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Quantum Entanglement Could Be the Result of Leptons, Quarks and Photons Simultaneously Experiencing 4-D Space as (3 + 1)-D Spacetime

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Abstract

We propose that quantum entanglement occurs because the fundamental particles, such as electrons, quarks, and photons, simultaneously experience both the 4^{th} real spatial dimension in R^4 as well as the time dimension in (3 + 1)-D spacetime. Consequently, the entangled particles can never become separated in the 4th spatial dimension no matter how far they have moved apart in the other 3 spatial dimensions. Because the quark and lepton families represent specific different discrete symmetry binary subgroups of SU(2), we can establish that the quantum states of the fundamental particles are defined in 4 spatial dimensions, so there is then no need for a spacetime communication from one detector (or particle) to inform the other detector (or particle) of the physical state of the first detected entangled particle. A clever experiment needs to determine whether the fundamental particles actually experience a 4th spatial dimension, and if so, whether they experience the 4th spatial dimension as the time dimension simultaneously. Apparently, if a Casimir-like test reveals that virtual particles have a non-zero mass, there are claims that a 4th spatial dimension does not exist.

Keywords

Quantum Entanglement, Four Dimensions, Particle Physics, Spacetime

1. Introduction

Quantum entanglement is a challenging behavior of Nature that begs for a geometrical understanding. We propose a geometrical explanation based upon the fundamental lepton and quark families representing specific discrete symmetry binary subgroups of SU(2) in four dimensional coordinate space R^4. Until such R^4 definitions became recognized and fully appreciated [1], this geometrical approach to quantum entanglement could not be considered. As a result of this definitive identification of the fundamental particle states, we propose that in our (3 + 1)-D spacetime world, the quantum entangled particles may appear to become separated as their R^3 spatial distance increases but there is no true separation in the R^4 world of the particles.

Quantum entanglement occurs because a group of particles can be generated in a way that the quantum state of each particle of the group cannot be described independently of the state of the others, even when the particles are separated by a huge distance. For two identical particles A and B that are quantum entangled, their total wave function $\Psi(X_A, X_B, t)$, where the X_i are their spatial coordinates, remains valid until the entanglement is lost and the wave function becomes the product $\Psi(X_A, t)\Psi(X_B, t)$.

The entanglement of quantum states does not occur as one would expect in classical mechanics [2], thereby emphasizing an essential difference between classical physics and quantum physics. Specifically, quantum entanglement dictates that the momentum, spin, polarization, and 3-D position measurements conducted on entangled particles will be perfectly correlated. For example, if a pair of entangled particles is generated with their total intrinsic spin known to be zero, and particle A is measured to have an up spin on a randomly chosen axis at a detector, then the spin of particle B, measured along the same axis at a second detector, will always be down, *i.e.*, the opposite spin.

These types of measurements, originally called Einstein, Podalsky, Rosen measurements, or EPR measurements, have always disagreed with the classical idea that each particle in the quantum entangled pair carries its unique fixed spin value along its trajectory from creation to its detection. But the EPR measurements support the quantum result that only the total spin value is fixed. An extensive physics research literature exists [3] from which more information can be learned about the variety of acceptable interpretations about entangled electrons, photons, neutrinos, molecules, buckyballs, as well as diamonds and Bose condensates.

This paradoxical behavior for quantum entangled particles, even if they are separated by distances of light-years before their measurements are achieved, has led to the abandonment of locality as one of the foundations of physics [4] [5]. The principle of locality is based upon the idea that influences occur locally and cannot travel faster than light speed. The EPR tests have included sufficiently separated detector locations such that the time interval between the measurements by the two detectors was more than 1000 times shorter than the time interval required for any communication to pass between them [6].

In Section 2, we provide an example of the quantum mechanical wave function and some limitations to the geometry that we consider important for discussing the possible origin of quantum entanglement. Section 3 reviews some important physical properties of the fundamental particles, the leptons, quarks, and electroweak (EW) interaction bosons, principally their quantum state definitions with regard to R^4. We also connect some of the geometric properties to the physical properties of the SM particles that we have established because they represent specific discrete symmetry subgroups of SU(2). Section 4 discusses some characteristics of a 4-D spatial geometry that emphasize its significant differences from our familiar 3-D spatial world R^3. We consider further whether the 4th spatial dimension and time can be two aspects of a single dimension for fundamental particles or whether there must exist a separate 4th spatial dimension in addition to the time dimension. Finally, in Section 5, we ask whether there is any independent evidence for a 4th spatial dimension and, if not, what kinds of planned experiments are in progress that may be able to check for a 4th spatial dimension in our physical world.

