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Supersymmetry in the Geometric Representation of the Early Universe Wave Function

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Abstract

The main goal of this article is to present a new result of a possible approach to the geometrical description of the birth and evolution of the universe. The novelty of the article is that it is possible to explain the nature of supersymmetry in terms of the geometric representation of the wave function and to propose a mechanism of spontaneous symmetry breaking of the excitation of the universe with different degrees of freedom. It is under such conditions that the well-known spontaneous symmetry breaking occurs and individual excitation acquires mass. At the same time, a phase transition of the first kind occurs with the formation of a new phase.

Keywords

Clifford Algebra, Wave Function, Early Universe, Supersymmetry, Spontaneous Symmetry Breaking

1. Introduction

The problem of the geometric representation of the universe and of how to incorporate it in the scheme of the general relativity theory is far from being solved. Modern ideas (rather hypotheses) about the cause of the formation of the state of the universe suggest the instability of some scalar field associated with the quantum nature of the matter [1]. The causes and physical mechanism of the appearance of this field, and hence the origin of the universe, have remained open for discussion for many years. Earlier, an approach was proposed [2] to describe the origin and evolution of the universe in terms of the first principles of statistical mechanics and quantum field theory. In this approach, we can answer the question of the probable emergence of such a field, but nothing can be said about its physical and geometric nature. The purpose of this paper is to describe the early universe in terms of some physical entity that has simple geometric interpretation and to use a known mathematical apparatus that may describe its appearance and probable changes.

We have to define a mathematical representation of the physical entity with which we could describe the initial state of the universe. The spinor representation of the wave function is rather unsuitable for the description of the origin of the universe as a quantum object [3]. It was shown [4] that there is no finite-dimensional representation of the complete linear coordinate group of transformations for spinors. Furthermore, Dirac spinors preserve the structure of a linear vector space rather than the structure of a ring. It was also shown [5] that there can exist only some associative algebras with a partition on the field of real numbers: the real number, the complex number, and the Clifford number algebras. Just these algebras possess the ring structure [6]. Arbitrary operations with them yield similar geometric objects.

The most suitable geometric structure is the Clifford number. The Clifford algebra is a vector space over the field of real numbers and is presented as an additive group where the multiplication of elements is distributive rather than commutative with respect to addition. This ring has ideals that may be obtained by multiplying an isolated element on the right or left by elements of the ring [6]. The ideals after this procedure are simple Dirac spinors in the standard approach. Thus, the Clifford algebra representation contains more information about the physical properties than the spinor representation. Clifford algebra may be extended [7] [8] to include a description of the origin and evolution of the universe.

As it was shown earlier [9]-[13], the application of the Clifford algebra covers all standard functions of quantum mechanics and provides [3] a unifying basis for the physical knowledge including the general relativity and electromagnetism. When we introduce the Clifford algebra in the scheme of quantum mechanics [11], we should not ignore the specifics of this formulation. Actually, in this case we obtain a quantum-mechanical theory that provides only an algebraic structure and does not contain any further specific requirements. It is possible to show [9]-[13] that Clifford's algebraic formalism is completely equivalent to the traditional approach to quantum mechanics.

First of all in cosmology, the question arises about the geometric nature of the fundamental field. It may be scalar as well as may have other geometric images. It is natural that its geometric characteristics should follow from the space that would be created due to the distribution of matter. In terms of relevant physical characteristics, the Clifford number is the most suitable at the moment [2] [3]. We do not detail focus on the basic properties of the Clifford algebra which are described in our previous articles [2] [14] [15] but stop on special aspects such geometrical presentation.

The main goal of this article is to present novel result a probable approach to the geometric description of the birth and evolution of the Universe which recent was published [14]. In this article the wave function as a fundamental field is represented by a Clifford number with the transfer rules that possess the structure of the Dirac equation for any manifold. The novelty of this article is as in terms of this geometric representation of the wave function, the nature of supersymmetry may be explained and probable mechanism of spontaneous symmetry breaking of the excitation of the universe with different degrees of freedom is proposed. Just under such conditions the well-known spontaneous symmetry breaking occurs and individual excitation gain mass.

