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Calculation of the Standard Model Parameters and Particles Based on a SU(4) Preon Model

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Abstract

This paper describes an extension and a new foundation of the Standard Model of particle physics based on a SU(4)-force called hyper-color, and on preon subparticles. The hyper-color force is a generalization of the SU(2)-based weak interaction and the SU(1)-based right-chiral self-interaction, in which the W- and the Z-bosons are Yukawa residual-field-carriers of the hyper-color force, in the same sense as the pions are the residual-field-carriers of the color SU(3) interaction. Using the method of numerical minimization of the SU(4)-action based on this model, the masses and the inner structure of leptons, quarks and weak bosons are calculated: the mass results are very close to the experimental values. We calculate also precisely the value of the Cabibbo angle, so the mixing matrices of the Standard model, CKM matrix for quarks and PMNS matrix for neutrinos can also be calculated. In total, we reduce the 29 parameters of the Standard Model to a total of 7 parameters.

Keywords

SU(4), Generalization of Weak Interaction, Extension of Standard Model, Numerical Minimization of Action, Hyper-Color, Preon

1. Introduction

The Standard Model of Particle Physics (SM) formulated in its final form in mid-seventies, is a very successful theory: in spite of repeated search for deviation from observation, after 50 years there is not a single experimental result contradicting it.

Still, it has several shortcomings, which make it hard to accept as a final theory, so it is generally considered to be incomplete.

SM has the following problems [1] [2] [3] [4]:

- SM does not fully explain baryon asymmetry (observed imbalance of matter and antimatter)

▪ SM does not explain the left-right-chiral asymmetry of the electro-weak force (spontaneous symmetry breaking $SU(2)_L xSU(1)_R$)

▪ SM does not explain the CP violation in kaons, it has to be introduced as a complex phase in the quark mixing Cabibbo-Kobayashi-Maskawa (CKM) matrix

▪ SM does not naturally incorporate neutrino oscillations and their non-zero masses, the masses are introduced by hand, and neutrino oscillations are inserted by introducing the purely experimental Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

▪ Pauli-SU(2) weak interaction is mediated by massive W- and Z-bosons, which is hard to accept from the relativistic point-of-view: all fundamental interactions should propagate with maximum velocity *c*, like gravitation, electromagnetism, and color interaction. Furthermore, this has remarkable parallels to the early interpretations of color interaction as a Yukawa force mediated by massive pions.

▪ SM does not contain any candidates for the dark matter particle required by observational cosmology

▪ SM has no explanation for the observed three generations of quarks and leptons

▪ SM has 29 parameters, which makes hard to accept as a complete theory

A starting point for an extended formulation of SM appears to be the fifth problem in the above list: Pauli-SU(2) weak interaction.

A plausible solution of the problem is the introduction of a SU(4) interaction with four charges and fifteen massless field bosons in analogy to the concept of the SU(3) color interaction with three charges (colors r g b), eight massless field-bosons (gluons) and eightfold symmetry introduced by Gell-Mann, Fritsch and Leutwyler in 1973.

SU(4) interaction, in the following called hypercolor, in analogy to the color interaction, yields a renormalizable quantum gauge field theory, with confinement and asymptotic freedom.

Pauli-SU(2) weak interaction becomes then the Yukawa weak force of the SU(4)-hypercolor interaction, and the mass of the Yukawa-bosons W and Z (~90 GeV) give the critical energy ($E_{hc} = 2m(Z) = 180\,\text{GeV}$) in analogy to the Callan-Symanzik color critical energy $E_{col} = 220\,\text{MeV}$.

So in reality the extended weak hypercolor force is roughly 1000 times stronger than the color force.

A plausible formulation of the four charges is hc = (*L*−, *L*+, *R*−, *R*+), where (+, −) is the electric charge, and (*L, R*) is the (left, right) chirality. The chirality χ is a fundamental invariant for spinors (left-chiral and right-chiral Weyl-spinors are components of a Dirac-bispinor).

This hc-charge definition is the only possible, because it has to encompass the electric charge (because of the electro-weak interaction) and chirality (because of the chiral asymmetry in SM).

With this hc-charge definition, there is a spontaneous symmetry breaking of the SU(4)-hc-interaction $SU(4)=SU(2)_L \otimes SU(1) \otimes SU(1)_{em}$

A remaining task is to find a sub-structure (preons), which unifies the basic components of SM: the 6 leptons and the 6 quarks. The simplest ansatz is introducing preons r and q with hc-charges, plus color-charge for q, with the parameters:

wave function $\Psi=(u_{L-},u_{L+},u_{R-},u_{R+})$

r-preons $(r_{L-},r_{L+},r_{R-},r_{R+})$, $Q(r)=-1/2$, $m(r)\ll 1\,\text{meV}$,

q-preons $(q_{L-},q_{L+},q_{R-},q_{R+})$, $Q(q)=+1/6$, $m(q)\sim 1\,\text{MeV}$, $Q_{col}(q)=(r,g,b)$

At first, such an ansatz based purely on symmetry aspects, seems risky to say the least.

Substructure ansatzes based on preons were proposed before (e.g. Harari [5]), and ended in speculations without concrete results.

Here enters the third component of a successful SM-extension: a new powerful and numerically relatively simple calculation method: direct minimization of action [6] [7]. This calculation method was introduced in [4] [7] and applied successfully in QCD for calculation of hadrons.

With these three ansatzes it is possible, as shown in the rest of this paper:

▪ to calculate numerically the mass hierarchy spectrum of the basic leptons and quarks in SM

▪ to explain naturally the huge differences of scale in energy-mass in SM, in particular the minuscule neutrino masses

▪ to explain naturally the three generations (simply by symmetry-compatible hc-boson configurations)

▪ to calculate in principle the mixing matrices CKM for quarks and PMNS for neutrinos (which explains also the neutrino oscillations)

▪ to reduce the number of parameters in SM from 29 to 7 parameters

Furthermore, reproducing by pure numeric calculation correctly the energy-mass spectrum of SM is as good as a direct experimental verification for proving the observational correctness of the extended SU(4)-preon-model (SU4PM).

Taken all this into account, it appears extremely lucky that such an ad-hoc model proved to be so successful both theoretically and experimentally. On the other hand, it is another example of the extreme importance and fundamental significance of symmetry aspects in physics.

In the following, we introduce in chap.2 the SU(4) gauge theory with 15 generalized Gell-Mann 4 × 4-matices as generators of the SU(4) Lie group.

In chap.3 we extend the SM to SU4PM by the introduction of the SU(4)-hypercolor interaction, and the two preons (*r, q*) as sub-particles of leptons and quarks.

In chap.4 the ansatz for wavefunctions, and the numerical algorithm are described.

In chap.5 we present the calculation results for energy-mass of the SM: the six leptons, the six quarks, and the interaction bosons W, Z, H (higgs), and some weakly interacting new particles, which arise from the ansatz.

In chap.6 we discuss some selected weak hadron decays.

2. SU(4) Gauge Theory

2.1. Gauge Theory

In the following, we consider the gauge theory QCD (quantum chromodynamics) based on SU(3) and the gauge theory QHCD (quantum hyper-color dynamics) based on SU(4) [8] [9].

The gauge invariant QCD Lagrangian is ($\hbar = c = 1$)

$$L = \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\psi - \frac{1}{4}F^{a}{}_{\mu\nu}F_{a}^{\mu\nu} \tag{1}$$

where $\psi_i(x)$ is the quark field, a dynamical function of spacetime, in the fundamental representation of the SU(3) gauge group, indexed by i j; $A^{a}{}_{\mu}(x)$ are the fields, also dynamical functions of spacetime, in the adjoint representation of the SU(3) or the SU(4) gauge group, indexed by a, b, … The γ^{μ} are Dirac matrices connecting the spinor representation to the vector representation of the Lorentz group.

The total field is $A^{a}{}_{\mu}(x) \equiv A^{a}{}_{\mu}(x)\lambda_a$ and the Dirac-conjugate $\bar{\psi}_i(x) = \psi_i^{c}(x)\gamma^{0}$, where ψ_i^{c} is the complex-conjugate.

D_{μ} is the gauge covariant derivative for calculation

$$D_{\mu} \equiv \partial_{\mu} - i\,g\,\tilde{A}^{a}_{\mu}\lambda_a \tag{2}$$

for simplicity, instead of $D_{\mu} \equiv \partial_{\mu} - i\,g\,A^{a}{}_{\mu}T^{a}$, with rescaled field $\tilde{A}^{a}_{\mu} \equiv A^{a}{}_{\mu}/2$, and where g is the coupling constant and $T^{a} = \lambda_a/2$ are the generators of the gauge group/algebra.

For the QCD based on SU(3) ([10] [11] [12] [13]), $A_{\mu}{}^{a}(x)$ is the (color) gluon gauge field, for eight different gluons $a = 1,\cdots,8$, $\psi(x)$ is a four-component Dirac spinor, and λ_a is one of the eight Gell-Mann matrices,

$$a = 1,\cdots,8$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{3}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

These matrices are traceless $Tr(\lambda_a) = 0$, Hermitian, and obey the extra trace orthonormality relation

$$Tr(\lambda_a\lambda_b) = 2\delta_{ab}$$

and commutation relations

$$[\lambda_a,\lambda_b]=2i\,\tilde{f}^{abc}\lambda_c,\quad \tilde{f}^{abc}=2f^{abc} \tag{4}$$

For the QHCD based on SU(4) $A_\mu{}^a(x)$ is the hc-boson field, for 15 hc-bosons and λ_a are the 15 generators of the SU(4), $a=1,\cdots,15$, the hc-matrices [14] [15] (in analogy to the 8 Gell-Mann matrices for the SU(3)):

$$\lambda_1=\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\quad \lambda_2=\begin{pmatrix}0&-i&0&0\\i&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\quad \lambda_3=\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} \tag{5}$$

$$\lambda_4=\begin{pmatrix}0&0&1&0\\0&0&0&0\\1&0&0&0\\0&0&0&0\end{pmatrix}\quad \lambda_5=\begin{pmatrix}0&0&-i&0\\0&0&0&0\\i&0&0&0\\0&0&0&0\end{pmatrix}$$

$$\lambda_6=\begin{pmatrix}0&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&0\end{pmatrix}\quad \lambda_7=\begin{pmatrix}0&0&0&0\\0&0&-i&0\\0&i&0&0\\0&0&0&0\end{pmatrix}\quad \lambda_8=\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-2&0\\0&0&0&0\end{pmatrix}$$

$$\lambda_9=\begin{pmatrix}0&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&0\end{pmatrix}\quad \lambda_{10}=\begin{pmatrix}0&0&0&-i\\0&0&0&0\\0&0&0&0\\i&0&0&0\end{pmatrix}$$

$$\lambda_{11}=\begin{pmatrix}0&0&0&0\\0&0&0&1\\0&0&0&0\\0&1&0&0\end{pmatrix}\quad \lambda_{12}=\begin{pmatrix}0&0&0&0\\0&0&0&-i\\0&0&0&0\\0&i&0&0\end{pmatrix}$$

$$\lambda_{13}=\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\quad \lambda_{14}=\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&-i\\0&0&i&0\end{pmatrix}\quad \lambda_{15}=\frac{1}{\sqrt{6}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-3\end{pmatrix}$$

The symbol $F^a{}_{\mu\nu}$ the gauge invariant field strength tensor, analogous to the electromagnetic field strength tensor, $F^{\mu\nu}$, in quantum electrodynamics. It is given by

$$F^a{}_{\mu\nu}=\partial_\mu A^a{}_\nu-\partial_\nu A^a{}_\mu+g\,f^{abc}A^b{}_\mu A^c{}_\nu,$$

rescaled $F^a{}_{\mu\nu}=\partial_\mu\tilde{A}^a_\nu-\partial_\nu\tilde{A}^a_\mu+g\,\tilde{f}^{abc}\tilde{A}^b_\mu\tilde{A}^c_\nu$

where f^{abc} resp. $\tilde{f}^{abc}$ are the structure constants of SU(3) or SU(4).

the generators $T^a=\lambda_a/2$ satisfy the commutator relations

$[T^a,T^b]=i\,f^{abc}T^c$, rescaled $[\lambda_a,\lambda_b]=i\,\tilde{f}^{abc}\lambda_c$

General Yang-Mills theory

Yang-Mills theories are a special example of gauge theory with a non-commutative symmetry group given by the Lagrangian [3]

$$L_{gf} = -\frac{1}{4} F^{a\mu\nu} F^{a}{}_{\mu\nu} \tag{6}$$

with the generators of the Lie algebra, indexed by *a*, corresponding to the *F*-quantities (the curvature or field-strength form) satisfying

$$Tr\left(T^a T^b\right) = \frac{1}{2}\delta^{ab} \quad \left[T^a, T^b\right] = i\, f^{abc} T^c \,,$$

where for SU(3) and SU(4) $T^a = \lambda_a/2$, and where the f^{abc} are structure constants of the Lie algebra, and the covariant derivative defined as

$D_\mu \equiv \partial_\mu - i\, g\, A^a{}_\mu T_a$ resp. $D_\mu \equiv \partial_\mu - i\, g\, \tilde{A}^a_\mu \lambda_a$, where $A^a{}_\mu$ is the field carrier, $\tilde{A}^a_\mu \equiv A^a{}_\mu/2$ is the rescaled field, and g is the coupling constant, and for a SU(N) group one has $N^2 - 1$ generators.

The relation for the field tensor

$$F^a{}_{\mu\nu} = \partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu + g\, f^{abc} A^b{}_\mu A^c{}_\nu$$

$$F^a{}_{\mu\nu} = \partial_\mu \tilde{A}^a_\nu - \partial_\nu \tilde{A}^a_\mu + g\, \tilde{f}^{abc} \tilde{A}^b_\mu \tilde{A}^c_\nu$$

follows from the commutator for the covariant derivative D_μ

$$\left[D_\mu, D_\nu\right] = -i\, g\, T_a F^a{}_{\mu\nu}$$

The field has the property of being self-interacting and equations of motion that one obtains are said to be semilinear, as nonlinearities are both with and without derivatives. This means that one can manage this theory only by perturbation theory, with small nonlinearities.

From the given Lagrangian one can derive the equations of motion given by

$$\partial^\mu F^a{}_{\mu\nu} + g\, f^{abc} A^{b\mu} F^c{}_{\mu\nu} = 0 \quad \text{(Yang-Mills-equations)}, \tag{7}$$

$$\text{resp.} \quad \partial^\mu F^a{}_{\mu\nu} + g\, \tilde{f}^{abc} \tilde{A}^{b\mu} F^c{}_{\mu\nu} = 0$$

which correspond to the Maxwell equations in electrodynamics $\partial^\mu F^a{}_{\mu\nu} = 0$, where $f^{abc} = 0$

Putting $F_{\mu\nu} = T^a F^a{}_{\mu\nu}$, these can be rewritten as

$$\left(D^\mu F_{\mu\nu}\right)^a = 0$$

The Bianchi identity holds

$$\left(D_\mu F_{\nu\kappa}\right)^a + \left(D_\kappa F_{\mu\nu}\right)^a + \left(D_\nu F_{\kappa\mu}\right)^a = 0$$

which is equivalent to the Jacobi identity

$$\left[D_\mu, \left[D_\nu, D_\kappa\right]\right] + \left[D_\kappa, \left[D_\mu, D_\nu\right]\right] + \left[D_\nu, \left[D_\kappa, D_\mu\right]\right] = 0 \quad \text{for Lie-groups}$$

since $\left[D_\mu, F^a{}_{\nu\kappa}\right] = D_\mu F^a{}_{\nu\kappa}$.

Define the dual strength tensor $\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$, then the Bianchi identity can be rewritten as

$$D_\mu \tilde{F}^{\mu\nu} = 0$$

A source current $J^a{}_\nu$ enters into the equations of motion (eom) as

$$\partial^\mu F^a{}_{\mu\nu} + g\, f^{abc} A^{b\mu} F^c{}_{\mu\nu} = -J^a{}_\nu$$

The Dirac part of the Lagrangian is

$$L_D = \bar{\psi}\left(i\hbar D_\mu \gamma^\mu - mc\right)\psi$$

with the resulting eom = gauge Dirac equation

$$\left(i\hbar D_\mu \gamma^\mu - mc\right)\psi = 0$$

2.2. The Running Coupling Constant of the QCD

We introduce the qq-potential (Cornell potential)

$$V(R,\beta) \approx V_0 - \frac{4}{3}\frac{\alpha\hbar c}{R} + \frac{\sigma R}{\hbar c} \quad \text{potential} = \langle q\bar{q}\rangle, \quad \sqrt{\sigma} \approx 440\ \text{MeV}$$

its measured values are shown below.

R_0 is the characteristic scale $R_0 \approx 0.49\ \text{fm}$, the scaling β-function is defined below.

Measured values of for different values of β are shown in **Figure 1**.

The data at β = 6.0, 6.2, 6.4 and 6.8 has been scaled by R_0, and normalized such that $V(R_0) = 0$. The collapse of the different sets of data on to a single curve after the rescaling by R_0 is evidence for scaling. The linear rise at large rR implies confinement [16] [17].

The color confinement results from $\lim\left(V(R), R \to \infty\right) = \infty$.

2.3. Callan-Symanzik Equation

The Callan-Symanzik equation describes the behavior of the transfer function of a Feynman diagram with n momentums [3]

$$G^{(n)}\left(x_1, x_2, \cdots, x_n; m, M, g\right),$$

where M = renormalization (cut-off) energy, g = coupling constant. ϕ = field strength, m = energy, with original and renormalized field $\phi = Z\phi_0$, transfer function $G^{(n)} = Z^{n/2} G_0{}^{(n)}$, under scaling transformation

$$g \to g + \delta g \quad M \to M + \delta M \quad \phi = Z\phi_0 \to Z'\phi_0 M = (1+\delta\eta)\phi$$

$$G^{(n)} \to (1 + n\delta\eta) G^{(n)}$$

From the cut-off independence

$$\frac{\partial}{\partial M} G_0{}^{(n)} = 0$$

we get the Callan-Symanzik equation

$$\left(M\frac{\partial}{\partial M} + \beta(g)\frac{\partial}{\partial g} + n\gamma + m\gamma_m \frac{\partial}{\partial m}\right) G^{(n)}\left(x_1, x_2, \cdots, x_n; m, M, g\right) = 0,$$

where $\gamma = -M\dfrac{\partial\eta}{\partial M} \quad \beta = M\dfrac{\partial g}{\partial M} \quad \gamma_m = \dfrac{M}{m}\dfrac{\partial\eta}{\partial M}$

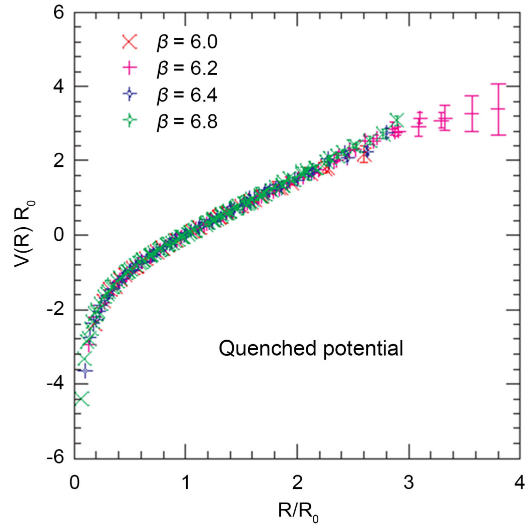


Figure 1. The static qq-potential in the quenched approximation obtained by the Wuppertal collaboration [16].

From the definition we get a differential equation for $g(M)$

$$M\frac{\partial g}{\partial M}+\beta(g)=0 \quad (8)$$

The running coupling for QCD is characterized by the β-function with colors $N = 3$, flavors $n_f = 3$, M = cut-off energy [16]

$$M\frac{\partial g}{\partial M}=-\beta(g)=-\left(\beta_0 g^3+\beta_1 g^5+\cdots\right)$$

$$\beta_0=\frac{1}{16\pi^2}\left(\frac{11}{3}N-\frac{2}{3}n_f\right)$$

$$\beta_1=\frac{1}{\left(16\pi^2\right)^2}\left(\frac{34}{3}N^2-\frac{10}{3}Nn_f-\frac{n_f}{N}\left(N^2-1\right)\right)$$

resulting in first order in $$g^2(m)=\frac{g^2(M)}{1+g^2(M)\beta_0\log\left(\frac{m^2}{M^2}\right)}$$

Which becomes for

$$m\to\infty \quad g(m)=\frac{1}{\sqrt{2\beta_0\log\left(\frac{m}{M}\right)}} \quad (9a)$$

$$\alpha_s(m)=\frac{g^2(m)}{4\pi}=\frac{1}{8\pi\beta_0\log\left(\frac{m}{\Lambda}\right)}=\frac{12\pi}{\left(11N-2n_f\right)\log\left(\frac{m^2}{\Lambda^2}\right)} \quad (9b)$$

α_s = coupling constant

where

$M = \Lambda \approx 220$ MeV critical energy of QCD, $\Lambda \approx m$(pion)2 = 280 MeV

$n_f = 3$: number of quark flavours

The corresponding critical length of QCD

$$r_{0c} = \frac{\hbar c}{\Lambda} = \frac{1.96 * 10^{-7}\ \text{eV} \cdot \text{m}}{220\ \text{MeV}} = 0.89 * 10^{-15}\ \text{m}$$

which is about the proton radius.