2. Proposed Source of Entanglement

So how could Nature be achieving quantum entanglement in our (3 + 1)-D physical world, a world in which the special theory of relativity (STR) forbids information transfer faster than the speed of light?

We propose that the physical world for the fundamental particles is simultaneously 4-dimensional spatial R^4 and (3 + 1)-D spacetime, meaning that leptons, quarks and the interaction bosons, such as the photon, experience both the 4th real spatial dimension of R^4 and the time dimension in (3 + 1)-D spacetime.

We artificially partition the particle world into a (3 + 1)-D spacetime and an internal symmetry space in which the fundamental particles are defined at every spacetime position. The dimension of the internal symmetry space is unknown, but we will show that leptons, quarks, and their interaction bosons require 4 spatial dimensions for their definitions. Hence, there is a one-to-one matching of the 4 internal space dimensions to the 4 dimensions of (3 + 1)-D spacetime.

Therefore, a particle wave function would depend upon the four coordinates as $\Psi(w/ict, x, y, z)$ for the fourth spatial coordinate w and the standard 3 spatial coordinates x, y, z, with the time coordiate being the ict for all 3-D observers. That is, the 4th dimension in the wave function would have a dual role for fundamental particles. An electron, for example, would be experiencing simultaneously both the 4th spatial dimension as well as the time dimension.

Consequently, entangled particles can never become separated in the 4th spatial dimension no matter how far they have moved apart in the other 3 spatial dimensions. The total quantum mechanical wave function of the entangled particles can never be partitioned into the individual wave functions because of this eternal connection in the 4th spatial dimension until the total quantum entanglement is lost. There is then no need for a spacetime communication from one detector (or particle) to inform the other detector (or particle) of the physical state of the first detected entangled particle.

Unlike the "hidden variables" conjecture, the quantum state of each particle will represent both possible spin values until the detector measures a quantum eigenvalue. That is, the wave function requires only the one spatial dimension, the 4th spatial dimension, to contain the one qubit of information for each particle. For example, the north and south poles along one direction of the Bloch sphere, or Poincaré sphere, would represent the normal basis states $|0\rangle$ and $|1\rangle$ for the spin-up and spin-down states of each electron. A pure qubit state for the spin of one electron would be

$$\frac{1}{\sqrt{2}}\left(|0\rangle+|1\rangle\right). \tag{1}$$

An example of quantum entanglement involving two electrons created with total spin zero can be represented by

$$\frac{1}{\sqrt{2}}\left(|01\rangle+|10\rangle\right), \tag{2}$$

in which the spin state of electron A occupies the first position in each ket and the spin state of electron B occupies the second position in each ket.

Therefore, in order to justify our proposed geometrical explanation of how quantum entanglement is achieved, two steps are required. First, the fundamental particles must be shown to have access to the 4th spatial dimension. Second, the ability of the fundamental particles to experience both R^4 and (3 + 1)-D simultaneously must be established.

Of course, we should consider all three possibilities: 1) A 4th spatial dimension that adds to the normal 3 spatial dimensions plus time dimension of Minkowski (3 + 1)-D spacetime to make (4 + 1)-D sacetime; 2) A 4th spatial dimension that must be extremely small in extent; 3) A 4th spatial dimension that simultaneously is the time dimension for the fundamental particles.

Possibility 1, the additional 4th spatial dimension creates several new problems. For one example, we would have easily detected such a large spatial dimension because there would have been a different formulation of special and general relativity, and we know that our present formulations are correct. Also, as we emphasize in Section 4 below, some vary strange behavior of 3-D objects can occur if an additional spatial dimension exists. For possibility 2, a very small 4th spatial dimension at each spacetime point may be possible, that is, not unreasonable. Whether one could correctly formulate special relativity and general relativity as we know them would need to be considered. Also, if such a very small 4th spatial dimension does exist, then how does one stop at adding only one additional small dimension instead of introducing 6 or 7 additional small spatial dimensions to make 10 or 11 total dimensions as suggested by superstring theory or M-theory, with either large dimensional spacetime being a continuous spacetime or a discrete spacetime?

Therefore, because the fundamental leptons, quarks, and Standard Model(SM) interaction bosons will be shown to access the 4th spatial dimension in the next section without the need for more than 4 spatial dimensions, we will consider only geometry issues associated with the SM particles experiencing R^4 and (3 + 1)-D simultaneously.

