

Disjointed Equivalence of Gravitational and Inertial Mass

Dirk J. Pons 

Department of Mechanical Engineering, University of Canterbury, Christchurch, New Zealand
Email: dirk.pons@canterbury.ac.nz

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Abstract

Problem—Contemporary physics offers no underlying reason for the equivalence of inertial and gravitational mass. **Approach**—The equivalence is examined from the new physics provided by the cordus theory, being a non-local hidden-variable (NLHV) theory. Mathematical formalisms are derived for masses and observers in different fabric densities. **Findings**—A disjointed equivalence is predicted, whereby inertial and gravitational masses are equivalent in any one situation, but a different equivalence holds when the fabric densities change. Consequently this theory predicts that the gravitational constant G varies with fabric density, and hence would be different across the universe and across time. Not only is the gravitational constant non-constant, but the formulation of gravitation changes with fabric density. Specifically, the theory predicts gravity is stronger at genesis (and the end of the universe) such that orbit velocity $v_B \propto \sqrt{r_B}$ (where r_B is orbit radius), compared to weaker gravitation at middle life epochs with $v_B \propto \frac{1}{\sqrt{r_B}}$. The current Earth location and epoch correspond to the latter case, *i.e.* Newtonian gravitation is recovered. The findings disfavour the existence of both dark energy and dark matter, and instead attribute these effects to differences in the fabric density. **Originality**—The work makes the contribution of deriving a mass equivalence relationship that includes fabric density, identifying a disjointed mass equivalence, and showing that the gravitation formulation itself changes with relative fabric densities.

Keywords

Identity of Mass, Gravitation, Inertia, General Relativity, Quantum Mechanics

1. Introduction

A deep issue for foundational and cosmological physics is the lack of an under-

lying principle for the equivalence of inertial and gravitational mass. The equivalence states that mass as determined from inertial behaviour (m_I), *i.e.* the resistance of a body to acceleration, is the same as determined from gravitational interaction with another body (m_G), where:

$$m_I = \frac{F_I}{a} \quad (1)$$

and

$$m_G = \frac{F_G r^2}{GM} \quad (2)$$

with force F , acceleration a , separation r between masses m and M , and gravitational constant G .

This equivalence is problematic since it seems an unnecessary coincidence. It is an assumption of Newtonian gravitation and general relativity (GR), but is not explained by them, nor by quantum mechanics (QM).

The present paper examines the equivalence from the non-local hidden-variable (NLHV) perspective, using the particle sub-structure proposed by the cordus theory [1]. We show that under this new physics the equivalence holds for each local gravitational frame, but is not universally identical in all situations. The key situational variable is identified as the fabric density. This is a vacuum attribute (described below) that has previously been identified as contributing to relativistic time dilation [2].

2. Background

Philosophically, there have been different ways to approach the equivalence problem. Newtonian and Einsteinian theories accepted the equivalence as real, and sought—unsuccessfully—the foundational principles thereof. Important points to consider in the debate are the multiple ways in which mass has physical effects, the origins of mass at a particle level (which primarily relates to the quantum theory interpretations), and the origin of gravitation (which relates to general relativity).

2.1. Multiple Attributes of Mass

Mass is the coupling for multiple physical effects that otherwise might be independent [3]:

1) Two masses attract each other (gravitational mass). The gravitation force or interaction has an unusual set of properties compared to the other forces: a) it only acts on particles with mass or energy; b) it always attracts, never repels (at least for matter-matter interactions, with antimatter it may be different [4]); c) it has infinite range; and d) it cannot be redirected or shielded. Mass is the fundamental strength variable for gravitation.

2) Resistance to acceleration (Newtonian or inertial mass). The greater the acceleration a or mass m of a body, the greater the force F required to change its speed v :

$$F = ma \quad (3)$$

or more generally

$$F = \frac{d(mv)}{dt} \quad (4)$$

3) Relativistic mass: as the speed v of a body of rest mass m_o approaches that of light c , so the effective mass m tends to infinity, or at least the resistance to acceleration does:

$$m = m_o \left(1 - \frac{v^2}{c^2} \right)^{0.5} \quad (5)$$

This effect applies even if there is no acceleration.