For energies $m \approx \Lambda$ we have the exact formula

$$g_c(m) = 4\pi \sqrt{\frac{3}{2(11N - 2n_f)\sqrt{\left(\log\left(\frac{m}{\Lambda}\right)\right)^2 + c_{GE0}{}^2}}}$$

$$= 4\pi \sqrt{\frac{1}{18\sqrt{\left(\log\left(\frac{m}{\Lambda}\right)\right)^2 + c_{GE0}{}^2}}}$$

for the numerical calculation we set $c_{GE0} = \frac{1}{\log\left(\frac{m(p)}{\Lambda_{QCD}}\right)} = 0.683$, which is consistent with the Callan-Symanzik relation for $m > 2\Lambda$, as shown in the plot **Figure 2** below.

2.4. The Running Coupling Constant of the QHCD

For the QHCD the Callan-Symanzik equation is still valid, as it is derived from the scale-independence of the theory.

The running coupling for QHCD with colors $N = 4$, flavors $n_f = 3$, Λ = transfer energy becomes in analogy to (9b)

$$\alpha_{hc}(m) = \frac{g^2(m)}{4\pi} = \frac{12\pi}{(11N - 2n_f)\log\left(\frac{m^2}{\Lambda_{hc}{}^2}\right)} \tag{10a}$$

Again, it must be corrected to avoid a singularity for

$$m = \Lambda_{hc} \tag{10b}$$

$$g_{hc}(m) = 4\pi \sqrt{\frac{3}{2(11N - 2n_f)\sqrt{\left(\log\left(\frac{m}{\Lambda_{hc}}\right)\right)^2 + c_{GE1}{}^2}}}$$

$$= 4\pi \sqrt{\frac{3}{76\sqrt{\left(\log\left(\frac{m}{\Lambda_{hc}}\right)\right)^2 + c_{GE1}{}^2}}}$$

we set $\Lambda_{hc} = 2m(Z_0) = 180\ \text{GeV}$ in analogy to the QCD, and $c_{GE1} = \frac{1}{\log\left(\frac{m(t)}{m(d)}\right)}$, with the masses of the top- and the d-quark: this should assess the logarithmic scale of the generation energy ratio.

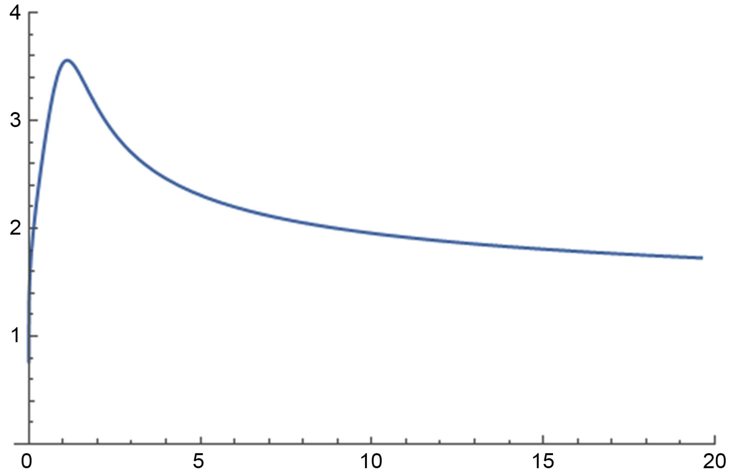


Figure 2. $g_c(m)$, m in E_0-units, E_0 = 196 MeV [18].

Both settings are of course only a plausible guess, but these values work very well for the preon model, as we will see.

The coupling constant g_{hc} for the QHCD is shown in the plot **Figure 3** below.

The peak is much higher than in QCD, which reflects the enormous span of the mass scale in the Standard Model.

The corresponding critical length of QHCD

$$r_{0hc} = \frac{\hbar c}{\Lambda_{hc}} = \frac{1.96 * 10^{-7}\,\text{eV} \cdot \text{m}}{180\,\text{GeV}} = 1.08 * 10^{-18}\,\text{m}$$

which is about 1/1000 of the proton radius: the energy scale of the QHCD is by a factor 1000 larger, and consequently the length scale by a factor 1000 smaller than in QCD. This agrees with the experimental assessment of the quark radius being about 1/1000 of the proton radius.

3. The Standard Model and QCD, the SU(4)-Preon Model and QHCD

The Standard Model of particle physics (SM) emerged in the mid 1970s as the universal theory of high-energy physics encompassing the electromagnetic, weak Pauli and strong color interactions, and based on a particle model with 6 basic lepton and 6 basic quark spinors in 3 generations (=flavors), plus field carrier bosons: 1 photon, 8 color gluons, 2 weak Pauli massive W-Z bosons, and scalar higgs H ([2] [3] [14] [20] [21] [22]).

The interactions of SM are described by SU(n) gauge theories: trivial SU(1) electromagnetic, SU(2) weak Pauli interaction, and SU(3) strong color interaction. The gauge charges are: n = 1 electromagnetic charge q, n = 2 the weak isospin $I_3 = \pm 1$, n = 3 the color c = (r, g, b).

The quarks form composite particles known as hadrons, among them the nucleons (p, n) which build the atomic nuclei, the leptons do not form composite particles.

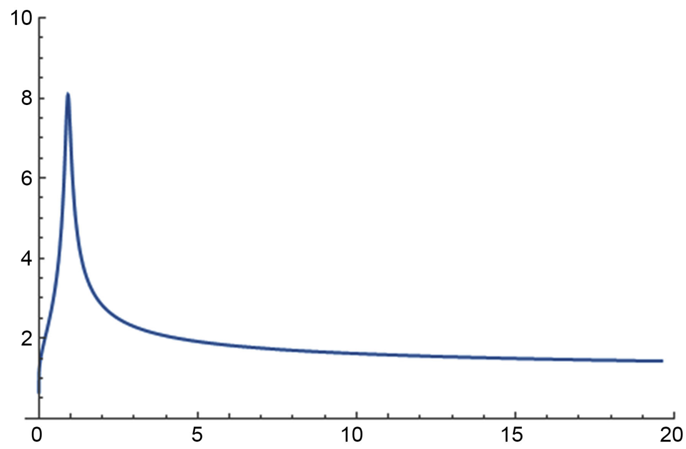


Figure 3. $g_{hc}(m)$, m in E_0-units, E_0 = 196 GeV [19].

The weak Pauli interaction breaks the chiral symmetry and becomes $SU(2)_L xSU(1)_R$ gauge interaction.

It combines via the Glashow-Weinberg mechanism with the electromagnetic interaction to become electroweak interaction $SU(2)_L(W)xSU(1)(Z)$ $xSU(1)(\gamma)$ with W-boson, Z-boson, photon.

Finally, the masses of the basic particles are generated via the Higgs mechanism through SU(n) symmetry breaking by the higgs H particle.

Based on this scaffold, the SM developped into a powerful theory, which describes all of particle physics correctly with no deviation from experiment until present.

3.1. Parameters of the Standard Model

Basic particles of the standard model [22]

The properties of the basic particles of the Standard Model are shown in **Table 1** below.

The quark radius: as of 2014, experimental evidence indicates they are no bigger than 10^{-4} times the size of a proton, *i.e.* less than 10^{-19} metres [23].

Field bosons

The following **Table 2** describes the basic bosons of the SM: 3 massive bosons W±, Z, H and 2 massless field-carriers: photon γ and gluon g.

Parameters Standard Model

The model has 28 parameters + fine-structure constant α_{em} [2] [21], as described in **Table 3** below.

3.2. The Basics of the Preon Model

The preon model describes the basic particles of the Standard Model (leptons, quarks and exchange bosons) as composed of smaller particles (preons), which obey a super-strong hyper-color interaction.

Examples are the rishon model (Harari 1979 [5] [24]) and the primon model (de Souza 2002 [25]).

Table 1. Basic particles of the Standard Model.

Fermion left-handed	Symbol	Electric charge	Weak isospin	Weak hyper-charge	Color charge	Mass
Generation 1						
electron	e^-	−1	−1/2	−1	1	511 keV
positron	e^+	+1	0	2	1	511 keV
e-neutrino	ν_e	0	+1/2	−1	1	<0.22 eV
e-antineutrino	$\bar{\nu}_e$	0	0	0	1	<0.22 eV
up-quark	u	+2/3	+1/2	+1/3	3	2.3 MeV
up-antiquark	$\bar{u}$	−2/3	0	−4/3	$\bar{3}$	2.3 MeV
down-quark	d	−1/3	−1/2	+1/3	3	4.8 MeV
down-antiquark	$\bar{d}$	+1/3	0	−2/3	$\bar{3}$	4.8 MeV
Generation 2						
muon	μ^-	−1	−1/2	−1	1	105.6 MeV
antimuon	μ^+	+1	0	2	1	105.6 MeV
mu-neutrino	ν_μ	0	+1/2	−1	1	<0.22 eV
mu-antineutrino	$\bar{\nu}_\mu$	0	0	0	1	<0.22 eV
charm-quark	c	+2/3	+1/2	+1/3	3	1275 MeV
charm-antiquark	$\bar{c}$	−2/3	0	−4/3	$\bar{3}$	1275 MeV
strange-quark	s	−1/3	−1/2	+1/3	3	95 MeV
strange-antiquark	$\bar{s}$	+1/3	0	−2/3	$\bar{3}$	95 MeV
Generation 3						
tau	τ^-	−1	−1/2	−1	1	1776.8 MeV
antitau	τ^+	+1	0	2	1	1776.8 MeV
tau-neutrino	ν_τ	0	+1/2	−1	1	<0.22 eV
tau-antineutrino	$\bar{\nu}_\tau$	0	0	0	1	<0.22 eV
top-quark	t	+2/3	+1/2	+1/3	3	173,210 MeV
top-antiquark	$\bar{t}$	−2/3	0	−4/3	$\bar{3}$	173,210 MeV
bottom-quark	b	−1/3	−1/2	+1/3	3	4180 MeV
bottom-antiquark	$\bar{b}$	+1/3	0	−2/3	$\bar{3}$	4180 MeV

Table 2. Field bosons of the Standard Model.

Particle	Charge	w.Isospin T	w.hcharge Y	Spin	Color	Lifetime	Mass
W±	±1	±1	0	1	0	3×10^{-25} s	80.4 GeV
Z	0	0	0	1	0	3×10^{-25} s	91.2 GeV
γ photon	0	0	0	1	0		0
g gluon	0	0	0	1	3		0
H higgs	0	0	0	0	0	10^{-22} s	125.1 GeV

Table 3. Parameters of the Standard Model [16], where electromagnetic fine-structure constant $\alpha_{em} = \frac{e_0^2}{4\pi} = \frac{1}{137}$.

Parameters of the Standard Model			
Symbol	**Description**	**Renormalization scheme (point)**	**Value**
m_e	Electron mass		511 keV
m_μ	Muon mass		105.7 MeV
m_τ	Tau mass		1.78 GeV
m_u	Up quark mass	μ_{MS} = 2 GeV	1.9 MeV
m_d	Down quark mass	μ_{MS} = 2 GeV	4.4 MeV
m_s	Strange quark mass	μ_{MS} = 2 GeV	87 MeV
m_c	Charm quark mass	$\mu_{MS} = m_c$	1.32 GeV
m_b	Bottom quark mass	$\mu_{MS} = m_b$	4.24 GeV
m_t	Top quark mass	On-shell scheme	172.7 GeV
θ_{12}	CKM 12-mixing angle	q flavor mixing	13.1˚
θ_{23}	CKM 23-mixing angle		2.4˚
θ_{13}	CKM 13-mixing angle		0.2˚
δ_{13}	CKM CP-violating Phase		0.995
θ_{12}	PMNS 12-mixing angle	ν flavor mixing	33.6˚ ± 0.8˚
θ_{23}	PMNS 23-mixing angle		47.2˚ ± 4˚
θ_{13}	PMNS 13-mixing angle		8.5˚ ± 0.15˚
δ_{13}	PMNS CP-violating Phase		4.1 ± 0.75
g_1 or g'	U (1) gauge coupling	$\mu_{MS} = m_Z$	0.357
g_2 or g	SU (2) gauge coupling	$\mu_{MS} = m_Z$	0.652
g_3 or g_s	SU (3) gauge coupling	$\mu_{MS} = m_Z$	1.221
Λ	crit. energy in SU (3)		220 MeV
c_{gE0}	additional log in col-coupling		0.69
θ_{QCD}	QCD vacuum angle		~0
v	Higgs vacuum expectation value		246 GeV
m_H	Higgs mass		125.36 ± 0.41 GeV
$m_{\nu e}$	electron neutrino mass		≤0.12 eV
$m_{\nu\mu}$	mu neutrino mass		≤0.12 eV
$m_{\nu\tau}$	tau neutrino mass		≤0.12 eV
α_{em}	fine-structure constant		1/137

The rishon model

In the rishon model, there are two preons (called rishons) T (charge +1/3e) and V (charge 0). Leptons and quarks and exchange bosons are built of 3 rishons. They obey a hc-interaction based on SU(3), the 3-rishon combinations have the (color)x(hyper-color) representation $SU(3)_c xSU(3)_{hc}$

TTT = antielectron

VVV= electron neutrino

TTV, TVT, VTT = three colours of up quarks

TVV, VTV, VVT = three colours of down antiquarks

$\overline{T}\overline{T}\overline{T}$ = electron

$\overline{V}\overline{V}\overline{V}$ = electron antineutrino

$\overline{T}\overline{T}\overline{V}$, $\overline{T}\overline{V}\overline{T}$, $\overline{V}\overline{T}\overline{T}$ = three colours of up antiquarks

$\overline{T}\overline{V}\overline{V}$, $\overline{V}\overline{T}\overline{V}$, $\overline{V}\overline{V}\overline{T}$ = three colours of down quarks

W^+ boson = $TTTVVV$

Generations are explained as excited states of the first generations, mass is not explained.

The primon model

In the primon model there are four preons (called primons) (p_1, p_2, p_3, p_4), which carry charge (+5/6, −1/6, −1/6, −1/6) and hc-charge, they obey a hc-interaction based on SU(2).

Quarks are built of two primons:

u (p_1, p_2), c (p_1, p_3), t (p_1, p_4), d (p_2, p_3), s (p_2, p_4), b (p_3, p_4), leptons are non-composite, there are 3 non-composite Higgs-bosons.

Generations are explained as primon-configuration, the mass spectrum is only qualitatively explained

Requirements for the preon model

The two basic ideas of the preon model (PM) are

-the basic particles of the Standard Model (SM) are composed of a few fundamental fermions

-there is a super-strong hyper-color interaction, with massless field bosons

A successful PM should uphold the symmetries and invariances of the SM and solve its main problems:

-PM should encompass the preservation of the baryon and lepton number

-PM should explain and derive the generations (flavor) of the SM and their energy scales

-PM should explain the allowed and not-allowed decay modes and the flavor-mixing of the SM

-PM should correctly calculate the mass spectrum, and explain the huge difference in mass scale between leptons and quarks, and between the generations: m(neutrino ν_e)~10^{-4} eV, m(top quark t) = 170 GeV, which makes a factor of 10^{15}

-PM should describe the weak exchange bosons W, Z, and the higgs H as Yukawa-bosons of the hc-interaction,

as all other fundamental field bosons graviton $A^{\mu\nu}$, photon A^{μ}, gluon A_c^{μ} are massless waves; the field bosons A_{hc}^{μ} of hc should be also massless

-hc interaction should be stronger the SU(3)-color interaction and should encompass the weak SU(2), also it should reproduce the spontaneous symmetry breaking of the electroweak symmetry group

$SU(2)_{L,ch\text{-}weak} \otimes SU(1)_{n\text{-}weak} \otimes SU(1)_{em}$ with their exchange bosons $\{W^\mu\} \otimes \{Z^\mu\} \otimes \{A^\mu\}$ and corresponding currents {charged-weak} ⊗ {neutral-weak} ⊗ {electromagnetic}.

-PM should reduce the 28 parameters of the SM to very few fundamental parameters.

3.3. Realization of the SU(4) Preon Model

The SU(4) preon model (SU4PM) is based essentially on two assumptions

-The SU4PM postulates two basic Weyl-spinors $\{r, q\}$ as the fundamental particles and the SU(4) as the gauge group of the hc-interaction, with spin $S = 1/2$, with electrical charge $Q_e = \{-1/2, 1/6\}$ and color charge $Q_c = \{0, 1\}$

-The field-bosons are the 15 generators $A_{hc}{}^\mu$ of the SU(4), described by the 15 standard generator 4 × 4 matrices λ_i of the SU(4). The SU(4) has 4 hc-charges: {chirality L, chirality R, electrical charge +, electrical charge - } in analogy to the 3 color charges of the SU(3): {r, g, b}.

From these assumptions follow the basic particle families of

-leptons $L = r \otimes r$ being a hc-tetra-spinor of a doublet of two r-preons, fermions with total spin $S = 1/2$

-quarks $Q = r \otimes q$ being a hc-tetra-spinor of a doublet of an r- and a q-preon, colored fermions with color $Q_c = 1$ with total spin $S = 1/2$

-(hypothetical) strong neutrinos $N_c = q \otimes q$ being a hc-tetra-spinor of a doublet of two q-preons, colored fermions with color $Q_c = 0$ with total spin S = 1/2

-weak bosons $B_w = r \pm r$ being a linear combinations of two or more r-preons, with total spin $S = 0$ (scalar like higgs H) or $S = 1$ (vector like W and Z)

-(hypothetical) strong bosons $B_c = q \pm q$ being a linear combinations of two or more q-preons, with color $Q_c = 0$ and total spin $S = 0$ (scalar like higgs H_q) or $S = 1$ (vector like Z_q)

A a hc-tetra-spinor is a hc-quadruplet with the hc-charges $\{L-, L+, R-, R+\}$.

Both preons can carry all four charges of SU(4), *i.e.* there are $\{rL-, rL+, rR-, rR+\}$ and $\{qL-, qL+, qR-, qR+\}$, where the spinor-anti-spinor pairs are $\{rL-, rR+\}$ and $\{rL+, rR-\}$.

The r-q-doublets, *i.e.* the quarks, have one more degree of freedom, as they consist of different fermions, and are therefore chiral-neutral, which is energetically more favorable.

A hc-doublet occupies two positions in a hc-tetra-spinor with indices (i, j), e.g the e-neutrino with the configuration $\{rL-, rL+, 0, 0\}$ has the hc-indices $(1,\bar{2})$, the bar over 2 signifies the anti-spinor.

One can show, that for two hc-indices $\{i, j\}$ there are three field-boson configurations, which are compatible with the SU(4) symmetry: one boson A_{ij} (corresponding to the non-diagonal hc-matrix $\tilde{\lambda}_{ij}$ interchanging i with j, e.g. for $(i,j)=(1,2)$ $\tilde{\lambda}_{ij} = \lambda_1$), four bosons $A_{ij}, \bar{A}_{ij}, A_{kl}, \bar{A}_{kl}$ (interchanging resp. (i,j), $(i,\bar{j})$, and the dual index pairs (k,l), $(k,\bar{l})$), and all 15 bosons as the third configuration. These correspond to the three generations (flavors) of the

SM, as the calculation shows.

Basic parameters of SU4PM

We have 6 parameters for SU4PM: 2 preon masses, and hyper-color/SU4 interaction the critical energy Λ_{hc} and the peak height constant c_{GE1}. Furthermore, we still have the corresponding 2 parameters of the color/SU3 interaction: the critical energy Λ_c and the peak height constant c_{GE0}.

The 4 interaction parameters have been derived in chap. 2.

For the mass of the r-preon, we make a guess of *m*(e-neutrino)/3: in the lightest lepton, the e-neutrino, there are two r-preons and one hc-boson, so *m*(*r*) will be approximately 1/3 of the assessed *m*(e-neutrino): this is assumed to be 1/1000 (1000 = approximate factor for flavor 3) of the best upper limit for *m* (tau-neutrino) = 0.1 eV.

