim 700$ GeV

schematic leptonic $\Sigma \to \Lambda\, l\, \nu$, $m_X \sim 700$ GeV

- high interaction energy kaon 3400 - 9200 GeV

schematic pionic-leptonic $K \to \pi l \nu$, $m_X \sim 2000$ GeV

schematic pionic $K_L^0 \to \pi\, \pi$, $m_X \sim 3400$ GeV

schematic pionic-leptonic $K \to \pi\pi l \nu$, $m_X = 7600...9200$ GeV

schematic pionic-photonic $K \to 3\pi\, \gamma$, $m_X \sim 3600$ GeV

schematic pionic-photonic $K \to 2\pi\, \gamma$, $m_X \sim 5700$ GeV

Characterization of radius

The calculated radius r of decaying particle is a parameter of the ingoing Lagrangian, and is measured in am = 10^{-18} m.

For weak decays we obtain values $r = 0.25...1.3$ am and quantum smear-out $\Delta r \sim r$.

For strong decays we obtain $r \sim 0.3$ am .

The following plot **Figure 18** presents the measured and calculated interaction energy described in the above table [2] [34].

The decays above 80 GeV (=m_W) are weak (hypercolor) decays, those below 25 GeV (=6m_b, m_b = 4.2 GeV) are strong (color) decays, the observed m_X is dark-blue, the calculated m_X is red (with calculation error bar), the color energy for weak decays, respectively electromagnetic energy for strong decays is cyan.

Another interesting decay parameter is the mean radius <r12> of the incoming system on the left side of the scheme, e.g. for the neutron decay

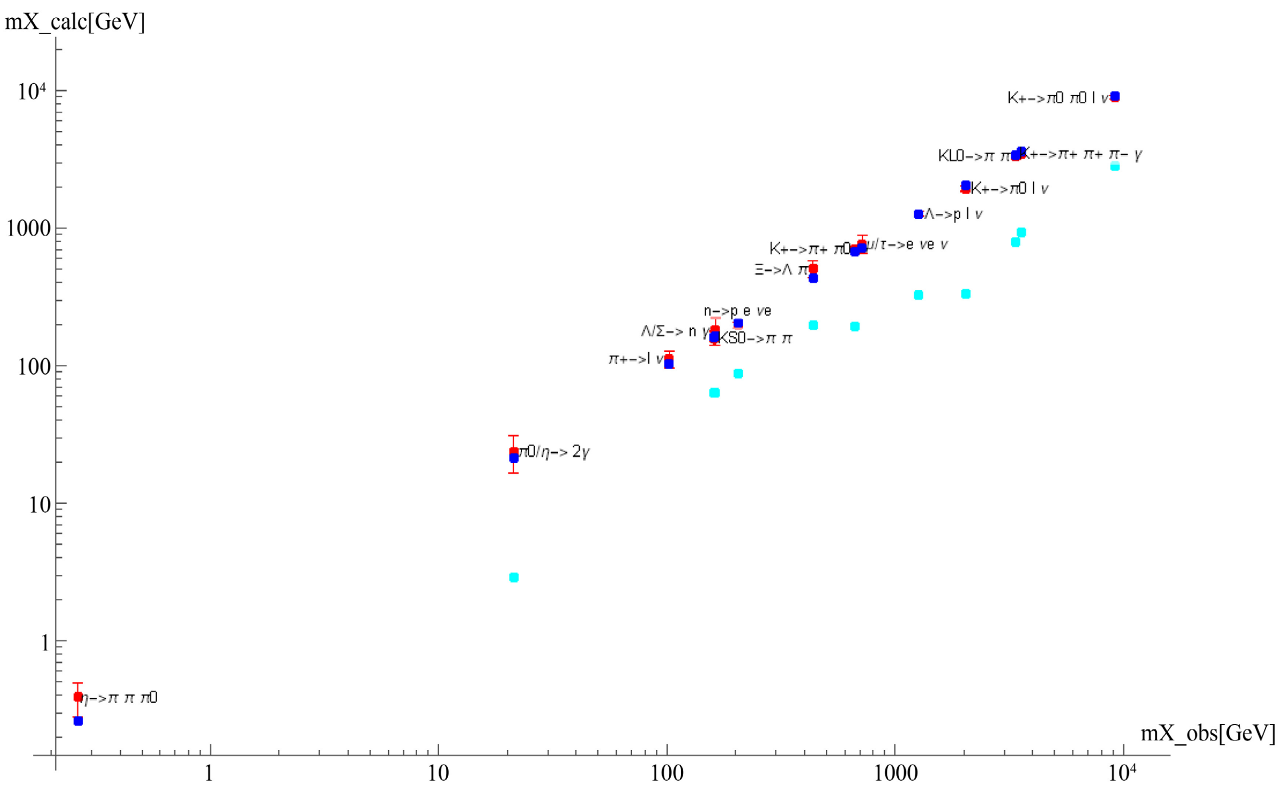


Figure 18. Measured and calculated interaction energy.

