Journal of Modern Physics, 2023, 14, 1672-1692
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

Mass of the Universe from Quarks: A Plausible Solution to the Cosmological Constant Problem

Kevin Oramah

Department of Mathematics and Physics, University of Stavanger, Stavanger, Norway
Email: oramahkc@gmail.com

How to cite this paper: Oramah, K. (2023) Mass of the Universe from Quarks: A Plausible Solution to the Cosmological Constant Problem. *Journal of Modern Physics*, **14**, 1672-1692.
https://doi.org/10.4236/jmp.2023.1412098

Received: October 12, 2023
Accepted: November 26, 2023
Published: November 29, 2023

Abstract

A framework to estimate the mass of the universe from quarks is presented, taking spacetime into account. This is a link currently missing in our understanding of physics/science. The focus on mass-energy balance is aimed at finding a solution to the Cosmological Constant (CC) problem by attempting to quantize space-time and linking the vacuum energy density at the beginning of the universe and the current energy density. The CC problem is the famous disagreement of approximately 120 orders of magnitude between the theoretical energy density at the Planck scale and the indirectly measured cosmological energy density. Same framework is also used to determine the mass of the proton and neutron from first principles. The only input is the up quark (u-quark) mass, or precisely, the 1st generation quarks. The method assumes that the u-quark is twice as massive as the down-quark (d-quark). The gap equation is the starting point, introduced in its simplest form. The main idea is to assume that all the particles and fields in the unit universe are divided into quarks and everything else. Everything else means all fields and forces present in the universe. It is assumed that everything else can be "quark-quantized"; that is, assume that they can be quantized into similar sizeable u-quarks and/or it's associated interactions and relations. The result is surprisingly almost as measured and known values. The proton structure and mass composition are also analysed, showing that it likely has more than 3 quarks and more than 3 valence quarks. It is also possible to estimate the percentage of dark matter, dark energy, ordinary matter, and anti-matter. Finally, the cosmological constant problem or puzzle is resolved by connecting the vacuum energy density of Quantum Field Theory (5.1E+96 kg/m^3) and the energy density of General Relativity (1.04E−26 kg/m^3). Upon maturation, this framework can serve as a bridging platform between Quantum Field Theory and General Relativity. Other aspects of natures' field theories can be successfully ported to the platform. It also increases the chances of solving some of the unanswered questions in physics.

Keywords

Cosmological Constant, Proton Mass-Structure, Quark-Quantization, Dark Matter, Dark Energy, Age of the Universe, Energy Density, Spacetime Quantization

1. Introduction

I am not a professional physicist; just a guy who attended physics classes at some point in life. These are just my thoughts that stemmed from my master thesis on chiral symmetry breaking of effective quantum chromodynamics (QCD) models. The results obtained from my thoughts are not far off from known values that I decided to share, in hope that it might help the professionals with an alternative approach or amend existing ones.

The approach used here is based on mass-energy conservation; a concept that should hold at all physical scales (classical, cosmological, or subatomic) thanks to Newtons', Einstein's, and Planck's postulates and theories. The fact that Newton's second law (classical) can be derived from the Schrodinger wave equation (quantum) infers that the latter is more fundamental than the former, while Einsteins mass-energy/momentum equivalence principle connects the classical and quantum scales to the cosmological. The importance of Planks energy-quanta postulate cannot be overlooked. It laid the foundation of quantum mechanics (QM) and quantum field theory (QFT) which helped physicists to come up with the standard model (SM)—one of the greatest achievements of science [1].

Hence this is an attempt to go from the quantum level to the cosmological level. The framework presented is not perfect but has potential to be adjusted appropriately with a great chance of success.

The framework is built on the premise that a primordial proton existed along with every other particle and fields in nature at the start-phase or zygote-phase of the universe. The conditions at this zygote-phase were strong enough to disintegrate the primordial proton and the quarks contained in the proton underwent some shredding and multiplication. The zygote process created quarks for the subsequent phases that lead to the formation of protons, neutrons, electrons, and other particles of nature.

For simplicity and convenience, we shall not be dealing with the SM langrangians. The method used is to assume that the u-quark is twice as massive as the down-quark (d-quark). The only input is the up quark (u-quark) mass, or to be more precise; the 1st generation quarks.

The gap equation [2] is the starting point, introduced in its simplest form. The main idea is to assume that all the particles and fields in the unit universe is divided into quarks and everything else. Everything else means all fields and forces present in the universe. It is assumed that everything else can be "quark-quantized", that is, assume that they can be quantized into similar sizeable u-quarks based on the mass-energy dependence described above. This will simulate the creation-annihilation process as well as the other necessary interactions. In probability theory, sometimes it is easier to do P(B) = 1 − P (A) instead of trying to solve P(B) = P(B1) + P(B2) + ⋯ + P(Bn). Similar approach is used in this article to help circumvent using the SM langrangians and its associated parameters since we are not focusing on all the properties of quarks, only its mass. The mass of the universe (matter/energy), as well as gravity has to be conserved [3].

The focus on mass-energy balance here is aimed at finding a solution to the Cosmological Constant (CC) problem by attempting to quantise space-time and linking the vacuum energy density at the beginning of the universe and the current energy density. The CC problem is the famous disagreement of 120 orders of magnitude [4] [5] between the theoretical energy density at the Planck scale and the indirectly measured cosmology energy density.

2. The Gap Equation

The gap equation is introduced in its simplest form and quark-quantized.

$$M = m + \sigma \tag{1}$$

$$1 = \frac{m}{M} + \frac{\sigma}{M} \tag{2}$$

$$1 = \frac{m}{m + xm} + \frac{xm}{m + xm} \tag{3}$$

$$1 = \frac{1}{1 + x} + \frac{x}{1 + x} \tag{4}$$

M is the effective mass, m is the up-quark mass, σ represents everything else including gluons, gravity, electromagnetic force, etc. x is quantization number; discreet, natural, and positive acting as the process counter.

Multiply across by dx and taking the integral

$$\int \mathrm{d}x = \int \frac{1}{1 + x}\mathrm{d}x + \int \frac{x}{1 + x}\mathrm{d}x \tag{5}$$

It is coincidentally easier to deal with the first item on the RHS which is the object of interest.

Equations (4) and (5) must be satisfied for every value of x. Meanwhile it is obvious from Equation (4) that the lower bound of the integral (Equation (5)) cannot be 0 (zero). If x is 0, Equation (4) then says that only quarks exist and Equation (5) losses its physical meaning. Hence it must start from 1.

It also signifies that "true vacuum" does not exist in our universe. However, it should be noted that it is a consequence of gravity (which holds everything together) being finite (because of initial assumption) from the above equations. Howbeit, this must be validated by showing that the equations herein provide a basis of natural mitosis and metamorphosis. Hence, it must produce the framework for proton division, structure, formation, and agglomeration (*i.e.* atomic nucleation).

3. Quark Mass Spectrum Generator

To explain my understanding of the natural logarithm and natural number e in a simple language, I came across this online write up that did better than I could have done [6]. It defined the natural logarithm ln(*x*) as TIME needed to GROW to *x* with 100% continuous compounding. The exponent e^y is defined as the amount of GROWTH after TIME *y*, also with 100% continuous compounding. The point here is that the natural logarithmic function inherently contains elements of time and growth to help analyse natural processes if formulated in the required format while adhering to established principles [7] [8].

It is assumed that there must have been a primordial proton from which all other protons were created as viewed by some physicists [9]. The quarks in this primordial proton must have been multiplied by some great and powerful process within a relatively very short period.

Logically, this means that ln(*x* + 1) obtained from the first term on the RHS of Equation (5) also signifies the time needed to grow to amount (*x* + 1). This also means, as stated earlier, that [*x* = 0] does not make physical sense since we should be talking about breaking 1 item or entity into 2, at least. Hence buttressing the point that the integral needs a lower limit of 1, not 0. Breaking 1 item (a quark from the primordial proton) into 2 serves as the quark creation process that will create quarks of different sizes; a minimum size of $2^{e^{-1}}$ and maximum size of $2^{e^{1}}$ for the 1st generation quarks. The base number 2 is chosen as the next number after 1, since we cannot do much with base 1 in this quantization process. All the other quark types must be built from the 1st generation quarks.