For the mass of the q-preon, we take 1/3 of mass(u-quark) the lightest quark, in analogy to the r-preon.

preon data

r-preons {*rL*–, *rL*+, *rR*–, *rR*+}

Q(*r*) = −1/2, *m*(*r*) = 0.033 meV

q−preons {*qL*–, *qL*+, *qR*–, *qR*+}

Q(*q*) = +1/6, *m*(*q*) = 0.77 MeV

coupling constant of hc-interaction

The coupling from the Callan-Symanzik equation must be corrected to avoid a singularity for $\mu = \Lambda_{hc}$

$$g_{hc}(m) = 4\pi \sqrt{\frac{3}{76\sqrt{\left(\log\left(\frac{m}{\Lambda_{hc}}\right)\right)^2 + c_{GE1}{}^2}}} \tag{11}$$

we set $\Lambda_{hc} = 2m(Z_0) = 180\ \text{GeV}$ in analogy to the QCD, and

$$c_{GE1} = \frac{1}{\log\left(\frac{m(t)}{m(d)}\right)} = 0.095$$

The configuration of the SM in the SU4PM

Every basic particle of the SM is assigned a preon and a hc-boson configuration.

The preon configuration of a fermion (leptons and quarks) occupies two of the 4 positions in a hc-quadruplet by a Dirac-bispinor, e.g. for electron with index pair (*1, 3*) we have $\begin{pmatrix} rL- \\ 0 \end{pmatrix}$ in position 1 and $\begin{pmatrix} rR- \\ 0 \end{pmatrix}$ in position 3, according to the hc-charge. The hc-quadruplet has the hc-charges (*L*–, *L*+, *R*–, *R*+).

There are 3 possible hc-boson configurations for an index-pair (*i, j*), which are consistent with the SU(4)-symmetry: 1 hc-boson *Aij* corresponding to first generation of flavor = 1, 4 hc-bosons $Aij + \bar{A}ij + Akl + \bar{A}kl$ corresponding to flavor = 2 (the bar specifies the conjugate coupler, and (*k, l*) is the complementary index pair, e.g. for electron it is (*2, 4*)), and finally all 15 hc-bosons corresponding to flavor = 3.

The fermions (leptons and quarks) have two independent preon-components *u*1 and *u*2, they form a bispinor with spin $S = 1/2$.

The bosons (weak boson W, Z, H) have only one independent preon-component u1, which is a linear combination of two preons, the spins add up to $S = 1$ for W and Z, or to $S = 0$ for H, e.g. for Z = Z0 $u1 = \left(\left(rL - \right) + \left(rR - \right) \right) / \sqrt{2}$ and $Z0 = \left(\begin{pmatrix} u1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ u1 \end{pmatrix} \right) / \sqrt{2}$. The weak bosons W and Z0 are carrier of the residual weak interaction, and the higgs H generates masses for all r-containing particles: leptons, quarks, weak bosons and the r-preon itself.

The SU4PM predicts the existence of hypothetical strong neutrinos, which consist of $q\bar{q}$ with electrical charge $Q = 0$ and color charge $Q_c = 0$. They are heavy (*m*(qnu) = 23.2 MeV) practically non-interacting particles: the interact only via very heavy q-boson Zq (*m*(Zq) = 644 GeV)), *i.e.* they interact only at high resonance energies with small cross-sections. There is a new hypothetical model for Dark Matter called SIMP with mass around 100 MeV and interacting strongly at high resonance energies [26]. The strong-neutrinos do fit into this category.

Furthermore, the SU4PM predicts the existence of strong bosons Zq and Hq, in analogy to weak bosons Z0 and H, built of q-preons instead of r-preons. the strong neutrinos interact with themselves via Zq, and Hq generates masses for strong neutrinos and the q-preon.

The decay of neutron and pion requires (to safeguard the conservation of hc-charge) the existence of further weak neutrinos: the non-chiral (sterile) neutrinos with masses similar to lepton neutrinos. The nc-neutrinos are neutral, non-chiral, and interact with themselves and lepton neutrinos via the weak ZL-boson similar to the Z0, but left-chiral.

The SU4PM SU(4) symmetry is spontaneously broken into the electroweak symmetry group

$SU(2)_{L,ch\text{-}weak} \otimes SU(1)_{n\text{-}weak} \otimes SU(1)_{em}$ with their exchange bosons $\{W^\mu\} \otimes \{Z^\mu\} \otimes \{A^\mu\}$ and corresponding currents {charged-weak} ⊗ {neutral-weak} ⊗ {electromagnetic}.

The basic particle families in the SU4PM representation of the Standard Model are shown in the schematic Table 4 below.

4. The Calculation Method of the SU(4)-Preon Model

We apply for the calculation of the parameters of SM particles the numerical minimization of action, using a Ritz.Galerkin expansion for the hc-bosons and a parameterized gaussian for the preons.

4.1. The Ansatz for the Wavefunction

Hc-boson wavefunction

Table 4. Particle configurations in the SU4PM representation of the Standard Model.

charged leptons {e, mu, tau}

$$x=\left(\begin{pmatrix} rL- \\ 0 \end{pmatrix},0,\begin{pmatrix} rR- \\ 0 \end{pmatrix},0\right)$$

$e=x+A13$ flavor F=1 one boson

$mu=x+A13+\bar{A}13+A24+\bar{A}24$ F=2: four bosons

$tau=x+A$ F=3: all bosons

lepton neutrinos {nue, num, nut}

$$x=\left(\begin{pmatrix} rL- \\ 0 \end{pmatrix},\begin{pmatrix} 0 \\ rL+ \end{pmatrix},0,0\right)$$

$nue=x+A12$

$num=x+A12+\bar{A}12+A34+\bar{A}34$

$nut=x+A$

u-quarks {u,c,t}

$$x=\left(0,\begin{pmatrix} (rL++qL+)/\sqrt{2} \\ (rL++qL+)/\sqrt{2} \end{pmatrix},0,\begin{pmatrix} (rR++qR+)/\sqrt{2} \\ (rR++qR+)/\sqrt{2} \end{pmatrix}\right)$$

$u=x+A24$

$c=x+A24+\bar{A}24+A13+\bar{A}13$

$t=x+A$

sterile neutrinos {nus1,nus2,nus3}

$$x=\left(\begin{pmatrix} rL- \\ 0 \end{pmatrix},0,0,\begin{pmatrix} 0 \\ rR+ \end{pmatrix}\right)$$

$nus1=x+A14$

$nus2=x+A14+\bar{A}14+A23+\bar{A}23$

$nus3=x+A$

d-quarks {d, s, b}

$$x=\left(\begin{pmatrix} (rL-+qL+)/\sqrt{2} \\ 0 \end{pmatrix},0,\begin{pmatrix} (rR-+qR+)/\sqrt{2} \\ 0 \end{pmatrix},0\right)$$

$d=x+A13$

$s=x+A13+\bar{A}13+A24+\bar{A}24$

$b=x+A$

weak massive bosons {W, Z0, ZL, H}
F=3, all A

$$W=\left(0,0,\begin{pmatrix} u1 \\ 0 \end{pmatrix},0\right)\sqrt{2} \qquad u1=((rR-)-(rR-))/\sqrt{2}$$

$$Z0=\left(\begin{pmatrix} u1 \\ 0 \end{pmatrix},\begin{pmatrix} 0 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ 0 \end{pmatrix},\begin{pmatrix} 0 \\ u1 \end{pmatrix}\right)/\sqrt{2} \qquad u1=((rL-)+(rR-))/\sqrt{2}$$

$$ZL=\left(\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ u1 \end{pmatrix},0,0\right)/\sqrt{2} \qquad u1=((rL-)+(rL+))/\sqrt{2}$$

$$H=\left(\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ u1 \end{pmatrix}\right)/2 \qquad u1=((rL-)+(rL+)+(rR-)+(rR+))/2$$

strong neutrinos {qnue, qnum, qnut}

$$x=\left(\begin{pmatrix} qL- \\ 0 \end{pmatrix},0,0,\begin{pmatrix} 0 \\ qR+ \end{pmatrix}\right)$$

$qnue=x+A14$

$qnum=x+A14+\bar{A}14+A23+\bar{A}23$

$qnut=x+A$

strong massive bosons {Zq, Hq}
F=3, all A

$$Zq=\left(\begin{pmatrix} u1 \\ 0 \end{pmatrix},\begin{pmatrix} 0 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ 0 \end{pmatrix},\begin{pmatrix} 0 \\ u1 \end{pmatrix}\right)/\sqrt{2} \qquad u1=((qL-)+(qR-))/\sqrt{2}$$

$$Hq=\left(\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ u1 \end{pmatrix},\begin{pmatrix} u1 \\ u1 \end{pmatrix}\right)/2 \qquad u1=((qL-)+(qL+)+(qR-)+(qR+))/2$$

For the hc-boson wavefunction we apply here the full Ritz-Galerkin series on the function system

$$f_k(r,\theta)=\left\{bfunc(r,r_0,dr_0)r^{k_1},k_1=0,\cdots,n_r\right\}\times\left\{\left(\cos^{k_2}\theta,\cos^{k_2}\theta\sin\theta\right),k_2=0,\cdots,n_\theta\right\}$$

with coefficients α_k, where $bfunc(r,r_0,dr_0)=\dfrac{1}{1+\exp\left(\dfrac{r-r_0}{dr_0}\right)}$ is a Fermi-stepfunction which limits the region $r \le r_0$ of the preon with "smearing width" dr_0.

$$Ag_i(t,r,\theta) = \left\{ \begin{pmatrix} Ag_{i1}(t,r,\theta)\cos aA_i \\ Ag_{i2}(t,r,\theta)\cos aA_i \\ Ag_{i1}(t,r,\theta)\sin aA_i \\ Ag_{i2}(t,r,\theta)\sin aA_i \end{pmatrix}, i = 1,\cdots,15 \right\} \tag{12}$$

where aA_i is the phase angle between the particle and the anti-particle part of the hc-boson, and with the Ritz-Galerkin-expansion

$$Ag_{kl}(t,r,\theta) = \sum_j \alpha[k,l,j] f_j(r,\theta)\exp(-it\,EA_k),\; k = 1,\cdots,15; l = 1,2$$

with energies EA_k

Because of hc-symmetry, the active (non-zero) hc-bosons are

$Ag = \{Ag_1,\cdots,Ag_{15}\}$ all hc-bosons: generation 3, flavor = 3

$Ag = \{Ag_{ij}, \bar{A}g_{ij}, Ag_{kl}, \bar{A}g_{kl}\}$ 4 hc-bosons: coupler and anti-coupler for hc-indices (*i*, *j*) and the corresponding 2 coupler-anti-coupler pair for the complementary indices (*k*, *l*): generation 2, flavor = 2

$Ag = \{Ag_{ij}\}$ one hc-boson for the hc-indices (*i*, *j*): generation 1, flavor = 1.

Preon wavefunction

The hc-quadruplet has 4 positions with the hc–charges {*L*–, *L*+, *R*–, *R*+}, and the particle wavefunction of a fermion (lepton or quark) has two positions occupied with indices (*i*, *j*)

$u = \{..(u_1)...(u_2)...\}$ u_1 and u_2 are preon Weyl spinors with 2 components.

For the preons we use here a model of a gaussian "blob"

$$u_k(t,r,\theta) = \begin{pmatrix} \exp(-it\,Eu_k)\exp\left(-\frac{(\vec{r}-\vec{r}_{u,k})^2}{2\,dr_{u,k}}\right)\cos a_k \\ \exp(-it\,Eu_k)\exp\left(-\frac{(\vec{r}-\vec{r}_{u,k})^2}{2\,dr_{u,k}}\right)\sin a_k \end{pmatrix} \tag{13}$$

where Eu_k is the energy, $\vec{r}_{u,k} = (ru_k, \theta u_k)$ and $dr_{u,k}$ is the position (*r*, *θ*) and its width, a_k is a phase.

A basic particle of the Standard Model consists of 2 preons u_i and 1, 4, 15 hc-bosons Ag_i for generation 1, 2, 3 respectively. The hc-boson number *i* of Ag_i is equal to the general Gell-Mann matrix λ_4.

For instance, the electron has one hc-boson $Ag_4 = A13$ corresponding Gell-Mann matrix λ_4, and the preon configuration

electron e = (*rL*–, *rR*–), occupied positions (1, 3)

electron configuration: $u = \left(\begin{pmatrix} r_{L-} \\ 0 \end{pmatrix}, 0, \begin{pmatrix} r_{R-} \\ 0 \end{pmatrix}, 0\right)$

Antiparticle positron configuration $\bar{u} = \left(0, \begin{pmatrix} 0 \\ r_{L+} \end{pmatrix}, 0, \begin{pmatrix} 0 \\ r_{R+} \end{pmatrix}\right)$

The SU(4) Lagrangian

From 2.1 we have for the SU(4) Lagrangian

$$L_{QHCD} = \bar{u}\left(i\gamma^{\mu}D_{\mu} - m\right)u - \frac{1}{4}F^{a}{}_{\mu\nu}F_{a}^{\mu\nu},$$

where u is the particle (lepton or quark) wave function defined above, and the covariant derivative is $D_{\mu} = \partial_{\mu} - i\,g\,Ag^{a}{}_{\mu}\lambda_{a}$ with SU(4) Gell-Mann 4 × 4 matrices λ_a ($a = 1,\cdots,15$) and the field tensor is

$$F_{a,\mu\nu} = \partial_{\mu}\left(Ag_{a}\right)_{\nu} - \partial_{\nu}\left(Ag_{a}\right)_{\mu} + g\,f^{abc}\left(Ag_{b}\right)_{\mu}\left(Ag_{c}\right)_{\nu},$$

where Ag_a are the hc-boson wavefunctions ($a = 1,\cdots,15$).

The action is $S = \int_V L_{QHCD}\left(x^{\mu}, u_i, Ag_i\right) r^2 \sin\theta\, dt\, dr\, d\theta\, d\phi$, which is to be integrated over the particle volume V and minimized in the parameters of u and Ag^{a}.

The parameters of the component preons and the hc-bosons within a particle are (see below):

$$par\left(u_i\right) = \left\{Eu_i, a_i, ru_i, \theta u_i, dru_i\right\},\quad par\left(Ag_i\right) = \left\{EA_i, aA_i\right\},$$

where Eu_i and EA_i are energies, a_i and aA_i are internal phases, $\left(ru_i, \theta u_i, dru_i\right)$ describe particle's location and smear-out.

The calculation method of minimization of SU(4) action is shown below for the electron in a schematic **Table 4(a)**.

4.2. The Numerical Algorithm

The energy, length, and time are made dimensionless by using the units: E ($E_0 = \frac{\hbar c}{1am} = 0.196\ \text{TeV}$), r(fm), t(am/c) am = 10^{-18} m. We can assume axial symmetry, so we can set $\varphi = 0$ and use the spherical coordinates

$$(t, r, \theta).$$

We choose the equidistant lattice for the intervals $(t, r, \theta) \in [0,1]\times[0,1]\times[0,\pi]$ with 21 × 21 × 11 points and, for the minimization 8x in parallel, 8 random sublattices [4] [19]:

$$l[ix, j] = \left\{\left\{\left(t_{i1}, r_{i2}, t_{i3}\right) \mid \left(i1, i2, i3\right) = random\left(lattice, j = 1,\cdots,100\right)\right\} \mid ix = 1,\cdots,8\right\}.$$

For the Ritz-Galerkin expansion in (r,θ) we use the 12 functions

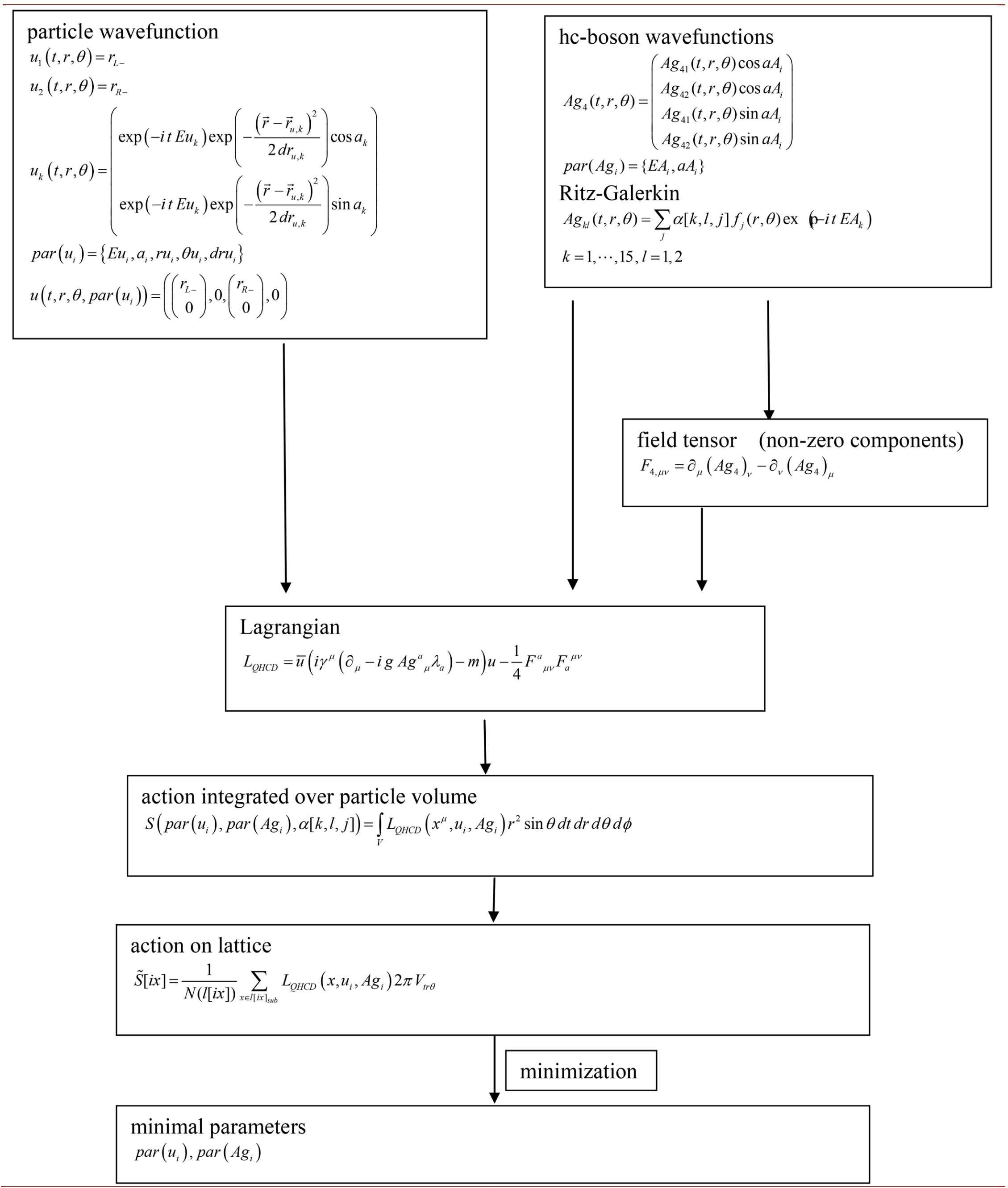
$$f_k\left(r,\theta\right) = \left\{bfunc\left(r, r_0, dr_0\right) r^{k_1}, k_1 = 0,\cdots,n_r\right\} \times \left\{\left(\cos^{k_2}\theta, \cos^{k_2}\theta\sin\theta\right), k_2 = 0,\cdots,n_{\theta}\right\}$$

The action $S = \int L_{QHCD}\left(x^{\mu}, u_i, Ag_i\right) r^2 \sin\theta\, dt\, dr\, d\theta\, d\phi$ becomes a mean-value on the sublattice $l[ix]$

$$\tilde{S}[ix] = \frac{1}{N\left(l[ix]\right)} \sum_{x \in l[ix]_{sub}} L_{QHCD}\left(x, u_i, Ag_i\right) 2\pi V_{tr\theta},$$

where $V_{tr\theta} = \pi$ the (t, r, θ)-volume and $N\left(l[ix]\right)$ is the number of points. We set $N\left(l[ix]\right) = 100$ for generation 1 and 2, $N\left(l[ix]\right) = 25$ for generation 3.

Table 4(a). Minimization of SU (4) action for the electron.



We impose the boundary condition for $Ag_i(r=r_0)=0$ via penalty-function (imposing exact conditions is possible, but slows down the minimization process enormously).

$\tilde{S}$ is minimized 8x in parallel with the Mathematica-minimization method "simulated annealing".