In this paper we show that the Hamiltonian of the universe in terms of the geometric interpretation of the wave function fully corresponds to the theory of supersymmetry. Thus it is possible to explain the asymmetry between the Bose and Fermi degrees of freedom of the universe and to obtain non-standard conditions of spontaneous symmetry breaking with the condensation of fields of different tensor dimensions. This opens up the possibility of another interpretation of the quantum phenomena in the early universe with the emergence of a new phase resulting from the first-order phase transition. The significance of this research lies in the application of an adequate mathematical apparatus for describing the possible properties of the universe formed at the quantum level.

2. Wave Function of the Universe

It may be assumed that each elementary formation at an arbitrary point of the manifold may be described in terms of a Clifford number. Then the wave function of an arbitrary excitation may be represented by a complete geometric object the sum of probable direct forms of the induced space of the Clifford algebra [2] [14]. In this case the full geometric entity may be written in terms of the direct sum of a scalar, a vector, a bi-vector, a three-vector, and a pseudo-scalar, *i.e.*, $\Psi = S \oplus V \oplus B \oplus T \oplus P$, that is given by

$$\Psi = \Psi_0 \oplus \Psi_{\mu}\gamma_{\mu} \oplus \Psi_{\mu\nu}\gamma_{\mu}\gamma_{\nu} \oplus \Psi_{\mu\nu\lambda}\gamma_{\mu}\gamma_{\nu}\gamma_{\lambda} \oplus \Psi_{\mu\nu\lambda\rho}\gamma_{\mu}\gamma_{\nu}\gamma_{\lambda}\gamma_{\rho} \tag{1}$$

With the reverse order of composition we may change the direction of each basis vector, and thus obtain $\bar{\Psi} = S \ominus V \oplus B \ominus T \oplus P$. Another element of symmetry is the change of multiplication of the basis vectors to the inverse in the representation of the Clifford numbers that turns its into $\tilde{\Psi} = S \oplus V \ominus B \ominus T \oplus P$. As long as the symmetry element is introduced, there should be present a mathematical operation on the field of Clifford numbers. The direct sum of the tensor subspace may be attributed with a ring structure with the use of a direct tensor product in the symbolic notation given by

$$\Psi\Phi = \Psi \cdot \Phi + \Psi \wedge \Phi, \tag{2}$$

where $\Psi \cdot \Phi$ is an inner product or convolution that decreases the number of basis vectors and $\Psi \wedge \Phi$ is an external product that increases the number of basis vectors. If each Clifford number is multiplied by a spatial fixed matrix that has one column with elements and other zeros, then we obtain the Dirac spinor with four elements. Making use of this we column can reproduce the spinor representation of each Clifford number. There occurs full correspondence between the spinor column and the elements of the exterior algebra.

Now let us determine the rule of comparing two Clifford numbers in different points of the manifold. For this purpose we have to consider the deformation of the coordinate system and the rule of translations on different manifolds. An arbitrary deformation of the coordinate system may be set in terms of the basis deformations $e_\mu = \gamma_\mu X$, where X is the Clifford number that describes arbitrary changes of the basis (including arbitrary displacements and rotations), that do not violate its normalization, *i.e.*, provided $\tilde{X}X = 1$. It is not difficult to verify that $e_\mu^2 = \gamma_\mu \tilde{X} \gamma_\mu X = \gamma_\mu^2 \tilde{X} X = \gamma_\mu^2$, and this relation does not violate the normalization condition [6]. Now, for an arbitrary basis, we may set, at each point of the space, a unique complete linearly independent form as a geometric entity that characterizes this point of the manifold. For a four-dimensional space, such geometric entity may be given by

$$\Psi = \Psi_0 \oplus \Psi_\mu e_\mu \oplus \Psi_{\mu\nu} e_\mu e_\nu \oplus \Psi_{\mu\nu\lambda} e_\mu e_\nu e_\lambda \oplus \Psi_{\mu\nu\lambda\rho} e_\mu e_\nu e_\lambda e_\rho \tag{3}$$