We can therefore choose to represent the quark spectrum with;

$$m_{1\_G} = 2^{\left(\alpha \cdot e^{2} - \alpha + 1\right) \cdot \frac{\beta}{e}} \quad \text{where} \quad \alpha = [0, \cdots, 1] \quad \text{and} \quad \beta = [1, \cdots, 38] \tag{6}$$

For the first generation of quarks. $\beta = 1$: min value is $2^{e^{-1}} = 1.29$; (with $\alpha = 0$) and max value of $2^{e^{1}} = 6.58$; (with $\alpha = 1$).

The distribution between the min and max values (*i.e.* the values in between $\alpha = 0$ and $\alpha = 1$) still may still need some fine tuning but I think that what is presented in **Table 1** is sufficient to get to the main point of this article: to introduce a framework to estimate the mass of the universe from quarks. Equation (6) was chosen because it roughly maintained the cumulative coupling constant of quarks while estimating the mass of the universe. Please see Equation (21a), Section 6. Also please refer to the next Section 4, for explanation of β stopping at 38.

It is shown from **Table 1** how the other generation of quarks systematically arise from the 1st generation quarks (the up and down quarks). It may seem that the range of the 1st generation quarks places a plausible barrier that prevents the generation of more generations of quarks. This means that the quark generations may have stopped at $\beta = 7$ because the next value becomes greater than 6.58. The model presented here produces results that are in line with this preposition, but

Table 1. Quark mass spectrum distribution.

β \ a	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1*	**1.29**	1.52	1.79	2.10	2.48	2.91	3.43	4.04	4.75	5.59	**6.58**
2	1.67	2.31	3.20	4.43	6.13	8.49	11.76	16.29	22.57	31.27	43.31
3*	2.15	3.50	5.71	9.31	15.18	24.75	40.35	65.78	107.23	174.82	285.01
4	2.77	5.32	10.21	19.59	37.59	72.12	138.38	265.51	509.45	977.51	1875.59
5	3.58	8.08	18.25	41.21	93.07	210.17	474.61	1071.79	2420.36	5465.77	12343.03
6	4.62	12.27	32.62	86.70	230.44	612.47	1627.82	4326.45	11498.90	30561.96	81228.08
7	**5.96**	18.64	58.31	182.40	570.58	1784.82	5583.08	17464.38	54630.14	170887.95	534552.75
8	7.69	28.31	104.23	383.74	1412.78	5201.26	19148.81	70497.72	259542.42	955524.09	3.52E+06
9	9.92	43.00	186.32	807.32	3498.12	15157.31	65676.42	284575.12	1.23E+06	5.34E+06	2.32E+07
10	12.81	65.31	333.05	1698.45	8661.52	44170.83	225256.43	1.15E+06	5.86E+06	2.99E+07	1.52E+08
11	16.53	99.19	595.34	3573.20	21446.35	128720.92	772582.57	4.64E+06	2.78E+07	1.67E+08	1.00E+09
12	21.33	150.65	1064.17	7517.32	53102.23	375113.49	2.65E+06	1.87E+07	1.32E+08	9.34E+08	6.60E+09
13	27.52	228.80	1902.23	15814.96	131483.77	1.09E+06	9.09E+06	7.56E+07	6.28E+08	5.22E+09	4.34E+10
14	35.51	347.50	3400.28	33271.57	325560.36	3.19E+06	3.12E+07	3.05E+08	2.98E+09	2.92E+10	2.86E+11
15	45.83	527.78	6078.08	69996.85	806103.71	9.28E+06	1.07E+08	1.23E+09	1.42E+10	1.63E+11	1.88E+12
16	59.14	801.59	10864.68	147259.64	2.00E+06	2.71E+07	3.67E+08	4.97E+09	6.74E+10	9.13E+11	1.24E+13
17	76.32	1217.44	19420.85	309805.38	4.94E+06	7.88E+07	1.26E+09	2.01E+10	3.20E+11	5.11E+12	8.14E+13
18	98.48	1849.03	34715.17	651769.73	1.22E+07	2.30E+08	4.31E+09	8.10E+10	1.52E+12	2.85E+13	5.36E+14
19	127.09	2808.29	62054.08	1.37E+06	3.03E+07	6.70E+08	1.48E+10	3.27E+11	7.22E+12	1.60E+14	3.53E+15
20	164.00	4265.19	110922.95	2.88E+06	7.50E+07	1.95E+09	5.07E+10	1.32E+12	3.43E+13	8.92E+14	2.32E+16
21	211.64	6477.91	198277.07	6.07E+06	1.86E+08	5.69E+09	1.74E+11	5.33E+12	1.63E+14	4.99E+15	1.53E+17
22	273.11	9838.57	354424.35	1.28E+07	4.60E+08	1.66E+10	5.97E+11	2.15E+13	7.75E+14	2.79E+16	1.01E+18
23	352.44	14942.69	633540.85	2.69E+07	1.14E+09	4.83E+10	2.05E+12	8.68E+13	3.68E+15	1.56E+17	6.62E+18
24	454.81	22694.77	1.13E+06	5.65E+07	2.82E+09	1.41E+11	7.02E+12	3.50E+14	1.75E+16	8.72E+17	4.35E+19
25	586.91	34468.53	2.02E+06	1.19E+08	6.98E+09	4.10E+11	2.41E+13	1.41E+15	8.31E+16	4.88E+18	2.86E+20
26	757.38	52350.35	3.62E+06	2.50E+08	1.73E+10	1.19E+12	8.26E+13	5.71E+15	3.95E+17	2.73E+19	1.89E+21
27	977.36	79509.04	6.47E+06	5.26E+08	4.28E+10	3.48E+12	2.83E+14	2.30E+16	1.87E+18	1.53E+20	1.24E+22
28	1261.24	120757.31	1.16E+07	1.11E+09	1.06E+11	1.01E+13	9.72E+14	9.30E+16	8.91E+18	8.53E+20	8.17E+22
29	1627.57	183404.65	2.07E+07	2.33E+09	2.62E+11	2.96E+13	3.33E+15	3.76E+17	4.23E+19	4.77E+21	5.37E+23
30	2100.31	278552.63	3.69E+07	4.90E+09	6.50E+11	8.62E+13	1.14E+16	1.52E+18	2.01E+20	2.67E+22	3.54E+24
31	2710.35	423062.15	6.60E+07	1.03E+10	1.61E+12	2.51E+14	3.92E+16	6.12E+18	9.55E+20	1.49E+23	2.33E+25
32	3497.58	642541.34	1.18E+08	2.17E+10	3.98E+12	7.32E+14	1.34E+17	2.47E+19	4.54E+21	8.34E+23	1.53E+26
33	4513.47	975883.52	2.11E+08	4.56E+10	9.86E+12	2.13E+15	4.61E+17	9.97E+19	2.16E+22	4.66E+24	1.01E+27
34	5824.43	1.48E+06	3.77E+08	9.60E+10	2.44E+13	6.22E+15	1.58E+18	4.02E+20	1.02E+23	2.61E+25	6.63E+27
35	7516.16	2.25E+06	6.74E+08	2.02E+11	6.05E+13	1.81E+16	5.42E+18	1.62E+21	4.87E+23	1.46E+26	4.36E+28
36	9699.27	3.42E+06	1.21E+09	4.25E+11	1.50E+14	5.28E+16	1.86E+19	6.56E+21	2.31E+24	8.15E+26	2.87E+29
37	12516.46	5.19E+06	2.15E+09	8.94E+11	3.71E+14	1.54E+17	6.38E+19	2.65E+22	1.10E+25	4.56E+27	1.89E+30
38*	16151.93	7.89E+06	3.85E+09	1.88E+12	9.18E+14	4.48E+17	2.19E+20	1.07E+23	5.22E+25	2.55E+28	1.24E+31

however raising the question of whether there are 3 generations (possibly due to symmetry) or if there should be 7 generations. Most of the quark-clusters (especially after $\beta = 7$) will be unstable for forming visible matter and eventually form the basis of dark matter and dark energy. On the other hand, another compelling argument for the SM's 3 quark generations may likely lie in the 3 distinct group (highlighted with asterisks) due to the calculation sequence to adhere to the Heisenberg uncertainty principle which is detailed in the next section. **1*** group representing the 1st generation; **3*** group representing the 2nd generation, and **38*** group representing the 3rd generation.