The proper parameters of the component preons and the hc-bosons within a particle are:

$$par(u_i)=\{Eu_i,a_i,ru_i,\theta u_i,dru_i\},\quad par(Ag_i)=\{EA_i,aA_i\}$$

Eu_i is the energy-mass of the preon u_i

a_i is sin(phase) of the preon u_i, where phase is the phase between the two spinor components

$(ru_i, \theta u_i)$ is the location of the preon u_i

dru_i is the uncertainty (stdev) of ru_i

EA_i is the energy of the hc-boson Ag_i

aA_i is sin(phase) of the hc-boson Ag_i, where phase is the phase between the two upper and the two lower components of the vector Ag_i

The complexities and execution times (on a 2.7 GHz Xeon E5 work-station) differ greatly for different generations.

For the generation 1 electron $e = \left(\begin{pmatrix} rL- \\ 0 \end{pmatrix}, 0, \begin{pmatrix} rR- \\ 0 \end{pmatrix}, 0 \right)$ with 1 hc boson A13:

complexity (Lagrangian) = 6.2 × 10^6 terms, minimization time t (minimization) = 37 s.

For the generation 3 tauon $\tau = \left(\begin{pmatrix} rL- \\ 0 \end{pmatrix}, 0, \begin{pmatrix} rR- \\ 0 \end{pmatrix}, 0 \right)$ with all 15 hc-bosons:

complexity (Lagrangian) = 283 × 10^6 terms, minimization time t (minimization) = 2500 s.

5. The Particles and Families of the SU(4)-Preon Model

Here we present the result of the calculation of the masses, inner structure, and some of the angles of the mixing matrices CKM and PMNS, using the minimization of the action described in chap.4.

5.1. Charged Leptons Electron, Muon, Tau

Spin S = 1/2, two free preons, occupying fixed positions in the hc-tetra-spinor

Preon configuration: $u = \left(\begin{pmatrix} rL- \\ 0 \end{pmatrix}, 0, \begin{pmatrix} rR- \\ 0 \end{pmatrix}, 0 \right)$

Boson configuration: flavor = 1: $(A13 = \lambda 4)$, flavor = 2: $(A13 = \lambda 4, \bar{A}13 = \lambda 5, A24 = \lambda 11, \bar{A}24 = \lambda 12)$

flavor = 3: all 15 bosons

The leptons are charged particles, they interact electromagnetically or weakly via Z and W bosons.

The leptons are spherically symmetric, and have therefore the gyromagnetic ratio g = 2 exactly, which is valid from the Dirac-equation for a point-like (or spherically symmetric) spin-1/2-particle.

The spherical symmetry arises from the fact, that all leptons consist of two r-preons, which differ only in the hc-charge, so it is plausible that their geometric parameters are equal (equal radius r_i, its uncertainty dr_i, equal phase angle a_i, and inter-preon-angle th = 0), as is shown in calculation.

In the energy distribution, the preons (shown in the first two values: i = (1, 2)) have considerably less energy than the hc-bosons in the case of the muon and the tauon, for the electron the only hc-boson carries almost all of the energy.

The calculated and observed masses of the charged leptons are shown in Table 5.

The energy of component preons and field bosons are shown in Figures 4-6.

The structure, *i.e.* calculated average distances of components with smear-out are shown in Figure 7.

The parameters of the three generations (flavors) are shown in Tables 6-8.

Table 5. Charged lepton masses.

	m (e)	m (mu)	m (tau)
exp.	0.511 MeV	106 MeV	1.78 GeV
calc.	0.29 ± 0.23 MeV	228 ± 150 MeV	2.26 ± 0.7 GeV

Table 6. Parameters of the electron.

Eu_i (MeV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
0.0256, 0.0256	0.241	−0.27, −0.27	−0.017	0.104, 0.104	0.276, 0.276	0
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
0.057, 0.044	0.121	.		0.058, 0.058	0.014, 0.014	

Table 7. Parameters of the muon.

Eu_i (MeV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
24.06, 24.06	0.00036, 0.0013, 46.33, 133.75	−0.48, −0.48	0.24, 0.266, −0.55, −0.632	0.648, 0.648	0.68, 0.68	0
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
18.32, 18.32	0.00045, 0.0011, 30.89, 87.17	.		0.045, 0.045	0.047, 0.047	

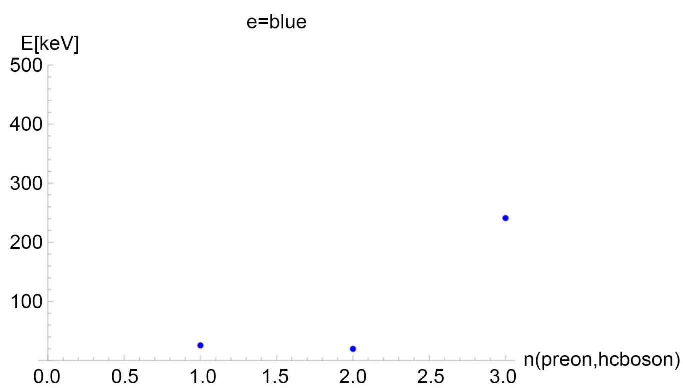


Figure 4. Energy distribution of electron: first preons ($u1$, $u2$), then bosons Ag_i.

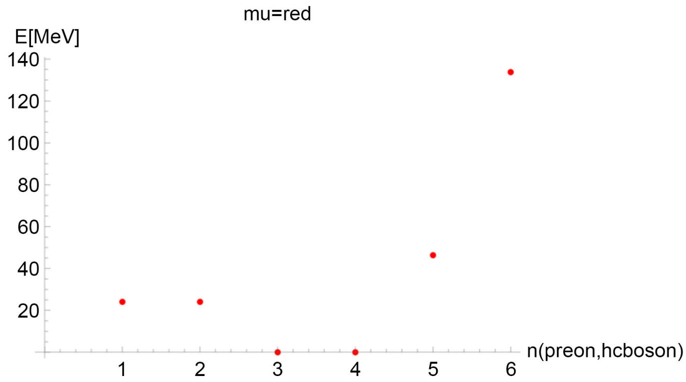


Figure 5. Energy distribution of muon: first preons ($u1$, $u2$), then bosons Ag_i.

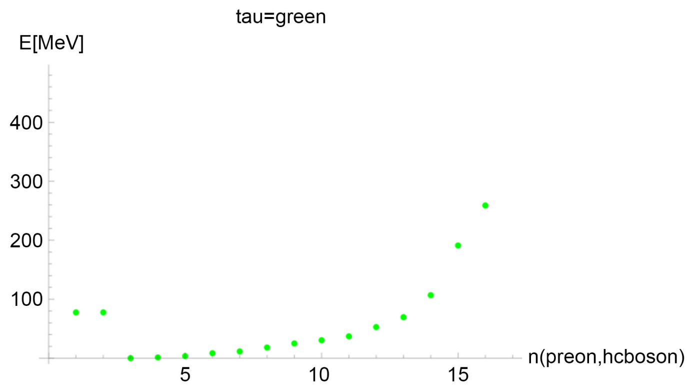


Figure 6. Energy distribution of tauon: first preons ($u1$, $u2$), then bosons Ag_i.

Table 8. Parameters of the tauon.

Eu_i MeV	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
77.68, 77.68	0.000258, 1.274, 3.51, 8.51, 11.45, 18.12, 25.0369, 30.46, 37.057, 52.78, 69.55, 106.83, 191.129, 259.009, 1297.48	0.216842, 0.216842	−0.33192, −0.0188942, −0.0449149, −0.325663, −0.0118209, \ −0.0943335, −0.226005, −0.149676, 0.143007, 0.0745547, 0.102575, −0.154493, −0.0987211, −0.161108, −0.0258635	0.19, 0.19	0.36, 0.36	0
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
77.66, 77.66	0.00028103, 1.68893, 2.36353, 5.65246, 6.56911, 9.40924, 11.9228, 11.9599, 15.7698, 30.2164, 34.4179, 17.5376, 107.57, 106.864, 180.17	.		0.033, 0.033	0.076, 0.077	

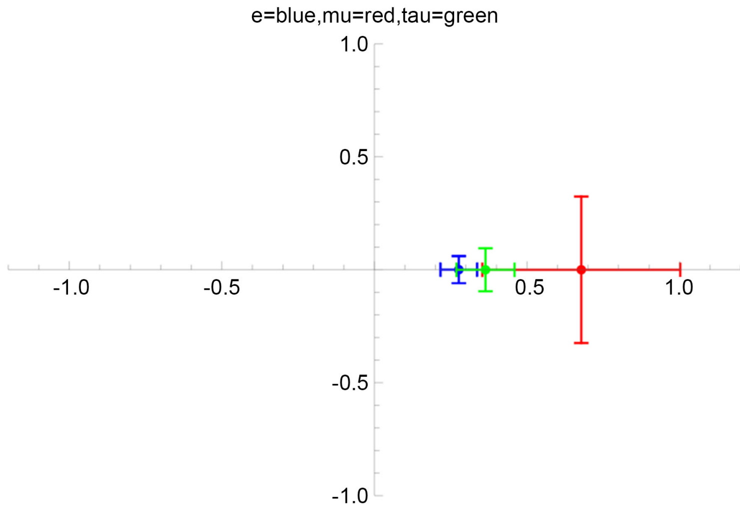


Figure 7. Structure of charged leptons: preons ($u1$, $u2$) radii r_i, uncertainty dr_i and angle th.

electron e = (*rL*−, *rR*−)

Preon configuration: $u = \left(\begin{pmatrix} rL- \\ 0 \end{pmatrix}, 0, \begin{pmatrix} rR- \\ 0 \end{pmatrix}, 0 \right)$

Antiparticle positron $\bar{u} = \left(0, \begin{pmatrix} 0 \\ rL+ \end{pmatrix}, 0, \begin{pmatrix} 0 \\ rR+ \end{pmatrix} \right)$

hc-boson $Ag_4 \triangleq \lambda 4$, as $A13 = \lambda 4$

$E_{exp} = 0.511$ MeV $Q = -1$

$E_{tot} = 0.29$ MeV, $\Delta E_{tot} = 0.23$ MeV

muon mu = (*rL*−, *rR*−)

hc-bosons

$Ag_4 = A13 \triangleq \lambda 4, Ag_5 = \bar{A}13 \triangleq \lambda 5, Ag_{11} = A24 \triangleq \lambda 11, Ag_{12} = \bar{A}24 \triangleq \lambda 12$

$E_{exp} = 106$ MeV $Q = -1$

$E_{tot} = 228$ MeV, $\Delta E_{tot} = 154$

tauon tau = (*rL*−, *rR*−)

hc-bosons: all 15 $Ag_1, \cdots, Ag_{15}$

$E_{exp} = 1.78$ GeV $Q = -1$

$E_{tot} = 2.26$ GeV, $\Delta E_{tot} = 0.70$.

5.2. Lepton Neutrinos ν_e, ν_{mu}, ν_{tau}

Spin $S = 1/2$, two free preons, occupying fixed positions in the hc-tetra-spinor

Preon configuration: $u = \left(\begin{pmatrix} rL- \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ rL+ \end{pmatrix}, 0, 0 \right)$

Boson configuration: flavor = 1: $(A12 = \lambda_1)$, flavor = 2: $(A12 = \lambda_1, \bar{A}12 = \lambda_2, A34 = \lambda_{13}, \bar{A}34 = \lambda_{14})$

flavor = 3: all 15 bosons

The lepton neutrinos [27] are spherically symmetric, as shown in the calculation, and have therefore zero magnetic momentum. The spherical symmetry arises from the fact, that all leptons consist of two r-preons, which differ only in the hc-charge, so it is plausible that their geometric parameters are equal (equal radius r_i, its uncertainty dr_i, equal phase angle a_i, and inter-preon-angle th = 0).

The lepton neutrinos are neutral, interact only weak via Z and W bosons.

As for mass, the best upper limit from cosmological data is $m < 0.12$ eV.

The calculated masses of the lepton neutrinos are shown in Table 9.

The energy of component preons and field bosons are shown in Figure 8.

The structure, *i.e.* calculated average distances of components with smear-out are shown in Figure 9.

The parameters of the three generations (flavors) are shown in Tables 10-12.

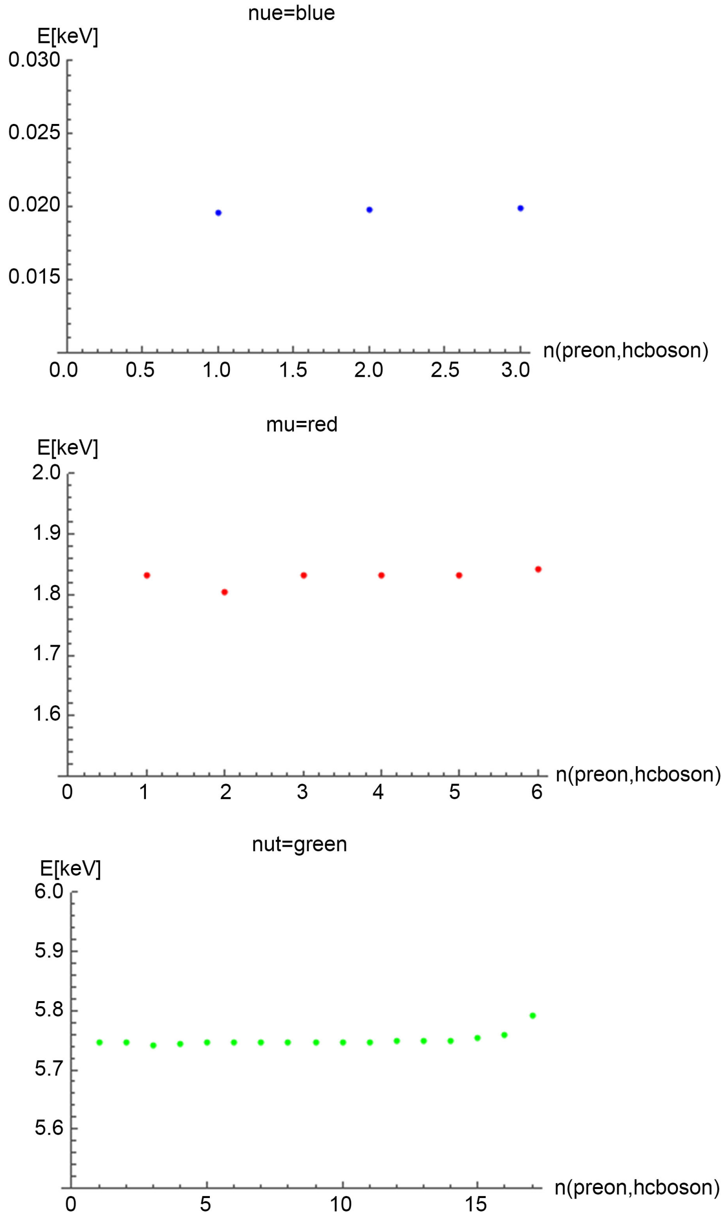


Figure 8. Energy distribution of lepton neutrinos: first preons ($u1$, $u2$), then bosons Ai.

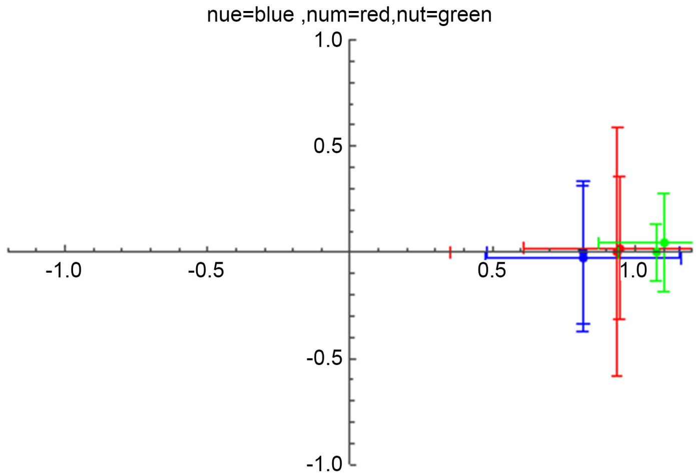


Figure 9. Structure of lepton neutrinos: preons ($u1$, $u2$) radii r_i, uncertainty dr_i and angle th.

Table 9. Lepton neutrino masses.

	m (nue)	m (num)	m (nut)
exp.			
calc.	0.30 meV	11 meV	98 meV

Table 10. Parameters of the electron neutrino.

Eu_i (meV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
0.0195789, 0.0198162	0.0198727	−0.00159052, 0.00281348	0.000719502	0.672092, 0.672795	0.817591, 0.817365	−0.0362275
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
0.000442384, 0.000217995	0.0000872723	.		0.0533686, 0.0533475	0.000416971, 0.00028167	

Table 11. Parameters of the muon neutrino.

Eu_i (meV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
1.83215, 1.80438	1.83322, 1.83333, 1.83335, 1.84298	0.00294051, 0.00304653	0.000719502	0.306423, 0.3312	0.943812, 0.936186	0.02
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
0.00234254, 0.0359295	0.000209844, 2.8895×10^{-6}, 0.0000362216, 0.0162998	.		0.111082, 0.111082	0.126494, 0.179059	

e-neutrino nue = (rL−, rL+)

Preon configuration: $u = \left(\begin{pmatrix} rL- \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ rL+ \end{pmatrix}, 0, 0 \right)$

Table 12. Parameters of the tauon neutrino.

Eu_i (meV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
5.74691, 5.74691	5.74263, 5.74519, 5.74578, 5.74647, 5.74688, 5.74707, 5.74725, 5.74761, 5.7479, 5.74861, 5.74951, 5.75005, 5.7531, 5.7595, 5.79127	0.00216278, −0.0145027	0.0645884, 0.0321258, 0.0714192, 0.0356015, 0.0665154, 0.0652989, 0.060689, 0.0555585, 0.0499117, 0.062275, 0.0407549, 0.0359398, 0.0666184, 0.0482816, 0.031136	0.306423, 0.3312	1.1011, 1.07371	0.0414724
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
0.110619, 0.110619	0.112495, 0.112474, 0.112249, 0.111351, 0.110999, 0.110905, 0.110818, 0.110445, 0.110137, 0.109776, 0.109065, 0.108836, 0.107668, 0.102724, 0.09513	.		0.207277, 0.197369	0.0609252, 0.06686	

Antiparticle right-chiral antineutrino $\bar{u} = \left(0, 0, \begin{pmatrix} rR- \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ rR+ \end{pmatrix}\right)$

$E_{exp} < 0.12$ eV $Q = 0$

$E_{tot} = 0.30$ meV, $\Delta E_{tot} = 0.038$

mu-neutrino num = (*rL*−, *rL*+)

$E_{exp} < 0.12$ eV $Q = 0$

$E_{tot} = 11.0$ meV, $\Delta E_{tot} = 0.055$

tau-neutrino nut = (*rL*−, *rL*+)

$E_{exp} < 0.12$ eV $Q = 0$

$E_{tot} = 98.0$ meV, $\Delta E_{tot} = 1.85$.

5.3. Non-Chiral Sterile (Hypothetical) Neutrinos *νs*1, *νs*2, *νs*3

Spin $S = 1/2$, two free preons, occupying fixed positions in the hc-tetra-spinor

Preon configuration: $u = \left(\begin{pmatrix} rL- \\ 0 \end{pmatrix}, 0, 0, \begin{pmatrix} 0 \\ rR+ \end{pmatrix} \right)$

Boson configuration: flavor = 1: $(A14 = \lambda_9)$, flavor = 2: $(A14 = \lambda_9, \bar{A}14 = \lambda_{10}, A23 = \lambda_6, \bar{A}23 = \lambda_7)$

flavor = 3: all 15 bosons

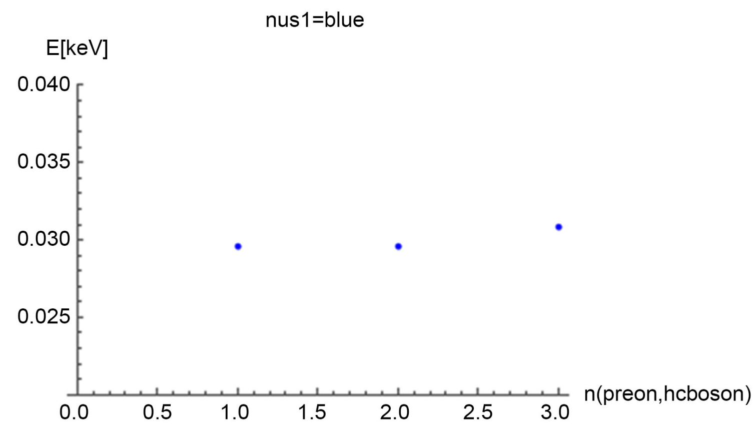
The hypothetical sterile neutrinos are involved in the neutron decay and interact only among themselves and with lepton neutrinos via the weak chiral boson ZL (see 4.1), so the denomination "sterile" is justified. They have similar masses as the lepton neutrinos, but they are Majorana particles: antiparticle = particle. Like lepton neutrinos, they are spherically symmetric and have zero magnetic momentum.