4) From the perspective of relativity, momentum p is a separate property to mass and the full energy-momentum formula is:

$$E = \left((pc)^2 + (m_o c^2)^2 \right)^{0.5} \quad (6)$$

5) Mass originates with particles, e.g. protons and neutrons (among others including the photon). There is no deeper explanation for why particles have the mass they do.

2.2. Origins of Mass

What determines the mass property of a particle? It is unclear whether mass is an intrinsic property of the particle, e.g. Copenhagen interpretation, or based on some other attributes. Neither QM nor GR directly address the question of the origin of mass. They also lack a common treatment of gravitation, having independent approaches based on particles and space-time. Much existing work has focused on proposing candidate mechanisms for the origins of mass for specific fundamental particles, with a particular focus on the see-saw mechanism for neutrino species e.g. [5] [6] [7] [8] [9], and quarks e.g. [10] [11] [12] [13], however none provide a comprehensive explanation. Mass arises, per QM, by interaction of the particle with a Higgs field [14] [15], the electroweak symmetry of which is broken at suitably low temperatures [16]. The particle that mediates this field is the Higgs boson [11] [17] [18] [19] [20]. The CERN findings of 2012-4 produced a signature at 125GeV consistent with such a boson [21] [22] [23]. However the mechanism for production of the Higgs boson is still unclear [24] [25]. There has been an expectation that new physics could arise from this direction [26], but this has been elusive. This is problematic because the mass generation mechanism is incompletely specified [27] [28], the mechanisms for stabilisation of the Higgs mass are unknown [23] [29], and there are discrepancies in the sizes of the fundamental interactions – the hierarchy problem [30] [31]. Possibly a deeper physics is needed [32], but if so it is unclear in what direction that lies.

A further problem is how mass arises as the aggregation of such particles. Related questions are the hierarchical structure of mass in the assembly of particles

(e.g. quarks into nucleons) [33] [34] [35], and the effect of binding energy or mass excess. Quantum chromodynamics predicts that the nucleon mass is caused by the breaking of chiral symmetry, but the process itself is unknown [36].

There are several other speculative lines of enquiry into the origins of mass, though none provide a compelling solution. These include attempts to identify the mass generation mechanisms and causes of electroweak symmetry breaking [27] [37] [38] [39], theories beyond the standard model [40], supersymmetry [30] [41] [42] [43] [44], extended or multi-Higgs frameworks [45] [46] [47] [48] [49], gravitons [50], axions [51], inflatons & dark energy [52] [53], connection of Higgs and dark matter sectors [28] [54] [55], analogy of Higgs with fluid resistance [56], technicolor particles [57], baryogenesis [54] [58], dynamics of mass-energy equivalence [59] [60], and whether there may be repulsion (negative mass) interactions between matter and antimatter [61]. Theories of mass also have implications in other areas such as variable speed of light theories [62] [63] and other gauge theories [64].

However the basic problem remains—the origins of mass are unknown for both inertial and gravitational mass. Additionally, it is unknown what attributes or structures of the particle represent the momentum vs. those represent the mass. Quantum theory is built on the premise that fundamental particles have no physical substructure and hence such questions can not be comprehended ontologically from within that theory. It is possible that QM may not have the requisite architecture to address these deeper questions, and hence there is value in exploring alternative paradigms.

2.3. Origins of Gravitation

Newtonian gravitation and then Einstein's general relativity (GR) represent gravitation as a continuous field effect, the latter based on the curvature of space-time. Hence space-time becomes an integrated structure. A foundational premise of GR is the *equivalence principle*: that gravitational and inertial effects are indistinguishable. This is related to the *relativity principle* that all inertial frames of reference are indistinguishable—no frame, neither its orientation nor velocity, is preferred to another. Hence general relativity requires, rather than proves, the equivalence of gravitational and inertial frames of reference [65]. This was Einstein's key insight and the principle of departure from Newtonian gravitation. One view might be that there has been no empirical refutation of the mass equivalence principle, hence the premise is worthy of elevation to a principle. Certainly there are not many theories that offer plausible alternatives. However it is still an assumption, and difficult to verify. Furthermore, there are conceptual difficulties with GR: the inability to describe singularities in space-time; and the lack of a particle interpretation that is consistent with quantum mechanics.