4. Proton Formation and Structure

Quantum theory requires that the Heisenberg uncertainty principle must be obeyed. We adhere to this principle in the manner and sequence of dealing with the equations above.

Keep LHS of Equation (5) constant; run the integration on the RHS starting from 1. The values of the LHS of Equation (5) is equivalent to the values β, which we are trying to ascertain here. Since the lower value of x needs to be 1, the upper value starts from 2 which validates the start from 1 on the LHS.

$$1 = \int_1^4 \frac{1}{1+x}\mathrm{d}x + \int \frac{x}{1+x}\mathrm{d}x \tag{6a}$$

$$1 = \ln(2.5) + \int \frac{x}{1+x}\mathrm{d}x \tag{6b}$$

A maximum is reached at $x = 4$ after which the first integral on the RHS becomes greater than 1. The same interval is now used on the LHS keeping the RHS constant.

$$\int_1^4 \mathrm{d}x = 3 = \int \frac{1}{1+x}\mathrm{d}x + \int \frac{x}{1+x}\mathrm{d}x \tag{7}$$

$$3 = \int_1^{39} \frac{1}{1+x}\mathrm{d}x + \int \frac{x}{1+x}\mathrm{d}x \tag{8a}$$

Again the LHS is kept constant while the RHS is taken to maximum. This happens at $x = 39$. Hence

$$3 = \ln(20) + \int \frac{x}{1+x}\mathrm{d}x \tag{8b}$$

The procedure is repeated likewise once more, and we get

$$\int_1^{39} \mathrm{d}x = 38 = \int_1^{w} \frac{1}{1+x}\mathrm{d}x + \int \frac{x}{1+x}\mathrm{d}x \tag{8c}$$

where $w = 63,711,863,514,227,512$.

This is already large. Computing EXP (63,711,863,514,227,511) for the next step does not seem to be feasible and even if there exists a computing cluster(s) that can handle it, the results or effects thereof is very negligible. Hence, we stop with 38 on the LHS.

If we hadn't gotten a clue from experiment, the equivalent up-quark mass number that will be calculated from the first equation on the RHS of Equation

(8c) will be 63711863514227512. It would not be rational to focus on Equation (8c) to fish for stable protons based on known proton mass used as guide. Hence it makes sense to use a value of 3 on the LHS (Equation (8b)) and see what value of proton mass it produces.

After restoring the effective mass *M*. we get;

$$3 \cdot M = \ln(20) \cdot M + (3 - \ln(20)) \cdot M \tag{9}$$

From Equation (9), the second term on the RHS $\{(3 - \ln(20)) \cdot M\}$ must lie between 1.29 and 6.58. It most likely would be much closer to 1.29 since the u-quark mass equivalent is the quantization parameter.

To determine the range of *M*:

$$2^{e^{-1}} \le (3 - \ln(20)) \cdot M \le 2^{e^{1}} \tag{10}$$

We can re-write Equation (9) as

$$M_N = M_P + \Delta_{40} \cdot M \tag{11}$$

where M_N is the neutron mass, M_P is the proton mass, and $\Delta_{40} = 3 - \ln(20)$

Comparing Equations (9) and (11) implies;

$$M_N = 3M$$

$$M_P = M_N \cdot \frac{\ln(20)}{3} = M \cdot \ln(20) \tag{12}$$

$$M_N - M_P = \Delta_{40} \cdot M$$

To estimate the proton mass in analogy to Equations (1) and (5):

$$M_P = 1 + \frac{\overline{M_{u,d}}}{\Delta_{40}} \tag{13}$$

where $\overline{M_{u,d}}$ is the average weighted mass of the u and d quarks; 1 represents the fact that the proton is not built on vacuum as explained earlier. It might help to look at it as 2^0 (see **Appendix**), since every quark must have a base value of 2 and there must be 1 or 2 free quark(s) locked-in in the proton structure that gives it the required stability and differentiate it from the neutron. The only un-verified possible explanation from the author for this lock-in-force is that it is a piece of vacuum energy.

After naively doing a little iteration around a linear average value of approximately 3.9 (average between 1.29 and 6.58), the value of 4 consistently showed up as solution; and this gave an almost precise mass of the proton.

It is already known from experiments that the unit of mass of subatomic particles is $\frac{\text{MeV}}{\text{C}^2}$.

Now we can, with a good enough certainty, take the average mass of u and d quarks $\overline{M_{u,d}} = 4\frac{\text{MeV}}{\text{C}^2}$.

This implies $M_u = \frac{8}{3}$ and $M_d = \frac{16}{3}$.

Using our assumption info from Equations (8a) and (8b), it means that the total quark mass in the proton is

$$N_q^P \cdot M_u = 40 \cdot \frac{8}{3} = \frac{320}{3} = 106.67 \frac{\text{MeV}}{\text{C}^2} \tag{13a}$$

where N_q^P is the number of equivalent u-quark in the proton.

This is almost the size of the strange quark. Hence, in disagreement with some suggestion that the proton might contain strange quarks [10]. Howbeit, it is shown here mathematically that the likelihood of strange quarks in the proton is very slim.

From Equation (13) $M_P = \frac{7-\ln(20)}{3-\ln(20)}$ and from Equation (12) $M_N = \frac{7-\ln(20)}{3-\ln(20)} \cdot \frac{3}{\ln(20)}$.

Hence $M = \frac{7-\ln(20)}{3-\ln(20)} \cdot \frac{1}{\ln(20)} \approx 313.2 \frac{\text{MeV}}{\text{C}^2}$ and $M_N - M_P = \frac{7-\ln(20)}{\ln(20)} \approx 1.337 \frac{\text{MeV}}{\text{C}^2}$.

The definition of coupling constants α_s might vary depending on the context [11]. Here it can be shown that α_s is the ratio of quarks in the proton because it also indicates the strength of interaction [12] or it's effect per quark, *i.e.*

$$\alpha_{s,p} = \frac{M_{u,d}^T}{M_p} = \frac{106.67}{938.27} = 0.1137 \tag{13b}$$

Hence, the quarks make up approximately 11.37% of the proton mass. $M_{u,d}^T$ is the total mass of u-quarks and d-quarks in the proton.

For the effective mass or neutron. the coupling constant is

$$\alpha_s = \frac{M_u^{13}}{M} = \frac{13 \cdot 8/3}{313.2} = 0.110685 \tag{13c}$$

where M_u^{13} is the mass of the 13 equivalent u quarks contained in the effect mass (lumps or clusters of quarks) that make up stable neutrons when they pair up in 3's (*i.e.* 3M).

Thus, it seems that the proton chose an average mass that is perfectly "quantized"; a whole natural number (4); no decimals. We can therefore define the proton as a perfect quantum object or rather composite to be precise.

The following expression from Equation (12) is used in calculating the values in **Table 2**, $x_1 = 1$ is the lower limit and $x_{12} = [2, \cdots, 39]$ is the upper limit.

$$\ln\left(\frac{x_1 + x_{12}}{2}\right) \cdot M \left[\frac{\text{MeV}}{\text{C}^2}\right] \tag{13d}$$

We have established the proton mass, the neutron mass and the neutron-proton mass difference. We have also established the value of *M*; the effective quark mass. It might even be tempting to equate the value $\Delta_{40}^{-1} \approx 234.32$ with the QCD energy scale. This scale varies across the $\beta = \{1, \cdots, 38\}$ levels from 11.9 to 1.44E+17 according to this framework.

Table 2. Proton metamorphosis.