If this point of the manifold is occupied by matter, then its geometric characteristics may be described by the coefficients of this representation. A product of arbitrary forms of this type is given by a similar form with new coefficients, thus providing the ring structure. This approach makes it possible to consider the mutual relationship of fields of different physical nature [3] [16]. If this point of the manifold is occupied by matter, then its geometric characteristics may be described by the coefficients of this representation. A product of arbitrary forms of this type is given by a similar form with new coefficients, thus providing the ring structure. This approach makes it possible to consider the mutual relationship of fields of different physical nature [3] [16]. To determine the characteristics of the manifold as a point function implies to associate each point of the set with a Clifford number and to find its value. If this function is differentiable with respect to its argument, we may introduce the differentiation operation. To define a transfer operation on an arbitrary manifold, we have to define a derivative operator, e.g., as given by $D = \gamma_\mu \frac{\partial}{\partial x_\mu}$ where $\frac{\partial}{\partial x_\mu}$ represents the change along the curves passing through a given point in the space. The action of this the operator for any Clifford number may be presented as

$$D\Psi = D \cdot \Psi + D \wedge \Psi \tag{4}$$

where $D \cdot \Psi$ and $D \wedge \Psi$ may be referred to as the "divergence" and the "rotor" of the relevant Clifford number Within the context of the definition of a differentiated variety, it is not enough to have one non-special coordinate system covering a variety whose topology differs from the topology of an open set in the Euclidean space.

Furthermore, by ascribing a given geometric interpretation to the wave function, we may obtain correct transfer rules for an arbitrary variety [3] and obtain new results concerning the geometric nature of the wave function. For the wave function as a geometric entity, we may write the first structure equation in the standard form

$$d\Psi - \omega\Psi = m\Psi \tag{5}$$

The latter equation reproduces the form of the Dirac equation but with fuller meaning than in the spinor representation. Such Dirac equation for the wave function may be obtained by minimizing the action constructed from the geometric invariant

$$S = \int \mathrm{d}\tau \left\{ \Omega\tilde{\Omega} + m\Psi\tilde{\Psi} \right\} \tag{6}$$

The Lagrange multiplier m provides the normalization condition for the wave function $\int \mathrm{d}\tau \left\{ \Psi\tilde{\Psi} \right\} = 1$. The above action is non-degenerate on the solution of the Dirac equation in contrast to the standard approach. In our approach the dynamic equation for the wave function is presented as a rule for the parallel translation for characteristics on an arbitrary manifold.

Let us return to the general expression for the action in the geometric representation and also to the definition of the general form of the wave function. It may be shown, within the context of the definition [6], that the wave function in the general case may be written as $\Psi = Q + V$ where $Q = q + iq'$ is a biquaternion, (here q and q' are quaternions), and the sum $V = \Psi_v \oplus \Psi_t$ contains a vector and a trivector or a pseudovector, $V = A + iB$, where A and B are four dimensional vectors.

