

Malus-Law Models for Aspect-Type Experiments

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Abstract

The inequalities of Bell, Clauser-Horne-Shimony-Holt (CHSH) and others are shown to be inconsistent with the Fundamental (Universal) Model of probability theory when combined with physics laws of the Malus-type. This combination permits the modeling of all results of quantum theory related to CHSH-Aspect-type experiments, while respecting Einstein's separation principle.

Keywords

Bell's Inequalities, Quantum Entanglement, EPR Experiments

1. Introduction

Einstein-Podolsky-Rosen (EPR) [1] have suggested a Gedanken-experiment for correlations of spatially separated measurements, proposing elements of physical reality as the cause of distant quantum-correlations. They intended to provide a firm logical framework for their discussions with Bohr about the nature of physical reality and the "entanglement" of quantum entities. Their framework of thought was, at least in principle, confirmed by Kocher and Commins [2] in experiments that involved especially prepared (entangled) photon pairs.

Bell [3] and Clauser-Horn-Shimony-Holt (CHSH) [4] presented mathematical models and inequalities related to these discussions which together with subsequent experimental results of Aspect and others [5] appeared to rule out the existence of Einstein's elements of physical reality (also referred to as Einstein's elements or just elements). The new twist of CHSH, Aspect and others beyond the Kocher-Commins experiments was the switching of the polarizers involved in the measurements between four different polarizer-angle pairs. The measurement results involving these specific polarizer-angle pairs, together with the

Bell-CHSH theories, seemed to favor influences at a distance instead of Einstein's elements.

The 2022 Nobel Prize in physics has led to waves of articles in the popular press that approve of and support instantaneous influences at a distance, based on the original theories of Bell-CHSH [3] [4] and confirmations of these theories by well-known scientists and mathematicians including Leggett [6], Mermin [7] and Gill [8]. These latter works appear, at first sight, entirely incontrovertible, understandable even for the non-expert and have never been refuted in a self-contained and condensed way, although numerous elaborate counterarguments have been published [9] [10] [11] [12].

It is the purpose of this paper to present a concise and self-contained refutation of Bell-CHSH-type inequalities, based on the following facts:

1) None of the well-known proofs of Bell-CHSH including [3]-[7] have modeled Einstein's elements by the Fundamental Model [13] of probability theory (choosing a real number between 0 and 1 at random and uniformly), which is recognized as being universal and emphasizes the possibility of all different elements. All Bell-CHSH proofs emphasize instead the repeated appearance of identical elements. Leggett [6], Peres [14] and many others have used counterfactual reasoning, Bell has used countable numbers of elements (Bertelmann's socks), while Mermin has used even small numbers of elements; all designed to model the repeated appearance of identical elements and all less general than the Fundamental Model.

2) The correlation of CHSH-Aspect-type pair-measurements may only be understood through a consistent evaluation of distant pair-events. One measurement must establish how Einstein's elements are being evaluated and the distant measurement needs to recognize the elements also and evaluate them with global consistency. Therefore, only the outcomes of such pairs relative to each other may indicate a physical law (such as the Malus law), not the single outcomes by themselves. Bell-CHSH have taken great efforts to guide the theory toward the single outcomes, because of Einstein's separation principle. However, their inequalities deal exclusively with the relative judgement of equal versus not-equal of the distant pair-events.

I present below a model based mathematically on the Fundamental Model of probability theory and physically on analogs to the Malus law. My model violates Bell-CHSH-type inequalities, agrees with quantum theory and does not involve instantaneous influences at a distance.

2. Aspect-Type Experiments Modeled by Bell's Functions

We consider applications of the Bell-CHSH model to Aspect-type experiments [5] with photon-pairs that are prepared in a certain way that is commonly named entangled. The photon-pairs emanate from a source and propagate toward two spatially distant measurement stations S_A and S_B that each use a polarizer followed by two detectors for measurement.

Aspect-type experiments [5] feature the fast switching of the polarizer angles j and j' . For simplicity, we consider only angles in the plane that is perpendicular to the photon propagation. The angle j is randomly switched in station S_A between a and a' , while j' in station S_B is randomly switched between b and b' . According to Einstein, this switching separates the two stations S_A and S_B and guarantees the stochastic independence of the emanated photon-pairs from the chosen polarizer angle-pairs, because it ensures that no signal can be of influence as long as it propagates slower than or equal to the speed of light c in vacuum.