	$x_1 + x_{12}$ from Equation (12)	M-I Pion Channel	M-II Meson Channel	M-III Baryon Channel
	1	-		
	2	-		
	3	126.99		
	4[G4]	217.09		
Pion	5	286.98		
	6		344.09	
	7[G3]		392.37	
	8[G3]		434.19	
	9		471.08	
	10		504.08	
	11		533.93	
	12		561.18	
	13		586.25	
Meson	14		609.46	
	15[G2]			631.07
	16[G2]			651.28
	17			670.27
	18			688.17
	19			705.11
	20			721.17
	21			736.45
	22			751.02
	23			764.95
	24			778.28
	25			791.06
	26			803.35
	27			815.17
	28			826.56
	29			837.55
	30			848.16
	31			858.43
	32[G1]			868.38
	33			878.02

Continued

	34	887.37
	35	896.44
	36	905.27
	37	913.85
	38	922.2
	39	930.34
Proton	40^{VP}	938.27

VP—Valence Porch (Valence Pool from 33 to 40); G1—Gate 1; G2—Gate 2; G3—Gate 3; G4—Gate 4. G2, G3 and G4 are locations where a change in sign occurs with regards to quark-quark/quark-gluon interactions. This is related to the S (normalisation factor) in Section splitting ratio. G1 is likely associated with some kind of symmetry in the proton. This indicates that the proton structure is complex and whatever is presented in this article is for illustrative purposes only and an attempt to make it more comprehensible.

Equations (12) and (13) show the importance of having M and Δ_{40} as fixed values in relation to proton-neutron calculations. An example of this importance is in calculating the proton charge-radius.

$$L_P = M_p \cdot C \cdot r_P^c \cdot \Delta_{40} = 1\left[\frac{\text{eV}}{\text{C}^2}\frac{\text{m}^2}{\text{s}}\right] \tag{14}$$

where L_P is the angular momentum in the reference frame of the proton, C is the speed of light, r_P^c is the proton-charge radius, Δ_{40} accounts for the effect of the strong force, $\left[\frac{\text{eV}}{\text{C}^2}\right]$ indicates that the calculation is done in the same reference of spin of the proton. The result from the above semi-classical calculation is close to measured values [13]. Hopefully, there is new information presented here to help resolve the proton radius puzzle and probably the proton spin crisis.

The presence of the one/two single quark(s) in the pion channel of the presented proton structure (**Table 2**) indicates that there must be some pretty strong barrier (for lack of proper qualification) that is capable of locking up such a high pressure [14] within such a rather small space in the pion channel. With reference to **Figure 1**, the effect of the gluons increases quite sharply for the pion channel and then gradually eases off in the meson channel. Hence, it is not evident that the high pressure on the first one/two equivalent quarks is attributed to just the gluon effect alone.

In **Figure 1**, the centre of the proton is from the 0 position. The gluon effect is approximately the inverse of the interquark coupling effects or the running coupling constants within the proton.

The mass distribution within the proton is as follows:

- due to quarks (11.4%)
- due to gluons (27.2%)

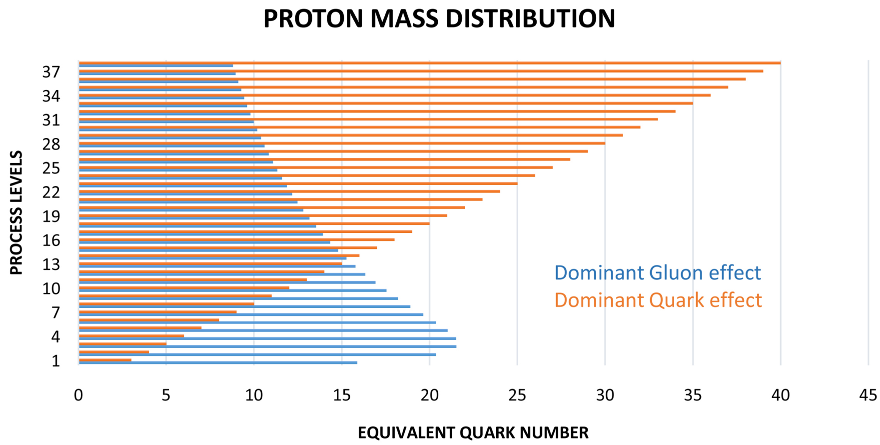

Figure 1. Illustration of proton mass distribution. Note that the equivalent quark number starts from 3. The first two equivalent quarks are omitted because I am not sure how to represent the effect on them because it would entail division by zero or very close to zero. This distribution is just for illustration purposes. Actual distribution is highly likely very complex.

- quark-quark/quark-gluon interaction (22%)
- due to internal momentum and other effects (39.4%)

The mass due to quarks is same as the coupling constant for the proton.

The mass due to gluons = $\left(1-\alpha_{s,p}\right)\cdot\left(1-\ln\left(2\right)\right)$

Please see the section on splitting ratio below to understand why the (1 − ln(2)) factor.

The mass due to interactions is approximately = $\frac{M}{M_p}-\alpha_{s,p}$.

At the accelerators, it is highly unlikely to temporarily dislodge more than 24 u-quarks equivalent (combination of 16 up-quarks and 8 down-quarks) per proton based on Figure 1, a value of 18 (12 up-quarks and 6 down-quarks) per proton is more realistic. The other quarks that are not accessible at the accelerators are hidden behind "safety valves" placed at strategic points within the proton; as indicated beneath Table 2 except Gate 1 that should still be accessible.

This rather simple description of the proton, together with other known aspects from experiments does not still mean we have cracked the mystery of the proton [15]. It is hoped that there is new information here that will aid to find solutions to some of the unanswered questions regarding the proton.

5. Dark Matter, Dark Energy, Antimatter

In this model it is possible to estimate the percentage of dark matter (DM), dark energy (DE) and antimatter (AM).

Consider a factory that manufactures or produces luxury goods at a very "high pace". A pace so high that it matches (or maybe at some point in history even exceeded) the speed of light. There is bound to be quite a number of defective goods. In the factory of nature, dark matter (DM) and dark energy (DE) are "defective items". I'll call them defectons. With reference to Section splitting ratio, Table 2 and current measured percentage values of DM and DE [16], we can analogically infer that:

DM is pseudo mesonic in nature: meso-defectons.

DE is pseudo baryonic in nature: baryo-defectons.

The very stringent regime of proton synthesis and its perfectionist characteristic is most likely the basis for DM and DE. The detailed nature of DM and DE however still needs further investigation.

Nature is very discreet and continuous at the same time. There are no voids or vacuum in space as long as gravity is involved [17]. Only the perfect works of nature are made visible. The rest are either defectons and/or "moulding tools" (vacuum energy inclusive). It is near impossible to separate vacuum energy from gravity. Vacuum energy is the latent potential that keeps every space enclosed by gravity alive. It works in tandem with gravity to create particles (quark splitting) and synthesize protons, with DE and DM as by products. There is high possibility that specific portions of vacuum energy are locked-in in the proton to maintain desired stability and possible continuity of the universe.

Splitting Ratio

From Equation (13d); $x_1 = 0$ (lower limit) and $x_{12} = 1$ (upper limit) are not part of the proton synthesis phase. As described earlier, one of the equations losses it's physical meaning. However, the upper limit $x_{12} = 1$ plays a very significant role in mass/energy allocation or what I would just call SPLITTING RATIO.

We write the solution to the Equation (5) as

$$1_x = \ln\left(\frac{x_2 + x_{21}}{2}\right)\cdot\frac{1}{S} + \left\{1 - \ln\left(\frac{x_2 + x_{21}}{2}\right)\cdot\frac{1}{S}\right\} \tag{15}$$

where $x_2 = 1$; $x_{21} = [1,\cdots,38]$; $x = [1,\cdots,38]$; $S = 1$ for $x = [1,\cdots,4]$; $S = 2$ for $x = [5,\cdots,13]$ and $S = 3$ for $x = [14,\cdots,38]$

For $x = 1$ and $x_{21} = 1$; Equation (15) → $1 = \ln(1) + (1 - \ln(1)) = A + B$ (15a)

We write Equation (4) as

$$y = \frac{1}{1 + x_{22}} + \frac{x_{22}}{1 + x_{22}} \quad (x_{22} = x_{21} + 1;\ x_{21}/x_{22} \rightarrow \text{start/end of time segments}) \tag{16}$$

where $y = [1,\cdots,38]$

For $y = 1$ and $x_{22} = 2$; Equation (16) → $1 = \frac{1}{3} + \frac{2}{3}$ (16a)

For $y = 2$ and $x_{22} = 3$; Equation (16) → $2 = \frac{1}{4} + \frac{7}{4} \rightarrow 1 = \frac{1}{8} + \frac{7}{8}$ (16b)

For $y = 3$ and $x_{22} = 4$; Equation (16) → $3 = \frac{1}{5} + \frac{14}{5} \rightarrow 1 = \frac{1}{15} + \frac{14}{15} = C + D$ (16c)

Equations (15) and (16) are repeated for all x and y.

where A = visible sector; B = hidden sector; C = excited wave/energy (that transforms to particle state); D = non-excited wave/energy.