It was proved in [6] that each even number $\Psi = \bar{\Psi} = Q$ for $\Psi\tilde{\Psi} = Q\tilde{Q} \neq 0$, and the Clifford number in the Euclidean space may be reduced to the canonical form, *i.e.*, $\Psi = Q = \left\{ \mu(x)\exp(i\beta) \right\}^{\frac{1}{2}} X$ where $X\tilde{X} = 1$ describes all the coordinate transformations associated with the translation and rotation of coordinates and with the Lorentz transformation in the Euclidean space. It is not difficult to show that $\int \tilde{\Psi}\Psi \mathrm{d}\tau$ is scalar and in the physical interpretation may be associated with the probability density of finding a particle in an arbitrary spatial point. The form of the wave function of an arbitrary ensemble of particles is analogous [8]. According to the proof in the book [6], the odd part of the general Clifford number may be presented as the even part multiplied by an individual element of this algebra and thus there is no problem with the manipulation of the full Clifford number. For an arbitrary wave function we take into account only the scalar part of this product that may be explicitly written as

$$\rho = \Psi\tilde{\Psi} = Q\tilde{Q} + V\tilde{V} \tag{7}$$

Thus it becomes possible to describe the intermediate states of the particle inasmuch as the form of the wave function of an arbitrary particle ensemble is analogous [8] [17] [18] [19] [20]. The structure equation thus obtained is written in terms of the introduced variable, is completely equivalent to the Dirac equation, and has well known solutions both for the calculation of the hydrogen atom spectrum and for the interpretation of the states [6].

To apply this equation to the description of the birth of the universe, we have to make another assumption. The new vacuum contains only the born formation. For this reason, all changes associated with the wave function are due only to its changes in vacuum. Therefore, its behavior may be influenced only by one characteristic of the new vacuum, namely, this wave function. In this case, the wave function itself acts as a field that changes its characteristics, or as a connectivity of the space with the new vacuum. The equation for the wave function looks natural as given by

$$d\Psi - \lambda\Psi\Psi = F \tag{8}$$

when the first structure equation is at the same time the second structure equation with the equation for the "curvature" given by

$$dF - F\lambda\Psi + \lambda\Psi F = J \tag{9}$$

It is assumed that $\omega \sim \lambda\Psi$. Among the new results, we indicate that the Dirac equation in the geometric representation in the general theory of relativity is just the equation of transfer on an arbitrary variety; therefore, its solution may be interpreted purely geometrically. Moreover, the geometric representation of the wave function yields other results that may simply reveal the geometric nature of the wave function [21] [22]. Later, these equations will be derived from the least action principle in the geometric interpretation. As follows from the previous analysis, a complete coordinate transformation group associated with the structure equation exists only in the Clifford-number representation of the wave function. The first structure equation for the wave function reproduces the form of the Dirac equation and, as it was shown in [6], its solutions are similar to those for the spinor representation. The latter observation solves the problem of the finite-dimensional representation of the wave function under the complete linear group of coordinate transformations.

3. Modernization of the Standard Cosmological Model

In the case of spontaneous generation of an additional field in vacuum, the ground state energy of the "new" vacuum for fields of different nature should be lower than the ground state energy of the "initial" vacuum [2]. Moreover, the interaction of this field with the fluctuations of any other field provides energy conservation for the new state of the system. We assume that occurrence in vacuum of the fundamental field that is generated spontaneously and interacts with the fluctuations of all other fields may be described in terms of the Clifford number [3]. The probable stationary states of the fundamental field are generated by the multiplicative noise produced by the nonlinear self interaction with fluctuations of this field. The generator of these fluctuations is the vacuum itself for each point of the Planck size on the manifold.

The present model differs from the widely studied scenario of the stochastic inflation of the Universe [1] that takes into account fluctuations of the fundamental field, but disregards the fluctuations of the unstable vacuum. The internal fluctuations generate the stochastic behavior of the system that may cause changes of its stationary state. The most significant point is that now the fundamental field is not a Clifford number rather than scalar and contains all the geometric characteristics of the space that may be born as a result of the emergence of the matter. Only the distribution of the matter can describe the space that arises.