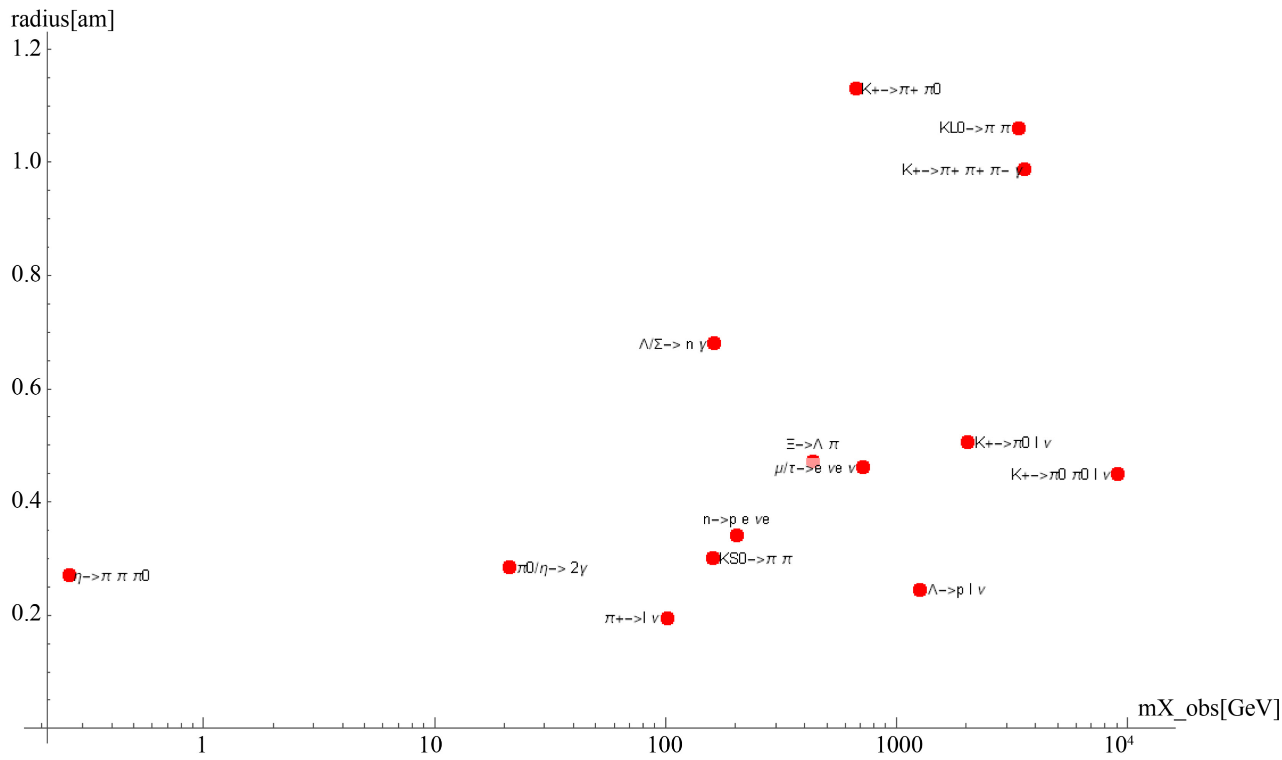


Figure 19. Mean radius in dependence of interaction energy.