$S = 1$, $S = 2$, $S = 3$ represents very distinct stages of the formation of the universe, that shows up as normalization factors in the equalities above.

A and B splits the visible matter sector (C) into matter and anti-matter portions; and splits the invisible matter sector (D) into DE and DM.

From stage 3 or 3rd period (where the protons were formed) in relation to the splitting ratio above we obtain (values in bracket → start of 3rd time period, x_{21}):

$$AC = \text{VM\_visible matter} = \ln(2)\cdot\frac{1}{15} \approx 0.0462\,(0.05776) \tag{17}$$

$$BC = \text{HAM\_hidden antimatter} = \frac{1-\ln(2)}{15} \approx 0.0204\,(0.02557) \tag{17a}$$

$$AD = \text{DE} = \frac{14\cdot\ln(2)}{15} \approx 0.647\,(0.63538) \tag{17b}$$

$$BD = \text{DM} = \frac{14\cdot(1-\ln(2))}{15} \approx 0.286\,(0.28128) \tag{17c}$$

It has been shown in this article that DM and DE exist in nature, and hence not just inferred to exist from observations. This is a step in the right direction towards solving the cosmological constant problem, fundamentally [4] [18].

Figure 2 (for x_{21}) and **Figure 3** (for x_{22}) depicts the results from the equations above in this section splitting ratio.

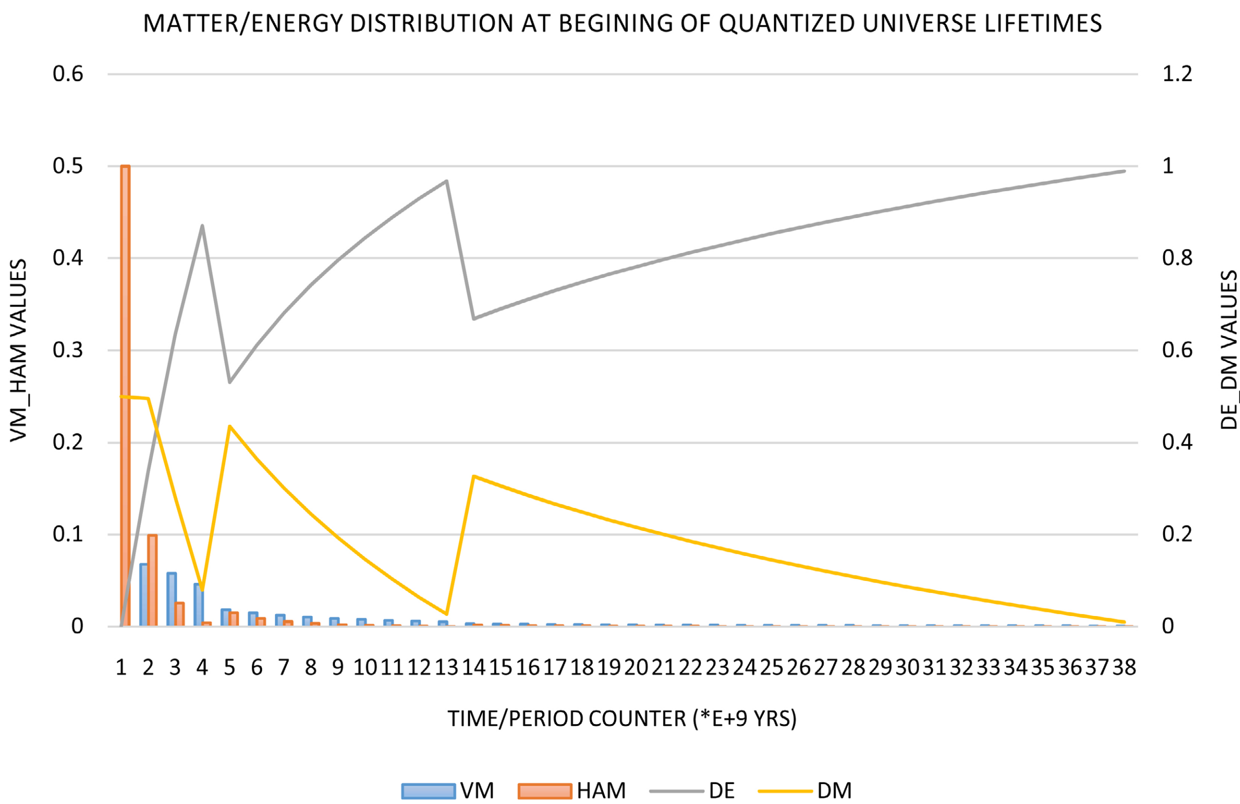

Figure 2. Mass-Energy formation and progression with time relative to the splitting ratio at beginning of each time segment.

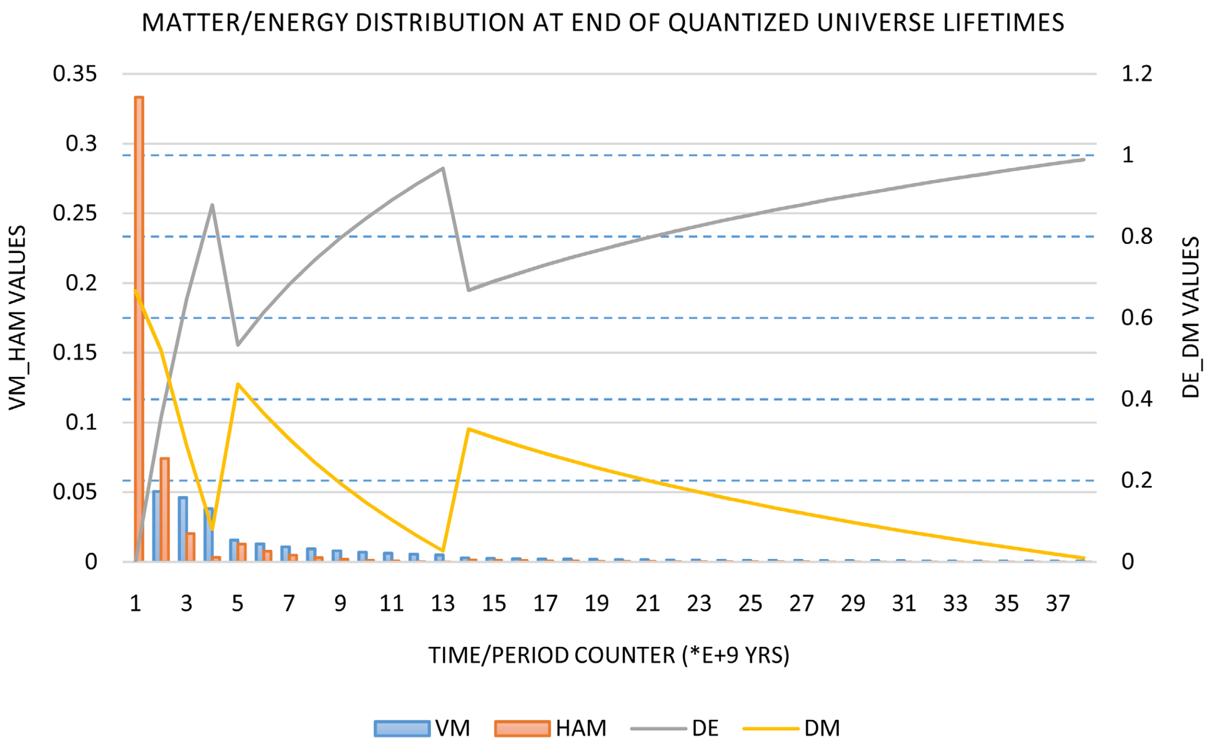

Figure 3. Mass-Energy formation and progression with time relative to the splitting ratio at the end of each time segment.