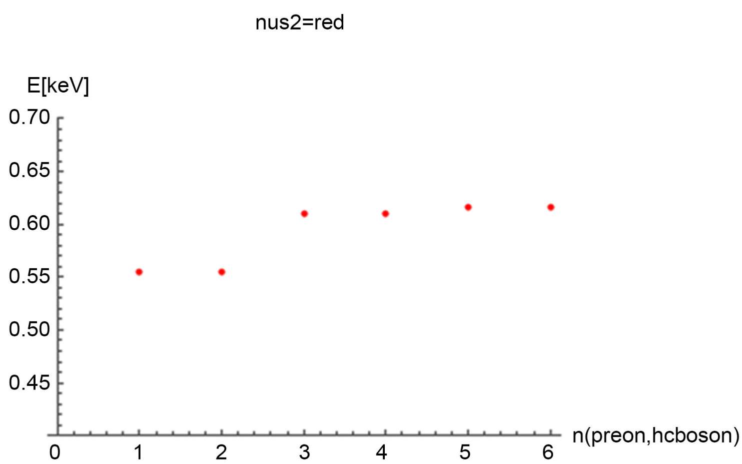
The calculated masses of the sterile neutrinos are shown in **Table 13**.

The energy of component preons and field bosons are shown in **Figure 10**.

The structure, *i.e.* calculated average distances of components with smear-out are shown in **Figure 11**.

The parameters of the three generations (flavors) are shown in **Tables 14-16**.





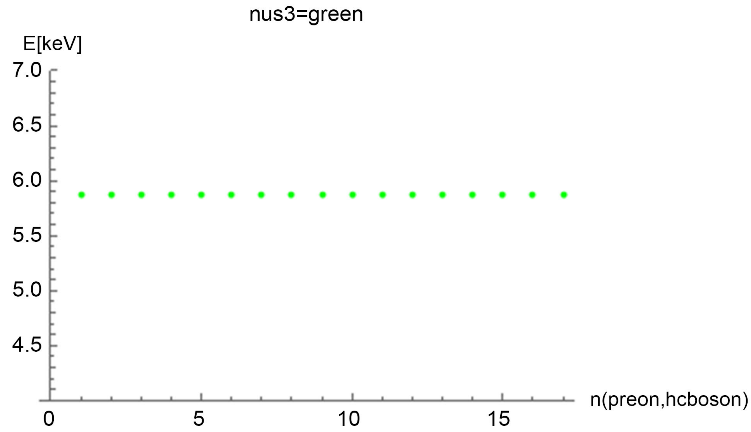


Figure 10. Energy distribution of sterile neutrinos: first preons ($u1$, $u2$), then bosons Ai.

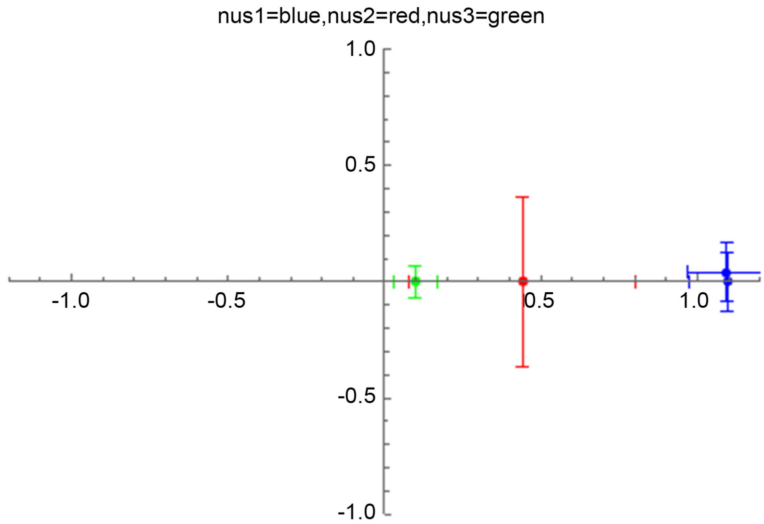


Figure 11. Structure of sterile neutrinos: preons ($u1$, $u2$) radii r_i, uncertainty dr_i and angle th.

Table 13. Masses of sterile neutrinos.

	m (nus1)	*m* (nus2)	*m* (nus3)
exp.			
calc.	0.09 meV	3.6 meV	100 meV

Table 14. Parameters of the sterile e-neutrino.

Eu_i (meV)	EA_i	a_i	aA_i	dru_i	ru_i	$\sin(\theta u_i)$
0.0295438, 0.0295438	0.03085	0.00981786, −0.00539754	0.000719502	0.247601, 0.245064	1.0941, 1.09465	0.0385823
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	$\Delta\sin(\theta u_i)$
0.000714214, 0.000714214	0.000840173	.		0.00802575, 0.00776682	0.00348974, 0.00362492	

Table 15. Parameters of the sterile mu-neutrino.

Eu_i (meV)	EA_i	a_i	aA_i	dru_i	ru_i	$\sin(\theta u_i)$
0.555866, 0.555866	0.610776, 0.610849, 0.616444, 0.616708	0.0837203, 0.0837203	0.524038, 0.145884, 0.584979, 0.615694	2.22087, 2.22087	0.439613, 0.439613	0.0
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	$\Delta\sin(\theta u_i)$
0.0579322, 0.0579322	0.029421, 0.0294231, 0.0244551, 0.0243638	.		1.8611, 1.8611	0.337827, 0.337827	

Table 16. Parameters of the sterile tau-neutrino.

Eu_i (meV)	EA_i	a_i	aA_i	dru_i	ru_i	$\sin(\theta u_i)$
5.87822, 5.87822	5.88029, 5.88029, 5.88029, 5.88029, 5.88029, 5.88029, 5.88029, 5.88029, 5.88029, 5.88029, 5.88029, 5.88029, 5.88029, 5.88029, 5.88029	0.0997489, 0.0997489	0.0517683, 0.0478681, 0.156694, 0.0480563, 0.0494643, 0.0577212, 0.0685586, 0.155112, 0.0500668, 0.050109, 0.0505401, 0.15493, 0.468362, 0.154732, 0.155897	0.0261638, 0.0261638	0.0974364, 0.0974364	0.0
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	$\Delta\sin(\theta u_i)$
0.00678084, 0.00678084	0.00339045, 0.00339045, 0.00339044, 0.00339043, 0.00339043, 0.00339043, 0.00339042, 0.00339042, 0.00339042, 0.00339042, 0.00339041, 0.00339011, 0.00338995, 0.00338953, 0.00338949	.		0.0738441, 0.0738441	0.0850158, 0.0850158	

nc-neutrino 1 nus1 = (*rL*−, *rR*+)

Preon configuration: $u = \left(\begin{pmatrix} rL- \\ 0 \end{pmatrix}, 0, 0, \begin{pmatrix} 0 \\ rR+ \end{pmatrix} \right)$

Antiparticle $\bar{u} = u$ (Majorana neutrino)

$E_{exp} = ?$ $Q = 0$

$E_{tot} = 0.090$ meV, $\Delta E_{tot} = 0.023$

nc-neutrino 2 nus2 = (*rL*−, *rR*+)

$E_{exp} = ?$ $Q = 0$

$E_{tot} = 3.56$ meV, $\Delta E_{tot} = 0.22$

nc-neutrino 3 nus3 = (*rL*−, *rR*+)

$E_{exp} =$ $Q = 0$

$E_{tot} = 100$ meV, $\Delta E_{tot} = 0.064$.

5.4. U-Quarks u, c, t

Spin $S = 1/2$, two free preons, occupying fixed positions in the hc-tetra-spinor

Preon configuration: $u = \left(0, \begin{pmatrix} (rL++qL+)/\sqrt{2} \\ (rL++qL+)/\sqrt{2} \end{pmatrix}, 0, \begin{pmatrix} (rR++qR+)/\sqrt{2} \\ (rR++qR+)/\sqrt{2} \end{pmatrix}\right)$

Boson configuration: flavor = 1: $(A24 = \lambda_{11})$, flavor = 2: $(A24 = \lambda_{11}, \bar{A}24 = \lambda_{12}, A13 = \lambda_4, \bar{A}13 = \lambda_5)$

flavor = 3: all 15 bosons

The U-quarks have the composition (*r*+, *q*+), and they are non-chiral, *i.e.* a superposition of (*rL*+, *qR*+) and (*rR*+, *qL*+). They are non-symmetric in r and q, so their internal structure is cylinder-symmetric or ring-symmetric, therefore there are corrections to the standard gyromagnetic factor 2, like for the nucleons. They carry the color charge, and do not appear separately, as the overall color must be zero (white).

The calculated and observed masses of the U-quarks are shown in **Table 17**.

The energy of component preons and field bosons are shown in **Figure 12**.

The structure, *i.e.* calculated average distances of components with smear-out are shown in **Figure 13**.

The parameters of the three generations (flavors) are shown in **Tables 18-20**.

Table 17. Masses of U-quarks.

	m (u)	*m* (c)	*m* (t)
exp.	2.3 MeV	1.34 GeV	171 GeV
calc.	2.35 ± 0.26 MeV	3.2 ± 1.87 GeV	163 ± 55 GeV

Table 18. Parameters of the up-quark.

Eu_i (MeV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
0.00100815, 0.00100963	1.58472	0.0674651, 0.100981	−0.538922	0.209696, 0.253259	0.0263, −0.280785	0.318731
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
0.000620367, 0.00057238	0.254744	.		0.0522386, 0.0483211	0.0472523, 0.0327625	

Table 19. Parameters of the c-quark.

Eu_i (MeV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
207.62, 158.774	84.6596, 281.775, 304.222, 2180.43	−0.0473157, −0.196647	0.187462, 0.228959, 0.152956, −0.33979	0.157295, 0.31158	0.0654933, 0.259696	0.15086
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
482.44, 296.717	281.296, 312.201, 159.539, 339.955	.		0.0332725, 0.0300652	0.00845404, 0.00406528	

Table 20. Parameters of the t-quark.

Eu_i (MeV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
16169.4, 10963.2	447.568, 1324.51, 1905.22, 3572.08, 4060.9, 5512.97, 7201.35, 8224.84, 8756.76, 9567.63, 11233.9, 12195.9, 14838.4, 19649.7, 27968.5	0.260102, −0.288355	0.0345205, −0.0889711, 0.117581, 0.0804355, 0.0439144, 0.0473357, −0.10843, 0.016335, −0.129588, −0.247394, −0.0279795, −0.18897, −0.337228, 0.0823711, −0.174481	2.30158, 2.56518	0.661335, −0.588081	0.381818
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
10545.1, 7710.93	650.619, 827.92, 845.732, 723.36, 260.622, 1147.26, 2692.84, 3336.08, 3111.95, 2532.61, 1738.6, 1466.69, 3647.34, 7499.15, 7115.09	.		0.896934, 0.609087	0.559172, 0.505538	

up-quark u = (*rL*+ + *qR*+)/ $\sqrt{2}$

Preon configuration: $u=\left(0,\begin{pmatrix}(rL++qL+)/\sqrt{2}\\(rL++qL+)/\sqrt{2}\end{pmatrix},0,\begin{pmatrix}(rR++qR+)/\sqrt{2}\\(rR++qR+)/\sqrt{2}\end{pmatrix}\right)$

Antiparticle $\bar{u}=\left(\begin{pmatrix}(rL-+qL-)/\sqrt{2}\\(rL-+qL-)/\sqrt{2}\end{pmatrix},0,\begin{pmatrix}(rR-+qR-)/\sqrt{2}\\(rR-+qR-)/\sqrt{2}\end{pmatrix},0\right)$

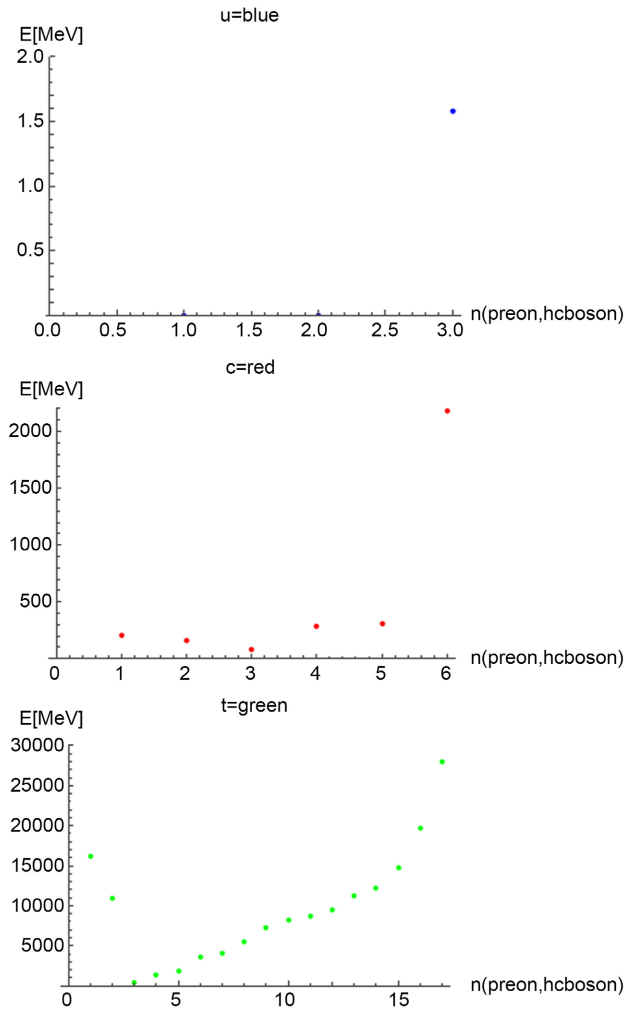


Figure 12. Energy distribution of U-quarks: first preons ($u1$, $u2$), then bosons Ai.

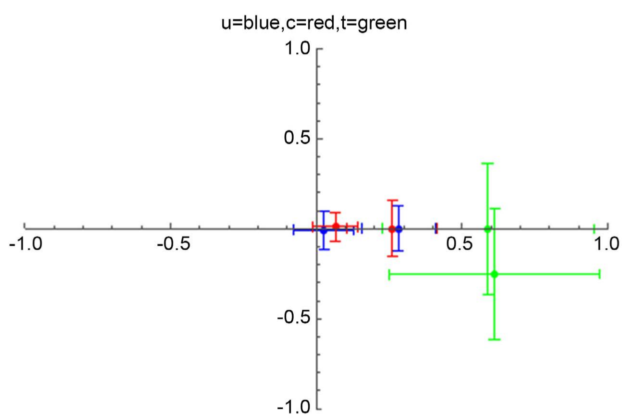


Figure 13. Structure of U-quarks: preons ($u1$, $u2$) radii r_i, uncertainty dr_i and angle th.

hc-boson $Ag_{11} \triangleq \lambda 11$

$E_{exp} = 2.3$ MeV $Q = +2/3$

$E_{tot} = 2.35$ MeV, $\Delta E_{tot} = 0.26$

c-quark c = (*rL*+ + *qR*+)/ $\sqrt{2}$

hc-bosons

$$Ag_{11} = A24 \triangleq \lambda 11, Ag_{12} = \bar{A}24 \triangleq \lambda 12, Ag_4 = A13 \triangleq \lambda 4, Ag_5 = \bar{A}13 \triangleq \lambda 5$$

$E_{exp} = 1.34$ GeV $Q = +2/3$

$E_{tot} = 3.2$ GeV, $\Delta E_{tot} = 1.87$

t-quark t = (*rL*+ + *qR*+)/ $\sqrt{2}$

hc-bosons: all 15 $Ag_1, \cdots, Ag_{15}$

$E_{exp} = 171$ GeV $Q = +2/3$

$E_{tot} = 163$ GeV, $\Delta E_{tot} = 55$.

5.5. D-Quarks d, s, b

Spin $S = 1/2$, two free preons, occupying fixed positions in the hc-tetra-spinor

Preon configuration: $u = \left(\begin{pmatrix} (rL - + qL +)/\sqrt{2} \\ 0 \end{pmatrix}, 0, \begin{pmatrix} (rR - + qR +)/\sqrt{2} \\ 0 \end{pmatrix}, 0 \right)$

Boson configuration: flavor = 1: $(A13 = \lambda_4)$, flavor = 2: $(A13 = \lambda_4, \bar{A}13 = \lambda_5, A24 = \lambda_{11}, \bar{A}24 = \lambda_{12})$

flavor = 3: all 15 bosons

The D-quarks have the composition (*r*–, *q*+), and they are non-chiral, *i.e.* a superposition of (*rL*–, *qR*+) and (*rR*–, *qL*+). They are non-symmetric in r and q, so their internal structure is cylinder-symmetric or ring-symmetric, therefore there are corrections to the standard gyromagnetic factor 2, like for the nucleons.

Apparently, the breaking of spherical symmetry is caused by flavor-mixing, as demonstrated in the dC-quark.

They carry the color charge, and do not appear separately, as the overall color must be zero (white).

D-quark flavors intermix via the CKM-matrix, its angles can be calculated (see dC-quark) by making a linear combination with variable CKM-angles, inserting into the hc-Lagrangian and minimizing. The solution is the energetically optimal CKM-mixture and yields the observed CKM-angles.

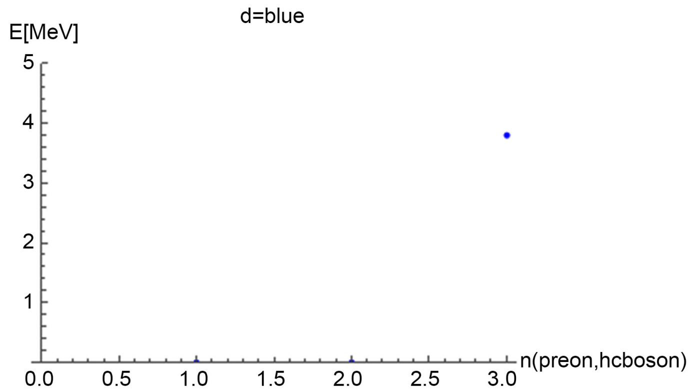
The calculated and observed masses of the D-quarks are shown in **Table 21**.

The energy of component preons and field bosons of the three flavors and Cabibbo-mixed quark (d, s) are shown in **Figure 14**.

The structure, *i.e.* calculated average distances of components with smear-out are shown in **Figure 15**.

Table 21. Masses of D-quarks.

	m (d)	*m* (dC), *α* (C)	*m* (s)	*m* (b)
exp.	4.8 MeV	4.8 MeV, 13.04˚	100 MeV	4.2 GeV
calc.	4.58 ± 0.3 MeV	4.74 MeV, 13.1˚	149 ± 15 MeV	6.1 ± 2.9 GeV



dC = d-part of Cabibbo-mixed quark (d, s), calculated Cabibbo-angle aC12 = 0.229 = 13.13˚ (exp. 13.04˚ + −0.05)

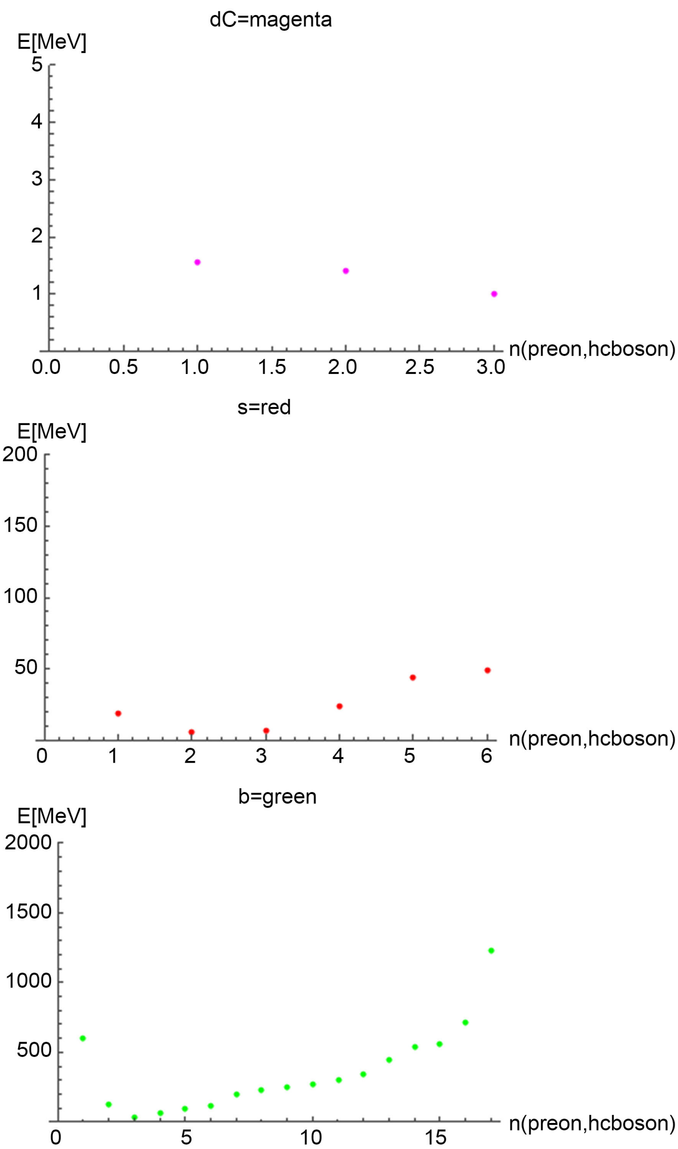


Figure 14. Energy distribution of D-quarks: first preons (*u*1, *u*2), then bosons Ai.