Possibly gravitation might in the future be recast as a particle-based force us-

ing virtual bosons. However a robust integration of gravitation with the other forces has not yet been achieved this way. Even so, this approach seems to require the abolition of the equivalence. GR has empirical shortcomings too, in the inability to explain anomalous rotation curves of galaxies [66] [67]. The theoretical consensus is to attribute this phenomenon to dark matter. Many different kinds have been proposed, but none have been empirically observed [68] despite considerable efforts and cost to do so.

A contrary and minority view is that anomalous galaxy rotation curves already provide an empirical confutation of mass equivalence, and that gravitation takes an alternative form. There is no absence of alternative theories. A candidate is modified Newtonian dynamics (MOND) [69] [70], which also requires the equivalence to be broken. MOND provides good quantitative fit to observations, but lacks an underpinning theory of causation. Another active area of theoretical exploration is torsional gravity [71]. There are several variants of this including teleparallel and metric-affine gauge formulations [68]. While these approaches have promise and can explain features of cosmology [68] [72], there is as yet no complete solution nor reconciliation with particle physics. Furthermore, they lack a deeper ontological explanation for the effect.

An alternative approach sees the equivalence as purely coincidental, per the anthropic principle or the multiverse [73]. That has its own difficulties as it relies on untestable metaphysics. Also, it does not explain the principles which set the presumably different ratios in the many universes.

2.4. Contributions from Hidden Variable Theories

The hidden-variable (HV) theories take a conceptual and logical approach to theory-development, and sometimes provide the vision and leadership for mathematical and empirical methods to follow afterwards. These theories tend to be premised on the assumption that particle phenomena that objectively exist are brought about by the existence of physical mechanisms at a deeper level of structure, and this implies that particles have sub-structure. This is a long-standing idea [74] that corresponds to the philosophical premise of physical realism [75]. Einstein and others expected HV theory to provide a deeper mechanics beneath QM [74], however this has been difficult to achieve.

There are two classes of HV theories: *local* and *non-local*. Locality in this context refers to the relationship between a particle and its environment: that a point object is only affected by fields at that location, not by remote values; and transmission of an effect occurs by at most the speed of light [76]. In contrast *non-locality* allows remote fields to affect the particle, and for these effects to be superluminal [77]. It is unlikely that *local* theories are viable, as they are incongruent with observable entanglement phenomena. The Bell inequalities [78] provide a mathematical formulation of this logic. However the validity of *non-local* theories is indeterminate. Some at least are excluded by the Bell-type inequalities, but not all [77]. There is a view in some parts of the literature, exem-

plified in the Colbeck and Renner (C&R) paper [79], that all types of hidden variable theories are excluded, *i.e.* that particles are zero dimensional (0-D) points without any substructure. Hence also that the attributes of the particle (mass, charge, etc.) are merely intrinsic rather than based on physical structures. However the conclusions of the C&R paper have been criticised as logically unsound [77]. Stripped of the mathematics, the logical structure of the C&R paper is a starting assumption that particles are 0-D points, that locality exists, and that quantum theory is correct. Then by deductive reasoning it was inferred that particles are 0-D points such that no substructure can exist. However that is an unsound conclusion because the output statements were already self-evident in the starting premises—the argument was self-confirming.

Even so, successful non-local hidden-variable (NLHV) theories are scarce. There have been many ideas [80] but few successful ones. Historically the most important is the de Broglie – Bohm pilot-wave theory [81] [82]. However this has not developed into a comprehensive new theory of physics, and does not extend to gravitation or the equivalence question.

A more recent NLHV theory is the cordus theory [83] which proposes that particles have a specific two-ended structure, hence *cordus*. This is not inconsistent with string theory, and the number of parameters required to define a cordus particle is the same as some variants of string theory, despite coming at the problem from completely different directions. The theory and makes specific predictions about the structures at the sub-particle level (reactive ends, fibril, discrete forces), and proposes mechanisms whereby these manifest the physical behaviour of the particle as a whole [4]. The theory has been applied to explain a wide variety of phenomena [83]. Of specific relevance to the equivalence question are its explanation and mathematical representation for particle motion [84], the relativistic Doppler & time dilation [2], the identity of matter [58], and gravitation with unification of the interactions [4] [85]. It explains the gravitation field as a torsionally handed emission of discrete forces from particles [2] [4] [84] [85]. This has some parallels with torsional gravity theory, specifically how the gradient of the field arises [68]. In torsional gravity theories [68] [72], the torsional effect is attributed to spin [86] but these theories lack a deeper ontological explanation for the effect. Nor does spin have any physical representation in quantum particle theory. In contrast the cordus theory provides a physical explanation for spin [83] and torsion in field emissions [4]. Theories that that involve both torsion and curvature are underpinned by absolute parallelism geometry [87], with coefficients that take discrete values reminiscent of quantum behaviour [88] [89], where the torsion term represents the interaction between the particle and the gravitational field [90]. The idea that the spin of a particle interacts in discrete steps with gravitational field was proposed on mathematical grounds by [89], and this is consistent with the cordus theory that also proposes a stepped type of motion (“gait”) [4]. The cordus theory is also compatible with the quantum hypothesis of the graviton, in that the torsional packet of discrete