Figure 2 and **Figure 3** has some shortcomings; 1) the 38E+9 yrs does not correspond to measured and accepted value for the age of the universe, 2) the missing transformation from values at 38E+9 yrs to values at 1E+9 yrs since the timeline is meant to be presented as cyclic, 3) the percentage values of DM and DE should have changed from those presented above but we still currently measure values from the proton formation stage.

However, they indicate that our universe is most likely born DM-antimatter symmetric, and that DE is expected to have higher percentage values than other forms of matter/energy.

6. Estimating the Mass of the Universe

Recall the Equations (1) to (6), and our assumption that σ (which includes gravity and other fields of nature) are quark-quantized. M, the effective mass/energy can be appropriated to estimate the mass of the universe. So far, we have no reason to doubt that the gravitational constant is not conserved [4] [5]. Hence, we introduce gravity as

$$G = \frac{V(\tau)}{M_U \tau^2} = constant \quad (18)$$

where $V(\tau)$ is the volume of the universe as a function of time; M_U is the constant mass/energy of the universe. Equation (18) is in analogy to replacing the constituents with the Planck equivalents to get the gravitational constant value.

We rewrite Equation (18)

$$M_U = \frac{V(\tau)}{G\tau^2} \tag{18a}$$

Take the ln of Equation (18a)

$$\ln(M_U) = \ln(V(\tau)) - \ln(G) - \ln(\tau^2) \tag{19}$$

$$\ln(M_U \cdot \tau^2) = \ln\left(\frac{V(\tau)}{G}\right) \tag{19a}$$

$$M_U = |M_q|^2 \tag{20}$$

where M_q is the aggregated quark mass. The value is squared because we are applying the Born rule to locate all masses/energies with certainty within the volume of the universe at a specific or specified time as well as accounting for imaginary (complex) values. In as much as we are trying to keep it simple with this quark-quantization method, we shall not neglect the basic QM rules and that we are dealing with wavicles or wave-like particles. To evaluate M_q, we integrate Equation (6) over $\alpha = (0, \cdots, 1)$ and then over $\beta = (1, \cdots, 39)$. Refer **Appendix** section for detail.

$$M_q = \int_1^{39} \frac{e\left(2^{2\cdot\beta\cdot e - \frac{\beta}{e}} - 2^{\frac{\beta}{e}}\right)}{\ln(2)(\beta\cdot e^2 - \beta)} \mathrm{d}\beta = \int_1^{39} Mq_\alpha \mathrm{d}\beta \tag{21}$$

Equation (21) can also be written as

$$M_q \approx \frac{1}{\bar{\alpha}_s} \sum_{i=\beta}^{\beta=1\cdots38} Mq_\alpha \tag{21a}$$

$\bar{\alpha}_s \approx \alpha_s$ was used as a control parameter for Equation (6).

Figure 4 shows the results from Equations (18) to (21). Gravitational constant is held constant as a function of density and scaled time. The scaled time is with reference to the end of each quantized time segment (*i.e.* corresponding to α = 1). The quantized time is used to estimate the volume created by gravity/energy/matter interaction, and then the scaled time is extracted with the effect of gravity and M_U. We shall have a closer look at this scaled time in Sections 6.1 and 6.2.

6.1. Time Quantization and Age of the Universe

This method or framework has churned out a life circle period of 38E+9 years for our universe. Unless I missed something, it is the time scale that makes most sense and connects the loose ends. It is possible that there should be some connection between the measured age of the universe and the value obtained herein.

With reference to Section 3, Equation (6) and **Table 1**:

$$\text{For } \alpha = 1;\ m_{1\_G} = 2^{e\beta};\ T_V = \ln\left(m_{1\_G} = 2^{e\beta}\right) = e\beta\ln(2) \tag{22a}$$

$$\text{For } \alpha = 0;\ m_{1\_G} = 2^{\frac{\beta}{e}};\ T_H = \ln\left(m_{1\_G} = 2^{\frac{\beta}{e}}\right) = \frac{\beta}{e}\ln(2) \tag{22b}$$

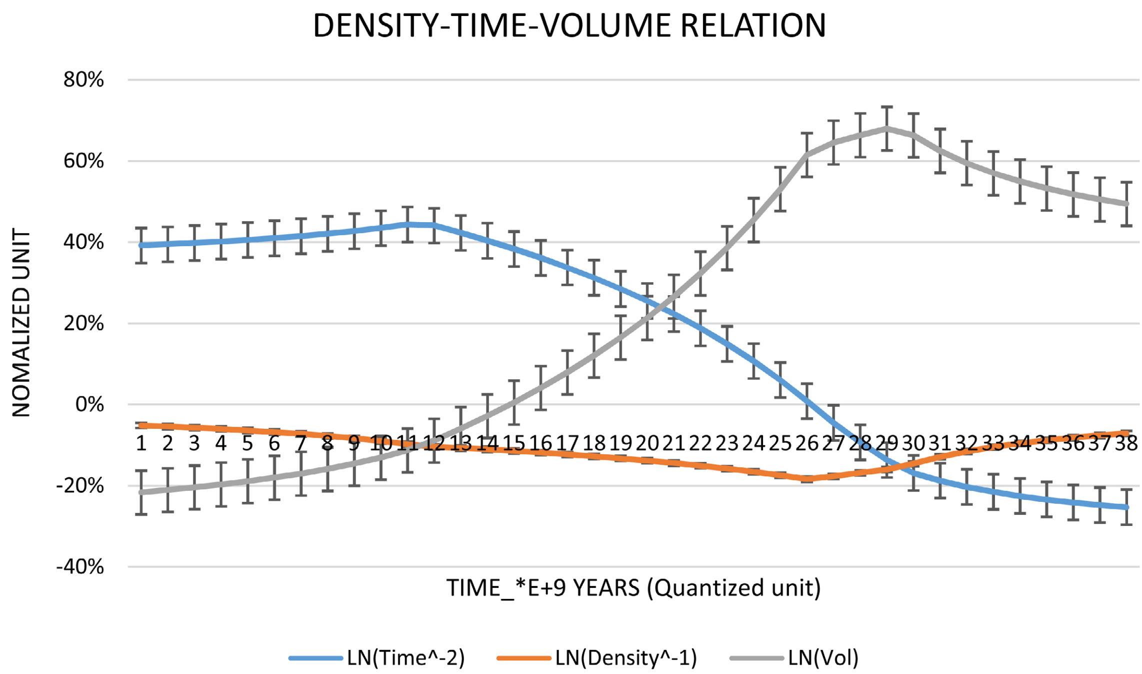

Figure 4. Normalised Log values of density, time and volume from Equation (18).

$$T_V = T_H \cdot e^2 \tag{23}$$

where T_V is reference to time across vertical section of **Table 1**;

T_H is reference to time across horizontal section of **Table 1**.

We now differentiate between β in T_V and β^* in T_H. β represents quantized value and β^* represents real value based on apparent measured value.

With respect to the scaled time (**Figure 5**):

$$T_V = T_H^* \cdot e^2 \tag{23a}$$

$$e \cdot \beta \cdot \ln(2) = \frac{\beta^*}{e} \cdot \ln(2) \cdot e^2 \tag{23b}$$

$$\beta = \frac{\beta^*}{e} \cdot e \tag{23c}$$

$$\beta^* = \Delta\tau \cdot e \tag{23d}$$

where $\Delta\tau$ is the apparent measured value. This is currently 13.832E+9 yrs [16].

β gives the range of the energy density and associated information. $\Delta\tau$ gives the β^* and hence a more accurate value of the energy density. This implies that our universe is approximately 37.6E+9 yrs based on the time description above. The above is the best explanation I can offer for the time data, though I stand to be corrected. However, the time distribution and scaling still needs professional assessment.