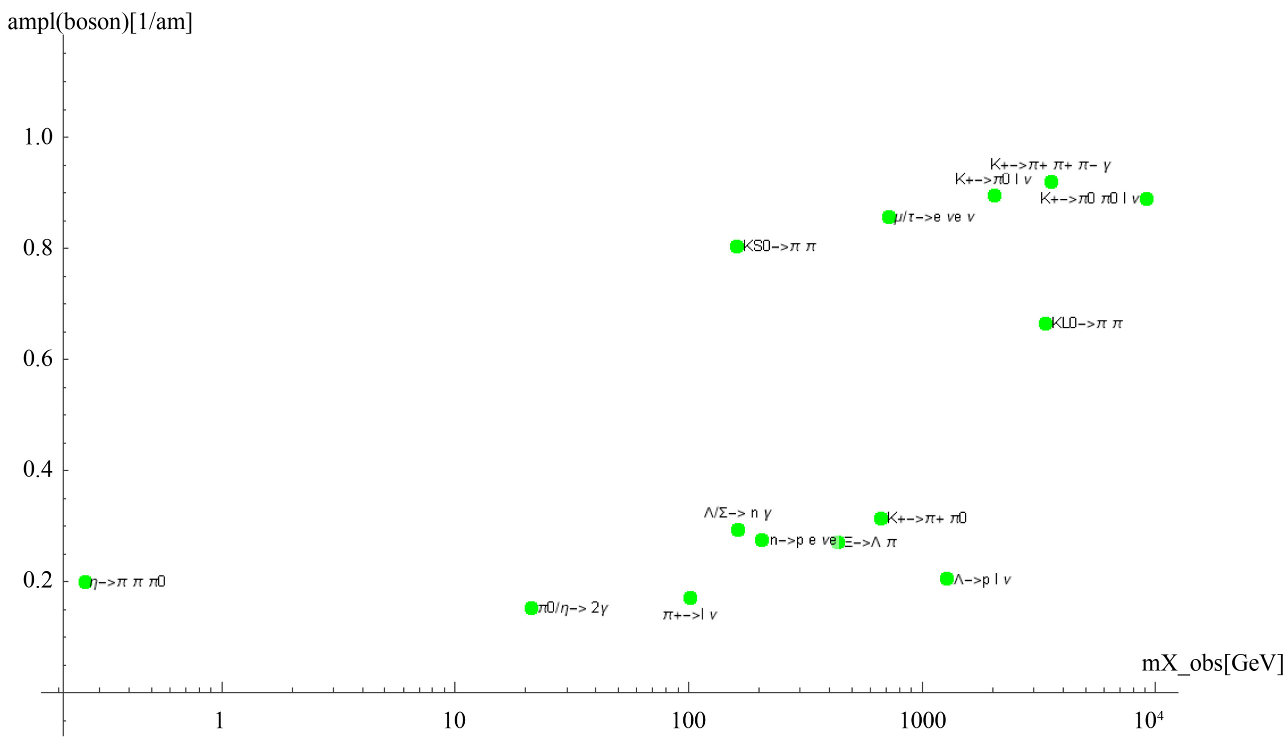


Figure 20. Mean field boson amplitude in dependence of interaction energy.

n->p e νe, the incoming system is $n\bar{p}$, in the decay scheme it is represented by $d\bar{u}$ [2] [34]. The plot of radius is shown in **Figure 19**.

It is interesting to see, that the mean radius separates basically into two groups: high-energy non-leptonic kaon-pion decays with <*r*12>>0.9 *am* and the remaining decays with <*r*12> <0.5 *am*, apart from the photonic *Λ/Σ* -> *n γ*.

The other important decay parameter is the mean (hypercolor, color, electromagnetic) field boson amplitude *Agi* for the weak decays, *Aci* for the color decays, of the incoming system, expressed in units am^{-1} [2] [34]. The mean field boson amplitude is shown in **Figure 20**.

Again, the amplitude separates into two groups, amplitude ≥ 0.6 for the kaon-pion decays an pure leptonic decays, and the remaining with amplitude ≤ 0.3, with the outlier *K+* -> *π+ π*0.

8. Conclusions

We introduce a two-step calculation method for calculation of general decay rates in the Standard Model, and apply it, producing results for a wide variety of decay processes, which are in good agreement with measurements.

The first step is an extended schematic formula by Chang [1], based on extended isospin. It supports the generalized model of a decay-mediating virtual particle with interaction energy m_X, in analogy to the weak interaction mediated by the W-boson with $G=\frac{G_F}{\sqrt{2}}=\frac{g^2}{8M_W^2}$, where $m_X=M_W$, g is the dimensionless weak interaction constant, and $G_F=1.1663787(6)\times10^{-5}\,\text{GeV}^{-2}$ is the Fermi weak coupling constant.

The second step is a numerical Lagrangian calculation method, which calculates the interaction energy m_X of the process numerically by minimization of action from the Lagrangian.

First we derive in chap.2 formulas in the conventional way for selected examples: neutron, muon, pions and kaons.