The two detectors are mounted after the polarizers in each station. Click of detector 1 in station S_A is registered as $A = +1$ (*horizontal*), click of detector 2 as $A = -1$ (*vertical*) and similarly we have $B = +1$ and $B = -1$, respectively, in station S_B . The distant detections are linked to a given pair of photons by measured clock-times that we denote by t_s , with $s = 1, 2, \dots, 4N$ enumerating the pair-measurements. I use $4N$ for the total number of pairs to account for the fact that the experimenters sort the detection data, after all is done, into 4 sets D_i , each containing the data for N pair-measurements labelled by $n = 1, 2, \dots, N$. The index $i = 1$ refers to measurements with angle-pair (a, b) , $i = 2$ to (a, b') , $i = 3$ to (a', b) and $i = 4$ to (a', b') . It is convenient to also relabel the $4N$ measurement times t_s into 4 sets T_i of measurement-times t_{in} corresponding to the 4 data-sets D_i . Note that, independent of the Aspect-switching, the angle differences $(j' - j)$ are equal to four values θ_i , which we call CHSH angles.

All the above definitions are operational. We need in addition a model for Einstein's elements of physical reality. Mach and Einstein defined these elements only by example. For our purpose, it is important to note that measurement times are certainly among these examples. Bell introduced the mathematical symbol λ and identified it with a "single variable or a set, or even a set of functions", which assume values that represent Einstein's elements. Using this complicated variable (or set of variables), Bell defined theoretical expectation values for the four data sets D_i . There exists a huge literature [9] [10] [11] discussing why Bell's theory does not agree with the actual experiments, and why this disagreement has led many scientists and science writers to suspect that Einstein's elements do not exist. A particularly vexing question is, what type of experiments Bell's theory really describes.

Avoiding these high-level theories, I concentrate on modeling exclusively the experimental averages and propose to use the Fundamental Model of probability theory to model Einstein's elements by numbers taken randomly and uniformly out of the real interval $[0, 1]$. One cannot present the actual Bell-CHSH inequalities and their theoretical expectation values this way, but one can present and describe Bell-CHSH-type inequalities that correspond to the data averages. Mermin [7], Gill [8] and many others have taken this path, except that they did not use the Fundamental and Universal Model of probability theory and they did not consider any physical law in addition to Einstein's separation principle.

In order to stay as close as possible to the Bell-CHSH notation and to put this notation into a one-to-one correspondence with the Aspect-type experimental data, I use the model-notation λ_{in} for the photon pairs that are measured at time t_{in} (which also may represent a pair of times (t'_{in}, t''_{in}) indicating the measured clock-times in the two stations S_A and S_B , respectively). We have then N of the λ_{in} in each of four sets L_i corresponding to the sets T_i of measurement times $t_{in} = (t'_{in}, t''_{in})$. The index i also indicates that the λ_{in} are all different, because they are randomly picked from $[0, 1]$. The actual data corresponding to detector clicks are modeled by Bell-type functions: when detector one clicks in station S_A the model-outcome is given by $A(j, \lambda_{in}) = +1$ and when detector two has clicked $A(j, \lambda_{in}) = -1$. In station S_B , we have $B(j', \lambda_{in}) = +1$ and $B(j', \lambda_{in}) = -1$, respectively. Using all these model functions in correspondence with the data D_i we get four sets of model-data that we denote by D_i^m .

3. Inequalities of the Bell-CHSH-Type

Instead of theoretical expectation values that Bell originally considered, we consider now averages $\mu(j, j', i)$ of model-data D_i^m for given polarizer angles (j, j') :

$$\mu(j, j', i) := \frac{1}{N} \sum_{n=1}^N A(j, \lambda_{in}) B(j', \lambda_{in}), \tag{1}$$

where j and j' represent a given value of the polarizer angles: a or a' in station S_A and b or b' in station S_B , respectively. Consider further the absolute value for the following combination of the four values of μ (corresponding to the four sets $\lambda_{in} \in L_i$):

$$|\langle Q \rangle| = |\mu(a, b, 1) + \mu(a, b', 2) + \mu(a', b, 3) - \mu(a', b', 4)| \tag{2}$$

Bell-CHSH and supporters have claimed that their EPR-model is exclusively based on Einstein's separation principle and assumptions that appear self-evident in Einstein's world of physical reality. The procedure of Bell-CHSH and supporters (see particularly [7]) to derive a constraint for $|\langle Q \rangle|$, is equivalent to neglecting the index i in the sums corresponding to Equations (1) and (2). This neglect is incommensurate with the Fundamental Model and implies that $\lambda_{in} = \lambda_n$ and that the sets L_i are, therefore, identical and independent of i , which results immediately in the Bell-CHSH-type inequality:

$$|\langle Q \rangle| \leq 2 \tag{3}$$

As is well known, this inequality is violated by Aspect-type experiments that use the CHSH angles θ_i .