forces is analogous to a quantised variable corresponding to the graviton. The theory includes the concept of fabric (described below) which is compatible (but also extends) the general relativity concept of space-time, and the quantum idea of quantum foam. Consequently it is interesting to examine what it implies for mass-equivalence.

In summary, the identity of mass is an important topic at foundational and cosmological levels. None of the existing theories satisfactorily explain why there should be an equivalence of gravitational and inertial mass, or even how mass arises.

3. Method

3.1. Objective

The purpose of this work was to explore the mass equivalence problem from the perspective of the new physics provided by the cordus theory, which is a type of non-local hidden-variable theory [1]. This is worth attempting for the potential to provide different insights to the problem.

3.2. Approach

A conceptual approach was taken, based on logical extension of the existing cordus theory. There was no a-priori expectation of whether equivalence did or did not hold. Instead we sought to identify the mechanisms that cause mass in this theory, and then infer the implications for equivalence.

This development used *inductive* reasoning. Unlike *deductive* reasoning which proceeds from explicit premises to decisive conclusions, the inductive approach starts with a limited set of premises and generalises to wider implications. One of the quality tests for this inductive methodology is whether it produces new principles that are congruent with principles arising from advancement of the theory in other directions. The anticipatory seeking of this congruence means that candidate new principles have to be checked for consistency with other aspects of the theory. In practice this means that the current results were reviewed, during development, by comparison with findings from all the other published work on the cordus theory [1] [2] [4] [58] [77] [83] [84] [85] [91]-[107]. No logical discontinuities were found. Hence the current findings are cogent with the wider cordus theory. Results are mathematical formalised, but the underlying approach is not primarily a mathematical one.

The precursor to this work was the cordus theory for the Lorentz transformation [2]. This derives the relativistic Doppler, Lorentz formulation, and time dilation from first principles from a particle perspective, which is novel. An unexpected term appeared in the formulation, in the form of the fabric density. Fabric density [93] [96] [102] [105] is a concept within the cordus theory that corresponds approximately to vacuum properties in electromagnetism.

For logical self-consistency within the cordus theory it is necessary to take the premise that mass is a contingent attribute. There is a basis to it determined by

particle identity, but the final expressed value is contingent on other variables. From this we reasoned that fabric density (to be explained) is one of those contingent variables. In the next step, we considered the implications of such an identity for mass. We concluded that a body ought to behave in internally consistent ways in both inertial and gravitational cases, because the underlying mechanisms of mass are related to a common cause in the discrete forces. This provided a rationale for believing the equivalence should hold, at least in any one fabric situation. A single fabric situation refers to all the masses and observers being immersed in the same fabric.

We then generalised to relative fabric situations where the masses and observers are in different fabric densities, and determined the corresponding mathematical formalisms. Mass interactions at cosmological scale, e.g. galaxy rotation curves, are in this category. We find that the fabric density changes the nature of the equivalence in a specific way. We conclude by exploring the implications of this.

4. Results

4.1. The Identity of Mass

4.1.1. Particle Emissions

In the cordus theory a fundamental particle comprises two reactive ends connected by a fibril, with the reactive ends energising in turn and emitting in three orthogonal directions when they do. Individual emissions comprise a sinusoidal varying pulse [84], hence these emissions may be approximated as “discrete forces”, “flux tubes”, or “continuous fields” depending on the context. The particle sub-structures for the electron and proton are shown in **Figure 1**. The emissions from a particle radially outwards and are diluted over a front comprising an expanding spherical area [85]. Hence the $1/r^2$ dependency of strength of field with separation distance r , which is a characteristic feature of Newtonian mechanics. The theory proposes that electrostatic force is carried by the direct linear action of these discrete forces, the magnetic by the bending of the flux tube, and the gravitational by the handedness of the three emissions, hence a unification is achieved [85].