In chap.2.8 and chap.2.9 we present the **fundamental Fermi golden rule** for 3-body and 2-body decays:

$$\mathrm{d}\Gamma_{3b}=\frac{\left|M\left(k,p_1,p_2,p_3\right)\right|^2}{2m}\frac{\mathrm{d}^3p_1}{(2\pi)^3\,2E_1}\frac{\mathrm{d}^3p_2}{(2\pi)^3\,2E_2}\frac{\mathrm{d}^3p_3}{(2\pi)^3\,2E_3}(2\pi)^4\,\delta^4\left(k-p_1-p_2-p_3\right)$$

$$\mathrm{d}\Gamma_{2b}=m\frac{\left|M\left(k,p_1,p_2\right)\right|^2}{2}\frac{\mathrm{d}^3p_1}{(2\pi)^3\,2E_1}\frac{\mathrm{d}^3p_2}{(2\pi)^3\,2E_2}(2\pi)^4\,\delta^4\left(k-p_1-p_2\right)$$

and derive kinematic factors for these processes:

$$I_{\Gamma 3s}\left(m,m_1,m_2,m_3\right)\ \text{and}\ I_{\Gamma 2s}\left(m,m_1,m_2\right).$$

In chap.3.1 we formulate the **phenomenological formula**:

$\Gamma=\tilde{G}^2m_i^k\left|P_l^m(x)\right|^2=\frac{G^2}{C_1}m_i^k\left|P_l^m(x)\right|^2$, where $P_l^m(x)$ Legendre polynomial $m=l$ or $m=l+1$, I = isospin I, $x=\frac{m_f}{m_i}$ mass ratio, $\tilde{G}=\frac{G}{\sqrt{C_1}}$ with G = *interaction constant*, m_i is the initial mass, k is the mass-power-coefficient.

The constant C_1 is process-dependent, standard value is $C_1 = 4\pi$.

The phenomenological scheme classifies decays into seven classes, according to the values of *k*, *l*, and *m*.

The interaction constant G is independent of masses m_i, m_f, and is in the same range within a class.

The seven classes discussed here are:

Strange hyperon-pion decays,

Two-body non-strange meson decays,

Three-four-body strange meson decays,

Three-body strange hyperon decays,

Non-strange leptonic three-body decays,

Three-body eta-pion decays,

Photon-radiative decays.

Also, we define and derive a formula for interaction energy m_X between the initial and final configuration of the decay process, which is the energy of the mediating boson in a weak decay.

In analogy to the weak interaction mediated by the W-boson $G = \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$,

we make the ansatz for the matrix element M and the interaction energy m_X of the mediating boson:

$M = \frac{m_i^2}{8m_X^2}$, so we obtain the decay width formula $\Gamma = |M|^2 I_\Gamma m = \left(\frac{m^2}{8m_X^2}\right)^2 I_\Gamma m$

The process can be weak (W-Z-mediated Pauli interaction, $G \sim G_F$), electromagnetic (interaction constant $G \sim \alpha = 1/137$) or strong $G \sim g_s$

The rest of chap. 3 deals with different special cases of the formula: muon, pions, kaon-pions, neutron, eta-pion, meson-2-photon decay, hyperon-photon.

In chap. 5 we show the actual form and results of the phenomenological formula for seven classes of decay processes, classified by the phenomenological scheme.

In chap. 6 we present the all calculation results from the phenomenological formula in tabular form and in graphic form of a plot.

In chap. 7.1 we describe the **theoretical background of the numerical Lagrangian calculation method**.

action $S = \int L\left(u_i\left(x^\mu\right), A_i\left(x^\mu\right)\right) \mathrm{d}^4 x = \min$, condition $E_{in} = E_{out}$

$L = L_{in} + L_{out} + L_{int}$ Lagrangian

for SU(3)-QCD and hypercolor-SU(4)-weak-QFT(=QHCD)

Dirac part $L_D = \bar{u}\left(i\hbar D_\mu \gamma^\mu - mc\right)u$, $D_\mu = \partial_\mu - igA^a{}_\mu \lambda_a$, with hc-field $A^a{}_\mu$, Lie structure constants f^{abc}

Field part $L_f = -\frac{1}{4} F^{a\mu\nu} F_{a\mu\nu}$, field tensor $F^a{}_{\mu\nu} = \partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu + gf^{abc} A^b{}_\mu \tilde{A}^c_\nu$

For SU(3)-QCD $\Psi = \left(u_1, u_2, u_3\right)$, e.g. proton $p = \left(u, u, d\right)$

For hypercolor-SU(4)-weak-QHCD $\Psi = \left(u_{L-}, u_{L+}, u_{R-}, u_{R+}\right)$, e.g. electron $e^- = \left(r_{L-}, 0, r_{R-}, 0\right)$

Generations $f_1 \simeq 1A_g$, $f_2 \simeq 4A_g$, $f_3 \simeq 15A_g$

For QED $D_\mu = \partial_\mu - igA_\mu$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$L_{QHCD} = L_{hc,D}\left(u_i, A_{g,j}\right) + L_{hc,f}\left(A_{g,j}\right)$, $\lambda_a = 15$ 4 × 4 Gell-Mann matrices