The parameters of the three of the three flavors and Cabibbo-mixed quark (d, s) are shown in **Tables 22-25**.

down-quark d = ($rL-$ + $qR+$)/ $\sqrt{2}$

Preon configuration: $u=\left(\begin{pmatrix}(rL-+qL+)/\sqrt{2}\\0\end{pmatrix},0,\begin{pmatrix}(rR-+qR+)/\sqrt{2}\\0\end{pmatrix},0\right)$

Antiparticle $\bar{u}=\left(0,\begin{pmatrix}0\\(rL++qL-)/\sqrt{2}\end{pmatrix},0,\begin{pmatrix}0\\(rR++qR-)/\sqrt{2}\end{pmatrix}\right)$

hc-boson: $Ag_4 \triangleq \lambda_4$

$E_{exp} = 4.8$ MeV $Q = -1/3$

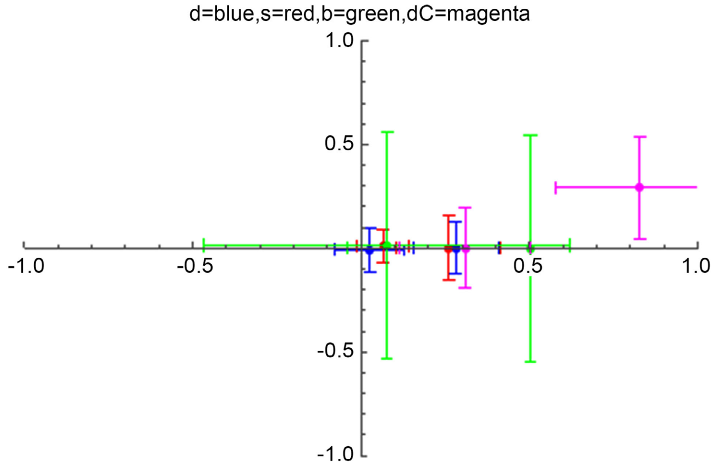


Figure 15. Structure of D-quarks: preons ($u1$, $u2$) radii r_i, uncertainty dr_i and angle th.

Table 22. Parameters of the down-quark.

Eu_i (MeV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
0.0011901, 0.000620564	3.81209	0.067465, 0.100981	−0.538924	0.209696, 0.253259	0.0263002, −0.280785	0.318731
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
0.000811471, 0.00070369	0.305601	.		0.0188066, 0.0900718	0.00476172, 0.00350625	

Table 23. Parameters of the s-quark.

Eu_i (MeV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
18.791, 5.99053	6.94284, 24.1632, 43.9623, 48.9406	−0.047311, −0.196639	−0.339778, 0.228951, 0.164457, 0.175962	0.157295, 0.311592	0.0654906, 0.259695	0.150859
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
1.73863, 1.93842	2.1682, 1.88257, 6.34742, 1.22757			0.018, 0.0088	0.0183405, 0.08854	

Table 24. Parameters of the b-quark.

Eu_i (MeV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
601.532, 130.4	35.4338, 69.6218, 92.0785, 120.049, 193.853, 224.967, 255.088, 266.136, 297.881, 348.389, 446.951, 535.473, 559.583, 713.301, 1232.01	−0.350658, 0.419618	−0.119199, 0.0701848, 0.0403467, 0.2601, 0.0412506, 0.175386, −0.0645038, 0.196578, 0.00791169, −0.0408362, −0.309195, 0.147146, 0.0139774, −0.126303, −0.178367	2.00585, 1.73462	0.0775948, 0.502463	0.186426
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
472.193, 67.3475	20.0937, 39.4015, 39.3106, 70.0438, 171.994, 191.423, 173.845, 173.003, 149.678, 106.309, 107.786, 107.91, 124.87, 228.263, 689.167			0.903552, 0.675784	0.0546897, 0.235836	

Table 25. Parameters of the Cabibbo-mixed down-quark.

Eu_i (MeV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
1.55842, 1.40699	1.00898	−0.624805, 0.263432	−0.649125	0.495338, 0.386903	0.877748, 0.308765	0.332405
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
1.38348, 0.700002	0.373778	.		0.00188066, 0.0900718	0.122162, 0.0502502	

$E_{tot} = 4.58$ MeV, $\Delta E_{tot} = 0.31$

s-quark s = $(rL- + qR+)/\sqrt{2}$

hc-bosons $Ag_4 = A13 \triangleq \lambda_4, Ag_5 = \bar{A}13 \triangleq \lambda_5, Ag_{11} = A24 \triangleq \lambda_{11}, Ag_{12} = \bar{A}24 \triangleq \lambda_{12}$

$E_{exp} = 100$ MeV $Q = -1/3$

$E_{tot} = 149$ MeV, $\Delta E_{tot} = 15$

b-quark b = $(rL- + qR+)/\sqrt{2}$

hc-bosons: all 15 $Ag_1, \cdots, Ag_{15}$

$E_{exp} = 4.2$ GeV $Q = -1/3$

$E_{tot} = 6.1$ GeV, $\Delta E_{tot} = 2.9$

Cabibbo-mixed down-quark dC = ($rL-$ + $qR+$)/ $\sqrt{2}$

$E_{exp} = 4.8$ MeV $Q = -1/3$

$E_{tot} = 4.74$ MeV, $\Delta E_{tot} = 2.45$.

5.6. Weak Massive Bosons W, Z0, ZL, H

Spin $S = 1$ or = 0, one preon u1: combination of one, two or four spinors

Preon configuration:

$$u = \left(0, 0, \begin{pmatrix} u1 \\ 0 \end{pmatrix}, 0\right) \text{ for weak exchange boson W, } S = 1$$

$$u = \left(\begin{pmatrix} u1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ u1 \end{pmatrix}\right) \text{ for weak exchange boson Z0, } S = 1$$

$$u = \left(\begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ u1 \end{pmatrix}, 0, 0\right) \text{ for (hypothetical) left-chiral Z-boson ZL, } S = 1$$

$$u = \left(\begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ u1 \end{pmatrix}\right) \text{ for higgs H, } S = 0$$

Boson configuration: only one flavor = 3: all 15 bosons

The weak massive bosons are the Yukawa bosons of the hc-interaction, *i.e.* they mediate the residual force of the hc-interaction in the form of a exponentially decreasing potential.

As shown below, they are spherically symmetric, the only preon is located approximately at radius $r \approx 1$ am.

The L-projections of leptons and quarks interact via SU(2) and (W, Z0) bosons, the R-projections of leptons and quarks interact via SU(1) and Z0.

This happens because of the SU(4)-symmetry breaking

$\mathrm{SU}(4) = \mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{SU}(1)_{\mathrm{R}} \otimes \mathrm{SU}(1)_{\mathrm{em}}$ with their exchange bosons $\{W\} \otimes \{Z0\} \otimes \{A_{em}\}$.

The higgs H is the only scalar among them, it generates mass for leptons and quarks, and also for the r-preon.

The sterile nc-neutrinos interact SU(2)-weakly with neutrinos via the (hypothetical) ZL-boson.

The calculated and observed masses of the weak massive bosons are shown in Table 26.

The energy of component preons and field bosons are shown in Figure 16.

The structure, *i.e.* calculated average distances of components with smear-out are shown in Figure 17.

Table 26. Masses of weak massive bosons.

	m (W)	m (Z0)	m (ZL)	m (H)
exp.	80.4 GeV	91.2 GeV		125.1 GeV
calc.	89 GeV	97 GeV	91 GeV	125 GeV

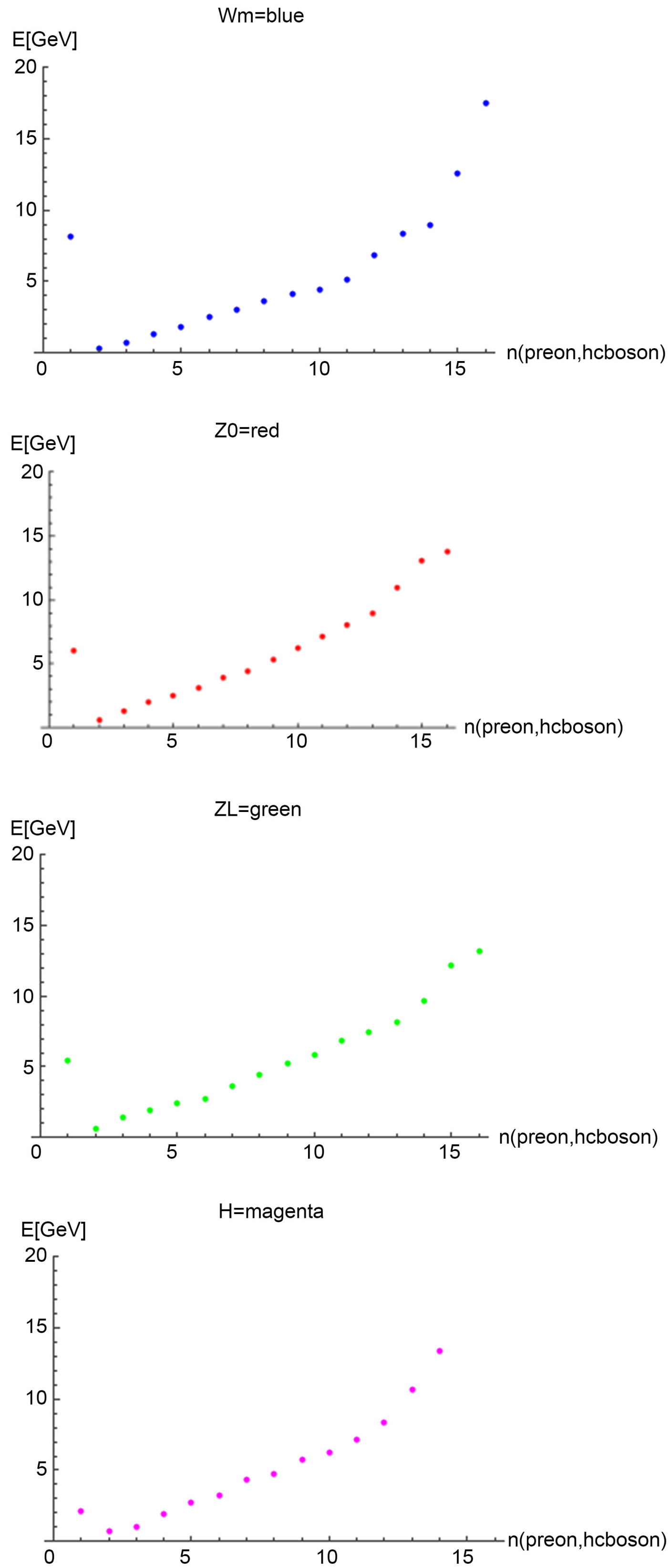


Figure 16. Energy distribution of weak massive bosons: first preons ($u1$), then bosons Ai.

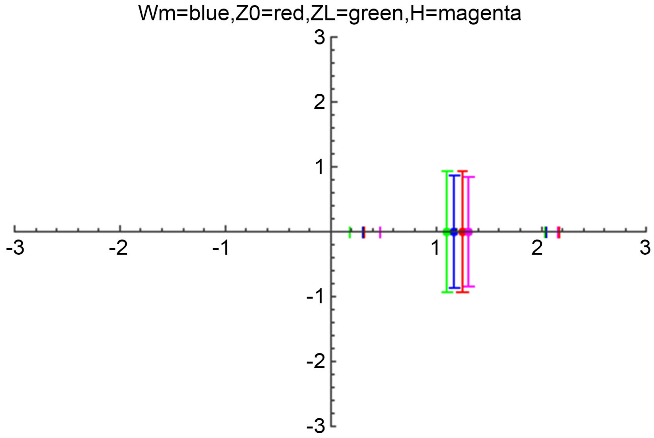


Figure 17. Structure of weak massive bosons: preons ($u1$) radii r_i, uncertainty dr_i and angle th, the only preon is located approximately at radius $r \approx 1$ am.

The parameters of the individual bosons are shown in **Tables 27-30**.

weak right-handed exchange boson W⁻⁻ W⁻⁻ = (rR− − rR−)/ $\sqrt{2}$, S = 1

Preon configuration: $u = \left(0, 0, \begin{pmatrix} u1 \\ 0 \end{pmatrix}, 0\right)\sqrt{2}$ $u1 = \left((rR-)-(rR-)\right)/\sqrt{2}$ antiparticle $\bar{W} = W^{+}$ configuration $u = \left(0, \begin{pmatrix} 0 \\ u1 \end{pmatrix}, 0, 0\right)$ $u1 = \left((rL+)-(rL+)\right)/\sqrt{2}$ hypothetical chiral counterpart: left-handed W* $u = \left(\begin{pmatrix} u1 \\ 0 \end{pmatrix}, 0, 0, 0\right)$

$u1 = \left((rL-)-(rL-)\right)/\sqrt{2}$

$E_{exp} = 80.4$ GeV $Q = -1$

$E_{tot} = 89$ GeV, $\Delta E_{tot} = 26$

neutral weak exchange boson Z0 Z0 = (rL− + rR− + rL+ + rR+)/2, S = 1

Preon configuration: $u = \left(\begin{pmatrix} u1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ Cu1 \end{pmatrix}, \begin{pmatrix} u1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ Cu1 \end{pmatrix}\right)/\sqrt{2}$

$u1 = \left((rL-)+(rR-)\right)/\sqrt{2}$ $Cu1 = \left((rL+)+(rR+)\right)/\sqrt{2}$

antiparticle $\bar{Z}_0 = Z_0$

$E_{exp} = 91.2$ GeV $Q = 0$

$E_{tot} = 97$ GeV, $\Delta E_{tot} = 30$

neutral left-handed weak (hypothetical) ZL ZL = (rL− + rL+)/ $\sqrt{2}$, S = 1

Preon configuration: $u = \left(\begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ u1 \end{pmatrix}, 0, 0\right)/\sqrt{2}$ $u1 = \left((rL-)+(rL+)\right)/\sqrt{2}$ antiparticle right-handed $\bar{Z}_L$ $\bar{u} = \left(0, 0, \begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ u1 \end{pmatrix}\right)/\sqrt{2}$ $u1 = \left((rR-)+(rR+)\right)/\sqrt{2}$

$E_{exp} = ?$ $Q = 0$

$E_{tot} = 91$ GeV, $\Delta E_{tot} = 28$

neutral mass-generating scalar higgs boson H H = (rL− + rL+ + rR- + rR+)/2, S = 0

Table 27. Parameters of the W-boson.

Eu_i (GeV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
8.20997	0.316331, 0.68873, 1.31464, 1.8232, 2.48807, 3.07844, 3.6289, 4.09488, 4.45176, 5.1892, 6.90223, 8.4103, 8.99396, 12.5852, 17.5486	−0.294831	0.0551789, −0.362417, −0.131927, 0.176835, −0.207657, 0.0407577, 0.0430164, 0.042737, −0.161912, 0.0364995, 0.056686, 0.0374209, 0.10742, −0.0329776, 0.0255881	2.6109	1.17267	0
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
10.1252	0.188613, 0.334553, 0.70658, 0.801391, 0.626902, 0.823354, 0.876158, 1.0928, 0.869573, 0.559216, 2.0035, 2.08725, 1.95618, 1.91668, 1.3873			0.81355	0.654887	

Table 28. Parameters of the Z0-boson.

Eu_i (GeV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
6.04329	0.601016, 1.31219, 2.03588, 2.57426, 3.10174, 3.96319, 4.46575, 5.33916, 6.22519, 7.11513, 8.06896, 8.94095, 10.9788, 13.0787, 13.777	−0.294831	0.0551789, −0.362417, −0.131927, 0.176835, −0.207657, 0.0407577, 0.0430164, 0.042737, −0.161912, 0.0364995, 0.056686, 0.0374209, 0.10742, −0.0329776, 0.0255881	2.6109	1.17267	0

Continued

ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	$\Delta \sin(\theta u_i)$
4.21067	0.42354, 0.63418, 0.928717, 0.946956, 1.1372, 1.30358, 1.4114, 1.20844, 1.02434, 1.25918, 1.27045, 0.93689, 2.58041, 5.49091, 5.57065			0.81355	0.654887	

Table 29. Parameters of the ZL-boson.

Eu_i (GeV)	EA_i	a_i	aA_i	dru_i	ru_i	$\sin(\theta u_i)$
5.41018	0.635455, 1.45762, 1.94515, 2.40743, 2.76174, 3.62666, 4.40736, 5.29138, 5.81184, 6.81575, 7.50969, 8.17982, 9.70438, 12.2009, 13.1613	−0.28215	−0.0634903, −0.0177523, 0.0393775, −0.0141295, 0.238785, 0.06813, −0.0828258, −0.0566217, 0.0147406, −0.0549006, −0.129071, −0.193776, 0.0224101, −0.196448, −0.0777609	4.20897	1.10542	0
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	$\Delta \sin(\theta u_i)$
3.61896	0.361193, 0.294054, 0.542048, 0.685343, 0.734258, 1.14914, 1.37386, 1.86499, 2.16942, 2.02409, 1.91406, 1.31147, 1.01549, 4.24462, 4.70292			0.896122	0.764349	

Table 30. Parameters of the higgs H.

Eu_i (GeV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
2.12256	0.687867, 1.06114, 1.89688, 2.72051, 3.1891, 4.31443, 4.70774, 5.75923, 6.2929, 7.21059, 8.37697, 10.7365, 13.3999, 22.669, 30.1505	0.242174	0.203185, 0.209845, 0.0797134, 0.249824, 0.098651, −0.0453497, 0.111729, 0.153663, 0.156595, 0.261526, −0.0971455, −0.0358294, 0.0815874, 0.0875567, −0.0353346	2.65352	1.31158	0
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
0.963583	0.596931, 0.840909, 0.733675, 1.05086, 1.1562, 1.75893, 1.94705, 1.83638, 2.30989, 2.54619, 2.87418, 4.01778, 2.02776, 10.3933, 8.6628			0.164707	0.599096	

Preon configuration: $u = \left(\begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ u1 \end{pmatrix}\right) \Big/ 2$

$u1 = \left((rL-)+(rL+)+(rR-)+(rR+)\right)/2$

antiparticle: itself $\bar{H} = H$

$E_{exp} = 125.1$ GeV $Q = 0$

$E_{tot} = 125$ GeV, $\Delta E_{tot} = 44$.

5.7. Strong Neutrinos (Hypothetical) q*ν*e q*ν*m q*ν*t

Spin S = 1/2, two free preons, occupying fixed positions in the hc-tetra-spinor

Preon configuration: $u = \left(\begin{pmatrix} qL- \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ qL+ \end{pmatrix}, 0, 0\right)$

Boson configuration: flavor = 1: $(A12 = \lambda_1)$, flavor = 2: $(A12 = \lambda_1, \bar{A}12 = \lambda_2, A34 = \lambda_{13}, \bar{A}34 = \lambda_{14})$

flavor = 3: all 15 bosons

The strong neutrinos are neutral spherically symmetric particles with composition ($q+$, $q-$) and have masses starting with 23 MeV. They can hc-interact via Zq strong bosons, but only for high energies

($E \sim m$(Zq) = 644 GeV), they are colorless and do not interact strongly.

The strong neutrinos are spherically symmetric, the two preons are located approximately at radius $r \approx 1$ am, as shown in the structure plot below.

They are candidates for dark matter, as they are in the appropriate mass range (around 100 MeV, according to the new SIMP-scheme for dark matter), and they interact with themselves at high energies, as was observed for dark matter in certain galaxies.

The calculated masses of the strong neutrinos are shown in **Table 31**.

The energy of component preons and field bosons are shown in **Figure 18**.

The structure, *i.e.* calculated average distances of components with smear-out are shown in **Figure 19**.