Leggett [6], Gill [8] and many others have arrived at the same result without using any λ or λ_{in} at all in their equations. Gill [8] did take the advantage of referring to experimentally observed averages instead of theoretical expectation values, but did not explicitly consider Einstein's elements or the Fundamental Model.

3.1. Paradoxical Consequences of Equation (3)

The disregard of the index i and the avoidance of any explicit use of the symbols λ_{in} , has far-reaching and paradoxical consequences. Consider four different Aspect-type experiments performed at four different places, Paris, Vienna, Urbana and the Canaries, respectively, and assume similar photon-pair sources. We use the N model-data for polarizer angles (a,b) to model the measurements in Paris, for (a,b') to model measurements in Vienna, for (a',b) in Urbana and for (a',b') in the Canaries. There exists no physics that implies any connections between the four model averages, because Bell-CHSH and followers claim to have used only Einstein's separation principle and self-evident physical aspects to justify neglect of the index i . We must then ask the question: how is it possible that the exclusive use of Einstein's locality conditions leads to a model that requires global interdependencies of worldwide scattered experiments? Obviously, the model-results obtained for Paris, Vienna and Urbana put a limit on the model averages for the Canaries if the Bell-CHSH-type inequalities are valid.

3.2. Cause and Resolution of the Paradox

This paradox raises the suspicion that more than self-evident assumptions have been used in addition to Einstein's separation principle in order to derive the inequalities. In the nascent status, the photon pairs and the λ_{in} must be independent of the polarizer settings (because of the fast switching). However, only interaction with a given polarizer angle provides meaning of what may be regarded e. g. as *horizontal* or *vertical* polarization of photons. Careful distinction of Einstein's elements before passing the polarizers and when actually detected, is definitely necessary and leads to possible resolutions of the paradox [10] [11]. Here, I present are solution based on the differences between data averages and theoretical expectation values with respect to the cardinality of the set of Einstein's elements versus the cardinality of the set of measurements (actual or model). All well-known proofs of Equation (3) use assumptions with respect to these cardinalities that are not self-evident at all as, for example, Mermin's addition of a "well known sampling theorem".

Mermin [7] derives the virtual identity of the sets L_i using Einstein's separation principle and the additional assumption of a countable number M of Einstein's elements. To be sure, the existence and emission of photon-pairs that possess only a countable (or even small) number M of different characteristics that determine their evaluation, must indeed lead to practically identical sets L_i . Because the photon-pairs and corresponding λ_{in} must not depend on the polarizer settings, they need to appear repeatedly if the number of measurements N is much larger than M . Therefore, for $N \gg M$, the sets L_i must be about identical.

To understand this fact and its consequences, it is useful to recall the definition of the model-expectation values E for a finite set of λ_m ($m = 1, 2, \dots, M$)

that are all emitted with equal probability for all j, j' . Instead of Equation (1) that describes model-data averages without additional assumptions, we have:

$$E(j, j') := \frac{1}{M} \sum_{m=1}^M A(j, \lambda_m) B(j', \lambda_m), \quad (1a)$$

For $M \ll N$, both expectation values and data averages fulfill the inequalities approximately. However, for $M \gg N$ only the expectation values approximate $|\mathcal{Q}| \leq 2$, while the data averages may significantly violate this inequality, because the L_i are not identical. A physical law, for example of Malus-type, may then rule and determine the relative function-values of A and B , as will be proven in detail below.

In addition, the assumption of a finite or countable M is, by itself, not self-evident at all. The λ_m are, of course, countable, because of the necessarily finite number N of measurements. However, these λ_m are sampled out of the set $M = [0, 1]$. The cardinality of the set M of the fundamental model is much larger than that of any countable sets of measurements and, therefore, $M \gg N$ and the reasoning of Mermin and all others does certainly not apply for the quadruple data averages $|\mathcal{Q}|$. From a mathematical point of view, one must realize that, in general, one just cannot express probabilities that are defined on the interval $[0, 1]$ by countable elements. From the physics point of view, one must realize that Bohr's ideas of complementarity certainly do not exclude the relation of physical entities to both a continuum and countable characteristics. It is the mathematical subtlety involving the cardinality of the involved sets of Einstein's elements versus the cardinality of measurement numbers that probably was not understood by Bell-CHSH, Mermin and others, although it had been noticed in reference [12].