There seems to be some strange connection between the structure of the proton; the age of the universe and the matter/energy content of the universe. In other words, there might be a connection between the proton and spacetime as shown on **Figure 6** by taking the ratio of the respective equivalent u-quarks to

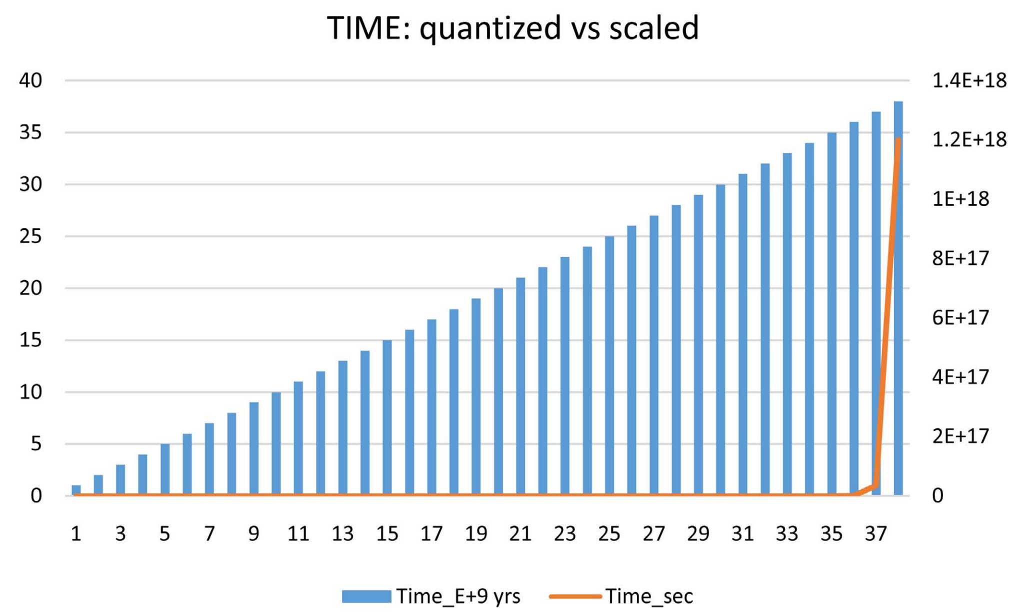

Figure 5. Timeline for the formation/lifespan of the universe.

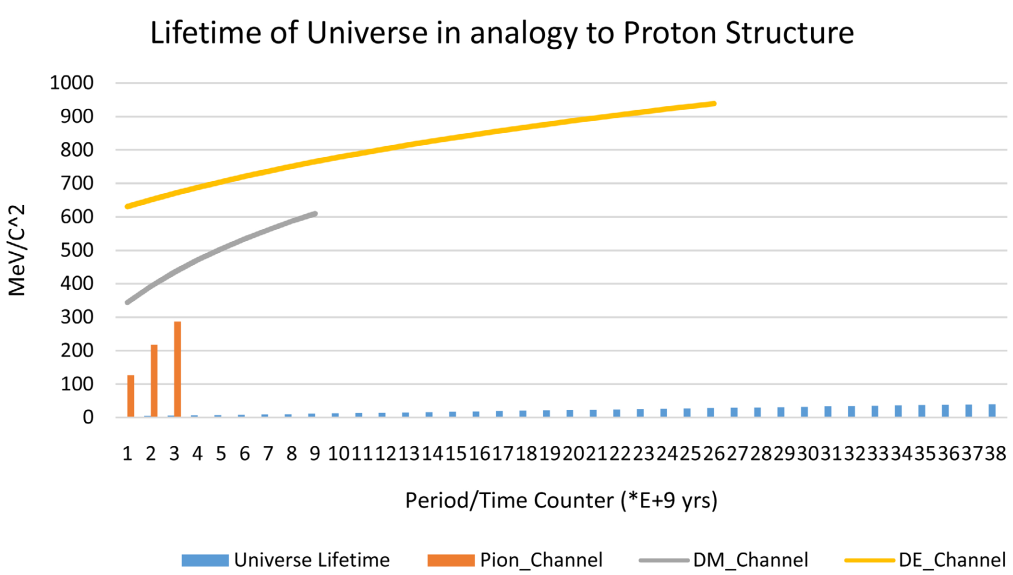

Figure 6. Analogy of proton structure and universe timeline.

the total u-quarks in the proton. It might as well be a result of the approach used. Maybe folks with better understanding can hopefully offer required explanation.

6.2. Reconciling the Energy Density of General Relativity and Qm

From the Equations (18)-(21), the following results are obtained assuming the universe is currently in its 38E+9 yr of existence (the last millennium).

- Mass of the universe (6.7E+54 kg)
- Volume of the universe (6.4E+80 m^3)
- density (1.04E−26 kg/m^3)

Table 3. Summary of scaled values.

Time_E+9 yrs	Time_sec	Density_kg/m³	Volume_m³
1	2.393E−39	2.627E+87	2.547E−33
2	1.075E−37	1.309E+84	5.111E−30
3	3.924E−36	9.751E+80	6.862E−27
4	1.384E−34	7.898E+77	8.481E−24
5	4.762E−33	6.605E+74	1.013E−20
6	1.631E−31	5.631E+71	1.188E−17
7	5.556E−30	4.854E+68	1.378E−14
8	1.886E−28	4.213E+65	1.588E−11
9	6.385E−27	3.674E+62	1.821E−08
10	2.159E−25	3.216E+59	2.081E−05
11	7.287E−24	2.821E+56	2.372E−02
12	2.458E−22	2.480E+53	2.698E+01
13	8.284E−21	2.183E+50	3.065E+04
14	2.790E−19	1.924E+47	3.477E+07
15	9.395E−18	1.697E+44	3.942E+10
16	3.162E−16	1.499E+41	4.464E+13
17	1.064E−14	1.324E+38	5.053E+16
18	3.578E−13	1.170E+35	5.717E+19
19	1.203E−11	1.035E+32	6.464E+22
20	4.045E−10	9.156E+28	7.307E+25
21	1.360E−08	8.103E+25	8.257E+28
22	4.570E−07	7.173E+22	9.327E+31
23	1.536E−05	6.352E+19	1.053E+35
24	5.161E−04	5.625E+16	1.189E+38
25	1.734E−02	4.983E+13	1.343E+41
26	5.825E−01	4.415E+10	1.515E+44
27	1.957E+01	3.912E+07	1.710E+47
28	6.573E+02	3.467E+04	1.930E+50
29	2.208E+04	3.073E+01	2.177E+53
30	7.416E+05	2.724E−02	2.456E+56
31	2.491E+07	2.415E−05	2.770E+59
32	8.364E+08	2.141E−08	3.125E+62
33	2.809E+10	1.899E−11	3.524E+65
34	9.433E+11	1.684E−14	3.974E+68
35	3.168E+13	1.493E−17	4.481E+71
36	1.064E+15	1.324E−20	5.052E+74
37	3.571E+16	1.175E−23	5.696E+77
38	1.199E+18	1.042E−26	6.422E+80

- radius of the universe (5.4E+26 m)
- timeline started at 2.39E−39 sec with volume of 2.55E−33 m^3 and density of 2.63E+87 kg/m^3

However, projecting the timeline down to the Planck time 5.39E−44 sec yields a density of 5.1E+96 kg/m^3 at a volume of 1.3E−42 m^3.

This shows that we can go from Quantum Mechanics energy density to the General Relativity energy density as depicted on **Table 3**, hence clearing up the cosmological constant puzzle. They are both correct, only differentiated by spacetime.

The conditions at the beginning of the timeline were VM = 0%; HAM = 50%; DE = 0% and DM = 50% according to the described splitting ratio (ref. **Figure 2**). However, VM is not actually 0% based on Equations (21) or (21a) and our assumption of the primordial proton. The percentage value will definitely be so small that it'll be quite close to zero.

Hence, we can now write Equation (18) as

$$G = \frac{1}{\rho_{GR}\tau_U^2} = \frac{1}{\rho_{QM}\tau_P^2} = constant \tag{24}$$

where ρ_{GR} is the General Relativity energy density; τ_U is the age of the universe at the time of measurement of ρ_{GR}.