4.1.2. Identity of Mass

At the macroscopic level, mass appears to be a fixed attribute determined strictly by the volume and density of the body, and ultimately by the number of protons, neutrons, and electrons in the body. However, the cordus theory explains mass differently, as the number of discrete force emissions convoluted with the frequency of the particle. The underlying principles are inferred as:

- §1 Heavier particles are those with more discrete forces to emit.
- §2 The theory also anticipates that having more discrete forces requires that the particle adopt a higher frequency to service the emissions, so the addition of discrete forces, e.g. antielectron vs. proton [58], has a disproportionately large effect on mass.

- §3 Furthermore, mass is predicted not to be fixed, but rather dependent on fabric density. Particle B seeks opportunity to emit its own discrete forces into the fabric, and the rate at which it does this is its frequency. The fabric density \emptyset affects its ability to emit. This is elaborated below. Consequently in this theory the mass of an object is an intrinsic property, but not fixed, irrespective of how it is measured.

4.1.3. Identity of Motion

Quantum theory explains force as the emission and receipt of bosons, but as both bosons and fundamental particles are assumed to be 0-D points, it is unclear how these are intercepted. Nor is a QM mechanism apparent for how the receipt of a boson causes force/displacement/velocity of the particle. In contrast the cordus theory does not suffer those limitations, because of its spatial dispersed and non-local structures, and is able to propose explanations for how a

Electron e

Characterised by one discrete force in each of the three directions. This balanced loading causes the structure to be stable against decay.

Physical structure

The discrete forces are released rather than retained as in the photon. Consequently there is an enduring succession of discrete forces in each of the three directions, which creates a long-ranged force effect.

New discrete forces continue to be created and sent down the flux tube at each frequency cycle

Inner Fibril provides instantaneous communication between reactive ends, hence a non-local effect

One reactive end energising and the other de-energising (180° out of phase)

Notation

The notation represents the quantities, charge, and matter-antimatter species of the discrete forces in the three orthogonal emission directions.

$e(a^1 . r^1 . t^1)$

The discrete approximation is illustrated here. The corresponding continuous orbital motion of the reactive end is [a, r, t] = [1, 1, 1]

Two reactive ends some distance apart (non-local)

↑ Energising reactive end

↓ De-energising reactive end

[r]

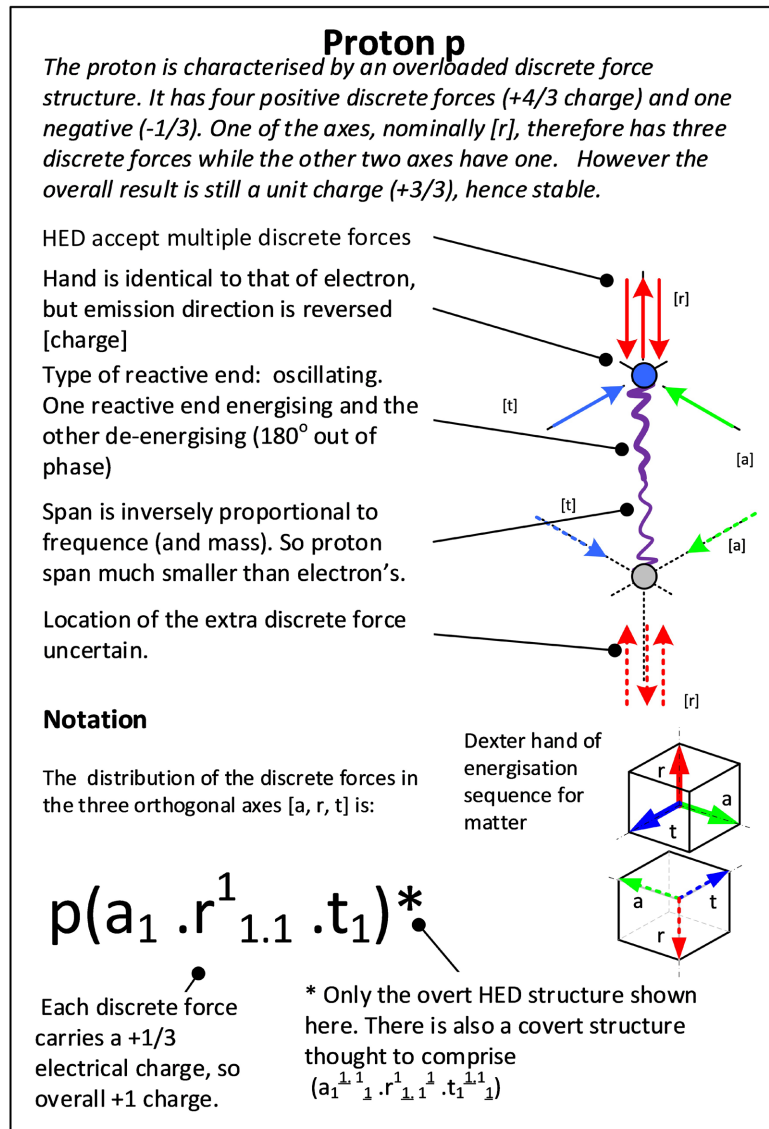
[t]