We start with the assumption that the phase transition from the "initial" vacuum to the new state of vacuum generates a new non-zero field. This means that the new field generates the "new" vacuum different from the "primary" vacuum for any field of an arbitrary geometric characteristic that may exist. The resulting field must reduce the energy of the "new" vacuum with respect to the energy of the "primary" vacuum. Therefore, the energy density of the ground state of the "new" vacuum may be supplied through $\varepsilon = \varepsilon_v - \frac{\mu_0^2}{2}\tilde{\Psi}\Psi$ where the second part is the field energy in terms of the wave function with the geometric representation as the Clifford number; the coefficient μ_0^2 describes the coupling of the new field and the "primary" vacuum, *i.e.*, the self-consistent interaction of the new field with the probable fluctuations that may exist in the "primary" vacuum. Here we have to make two remarks. The first one concerns the decrease in the initial energy of the ground state with the appearance of a new field, and the second one is related to the coupling coefficient that is now positive, so that explanations of the appearance of such a sign used in the standard approach are not needed. The energy of the new system may be presented in the form

$$E = E_v - \int \frac{\mu_0^2}{2}\tilde{\Psi}\Psi \mathrm{d}\tau, \tag{10}$$

If we want to describe the evolution of the system $\langle out | \exp iHt | in \rangle$, we still need to average all probable fluctuations with which the new field can interact. For this purpose it is sufficient to present the nonlinear coupling in the form $\mu_0^2 = \mu^2 + \xi$, where $\langle \xi(t)\xi(0) \rangle = \sigma^2$ and σ^2 is the dispersion of the coupling coefficient fluctuations, and to carry out averaging

$$\langle out | \exp iHt | in \rangle \sim \langle \tag{11}$$

$$\int D\Psi \int D\xi \exp i\left\{E_v - \frac{1}{2}\mu^2\tilde{\Psi}\Psi + \frac{1}{2}\xi\tilde{\Psi}\Psi + \frac{\xi^2}{\sigma^2}\right\} | in \rangle \tag{12}$$

$$out | \sqrt{4\pi\sigma} \int D\Psi \exp i\left\{E_v - \frac{1}{2}\mu^2\tilde{\Psi}\Psi + \frac{\sigma^2}{4}\left(\tilde{\Psi}\Psi\right)^2\right\} | in \rangle$$

after integration over fluctuation fields yields.

This implies that we have a new system with the effective energy (averaged over the fluctuations of other fields) given by

$$E = E_v - \int \left[\frac{1}{2}\mu^2\tilde{\Psi}\Psi - \frac{\sigma^2}{4}\left(\tilde{\Psi}\Psi\right)^2\right] \mathrm{d}\tau \tag{13}$$

where we may introduce the effective potential of the fundamental field in the geometric interpretation

$$V(\Psi)=-\frac{1}{2}\mu^2\tilde{\Psi}\Psi+\frac{\sigma^2}{4}(\tilde{\Psi}\Psi)^2 \tag{14}$$

that reproduces the well-known expression for the energy of the fundamental scalar field but with the nonlinear coefficient determined by the dispersion of fluctuations. This implies that with no field $\varphi=0$, $E=E_v$ while for $\tilde{\Psi}\Psi=\frac{\mu^2}{\sigma^2}$ the expression for the effective ground state energy of the "new" vacuum reduces to $E=E_v-\frac{\mu^4}{4\sigma^2}\tau$. As follows from the latter relation, the energy of the "new" vacuum is lower than the energy of the primary vacuum, *i.e.* the phase transition results in the formation of a new vacuum ground state. If σ^2 tends to infinity, then the energy of the new state tends to the initial energy of the ground state. The energy of the new state may vanish for $E_v=\frac{\mu^4}{4\sigma^2}$. This relation may be applied to estimate the maximum dispersion of the field fluctuations if provided the vacuum temperature is given by $\Theta_v=\frac{\mu^4}{2\sigma^2}$.