3. Lepton/Quark Discrete Symmetries

In order for a geometrical explanation of how Nature achieves quantum entanglement to apply, we first need to establish whether all the fundamental particles experience 4 spatial dimensions and not just 3 spatial dimensions.

The SM of leptons, quarks, and their interactions is a quantum field theory (QFT), which requires a continuous symmetry. Before we introduced specific discrete symmetry binary subgroups of SU(2) for the lepton and quark families, all the families were considered to be representing SU(2) × U(1). Recently, however, various attempts to derive the mass values of the leptons and quarks, *i.e.*, solving the flavor problem, has led other researchers to consider discrete symmetry modular groups [7], many of which are the discrete symmetry binary subgroups we assigned to the lepton and the quark families.

Having discrete symmetries for the families is a violation of the QFT requirement. However, we determined that the QFT continuous symmetry requirement is satisfied because the discrete symmetry groups for the lepton families act together to mimic SU(2) and the quark families separately do likewise. This proposed collective behavior to mimic SU(2) leads directly to the correct mixing matrices independently for the leptons and the quarks, in complete agreement with empirical results.

The ability of a lepton, such as an electron, to experience 4 spatial dimensions instead of just 3 spatial dimensions needs explanation because we know [1] that the 3 lepton families represent 3 different specific discrete symmetries in R^3, namely, the binary subgroups of SU(2) also called the double groups 2T, 2O, 2I, for the regular tetrahedron, the regular octahedron, and the regular icosahedron. The lepton families defined by these familiar symmetries, which define the Platonic solids, indicate that the leptons are 3-D non-pointlike objects at the Planck scale of about 10^{-35} meters.

How do we know that the lepton families represent these specific discrete symmetries? Because they predict [8] the correct lepton mixing angles from the linear superposition of their group generators and produce entries of the 3 × 3 PMNS neutrino mixing matrix that agree with values determined experimentally. Therefore, the lepton family states might require only 3 spatial dimensions for their SM weak isospin definitions, not 4. This result conflicts with our proposal that 4 spatial dimensions are needed for particles exhibiting quantum entanglement. However, their geometry also provides the resolution of this apparent discrepancy.

Geometrically, one can represent the two particle states in each lepton family as vectors from the origin to the 2-D surface of a 3-D sphere. The locations of two points on a 2-D surface require 4 d.o.f., *i.e.*, degrees of freedom for defining their unique locations on the surface. So, for each lepton family Nature had a choice in partitioning the 4 d.o.f., either as the family pair with each particle state having 2 d.o.f. or as the pair with one particle having 3 d.o.f. and its partner particle having only 1 d.o.f.

However, recall that STR requires that a non-zero mass particle must have 3 d.o.f. so that all observers will experience the same physics, and that a particle with fewer d.o.f. will be massless. Thus, according to the SM weak isospin state definitions, Nature seems to have chosen the 3/1 partitioning for the leptons, with its weak isospin electron-like state having a mass and its partner neutrino state being massless. Quite possibly, if Nature had ever chosen two massless particle states with 2 d.o.f. each for the lepton families, they could be the elusive sterile neutrinos proposed to explain some experimental data.

Before analyzing the charged-lepton states and the neutrino states in more detail, we must consider the quark family symmetries. Building upon our successful identification of the lepton family discrete symmetry assignments in R^3, we have proposed [1] [9] [10] that each quark family represents a different specific discrete symmetry in R^4. There are 4 possible regular discrete symmetries in R^4, so we have predicted 4 quark families. These 4-D symmetries can be built up geometrically from the 3-D discrete symmetries that the leptons represent, so the two sets are related. Again, we are proposing that the quarks are not pointlike objects but actually are 4-D objects with regular symmetries at the Planck scale.

Our quark family discrete symmetry group assignments are: (u, d) represents [3, 3, 3], (c, s) represents [4, 3, 3], [t, b] represents [3, 4, 3], and the 4th quark family [t', b'] represents [5, 3, 3]. Again, these symmetry group assignments produce the correct mixing angles for a 4 × 4 CKM4 quark mixing matrix as well as for the normal CKM matrix, *i.e.*, the 3 × 3 submatrix of CKM4. There is one discrepancy, however, for the 3 × 3 CKM matrix at the V_{ub} position in the first row, and we are attempting to determine its source. There is a phase factor in the CKM mixing matrix at this position that can reduce the predicted real value of $V_{ub} = 0.0098$ to almost the empirical value of 0.0039, but the source of this factor in our CKM4 mixing matrix remains to be determined.