[a]

Fractional charge (1/3) negative/positive corresponds to outward/inward direction of effect indicated by super/subscript. Matter/antimatter species corresponds to energisation sequence, the latter indicated by underscore (not applicable here).

Dexter hand of energisation sequence for matter: [a] → [r] → [t].

(a)



(b)

Figure 1. Particle sub-structures for the electron and proton. (a) The electron, as a fundamental particle, has a clean architecture of discrete field emissions symbolised by sequential emissions in the $[a, r, t] = [1, 1, 1]$ plane, where $[a, r, t]$ is the Cartesian coordinate system for a reactive end. All these emissions are of the same sign. Image adapted from [108] Creative Commons Attribution 4.0 International license. (b) The proton emissions are more complex as they include both positive and negative charged elements, and covert emission components (not illustrated). Image adapted from [58].

particle detects and moves in a field [4]. The results show that under this theory each of the two reactive ends moves in an inclined orbit around its nominal central location. This has two important consequences. First, it allows the reactive end to sample the space around it and hence intercept & interact with external fields. This addresses the targeting problem of theories that rely on 0-D point particle exchange. It also explains why locality is not strictly preserved at small scales. Second, the reactive end emits forces during its orbit. These may be con-

considered discrete pulses (the earlier interpretation of the cordus theory) or sinusoidally variable fields – both interpretations are correct depending on the perspective. The orbital motion thus corresponds to the handed emission of fields/discrete forces. In turn this handed emissivity underpins the gravitational interaction. Importantly, incoming fields/handed discrete forces retard or advance the reactive end's own emissions, and hence also its orbital motion. The orbital motion and emissions are coupled phenomena. This orbit is circular in shape for a particle at rest, but becomes warped into an asymmetrical spiral when the particle moves or is in the presence of a field, see **Figure 2**. In turn this causes the reactive end (hence the particle as a whole) to move in the direction of the favourable field gradient [4]. As a consequence, motion and velocity of the particle arise [84], which is relevant to the present discussion because this corresponds to the inertial response of the particle.

The reactive end moves to the degree that it is de-energised, and is stationary when fully energised. Consequently, for linear motion, each reactive end moves forward in turn. Hence motion is intermittent at the level of an individual reactive end, but continuous when considering the overall effect of both reactive ends. The presence of external discrete forces, whether from fields or the background fabric, affects this ability to emit. This provides the reactive end with a mechanism to sample the region of space around its nominal location [4], and

Cumulative displaced reactive end position [red] vs undisplaced [green]