ρ_{QM} is Quantum Mechanics energy density; τ_P is the age of universe = Planck time.

7. Summary/Conclusions

The framework presented in this article shows that we can estimate the mass of the universe from quarks. This is a link currently missing in our understanding of physics/science. Except for Equation (14), the entire manuscript revolves around three main Equations (1), (6), (18). The electron mass can be estimated as well from the framework. Upon maturation, it can serve as a bridging platform between QFT and GR. Other aspects of natures' field theories can be successfully ported to the platform. It also increases the chances of solving some of the unanswered questions in physics. The main takeaways from this framework are:

- Seems our universe was born DM-Antimatter symmetric.
- No living generation will likely have the opportunity to witness proton decay; at least not in flesh and blood.
- It supports the localised big bang thesis.
- Higher dimensions or symmetries might not really be necessary for our universe.
- Hope that some information in this manuscript triggers something new in some way of reconciling GR and QM via spacetime quantization.
- DM and DE are likely quark-like (pseudo-quarks) in nature.
- Analysing the proton from a different perspective.

On a personal note, I don't think gravity is as weak as it is perceived to be. It is likely at work in more ways than we can fathom. Gravity is the most perfect and powerful spring in nature, that creates the boundary of the universe and space for matter and energy.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Roberts, C.D. (2023) *EPJ Web of Conferences*, **282**, Article Number 01006. https://doi.org/10.1051/epjconf/202328201006

[2] Adler, S.L. (1986) *Progress of Theoretical Physics Supplement*, **86**, 12-17. https://doi.org/10.1143/PTPS.86.12

[3] Wilczek, F. (2003) *MIT Physics Annual*. https://physics.mit.edu/wp-content/uploads/2021/01/physicsatmit_03_wilczek_originofmass.pdf

[4] Weiberg, S. (1989) *Reviews of Modern Physics*, **61**. https://doi.org/10.1103/RevModPhys.61.1

[5] Margan, E. (2012) Estimating the Vacuum Energy Density—An Overview of Possible Scenarios. Jozef Stefan Institute, Ljubljana.

[6] Azad, K. (2016). https://betterexplained.com/articles/demystifying-the-natural-logarithm-ln/

[7] Bhattacharya, R. (2023) *The Journal of Young Physicists*. https://www.journalofyoungphysicists.org/quantum-gravity-and-space-time/

[8] Elitzur, A.C. and Dolev, S. (2005) Becoming as a Bridge between Quantum Mechanics and Relativity. World Scientific Publishing Co. https://doi.org/10.1142/9789812701596_0031

[9] Luminet, J.-P. (2014) Lemaître's Big Bang. Frontiers of Fundamental Physics. Aix Marseille University (AMU), Marseille.

[10] Ball, R.D., *et al.* (2022) The NNPDF Collaboration. Evidence for Intrinsic Charm Quarks in the Proton.

[11] Hinchliffe, I. (2005) Quantum Chromodynamics and Its Coupling. https://pdg.lbl.gov/2006/reviews/qcdrpp.pdf

[12] Deur, A., Brodsky, S.J. and de Teramond, G.F. (2016) *Progress in Particle and Nuclear Physics*, **90**, 1-74.

[13] Peset, C., Pineda, A. and Tomalak, O. (2021) The Proton Radius (Puzzle?) and Its Relatives. FERMILAB-PUB-21-254-T; TUM-HEP 1340/21.

[14] Burkert, V.D., Elouadrhiri, L. and Girod, F.X. (2018) *Nature*, **557**, 396-399. https://doi.org/10.1038/s41586-018-0060-z

[15] Hadhazy, A. (2021). https://www.kavlifoundation.org/news/the-enduring-quest-for-proton-decay

[16] Aiola, S., *et al.* (2021) The Atacama Cosmology Telescope: DR4 Maps and Cosmological Parameters. Draft Version.

[17] Macken, J.A. (2015) Spacetime Based Foundation of Quantum Mechanics and General Relativity. *Progress in Theoretical Chemistry and Physics*, Springer, Switzerland, 219–245. https://doi.org/10.1007/978-3-319-14397-2_13

[18] Koksma, J.F. and Prokopec, T. (2011) The Cosmological Constant and Lorentz Invariance of the Vacuum State. ITP-UU-11/16, SPIN-11/10.

Appendix

Detailed information regarding estimation of aggregated quark mass M_{q}, based on my understanding and interpretation of event.

Using Equation (6)

$$Mq_{\alpha} = \int m_{1\_G} \mathrm{d}\alpha = \int 2^{\alpha\beta \cdot e - \frac{\alpha\beta}{e} + \frac{\beta}{e}} \mathrm{d}\alpha \tag{A1}$$

Apply substitution $u = \alpha\beta \cdot e - \frac{\alpha\beta}{e} + \frac{\beta}{e}$; $\frac{\mathrm{d}u}{\mathrm{d}\alpha} = \beta \cdot e - \frac{\beta}{e} = \frac{\beta\left(e^2 - 1\right)}{e}$

$$Mq_{\alpha} = \frac{e}{\beta\left(e^2-1\right)} \int 2^u \mathrm{d}u = \frac{e}{\beta\left(e^2-1\right)} \cdot \frac{2^u}{\ln(2)} \cdot C \Bigg|_{\alpha=0}^{\alpha=1} \tag{A2}$$

where $C = 2^{u'} = 2^{\int \mathrm{d}u = \frac{\beta\left(e^2-1\right)}{e} \int \mathrm{d}\alpha} = 2^{\frac{\beta\left(e^2-1\right)}{e} \cdot \alpha}$

$$Mq_{\alpha} = \frac{e}{\beta\left(e^2-1\right)} \cdot \frac{2^{2 \cdot e \cdot \beta - \frac{\beta}{e}} - 2^{\frac{\beta}{e}}}{\ln(2)} \tag{A3}$$

Notice that the integral constant required special attention. Taking it like the usual + C did not yield the expected result if Equation (A1) was integrated within the $\alpha = \{0, \ldots, 1\}$ range for $\beta = 1$ because it (+C) has the wrong interpretation or physical meaning. This is the "quark shredding" or quark creation/multiplication process, while the second integration over β is the moulding or forming process.

Let us examine the integral constant obtained above at $\beta = 1$:

$$C_{\alpha=1} = 2^{\frac{e^2-1}{e}} = 2^{e - \frac{1}{e}} = \frac{2^e}{2^{\frac{1}{e}}} \quad \text{and} \quad C_{\alpha=0} = 2^0 = 1 \tag{A4}$$

We can see that C is a dimensionless constant because it is just the ratio of the max/min value of the 1st generation quarks. Dimensionless 1 for $\alpha = 0$ indicates the that vacuum energy is not included in this particular process or part of the summation.

Alternatively, the following method might be used, especially when it is difficult to ascertain the integral constant. Instead of the $\alpha = \{0, \ldots, 1\}$ range, we use $\alpha = \{1, \ldots, 2\}$ range and let the integral constant be 1. There is some small difference that doesn't mean much cumulatively in this case.

$$Mq_{\alpha} = \int_1^2 m_{1\_G} \mathrm{d}\alpha = \int_1^2 2^{\alpha\beta \cdot e - \frac{\alpha\beta}{e} + \frac{\beta}{e}} \mathrm{d}\alpha \tag{A5}$$

Apply the u substitute as above,

$$u_{\alpha=1} = \beta \cdot e = u1; \; u_{\alpha=2} = 2\beta \cdot e - \frac{\beta}{e} = u2$$

$$Mq_{\alpha} = \frac{e}{\beta\left(e^2-1\right)} \int_{u1}^{u2} 2^u \mathrm{d}u = \frac{e}{\beta\left(e^2-1\right)} \frac{2^u}{\ln(2)} \Bigg|_{u1}^{u2} = \frac{e}{\beta\left(e^2-1\right)} \frac{2^{2 \cdot e \cdot \beta - \frac{\beta}{e}} - 2^{\beta \cdot e}}{\ln(2)} \tag{A6}$$