The effective potential may now be rewritten in terms of the probability density $\tilde{\Psi}\Psi=\rho$ of the material field

$$V(\rho)=-\frac{1}{2}\mu^2\rho+\frac{\sigma^2}{4}\rho^2 \tag{15}$$

that may be useful for interpreting different compositions of the energy and matter produced by the spontaneous symmetry breaking. It should be noted that it is the full probability density of the material field, and whether it is "dark", depends on the tensor characteristics of the field in which we experience it. It may be invisible in the vector electromagnetic field, but it should definitely be felt in the gravitational field and, possibly, in the fields of other tensor representations. This expression corresponds to the free energy of the system in the mean field approximation. In such a system, the first-order phase transition can occur due to the spontaneous symmetry breaking. The value of the density of the new phase [23] after the phase transition $\rho_c=\frac{\mu^2}{\sigma^2}$ as well as the probability of creating a new phase. As it is not difficult to notice, the entropy $S=-\int \mathrm{d}\tau\rho\ln\rho$ of the initial state $\rho=0$ is equal to zero, and with a spontaneous symmetry breaking due to the phase transition, it is different from zero and increases with the decrease in the noise dispersion of the initial vacuum state. It will be shown below that this fact corresponds to the physical picture when the amount of overturning between different degrees of freedom is fixed in the system and the symmetry between the bosonic and fermionic subsystems is broken. This is the probable reason for the baryon asymmetry of the universe. At the same time, this leads to the usual spontaneous violation of symmetry that is necessary in the standard model. This can be seen in the consideration below.

4. Supersymmetry in the Geometric Representation and Probable Predictions

Now we propose a slightly different scenario for the birth of the Universe based on the representation of its wave function as a geometric entity. What appears as the result of the birth of matter should contain a geometric image. Only the birth of the matter and its distribution may be interpreted in terms of geometry. Such an entity might be a Clifford number with an appropriate physical interpretation. An additional field is required for the emergence of the matter, whose spontaneous excitation leads to the emergence of elementary particles. Solving the question of the impact of the early supersymmetric quantum cosmological era on current cosmological observations was the purpose of the paper [24] [25]. The prospects of quantum cosmology are given in the comprehensive review [26]. In our case, such a field is the wave function Ψ in terms of different tensor representations, *i.e.*, it has all probable tensor representations with the dimensions of the space to be created. That is, the geometry is laid down from the very beginning in the characteristics of the point of the manifold on which we describe it.

Minimizing the expression for the energy of the system 10 by independent functions Ψ and $\tilde{\Psi}$, yields for the wave function in the homogeneous case the Gross-Pitaevskii equation with the physical consequences on its solution, *i.e.*,

$$\frac{\delta E}{\delta \tilde{\Psi}} = \left[-\mu^2 + \sigma^2\left(\tilde{\Psi}\Psi\right)\right]\Psi = 0 \tag{16}$$

Similar in content, but richer in nature equations may be obtained for the dynamics of changes in the wave function in the geometric interpretation. To do this, we consider the action written for the wave function of the universe in the presence of the matter. As it was mentioned earlier, the action in terms of the geometric invariant may be given by

$$S = \int \mathrm{d}\tau \left\{\frac{1}{2}\left(F\tilde{F} + \tilde{F}F\right) + m\Psi\tilde{\Psi}\right\} \tag{17}$$

The Lagrange multiplier m provides the normalization condition for the wave function $\int \mathrm{d}\tau\left\{\Psi\tilde{\Psi}\right\} = 1$ and the "general" curvature in the representation of Clifford numbers takes the form $F = d\Psi - \Psi\Psi$. Minimization of this functional yields an equation that at the same time is the second structural equation, *i.e.*,

$$dF - F\lambda\Psi + \tilde{\lambda}\Psi F = J \tag{18}$$

for the change of the "curvature" under the parallel transfer under the influence of the complete group of transformations of the coordinate system. In the homogeneous case $d\Psi = 0$ and inasmuch as $dF = 0$ this equation is transformed to the previous Gross-Pitaevskii equation, *i.e.*,