In all of the Bell-CHSH-type proofs including [3]-[8], there exists a basic identification of the sets L_i or equivalent sets. That identity may not be deduced from locality considerations alone and is a non-sequitur for the averages of the model-data if the cardinality of Einstein's elements is that of a continuum.

4. Aspect-Type Experiments and the Fundamental Model of Probability Theory

The Fundamental Model [13] is, as mentioned, based on the elementary events of picking randomly and uniformly a number from the interval $[0, 1]$ of the reals. Elementary events are usually denoted in probability theory by ω , while we denote them in our model by λ_m to make their relation to actual measurements visible. The indices i and n do not indicate that the sets of all possible λ_m are countable. They only indicate that a countable number is selected from the interval $[0, 1]$ for any given model-sequence corresponding to Aspect-type measurement sequences. Consequently, all λ_m are different with probability 1. As mentioned above and shown in [13] the Fundamental Model is truly universal in that every other experiment of probability theory is contained in it. It also permits the description of both finite and countable N [13]. For these reasons, it is

ideal to model Aspect-type experimental averages.

In order to invalidate inequality (3), all we need is a guarantee that the sets L_i are not identical, while independent of the polarizer angles. That guarantee is provided automatically by use of the Fundamental Model to represent Einstein's elements of physical reality and the corresponding λ_m .

As an aside, the use of the Fundamental Model also invalidates the counterfactual reasoning of Peres [14], Leggett [6] and others, who have argued as follows: Consider a measurement with a given m and pair of polarizer angles. Had we used a different pair of polarizer angles, we would have obtained results for the identical λ_m . This author responds: had they used the Fundamental Model with λ_m as elementary events, the sets L_i would not have been identical for different i , and they could not have dropped the index i , without additional assumptions.

We, therefore, may use the randomly and uniformly chosen λ_m to search for a more complete model that obtains the experimental averages in agreement with quantum theory.

5. A Model Obeying Einstein's Separation Principle and Violating Bell-CHSH

It is important to realize, before starting with a more detailed modeling-approach, that the Bell-CHSH-type inequalities of the present discussions depend only on the number of positive versus negative signs that the products AB assume for the randomly chosen λ_m . In other words, it matters only how many pair outcomes (A, B) are judged as being equal or different. That judgement, in turn, depends on both the λ_m and the global physical law that relates to their evaluation by the two polarizers and corresponding functions A and B .

Constraints due to what experimenters Alice (in S_A) and Bob (in S_B) may or may not know, apply only to the separated (also called "marginal") outcomes A and B . Indeed, if Alice, Bob and involved theoreticians know exclusively local facts, they may only deduce the random marginal model-outcomes of ± 1 in the separate stations. Again, these separate outcomes are not what the Bell-CHSH-type inequalities deal with; they deal exclusively with the products AB for given λ_m .

Theoreticians developing a model must further be able to use a coordinate system and the corresponding macroscopic equipment configurations in space and time; physical models have not yet exorcised spook in any other way. Theoreticians must also agree on a globally consistent meaning of the measurement outcomes. For example, the polarizer angle a together with a click of a designated detector in both experimental wings, means that the measurements indicate a global value of (for example) "horizontal" or "right-circular" or just $A = B = +1$ (alternatively "vertical" or "left-circular" or just $A = B = -1$).

With the above facts and global conventions, we first develop model sets D_1^m, D_2^m corresponding to the actual data-sets D_1, D_2 . We introduce a random

function $rm_{12}(t_{in}) = \pm 1$ of the measurement times and use the following functions A, B that define the model outcomes:

$$A(a, \lambda_{in}) = A'(a, \lambda_{in})rm_{12}(t_{in}) \quad \text{and} \quad B(j', \lambda_{in}) = B'(j', \lambda_{in})rm_{12}(t_{in}),$$

With $j' = b$ or alternatively $j' = b'$ or any angle at all.

We now impose a globally consistent evaluation on the λ_{in} by letting $A'(a, \lambda_{in}) = +1$ and $B'(a, \lambda_{in}) = -1$ for all $\lambda_{in} \in [0, 1]$. In addition, we consider the case that the experimenters in S_b rotate the polarizer setting from a to j' and that this rotation results locally in a Malus-type evaluation-law for the λ_{in} : we then have $B'(j', \lambda_{in}) = -1$ if and only if $\lambda_{in} \leq \cos^2(j' - a)$ and $B'(j', \lambda_{in}) = +1$ otherwise. Such a possibility contradicts the Bell-CHSH inequality and has already previously been considered (see e. g. [11] and [15]). Now, however, the inequality is invalidated to start with by the use of the Fundamental Model of probability theory for the λ_{in} , as well as the consideration of model-data averages.