The parameters of the three generations (flavors) are shown in **Tables 32-34**.

Table 31. Masses of strong neutrinos.

	m (qnue)	m (qnum)	m (qnut)
exp.			
calc.	23.2 MeV	205 MeV	2.4 GeV

Table 32. Parameters of the qe-neutrino.

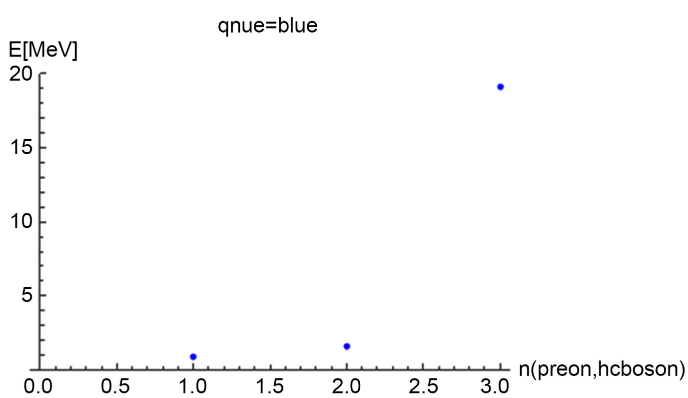
Eu_i (MeV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
0.916713, 1.57978	19.1558	0.0499768, 0.0499806	0.0499709	0.218706, 0.217761	1.08906, 1.08886	0.0495826
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
2.59139, 4.46489	6.42353			0.00260392, 0.0000482519	0.000467796, 0.0000799548	

Table 33. Parameters of the qm-neutrino.

Eu_i (MeV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
2.31669, 2.10932	3.2139, 27.2516, 36.8587, 131.637	0.049974, 0.0499723	0.0499795, 0.0499777, 0.0499851, 0.0499601	0.218962, 0.217768	1.08916, 1.08885	0.0494963
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
4.18504, 4.14824	4.03572, 16.4507, 20.6083, 43.8355			0.00272481, 0.0000218384	0.000633244, 0.0000799629	

Table 34. Parameters of the qt-neutrino.

Eu_i (MeV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
62.9487, 61.5266	6.27604, 9.78005, 14.0006, 17.2518, 26.4587, 32.2502, 44.8203, 62.4957, 71.6555, 88.2316, 105.198, 154.92, 251.417, 406.445, 980.267	0.0498284, 0.0496889	0.0499212, 0.0499565, 0.0499232, 0.0499843, 0.0500119, 0.0499806, 0.0499806, 0.0500343, 0.0499183, 0.0495368, 0.0499496, 0.0501089, 0.0500246, 0.0500326, 0.0499384	0.250849, 0.21778	1.09488, 1.08809	0.0362321
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
80.6687, 82.6461	7.47768, 7.63514, 11.768, 12.944, 23.1368, 23.3382, 31.1644, 43.8489, 52.4387, 59.1117, 70.624, 56.9479, 109.749, 231.239, 579.301			0.0345065, 0.000493132	0.00516914, 0.000793051	



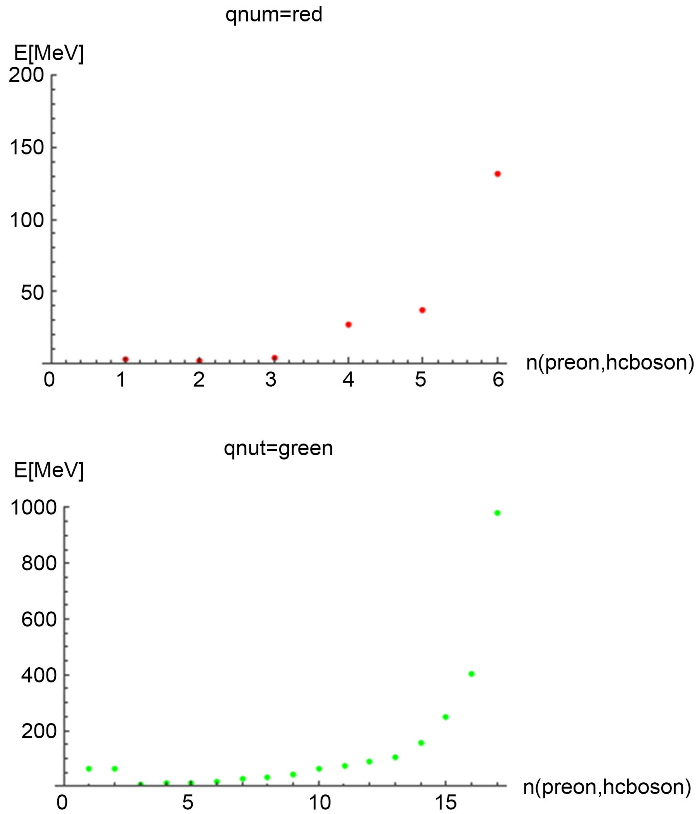


Figure 18. Energy distribution of strong neutrinos: first preons ($u1$, $u2$), then bosons Ai.

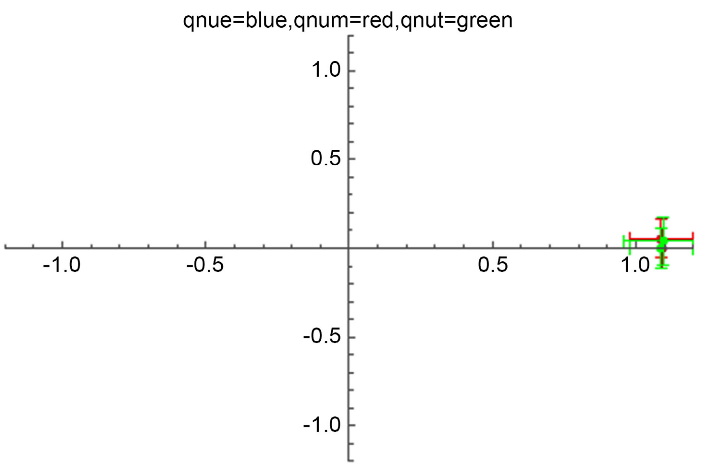


Figure 19. Structure of strong neutrinos: preons ($u1$, $u2$) radii r_i, uncertainty dr_i and angle th.

qe-neutrino qnue = ($qL-$, $qL+$)

Preon configuration: left-handed q-neutrino $u = \left(\begin{pmatrix} 0 \\ qL- \end{pmatrix}, \begin{pmatrix} qL+ \\ 0 \end{pmatrix}, 0, 0 \right)$

Antiparticle right-handed anti-q-neutrino $\bar{u} = \left(0, 0, \begin{pmatrix} 0 \\ qR- \end{pmatrix}, \begin{pmatrix} qR+ \\ 0 \end{pmatrix} \right)$

$E_{exp} = ?\ Q = 0$

$E_{tot} = 23$ MeV, $\Delta E_{tot} = 13.5$

qm-neutrino qnum = (*qL*−, *qL*+)

$E_{exp} = ?\ Q = 0$

$E_{tot} = 205$ MeV, $\Delta E_{tot} = 93$

qt-neutrino qnut = (*qL*−, *qL*+)

$E_{exp} = ?\ Q = 0$

$E_{tot} = 2.40$ GeV, $\Delta E_{tot} = 1.48$.

5.8. Strong Bosons (Hypothetical) Zq, Hq

Spin $S = 1$ or = 0, one free preon u1: combination of four spinors

Preon configuration:

$$u = \left(\begin{pmatrix} u1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ u1 \end{pmatrix} \right) \text{ for strong exchange boson Zq}$$

$$u = \left(\begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ u1 \end{pmatrix} \right) \text{ for q-higgs Hq}$$

Boson configuration; all hc-bosons active flavor = 3

The strong bosons are color-neutral and do not interact by color.

They are spherically symmetric, the only preon is located approximately at radius $r \approx 1$ am, as shown in the structure plot below.

The strong boson Zq is the Yukawa-boson for the hc-interaction of q-neutrinos.

The strong higgs Hq generates masses for the q-neutrinos and for the q-preons.

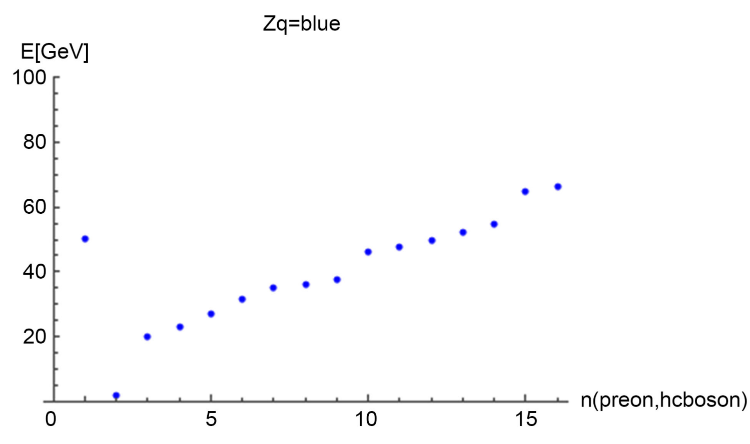
The q-neutrinos interact very weakly, because the masses of the strong bosons are very large.

The calculated masses of the strong bosons are shown in **Table 35**.

The energy of component preons and field bosons are shown in **Figure 20**.

The structure, *i.e.* calculated average distances of components with smear-out are shown in **Figure 21**.

The parameters of the individual bosons are shown in **Table 36**, **Table 37**.



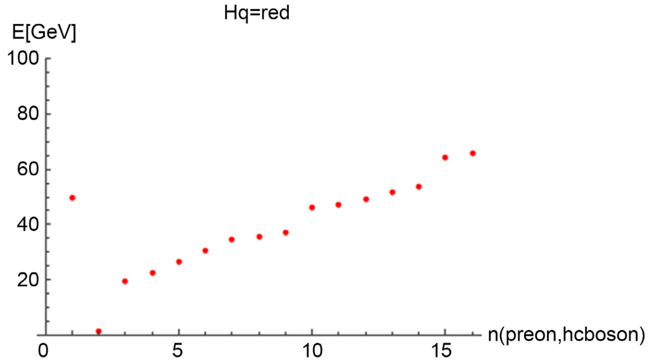


Figure 20. Energy distribution of strong bosons: first preon ($u1$), then bosons Ai.

Table 35. Masses of strong bosons.

	m (Zq)	m (Hq)
exp.		
calc.	644 GeV	637 GeV

Table 36. Parameters of the strong boson Zq.

Eu_i (GeV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
50.1031	1.75913, 20.0747, 22.9369, 27.0332, 31.3827, 35.2293, 36.2947, 37.6842, 46.383, 47.6871, 49.7122, 52.4871, 54.6914, 64.7501, 66.1951	0.242169	0.231796, −0.207073, 0.131049, −0.253369, 0.15414, 0.199737, 0.161236, 0.266433, −0.269026, 0.131364, 0.155354, 0.203886, 0.235986, 0.226728, 0.056805	2.90034	0.953641	0
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
0.501804	1.40428, 2.2256, 2.1451, 4.24188, 3.13026, 1.44886, 1.19789, 1.53643, 1.07209, 0.567924, 0.839207, 1.81534, 1.76197, 1.38173, 1.23064			0.0598953	0.243724	

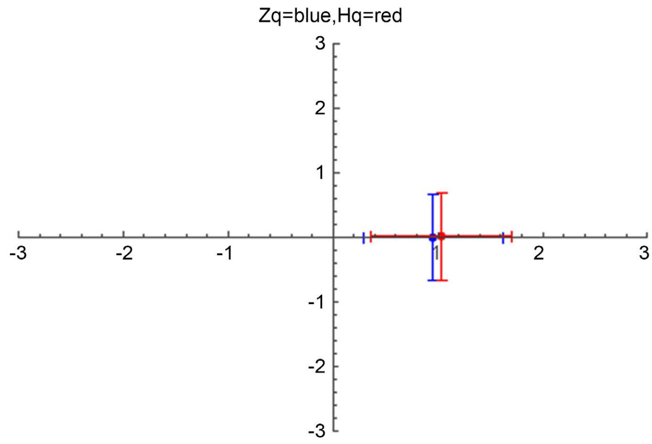


Figure 21. Structure of strong bosons: preon ($u1$) radii r_i, uncertainty dr_i and angle th, the only preon is located approximately at radius $r \approx 1$ am.

Table 37. Parameters of the strong higgs Hq.

Eu_i (GeV)	EA_i	a_i	aA_i	dru_i	ru_i	sin (θu_i)
49.8974	66.1951}, {49.8974, 1.49444, 19.6994, 22.5362, 26.3583, 30.6179, 34.632, 35.8439, 37.1908, 46.1384, 47.4992, 49.4017, 51.9202, 54.0522, 64.3069, 65.783	0.242181	0.207549, −0.304129, 0.131516, −0.254004, 0.253908, 0.206301, 0.161453, 0.252253, −0.272395, 0.131765, 0.163953, 0.204921, 0.242696, 0.221589, 0.0809426	2.97112	1.03787	0
ΔEu_i	ΔEA_i	Δa_i	ΔaA_i	Δdru_i	Δru_i	Δsin (θu_i)
0.0563816	0.958115, 1.67958, 1.65813, 3.0444, 1.70715, 0.281763, 0.812278, 0.540787, 0.748368, 0.324524, 0.292485, 2.08406, 0.685153, 0.707936, 1.09514			0.071377	0.253642	

strong exchange boson Zq Zq = (*qL*− + *qR*− + *qL*+ + *qR*+)/2

Preon configuration: $u = \left(\begin{pmatrix} u1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ Cu1 \end{pmatrix}, \begin{pmatrix} u1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ Cu1 \end{pmatrix} \right) \Big/ \sqrt{2}$

$Cu1 = \left(\left(qL - \right) + \left(qR - \right) \right) / \sqrt{2} \quad u1 = \left(\left(qL + \right) + \left(qR + \right) \right) / \sqrt{2}$

antiparticle itself $\bar{Z}_q = Z_q$

E_{exp} = ? $Q = 0$, $S = 1$

E_{tot} = 644 GeV, ΔE_{tot} = 26

strong higgs scalar boson (hypothetical) Hq, Hq = (*qL*− + *qL*+ + *qR*− + *qR*+)/2

Preon configuration: $u = \left(\begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ u1 \end{pmatrix}, \begin{pmatrix} u1 \\ u1 \end{pmatrix} \right) \Big/ 2$

$u1 = \left(\left(qL - \right) + \left(qL + \right) + \left(qR - \right) + \left(qR + \right) \right) / 2$

antiparticle: itself $\bar{H}_q = H_q$

E_{exp} = ? $Q = 0$, $S = 0$

E_{tot} = 637 GeV, ΔE_{tot} = 17.

5.9. Mass Hierarchy and the Koide Formula

In 1982 Koide set up a formula for the 3 generations of charged lepton masses [28]

$m_1 + m_2 + m_3 = \frac{2}{3}\left(\sqrt{m_1} + \sqrt{m_2} + \sqrt{m_3}\right)^2$, where $m_1 = m_e, m_2 = m_\mu, m_3 = m_\tau$ or for the Koide function $k\left(m_1, m_2, m_3\right) = \frac{2}{3}\frac{\left(\sqrt{m_1} + \sqrt{m_2} + \sqrt{m_3}\right)^2}{m_1 + m_2 + m_3}$ we get $k(e) = k\left(m_e, m_\mu, m_\tau\right) = 1$ for charged leptons = $l_e = (e, \mu, \tau)$.

Calculation with observed values for basic particles yields [6] for the Koide value for charged leptons, U-quarks, and D-quarks

$$k(e) = 0.9998,\ k(u) = 1.2673,\ k(d) = 1.0891$$

and for neutrinos with SU4PM calculated values

$$k(\nu) = 0.8654$$

The masses of the 3 generations of the basic particles of the Standard Model are given in **Table 38** below, where the neutrino masses are taken from the SU(4)-preon calculation above, the remaining values are measured.

Table 38. Masses of the 3 generations of the basic particles of the Standard Model.

	m_1	m_2	m_3
neutrino (ν_e, ν_μ, ν_τ)	0.30 meV	11 meV	98 meV
ch.lepton (e, μ, τ)	0.511 MeV	106 MeV	1.78 GeV
u-quark (u, c, t)	2.3 MeV	1.34 GeV	171 GeV
d-quark (d, s, b)	4.8 MeV	100 MeV	4.2 GeV

Nan Li [28] gives the assessment for $k(\nu)$: $0.50 < k(\nu) < 0.85$, which is roughly in agreement with the above value for $k(\nu)$.

The Koide formula is approximately $k \approx 1$ for all basic particles, with a deviation of about 20% for neutrinos and u-quark generations.

In the SU(4)-preon model, the generations are due to the 3 configuration of hc-bosons (hcb) $N_i = (1, 4, 15)$ which are compatible with the symmetry of SU(4) (are invariant under an automorphism subgroup).

We make an ansatz for the mass-energy of generations u_i:

$M(u_i) = E_{ui} + m_{ui} N_i^{a_{ui}}$, where E_{ui} is the non-hcb energy contribution, m_{ui} is the first-generation-energy, a_{ui} is the hcb-exponent, and $N_i = (1, 4, 15)$ is the number of hcb's in a generation i.

Fitting the mass table with this ansatz gives

$E_{u_1} = -28.18 \quad E_{u2} = -139.84 \quad E_{u3} = -550.62 \quad E_{u4} = -61.19$

$m_{u_1} = 5.06 \quad m_{u_2} = 10.79 \quad m_{u_3} = 19.16 \quad m_{u_4} = 6.99$

$a_{u_1} = 1.11 \quad a_{u_2} = 1.20 \quad a_{u_3} = 1.50 \quad a_{u_4} = 1.34$

The resulting exponents a_{ui} vary from $a_{u1} = 1.11$ for neutrinos to $a_{u3} = 1.50$ for u-quark generations with a mean

$E(a_{ui}) = 1.292$ and standard deviation $Std(a_{ui}) = 0.1720$.

If we approximate the mass formula $\hat{M}(u_i) = m_{ui} N_i^{a_{ui}}$ neglecting the non-hcb energy E_{ui}, then the scale factor cancels out, and the Koide function depends only on the exponent a_{ui} of the family (u_i).

We get the following approximate values k' for the Koide value k of the 4 families:

$$k(\nu) = 0.8106,\ k(e) = 0.9177,\ k(u) = 1.242,\ k(d) = 1.091$$

which is a good approximation.

So we can conclude:

the **approximate validity of the Koide formula $k \approx 1$** for the 4 families is the result of the power law of the generation mass hierarchy with the **exponent $a_{ui} \approx$ 1.3 approximately constant** across the 4 families.

5.10. Assessment of the Quark and Lepton Mixing

It is possible to assess roughly the values of the CKM matrix for quark mixing and the PMNS matrix for neutrino mixing based on the SU(4) preon model.

Quark mixing

In 4.5 we calculated the CKM 12-element for the d → u decay (Cabibbo angle) as aC12 = 0.229, which agrees well with the experimental value. The calculation for the other elements of the CKM matrix can be carried out correspondingly. However, one can assess these elements roughly, based on the number of hc-bosons per generation.

The particle configuration for the generations (=flavors) is

flavor 1: 1 hc-boson+2 preons e.g. $A13$, $rL-$, $rR-$ for electron e^-

flavor 2: 4 complementary hc-bosons with conjugates +2 preons e.g. $A13, \bar{A}13, A24, \bar{A}24$, $rL-$, $rR-$ for electron e^-

flavor 3: all 15 hc-boson +2 preons

We expect naively that the coupling between generations scales roughly with the Boltzmann factor (kB = 1)

$$c_{i,j} = C_1 \exp\left(\frac{E_0 N(i)}{T}\right) = C_1 \beta^{N(i)}$$

where $N(i)$ = number of particles in *i*-th generation T the temperature and C_1, β constants.

With $\beta = 1.34$ and $C_1 = 0.5$ we get $c_{1,2} = 0.206$ $c_{2,3} = 0.019$ $c_{1,3} = 0.0080$ in comparison with CKM values $(0.22, 0.041, 0.0035)$

Lepton mixing

With quarks, quark transformations run according to the scheme $q_1 \to q_2 + W$, with a W-boson, which consists of *r*-preons.

With electrons and neutrinos, transformations $e \to \nu + X$ or $\nu \to e + X$ are impossible because of preon conservation.

Transformations between neutrino flavors $\nu_i \to \nu_j$ are described by the PMNS matrix, according to the above formula $c_{i,j} = C_1 \exp\left(\frac{E_0 N(i)}{T}\right)$. Normally neutrinos have kinetic energies much higher than their rest mass, e.g. solar neutrinos in MeV range, and $m(\nu) \approx E_0 N(i) \ll T$, so the exponent is around zero, and we expect the $c_{i,j}$ to be in the same range, which is the case.