$$\left[-\mu^2 + \sigma^2\left(\tilde{\Psi}\Psi\right)\right]\Psi = 0 \tag{19}$$

with $-\mu^2 = m$ and $\sigma^2 = \tilde{\lambda} - \lambda$

It was previously proved that in the presence of a spontaneously generated fundamental field, the energy of the vacuum state for any other field is lower than the energy of the ground state of the primary vacuum, and that the energy of the fundamental field is affected by its nonlinear interactions with fluctuations of physical fields of different nature. To avoid the problem of the influence of the gravitational field on the evolution of the universe at the stage of spontaneous nucleation of the fundamental field, we note that the energy of the primary vacuum is not contained in the Einstein equation, and the dynamics of the universe is determined only by the potential energy of the fundamental field that produces the matter. The distribution of the matter, in turn, determines the geometry.

According to Dirac's theory, we may move from the classical Poisson brackets to the quantum ones and rewrite the Hamiltonian in terms of the secondary quantization, where instead of classical geometric representations of the wave function we introduce the operators of birth and annihilation of quanta of this field. In the operator form the Hamiltonian of the Universe may be written as:

$$H = E_v - \frac{1}{2}\mu^2\hat{\Psi}^+\hat{\Psi} + \frac{\sigma^2}{4}\hat{\Psi}^+\hat{\Psi}\hat{\Psi}^+\hat{\Psi} \tag{20}$$

For the field operators of the general form thus introduced, the commutation relations may be unusual. We note that this field has by definition representation of both boson and fermion fields that should include probable transformations of bosons into fermions and vice versa. For this reason, we may assume that the product of the birth and annihilation operators of the field in the general representation may be written in the form

$$\hat{\Psi}^+\hat{\Psi} = S = \hat{B}^+\hat{F} + \hat{F}^+\hat{B} \tag{21}$$

where $\hat{B}^+$ and $\hat{B}$ denote Bose birth and annihilation operators and let $\hat{F}^+$ and $\hat{F}$ denote Fermi birth and annihilation operators with the (anti) commutation relation $\left[\hat{B},\hat{B}^+\right] = \hat{F},\hat{F}^+ = 1$, $\left[\hat{B},\hat{F}^+\right] = \left[\hat{F},\hat{B}^+\right] = \hat{F}^2 = \left(\hat{F}^+\right)^2 = 0$. As it was shown earlier, the value of the density ρ plays the role of the supersymmetric charge Q that describes the intensity of the transfer between different degrees of freedom in the general representation of the wave function. Now it is not difficult to verify that the Hamiltonian may be written in the supersymmetry form as given by

$$H = E_v - \frac{1}{2}\mu^2 S + \frac{1}{4}\sigma^2 S^2 \tag{22}$$

where the square of the supercharge is $S^2 = \hat{B}^+\hat{B} + \hat{F}^+\hat{F}^+ = n_B + n_F$ and presents the total number of bosons and fermions. It is obvious that all the elements of the supersymmetry with the commutation relations $\left[\hat{S},\hat{S}^2\right] = 0, \hat{S} = \hat{S}^+$ are contained in the presented form, where the individual parts of the Hamiltonian are associated with the integrals of motion and are preserved both separately and together. The interpretation of the supercharge in our case is that the geometric representation of the wave function provides a possibility to consider a physical mixture of bosons and fermions, and the charge itself describes the probable transformation of particles into each other. That is, the initial wave function describes a mixed state of bosons and fermions with probable mutual transformations of individual components. If we calculate the partition function [27]

$$Z = Tr\exp\beta\left[\frac{1}{2}\mu^2 S - \frac{1}{4}\sigma^2 S^2\right] \tag{23}$$

then we obtain the thermodynamic quantities of interest as given by

$$\langle S\rangle = \frac{2}{\beta Z}\frac{\partial Z}{\partial\mu^2}, \langle S^2\rangle = \frac{1}{Z}\frac{\partial Z}{\partial\beta} + \mu^2\langle S\rangle \tag{24}$$

with the average value of the number of fermions and for bosons in the Universe being given by

$$\langle n_F\rangle = \frac{1}{2}\left(1 - Z^{-1}\right), \langle n_B\rangle = \frac{\langle Q^2\rangle}{E} - \langle n_F\rangle \tag{25}$$