Quantum mechanics and the experimental results require furthermore that the averages of the marginals of the products are $\langle A \rangle = \langle B \rangle = 0$, which we fulfill by letting the function $rm_{12}(t_{in})$ randomly assume values of $rm_{12}(t_{in}) = \pm 1$ in both stations, which may be accomplished without the introduction of any non-local effects by associating the time t_{in} with the signals received at this time (or the pair of times $t_{in} = (t'_{in}, t''_{in})$). For example, the two values ± 1 of the function $rm_{12}(t_{in})$ may actually be linked to the physical existence of different kinds of photon-pairs (e. g. righthanded-lefthanded, righthanded-righthanded etc.) that are randomly emanated from the source and evaluated by the polarizers and detectors to exhibit opposite signs. We may then regard the function rm_{12} to be a function of the λ_{in} instead of the t_{in} .

Using all above model-conventions and elementary trigonometry, we obtain:

$$\frac{1}{N} \sum_{n=1}^N A(a, \lambda_{in})B(j', \lambda_{in}) \cong \sin^2(j' - a) - \cos^2(j' - a) = -\cos 2(j' - a), \quad (4)$$

as well as $\langle A \rangle = \langle B \rangle \cong 0$, which reproduces experimental results that agree closely with the results of quantum mechanics for properly prepared (entangled, operationally speaking) photon pairs. Beyond the results of quantum mechanics, we also have modeled the values for the single outcomes of each measurement. We may even obtain all the measurement-values of the actual experiment by reverse engineering the values of the function $rm_{12}(t_{in})$ and putting them equal to -1 precisely when the actual outcomes of number n in station S_a are equal to -1 .

Those who still suspect the involvement of a non-locality in such Malus-type arrangements, may wish to contemplate the dependence of the twins age on the velocity differences in the theory of relativity. Be all that as it may, however, my model does certainly not require any instantaneous influences, for it derives an infinitude of model results in agreement with actual experiments by use of a random number out of $[0, 1]$ for modeling Einstein's elements and one single parameter: $(j' - a)$.

The model for the remaining data sets D_3 and D_4 , namely the sets D_3^m and D_4^m , is easy to derive after the following considerations. It was somehow lost in the transition from Bell's analysis to the experiments of the CHSH-Aspect-type that once we switch the polarizer to an angle a' different from the angle a in station S_A , we must not use the same detector-outcomes for the definition of *horizontal* (+1) or *vertical* (-1) in station S_B , without risking logical contradictions regarding the global physical or geometric characterization of the photon pairs. In order to avoid this problem, we rotate the coordinate system around the axis of photon propagation such that a' turns into a and postulate that such a rotation permits the evaluation of the λ_{in} exactly as before, but now using the rotated polarizer angles. With this new coordinate system, we may use the same model that we have developed for D_1, D_2 now for D_3, D_4 . Aspect's polarizer switching has no effect at all within the so described model. Indeed, Aspect and many others have emphasized the very fact that the switching has no influence on the data averages. The author is, of course aware that the above model is reverse engineered and does not necessarily identify any actual physical "machinery". However, the model certainly does relieve us from the necessity of instantaneous influences and encourages the search for the machinery related to Einstein's elements.

6. "Freedom" and the Sets L_i

Gerard 't Hooft's suggestion [16] that "freedom" and "free will" do not apply to models of CHSH-Aspect-type experiments is, thus, mathematically confirmed by the use of the Fundamental Model of probability theory [13] and without any "conspiracies". We only need to permit that the cardinality of Einstein's elements be that of a continuum. Counterfactual reasoning, on the other hand [6], [14], is refuted, because we have no freedom to demand identical elements for different experiments; model or actual.

Along the same line one finds that the "Bell-Game" [8] cannot be played by Alice (in station S_A) and Bob (in S_B), who are asked to predict the possible outcomes for the other station, given only one value λ_{in} for four polarizer angles. It is not possible to obtain 4 consistent outcomes that obey Malus-type laws for one given λ_{in} . Therefore, the Bell-Game cannot be played, independent of any considerations of locality and of what Alice and Bob may or may not know.

7. Conclusion

I have proven the existence of violations of Bell-CHSH inequalities and the possibility to derive experimental averages close to quantum theory by a model that respects Einstein's separation principle, applies Malus-type laws and uses the Fundamental Experiment of probability theory.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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