$L_{QCD} = L_{c,D}\left(u_i, A_{c,j}\right) + L_{c,f}\left(A_{c,j}\right)$, $\lambda_a = 8$ 3 × 3 Gell-Mann matrices

$L_{QED} = L_{e,D}\left(u_i, A_e\right) + L_{e,f}\left(A_e\right)$

$$L_{JJ}\left(J\left(d\bar{u}\right), J\left(W\right)\right) = \frac{\left(u_1^{+}\gamma^\mu u_2\right)\left(u_3^{+}\gamma_\mu u_4\right)}{m_X^2}$$

$S = min$ yields solution $u_i, A_{g,j}, A_{c,j}, A_{e,j}$ with energy $E\left(u_i\right), E\left(A_{g,j}\right), E\left(A_{c,j}\right), E\left(A_{e,j}\right)$, total energy $m_{Xcal} = E_{in} = E_{out}$

In chap. 7.2 we give the details of the numerical action minimization procedure.

In chap. 7.3 we present the calculated parameters of the decay process

- the calculated and the experimental values of the interaction energy m_X in tabular form and in a graphical plot.
- in the ingoing particle values of color and electromagnetic energy E_{col}, E_{em}.
- field boson amplitudes weak-hcolor, strong-color. electromagnetic $\left(A_{g,i}, A_{c,i}, A_{e,i}\right)$.
- radius r and its smear-out Δr.

The scheme of the decay process is formulated as follows:

$q_1\bar{q}_2 \to b \to p$ or $\bar{q}_1 q_2 \to b \to p$, where q_1, q_2 are quarks in the incoming quark-current, b is the mediating boson $b = W, Z, \pi^0$, p are the outgoing particles, $p = \pi^0, \pi, W, \gamma$, where p can be represented as one or more quark-currents except for the photon γ, which is itself the electromagnetic current.

The resulting interaction energy m_X in the table above is not distributed uniformly, but accumulates around certain values, the energy classes.

$E_{h1} \approx 150$ GeV for 1 hc-boson

$E_{h2} \approx 400$ GeV for 2 hc-bosons

$E_{h4} \approx 700$ GeV for 4 hc-bosons

$E_{h6} \approx 1500$ GeV for 6 hc-bosons

$E_{h12} \approx 3500$ GeV for non-diagonal 12 hc-bosons outgoing W (1 hcb)

$E_{h12,3h} \approx 5700$ GeV for non-diagonal 12 hc-bosons outgoing W (3 hcb)

$E_{h15} \approx 7500$ GeV for all 15 hc-bosons outgoing W (3 hcb)

$E_{h15,3h} \approx 9000$ GeV for all 15 hc-bosons outgoing W (6 hcb)

$E_{c1} \approx 0.3$ GeV for 3 gluons (color interaction, factor 1000 weaker than hc-interaction)

$E_{c6} \approx 7$ GeV for 6 non-diagonal gluons; $E_{c8} \approx 20$ GeV for all 8 gluons

For weak decays the energy span in m_X is roughly: $\dfrac{E_{h15}}{E_{h1}} = \dfrac{9000\ \text{GeV}}{150\ \text{GeV}} = 60$,

So the energy span scales like $\dfrac{E_{h15}}{E_{h1}} = 60 \approx \left(n_h\right)^{3/2}$

The classification according to interaction energy m_X

For weak decays is as follows.

- Low interaction energy 100 - 400 GeV

$n \to pe\bar{\nu}$, $m_X \sim 200$ GeV

Schematic photonic $\Lambda/\Sigma/\Xi \to \gamma...$, $m_X = 130...260$ GeV

Schematic pionic $\Lambda/\Sigma/\Xi \to \pi...$, $m_X \sim 400$ GeV

Schematic one-pion $K \to \pi$, $m_X \sim 160$ GeV

Schematic leptonic $K/\pi \to l\nu$, $m_X = 130...180$ GeV

- Middle interaction energy 700 - 1700 GeV

Schematic nucleonic $\Lambda/\Sigma \to (p/n)\nu$, $m_X = 1300...1700$ GeV

Schematic pure leptonic $\mu/\tau \to l\,2\nu$, $m_X \sim 700$ GeV

Schematic leptonic $\Sigma \to \Lambda\, l\, \nu$, $m_X \sim 700$ GeV

The classification according to interaction energy m_X

For strong decays is as follows.

There are here 3 strong (color) decays:

Pion and eta decays, with scales $m_X \approx (0.3, 7.5, 20)$ GeV, mediated by a pion.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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