The proposed existence of 4 quark families but only 3 lepton families conflicts with the standard method by which the triangle anomalies have been cancelled throughout the decades [11]. However, with our collective behavior for the 3 lepton families mimicking SU(2) and the 4 quark families mimicking SU(2) also, there is now a direct one-to-one cancellation of opposite sign contributions. We no longer have the question of which quark family will cancel the contribution of the electron family, *i.e.*, the up/down family or the charm/strange family, etc.

Note that the proposed 4th quark family has not been detected yet, but its existence may be difficult to detect even at the LHC because its quarks may have extremely short half-lives and decay into numerous background b quarks, for example. Hopefully, the two quarks b' and t' will be detected soon because several important physical phenomena may be explained by their contributions. One such application would be to explain the baryon asymmetry of the Universe (BAU), for which a very high mass t' quark at about 3 TeV would increase the present estimated value of the Jarlskog constant [12] by a needed factor of about 10^{13}.

Geometrically, a 4-D sphere with its 3-D surface will have 6 d.o.f. for the two quark states per family. Nature has partitioned these 6 d.o.f. into 3 d.o.f and 3 d.o.f., dictating two massive quark states per quark family. Also, because quarks are 4-D particles, they remain confined, *i.e.*, a free quark cannot exist in a 3-D spatial world. Therefore, we previously proposed a geometrical explanation of quark confinement, meaning that a quark (antiquark) must join in combination with an antiquark (quark) to form a 3-D meson or combine with other quarks (antiquarks) to form a 3-D baryon (anti-baryon).

The existence of antiparticles also has a geometrical explanation. According to Bott Periodicity in mathematics, any normal real space of dimension 4N has an equivalent conjugate real space of the same dimension, true for integers N only. Thus, the SM has fundamental particles, *i.e.*, the quarks in the 4-D normal space R^4, so their antiparticles occupy the 4-D conjugate space R'^4. The geometry suggests that the particles, even those defined in the subspace R^3 of R^4 such as the leptons, actually participate in R^4 because they have antiparticles. Further discussion below utilizes this inclusion.

Now we return to examine the SM neutrino weak isospin states that were originally considered to be massless. Experiments have established [13] that the three SM neutrino states (ν_e, ν_μ, ν_τ) mix to form three different neutrino mass states (m_1, m_2, m_3), each with very small mass values in the meV range instead of the MeV range. We have predicted [14] their mass values to be 0.3 meV, 8.9 meV, and 50.7 meV, respectively.

However, having neutrino non-zero mass states would require 3 d.o.f. for each, not just the 1 d.o.f. presently allotted to the SM weak isospin states. Therefore, with the charged-lepton states and the neutrino mass states both having non-zero mass values, we suspect that all the lepton states participate in the 4th spatial dimension even though they require only three spatial dimensions for their lepton family definitions. That is, by analogy to the two quark family states, each with 3 d.o.f., the two lepton states per family require R^4 also. Thus we propose that all the fundamental particles in Nature, the leptons and the quarks that comprise all matter in the Universe, exist in 4 spatial dimensions. Consequently, we can try to use the 4th spatial dimension to resolve the quantum entanglement paradox.

We still have the photon to consider. Can we establish that a photon also participates in the 4th spatial dimension? And the weak interaction bosons, too? Yes, because a photon γ is one of the four EW bosons of the SM, along with the W^+, W^-, and Z^0, that mediate the electroweak interaction, which includes interacting with the electrically charged 4-D quark states as well as with their weak charges. Therefore, the photon is participating in R^4 and R'^4. In addition, recall that STR dictates that no time elapses for the photon in its own reference frame! And, because numerous experiments reveal that quantum entangled photons behave in the same way as quantum entangled electrons, photon wave functions must include the 4th spatial dimension also.

We have established therefore that the fundamental particles of the SM, *i.e.*, the leptons, the quarks, and their boson mediators for the electromagnetic, weak, and color interactions, all participate in 4 spatial dimensions.

4. Nature of Time and Space

The time "dimension" and the 4th spatial dimension have different mathematical and physical properties. For a fundamental particle in (3 + 1)-D spacetime, time goes only in one direction [10], *i.e.*, forward for the particles in the normal space we experience. In contrast, the particles can move forward or backward in each spatial dimension.