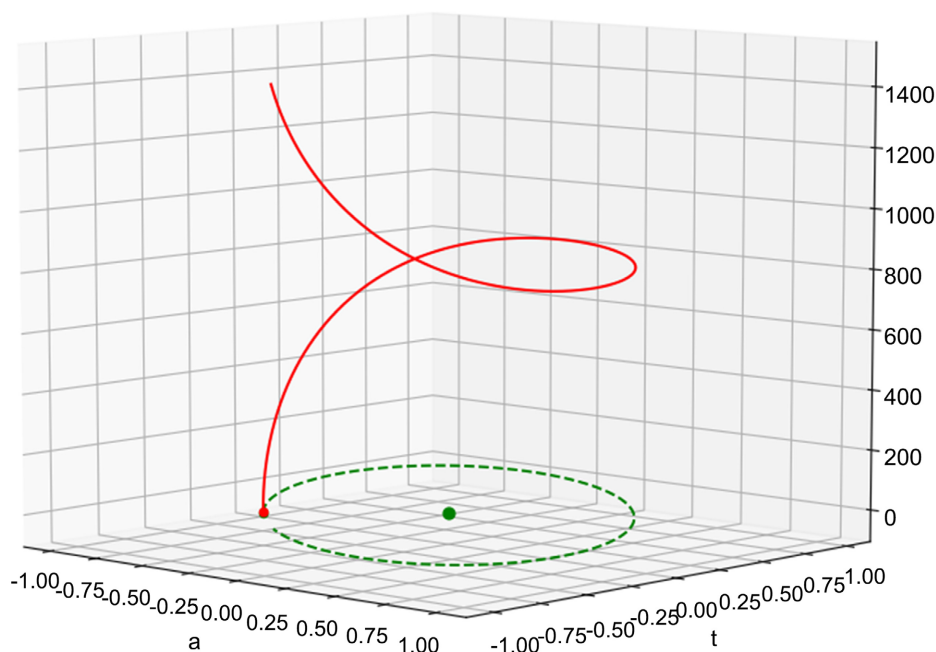


Figure 2. Locus of a single reactive end, undisturbed and under the effect of an electrostatic force in the r direction. Dashed (green) curve shows motion of the undisturbed reactive end. This is a circle in the $[a, r, t] = [1, 1, 1]$ plane, where $[a, r, t]$ is the Cartesian coordinate system for a reactive end. Solid (red) curve shows cumulative locus of the reactive end under the effect of a force in the r direction. This is not a regular spiral. Image reproduced from [4] with permission.

hence both detect and respond to field gradients [85]. This also provides a mechanism for the particle to change its own energisation in response to fabric density, and it is proposed that frequency slows in response to higher fabric density.

4.1.4. Equivalence from a Velocity Perspective

Consider a small test mass B as a satellite in a gravitational interaction with another larger body A. We assume that both bodies are physical objects at our macroscopic level of existence and hence comprise internally decoherent assemblies of matter, as opposed to having quantum coherence between the particles. This is a reasonable assumption to make at the level at which relativity and gravitation occur. We are primarily interested in small body B in the following analysis. We assume B is of negligibly small mass compared to A, *i.e.* we do not consider the reciprocal gravitational effect of B on A.

From the perspective of particle B, its identity is expressed by the emissions of its discrete forces, with the frequency of emissions moderated by the fabric density in which it finds itself (described below). Those emissions are intricately linked to both its tangential motion and its gravitational response. Previous work in the cordus theory has established the theoretical grounds for both. The forward motion has been shown to arise from periodic movement of the reactive ends during their de-energised phase [84]. The motion is not continuous, but inversely related to the strength of energisation, which is sinusoidal. Other work showed that the gravitational response arises from distortion of the locus of the reactive end, which also occurs to the extent that the reactive end is not energised [4] [85]. The gravitational response is therefore one of discrete displacements, the linear motion likewise, and both are affected by frequency. This is an important finding because it implies, from the perspective of self-consistency of the particle's emissions, that both its linear and orbital motion are affected by frequency. This relationship is not anticipated by either GR or QM.

This line of reasoning implies that the centripetal acceleration relation is applicable, which relates radial and tangential motion Hence:

$$a_{B2} = \frac{v_{B2}^2}{r_B} = \frac{F_{rB}}{m_B} \quad (7)$$

where a_{B2} is the radial acceleration of body B moving at velocity v_{B2} at orbital radius r_B , with radial force F_{rB} and mass m_B .

In the orbit case the radial force is the gravitational force $F_{rB} = F_{gB}$ and the tangential force is the inertial.

For clarity, note that in the cordus theory per §2 the discrete force interactions of the ranged forces are fundamentally unidirectional [4]. The fact that remote particle B moves in response to the discrete forces emitted by basal particle A does not change the behaviour of A. This is a logical necessity to preserve temporal causality: the discrete forces received by B were emitted by A some time in the past and have travelled out at the speed of light. If retrocausality is to be denied, as physical realism expects, then it is a logical necessity that the ranged

forces must all be unilateral. It is only when considering discrete forces *en-masse* that the familiar bi-directional force interaction emerges. Under the cordus theory the gravitational interaction experienced by B is a consequence of discrete forces it receives from A. The interaction only changes B, the recipient, not A. The corollary is that B can only change A by sending discrete fields back to A. The above comments apply to decoherent bodies. Coherent assemblies of matter have access to the synchronous interaction [85] [95] and thus to entanglement which can result in superluminal conveyance of information [77], but this is not relevant to the mass equivalence question.