Transformation between charged leptons with different flavors, e.g. $\mu \to e + X$ run with flavor conservation

$\mu \to e + \bar{\nu}_e + \nu_\mu + \Delta E$ or in preon formulation

$$\left(A13, \bar{A}13, A24, \bar{A}24, rL-, rR-\right) \to \left(A13, rL-, rR-\right) + \left(\bar{A}13, rR-, rR+\right) + \left(A13, \bar{A}13, A24, \bar{A}24, rL-, rL+\right) + \Delta E$$

here two hcb's $A13$ $\bar{A}13$ are emitted, $rR-, rR+, rL-, rL+$ are created as pairs, and $A13, \bar{A}13, A24, \bar{A}24$ are simply "passed".

The flavor-violating transformation $\mu \to e + \bar{\nu}_e + \nu_e + \Delta E$ is not forbidden by conservation laws, but strongly suppressed in comparison to the flavor-conserving transformation because of the very small neutrino mass.

In preon formulation

$$\left(A13, \bar{A}13, A24, \bar{A}24, rL-, rR-\right) \to \left(A13, rL-, rR-\right) + \left(\bar{A}13, rR-, rR+\right) + \left(A13, rL-, rL+\right) + \Delta E$$

In the inverse transformation, which is equivalent, the hcb quartet $A13, \bar{A}13, A24, \bar{A}24$ with muon energies has to be emitted in the neutrino ν_e. If we assume the temperature of the neutrinos to be about in the order of the electron mass, the process will be suppressed by the Boltzmann factor

$$f\left(\mu \to e + \bar{\nu}_e + \nu_e\right) = \exp\left(-\frac{4E(Aij, \mu)}{m(e)}\right) \approx \exp\left(-\frac{m(\mu)}{m(e)}\right) = \exp\left(-\frac{100\ \text{MeV}}{0.511\ \text{MeV}}\right) = 1.0 \times 10^{-85}$$

5.11. Deviations from the Standard Model

We can assess the deviation of the SU(4) hypercolor model from the standard model by the energy ratio

$$f_{dev} = \left(\frac{mc^2}{E_{hc}}\right),$$

where m is the mass of the corresponding particle, and $E_{hc} = 180\,\text{GeV}$ is the hypercolor energy scale. As an example, let us consider the magnetic moment off the muon, where we measure a deviation from the Standard model result [29].

Assessed deviation of the muon and electron magnetic moment

The muon mass is $m_\mu = 105.6\,\text{MeV}$, the measured relative deviation $\frac{\Delta a_\mu}{a_\mu} = \frac{2.3}{1855900} = 1.2\times10^{-6}$ [29], the assessed deviation of the muon magnetic moment $\frac{\Delta a}{a} \sim \left(\frac{\Delta r}{r}\right)^2 \sim \left(\frac{\Delta E}{E}\right)^2$, so $\frac{\Delta a_\mu}{a_\mu} \approx \left(\frac{m_\mu c^2}{E_{hc}}\right)^2 = 0.34\times10^{-6}$, which is in the scale of the measured deviation.

For the electron we get the assessment $\frac{\Delta a_e}{a_e} \approx \left(\frac{m_e c^2}{E_{hc}}\right)^2 = 8\times10^{-12}$, where the current measurement precision is $\frac{\delta a_e}{a_e} = 3\times10^{-10}$, well above the assessed deviation.

6. Weak Hadron Decays in the SU(4)-Preon Model

6.1. Neutron Decay

The neutron decay obeys the scheme $dd \to ud + e^- + \bar{\nu}_e$, *i.e.* for free neutrons

$$n \to p + e^- + \bar{\nu}_e \tag{14}$$

with the mean lifetime of $\tau = 881.5 \pm 1.5$ s and energy $\Delta E = 0.782343$ MeV

In the SM it is described by the interaction of a virtual W-boson

$$n \to p + W^- \to p + e^- + \bar{\nu}_e \tag{14a}$$

With the probability of about $p = 0.001$, an additional photon is emitted

$$n \to p + W^- \to p + e^- + \bar{\nu}_e + \gamma$$

Currently, there is a "neutron lifetime puzzle": the lifetime measured by proton-counting (beam-method lifetime τ_1) yields $\tau_2 = \tau_1 + 8$ s, compared to the bottle-method (lifetime τ_2) of counting the remaining neutrons.

A possible explanation is the possibility of other decay channels for n.

In the SU4PM the decay proceeds as follows

$$d(rR-,qL+) \to u(rL+,qR+) + W^-(rR-,rR-) + Z_q(qL-,qL+) \tag{15}$$

$$d(rL-,qR+) \to d(rR-,qL+) + Z_L(rL-,rL+) + \bar{Z}_q(qR-,qR+)$$

with the immediate decay $W^-(rR-,rR-) \to e^-(rL-,rR-) + \bar{\nu}_e(rR-,rR+)$ and the decay $Z_L(rL-,rL+) \to \nu_e(rL-,rL+) + \nu_{s1}(rL+,rR-)$, *i.e.* the total reaction is $n \to p + e^{-} + \bar{\nu}_e + \nu_e + \nu_{s1}$, with the additional emission of a neutrino and a sterile neutrino, which are undetectable and carry away a small fraction of the total energy, ascribed to the antineutrino.

The neutrino and the antineutrino annihilate in a small fraction of events, producing an additional photon.

The virtual Z_q and $\bar{Z}_q$ annihilate immediately and carry no energy away.

6.2. Transitions of Quarks

A quark can make a transformation, which swaps the chirality of its components. This is seen at the example of a d-quark transition (16)

$$d(rR-, qL+) \to d(rL-, qR+) + \bar{Z}_L(rR-, rR+) + Z_q(qL-, qL+)$$
$$\to d(rL-, qR+) + \bar{\nu}_e(rR-, rR+) + \nu_q q(L-, qL+)$$

$$d(rR-, qL+) \to d(rL-, qR+) + \bar{Z}_L(rR-, rR+) + Z_q(qL-, qL+)$$
$$\to d(rL-, qR+) + \bar{\nu}_e(rR-, rR+) + \nu_q(qL-, qL+)$$

Both transitions take at least the energy $\Delta E = 23$ MeV for the mass of ν_q.

This transition can serve as an additional channel for the neutron decay:

$n \to n + \bar{\nu}_e + \nu_e + \bar{\nu}_q + \nu_q$, which takes away $\Delta E = 2 \times 23$ MeV and makes fast neutrons slow, making them undetectable by the usual scintillation method. This would explain the "neutron lifetime puzzle".

6.3. Pion Decay

The pion decay is the other major source of weak hadron decays, in the SM it is described as

$$u\bar{d} \to e^{+} + \nu_e \tag{17}$$

In the SU4PM the decay proceeds as follows

$$u(rR+, qL+) \to u(rL+, qR+) + \bar{Z}_L(rR-, rR+) + \nu_q(qL-, qL+) \tag{18}$$
$$\bar{d}(rL+, qR-) \to \bar{u}(rR-, qL-) + W^{+}(rL+, rL+) + \bar{\nu}_q(qR-, qR+)$$

the virtual W-boson and ZL-boson decay into electron and neutrinos

$$W^{+}(rL+, rL+) \to e^{+}(rL+, rR+) + \nu_e(rL-, rL+)$$
$$\bar{Z}_L(rR-, rR+) \to \bar{\nu}_e(rR-, rR+) + \nu_{s1}(rL-, rR+)$$

so the overall reaction is (19)

$$u(rR+, qL+) + \bar{d}(rL+, qR-) \to u(rL+, qR+) + \bar{u}(rR-, qL-)$$
$$+ e^{+}(rL+, rR+) + \nu_e(rL-, rL+) + \bar{\nu}_e(rR-, rR+) + \nu_{s1}(rL-, rR+), \textit{ i.e.}$$

$u\bar{d} \to e^{+} + \nu_e + \bar{\nu}_e + \nu_{s1}$, the pion decays into an electron and antineutrino plus the (undetectable) neutrino and sterile neutrino.

7. Conclusions

Formulation of the extended model

In the first three chapters we describe SU4PM, the extended SM.

The extension happens in four steps:

-in chap.2: extending the Pauli-SU(2) weak interaction to SU(4)-hypercolor interaction, which is renormalizable quantum gauge field theory, with confinement and asymptotic freedom, with charges $hc = (L-, L+, R-, R+)$.

Pauli-SU(2) weak interaction becomes then the Yukawa weak force of the SU(4)-hypercolor interaction, after a spontaneous symmetry breaking of the SU(4)-hc-interaction $\mathrm{SU}(4) = \mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{SU}(1)_{\mathrm{R}} \otimes \mathrm{SU}(1)_{\mathrm{em}}$.

-in chap.3: introducing sub-particles as constituents of basic particles of SM: preons r and q with hc-charges, plus color-charge for q, with the parameters:

wave function $\Psi = (u_{L-}, u_{L+}, u_{R-}, u_{R+})$

r-preons $(r_{L-}, r_{L+}, r_{R-}, r_{R+})$, $Q(r) = -1/2$, $m(r) \ll 1\,\mathrm{meV}$,

q-preons $(q_{L-}, q_{L+}, q_{R-}, q_{R+})$, $Q(q) = +1/6$, $m(q) \sim 1\,\mathrm{MeV}$, $Q_{col}(q) = (r, g, b)$

-in chap.4: adding a new powerful calculation method: direct minimization of action. This calculation method was introduced in [4] [7] and applied successfully in QCD for calculation of hadrons.

-in chap.5: formulating the ansatz for wavefunctions.

The calculated results for energy-mass of basic particles are presented in chap.5.

Systematics

The systematics is described at the example of charged leptons.

For each particle family (generations), are presented:

-preon configuration and hc-boson configuration

Preon configuration: $u = \left(\begin{pmatrix} rL- \\ 0 \end{pmatrix}, 0, \begin{pmatrix} rR- \\ 0 \end{pmatrix}, 0 \right)$

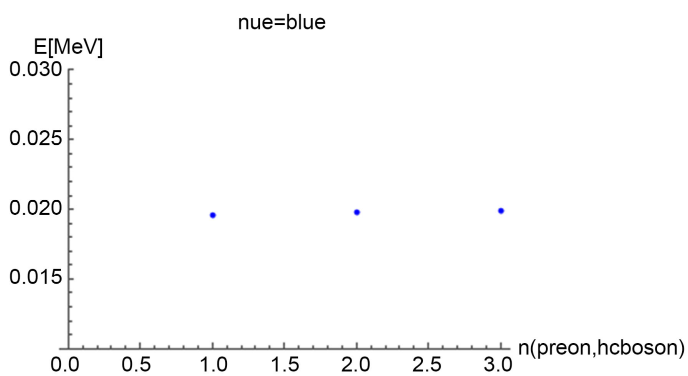
Boson configuration: flavor = 1: $(A13 = \lambda 4)$, flavor = 2: $(A13 = \lambda 4, \bar{A}13 = \lambda 5, A24 = \lambda 11, \bar{A}24 = \lambda 12)$

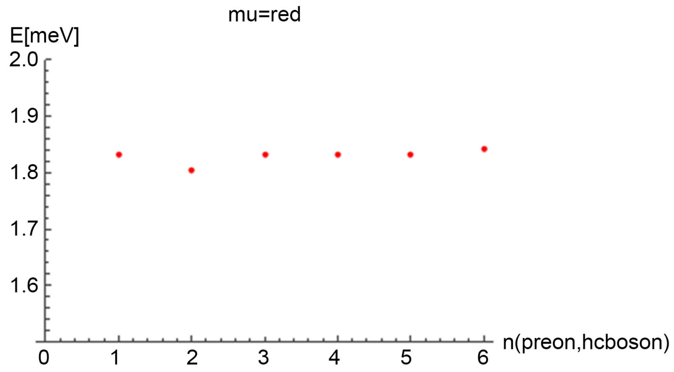
flavor = 3: all 15 bosons

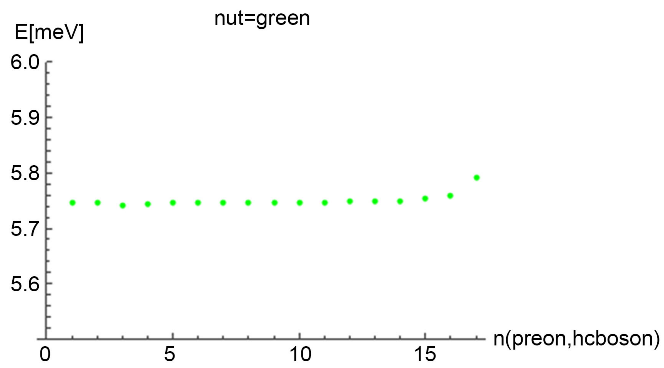
-calculated and observed mass

	m (e)	*m* (mu)	*m* (tau)
exp.	0.511 MeV	106 MeV	1.78 GeV
calc.	0.293 ± 0.22 MeV	228 ± 150 MeV	2.26 ± 0.7 GeV

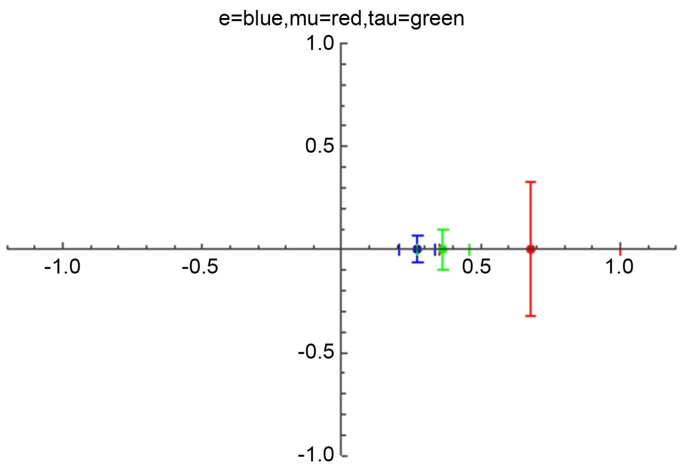
-energy distribution for three generations







-spatial preon configuration in (r,θ):



Mass hierarchy and the Koide formula

If we take for the neutrinos the calculated values, and for the rest the observed values, we get the following mass table for leptons and quarks

	m_1	m_2	m_3
neutrino (ν_e, ν_μ, ν_τ)	0.30 meV	11 meV	98 meV
electron (e, μ, τ)	0.511 MeV	106 MeV	1.78 GeV
u-quark (u, c, t)	2.3 MeV	1.34 GeV	171 GeV
d-quark (d, s, b)	4.8 MeV	100 MeV	4.2 GeV

The Koide formula [28] $k(m_1,m_2,m_3)=\frac{2}{3}\frac{\left(\sqrt{m_1}+\sqrt{m_2}+\sqrt{m_3}\right)^2}{m_1+m_2+m_3}\approx 1$ is approximately valid for the generations (1, 2, 3) of basic particles. The precise values are $k(\nu) = 0.8654$, $k(e) = 0.9998$, $k(u) = 1.2673$, $k(d) = 1.0891$ for the four basic families neutral leptons, charged leptons, u-quarks, d-quarks.

There is an approximate scaling law for the generation mass scale.

We make an ansatz for the mass-energy of generations u_i:

$M(u_i)=E_{ui}+m_{ui}N_i^{a_{ui}}$, where E_{ui} is the non-hcb energy contribution, m_{ui} is the first-generation-energy, a_{ui} is the hcb-exponent, and $N_i = (1, 4, 15)$ is the number of hcb's in a generation i.

Fitting the formula yields the exponents $a_{u_1}=1.11$ $a_{u_2}=1.20$ $a_{u_3}=1.50$ $a_{u_4}=1.34$, so $a_{u_i}\approx 1.3$.

We have the result: the approximate validity of the Koide formula $k \approx 1$ for the 4 families is the result of the power law of the generation mass hierarchy with the exponent $a_{ui} \approx 1.3$ approximately constant across the 4 families.

Calculated and observed masses of basic SM particles

Leptons and pure quarks

	m (e)	*m* (mu)	*m* (tau)
exp.	0.511 MeV	106 MeV	1.78 GeV
calc.	0.293 ± 0.22 MeV	228 ± 150 MeV	2.26 ± 0.7 GeV
	m (nue)	*m* (num)	*m* (nut)
exp.			
calc.	0.30 meV	11 meV	98 meV
	m (u)	*m* (c)	*m* (t)
exp.	2.3 MeV	1.34 GeV	171 GeV
calc.	2.35 ± 0.26 MeV	3.2 ± 1.87 GeV	163 ± 55 GeV
	m (d)	*m* (s)	*m* (b)
exp.	4.8 MeV	100 MeV	4.2 GeV
calc.	4.58 ± 0.3 MeV	149 ± 15 MeV	6.1 ± 2.9 GeV

dC = Cabibbo-mixed d-quark

	m (dC), *α* (C)
exp.	4.8 MeV, 13.04˚
calc.	4.74 MeV, 13.1˚

Weak massive bosons W, Z0, H (higgs), ZL (weakly interacting left-chiral Z-boson)

	m (W)	*m* (Z0)	*m* (ZL)	*m* (H)
exp.	80.4 GeV	91.2 GeV		125.1 GeV
calc.	89 GeV	97 GeV	91 GeV	125 GeV

new weakly interacting particles
sterile neutrinos $\nu s1$, $\nu s2$, $\nu s3$;
strong neutrinos νqe νqm νqt
strong bosons Zq Hq

	m (nus1)	*m* (nus2)	*m* (nus3)
exp.			
calc.	0.09 meV	3.6 meV	100 meV
	m (nuqe)	*m* (nuqm)	*m* (nuqt)
exp.			
calc.	23.2 MeV	205 MeV	2.4 GeV
	m (Zq)	*m* (Hq)	
exp.			
calc.	644 GeV	637 GeV	

Structure of basic SM particles

Symmetry and inner structure of particles is determined by the spatial distribution of preons.

Length is specified in units $r0 = 0.2 \times 10^{-18}$ m

Mean location (r(gi), θ(gi)) of preons in generation i = 1, 2, 3

	r (g1)	*r (g2)*	*r (g3)*	*θ (g1)*	*θ (g2)*	*θ (g3)*
e	*0.25*	*0.35*	*0.5*			
ν	*0.9*	*1.*	*1.1*			
u	*0, 0.3*	*0.1, 0.3*	*0.6, 0.6*			*0, π/6*
d	*0, 0.3*	*0, 0.3*	*0.1, 0.5*			
dC	*0.3, 0.8*			*0, π/8*		

Structure characteristics

We have the following structure characteristics:

-charged leptons (e, μ, τ) are spherically symmetric, with increasing radii (0.25, 0.35, 0.5)

-neutral leptons (νe, $\nu\mu$, $\nu\tau$) are spherically symmetric, with roughly equal radius ≈ 1

-pure u-quarks (u, c, t) have double-peaked structure with increasing radii ((0, 0.3), (0.1, 0.3), (0.6, 0.6)), the first two are spherically symmetric, and only the t-quark is slightly axial $\theta = (0, \pi/6)$

-pure d-quarks (d, s, b) have double-peaked structure with increasing radii ((0, 0.3), (0, 0.3), (0.1, 0.5)), and are spherically symmetric

-Cabibbo-mixed d-quark dC has double-peaked structure (0.3, 0.8) and is slightly axial $\theta = (0, \pi/8)$

Consequences from the calculated structure

-Cabibbo-mixing breaks the spherical symmetry

The observed first generation quarks (uC, dC) are Cabibbo-mixed with the CKM matrix, the higher generation quarks can be considered as approximately pure.

Cabibbo-mixing breaks the spherical symmetry, as shown for dC, and makes both first-generation quarks (uC, dC) axial.

-neutrino-mixing with large angles

Neutrino generations are one-peaked spherically symmetric, with approximately equal radius. Therefore it is plausible that mixing by PMNS matrix is easy, *i.e.* with large angles (neutrino oscillations).

-comparison of PMNS and CKM matrix

Quark mixing by CKM matrix is of type $(u,c,t)\times(d,s,b)$, where the first list labels the rows and the second list labels the columns, *i.e.* it is "partner-oriented" mixing.

Neutrino mixing by PMNS matrix is of type $(\nu_e,\nu_\mu,\nu_\tau)\times(\nu_e,\nu_\mu,\nu_\tau)$, *i.e.* it is "self-oriented" mixing.

Partner-oriented mixing of leptons according to the CKM scheme is not allowed (or energetically unfavorable), because neutrinos are chiral, and electrons are not.

Self-oriented mixing of quarks is allowed, but energetically unfavorable, which could be shown numerically by calculating a combination of both mixing schemes.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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