When time and space are understood via 8-dimensional Clifford algebra, their interconnections are more complicated than usually considered because the 8 dimensions exhibit two sets of 4 dimensions that agree with both special relativity (STR) and general relativity (GTR) [15]. From 4 spatial dimensions, one does a spacetime split to create (3 + 1)-D spacetime, realizing that the 4th spatial dimension $w \leftrightarrow ict$.

Briefly, the 4th spatial dimension is connected to the time dimension such that a particle defined in 4 spatial dimensions can simultaneously exhibit the passage of time in (3 + 1)-D, thereby not requiring a separate 4th spatial dimension. Thus, the particle not only exists in its 4 spatial dimensions but also simultaneously exhibits (3 + 1)-D behavior.

Particle decay must be included in this dual behavior. The geometrical reason for particle decay is explained via Kuratowski's Theorem, *i.e.*, the only stable graphs in dimensions greater than 2-D are K_5 and $K_{3,3}$. Remarkably, the 4-D symmetry group [3, 3, 3] for the first quark family (u, d) is the K_5 graph, so all other quark families must decay eventually to the first quark family. The first lepton family (ν_e, e^-) symmetry group 2T is geometrically related to [3, 3, 3] and behaves similarly. Consequently, a fundamental particle defined in R^4 can achieve physical outcomes not possible for an object defined in and confined solely to the smaller dimensional space R^3.

The spatial world of R^4 is remarkably different and quite unfamiliar to us (3 + 1)-D spacetime entities. For example, a person living in a 3-D spatial world cannot experience the 4th spatial dimension. One can understand this limited behavior by considering the direct analogy to the fact that a 2-D person living in a 2-D "flatland" cannot experience the 3rd dimension! That is, imagine you are a 2-D person living in a 2-D world such as in a tabletop surface that exists in a 3-D spatial world, which you cannot experience. You cannot reach outside your 2-D surface, but a 3-D person can reach to the 2-D tabletop and add or subtract a 2-D object, an event which you might comsider as the sudden "magical" appearance or disappearance of the 2-D object.

In order to appreciate better an R^4 world, consider how much bigger the 3-D spatial world is compared to the 2-D world even when the 2-D world's two orthogonal coordinate directions can extend to infinity. Some additional examples of the behavior differences between objects in the two spaces R^3 and R^4 can be thought provoking:

1) With a standard 3-D rope, a secure knot cannot be tied in R^4.

2) Two interconnected solid rings in R^3 can be separated in R^4 without the rings touching each other.

3) A 4-D entity can reach into a 3-D refrigerator without going through its

door or walls to remove a piece of food from the refrigerator.

Such remarkable behavior is seldom considered or appreciated by us 3-D entities. And note also that in a 4-D space by removing a 3-D object from a 3-D container without going through the walls of the container, the 3-D space of the object "goes along" with the object.

5. Future Tests for a 4th Spatial Dimension

Can one test independently from the proposed quantum entanglement behavior for the existence of 4 spatial dimensions? Although we could not find evidence for the existence of a physical 4th spatial dimension in any research articles, future Casimir-like tests attempting to measure virtual particle mass values might resolve the issue [16]. Supposedly, if virtual particle mass values are not zero, then one proposed explanation for this result relies upon there not being additional spatial dimensions beyond three. However, if the virtual particle mass is zero, then more than three spatial dimensions are possible.

Obviously, our proposed existence of a 4th spatial dimension as the principal means by which quantum entanglement exists needs to be investigated by clever future experiments. Until the existence of a 4th spatial dimension acting simultaneously as the time dimension is firmly established as a definite yes or no, our suggested explanation remains a possible step toward a better understanding of quantum entanglement in Nature.

6. Conclusions

We propose a geometric explanation for the origin of quantum entanglement behavior: All quantum entangled particles experience a 4th spatial dimension and maintain their total connection in this 4th spatial dimension as they separate in (3 + 1)-D spacetime, thereby having an eternal connection until total quantum entanglement is lost. There is then no need for a spacetime communication from one detector (or particle) to inform the other detector (or particle) of the measured physical state of the first detected entangled particle.

We consider the 4th spatial dimension for the fundamental particles and the time dimension in our (3 + 1)-D world to be two aspects of the same geometrical dimension. Hopefully, a future experiment to check for the existence of a 4th spatial dimension experienced by fundamental particles will determine whether quantum entanglement is explained by our geometric explanation.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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