As it was shown in the paper [27] for small μ^2 the relation between bosons and fermions in the universe may be presented as: $\frac{\langle n_B\rangle}{\langle n_F\rangle} = \coth\frac{\beta\sigma^2}{2}$ and may take arbitrary predetermined values of this relation, depending on when the phase transition of the condensation of the bosonic part of the general representation of the wave function occurs. This observation indicates the reason for the bosonic asymmetry of the universe that is closely related to the baryonic asymmetry. This corresponds to the physical picture when the amount of overturning between different degrees of freedom is fixed in the system and the symmetry between the bosonic and fermionic subsystems is broken. This is the probable reason for the baryon asymmetry of the universe. At the same time, this leads to the usual spontaneous violation of symmetry that is necessary in the standard model. Now, if we remember that the supercharge by definition corresponds to the density $S = \rho$, it becomes obvious that the violation of the supersymmetry and the fixation of its relevant value leads to the birth of the matter.

If we represent the explicit form of the wave function in terms of quaternions and vectors, then we obtain the Lagrangian in the form

$$\begin{gathered} L = \frac{1}{2}\Big(d\left(Q + A + iB\right) + \left(\tilde{Q} + A - iB\right)\left(Q + A + iB\right)\Big) \\ \Big(d\left(\tilde{Q} + A - iB\right) + \left(Q + A + iB\right)\left(\tilde{Q} + A - iB\right)\Big) \end{gathered} \tag{26}$$

If we recall the full definition of the biquaternion $Q = \phi + i\tilde{\phi} + b$ in terms of the scalar ϕ, the pseudo scalar $Q = i\tilde{\phi}$, and the bi-vector *b*, then this Lagrangian contains all the requires information about the standard approach of spontaneous symmetry breaking and obtaining the masses of the relevant particles. In order to avoid cumbersomeness we do not give it in the explicit form, but it should be emphasized that to separate the characteristics of particles and the manifold is not necessary since in this case the non-linearity in the scalar and pseudo-scalar fields may disappear. Moreover, it is possible to spontaneously break the symmetry separately by different tensor representations of the complete sum of the wave functions, say a scalar field or a pseudo scalar field. In this case, it is possible to independently break the symmetry for different scalar and vector fields and obtain the relevant parameter values, all of these being determined through one unknown parameter that is obtained in the previous section through the relation $\frac{\mu^4}{\sigma^2}$. That is, due to the value of the coupling of the relevant field with initial vacuum μ^2 and the noise dispersion σ^2 of such vacuum state determined all necessary parameter.

5. Conclusion

We consider the Clifford algebraic formalism as a suitable method for describing the initial state of a vacuum with probable birth of a fundamental field. This field should contain probable geometric characteristics and be completely equivalent to the traditional approach to the quantum field theory. The approach makes it possible to explain the existence of supersymmetric properties of the initial fundamental field, as well as the spontaneous violation of symmetry between bosons and fermions in the universe. Moreover, it provides a possibility to explain the appearance of the "dark matter" due to the influence of fields of tensor dimensions different from the electromagnetic field. Unfortunately, a rigorous mathematical proof of such an approach does not exist at the moment, but for purely physical reasons, such representations may favor better understanding of the scenario of the birth and evolution of the universe. After everything said above, it can be assumed that the energy of the initial vacuum state can be taken as zero. That is, due to the value of the coupling of the relevant field with unstable initial vacuum and the noise dispersion of such vacuum state determined all necessary parameter.

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Conflicts of Interest

The author declares no conflict of interest.

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