Also, note that per §3 discrete forces are not consumed in the interactions with other particles (in contrast to the colour change of QCD or the virtual bosons of QED), but instead continue to propagate away from their emitting particle. Therefore they act progressively on *every* particle within a macroscopic body as they travel through that body. Hence whether body B comprises one electron, or a whole star, the results are the same, *i.e.* the gravitational interaction generalises for decoherent assemblies of matter.

To address the equivalence question, it is necessary to substitute the gravitational force F_{gB} in the above. However before doing so it is necessary to introduce the fabric variable.

4.2. Fabric Properties

The *fabric* refers in this theory to space between matter, which contains the field emissions of other particles [93]. The nature of these emissions is described in [4] and [84]. The fabric density \emptyset depends on the spatial distribution of matter in the accessible universe. The universe itself expands by spatial divergence, and bodies move about, hence \emptyset is non-isotropic and spatially and temporarily variable across the universe [105]. At any one location in the universe, the fabric takes a local value that may be different to other locations because of the different exposure to the emissions of remote masses.

The fabric is not the same as an aether. The Michelson-Morley experiment [109] disproved the existence of a directional aether wind affecting the speed of light due to the relative movement between the Earth and the medium. However such experiments do not disprove the present proposition, where the speed is isotropic but varies with fabric density [105]. The fabric is not a static medium or fluid such that it has motion of its own to contribute to photons traversing it. This fabric is discrete at the fundamental scale, but approximately smooth at coarser scales. The local fabric density determines the electrical and magnetic constants of the vacuum, which by this theory would not be universally constant [105]. Per this theory the speed of light depends on the fabric density [105]. This may be unconventional but is consistent with the observation that general relativity already accepts that the speed of light is dependent on gravitational field strength, and likewise optics accepts a speed of light that is dependent on refractive index (which is a density dependent property). The emissions from a par-

ticle propagate out at the local speed of light to affect others elsewhere in space. The fabric thus provides the mechanism whereby remote regions are causally connected temporally [96]. Consequently it may be shown that time in this theory is an emergent property of the fabric rather than a dimension [2] [96].

We use the term “*situation*” to refer to a location in space with its local fabric. This is similar to “*frame*” in relativity, but with the inclusion of the fabric parameter. GR does not admit the possibility that inertial frames of reference may not be identically alike. In contrast the cordus theory asserts that equivalence of locations only arises when *both* the inertial conditions and fabric density are the same.

4.2.1. Fabric Properties: Density, Vector, and Gradient

There are multiple attributes to the fabric. It has a scalar magnitude (“density”) value, and also a vector attribute. The magnitude of the fabric density \emptyset at a location x is the summed magnitude of all contributions of masses m_i at range r_i in the observable universe, hence:

$$\emptyset = \sum_x \left| \frac{m_i}{r_i^2} \right| \quad (8)$$

where the computation is situationally-centric to the location, *i.e.* the radial separation r_i is measured from the location under examination. The reason for the r_i^2 dependency is due to spatial dilution of the field. Discrete forces travel radially outwards and are diluted over a front comprising an expanding spherical area [85].

The fabric vector $\vec{\emptyset}$ is the vector sum of the mass contributions:

$$\vec{\emptyset} = \sum_i \frac{m_i}{r_i^2} \quad (9)$$

This vector arises because of the asymmetrical distribution of masses around situation i . The magnitude of the resultant vector is $|\vec{\emptyset}| = \nabla \emptyset$, which is the gravitational field intensity at situation i . This gradient is not a flow of discrete forces – there is no wind or movement of any aether.

The equations for $\vec{\emptyset}$ and \emptyset appear to have a singularity for vanishingly small r . This does not occur as the expressions are only for the far field when the masses m_i are far from each other, e.g. in a galaxy. In the near field the fabric effect becomes one of individual negotiation between particles for emission rights, and this corresponds to alignment of particle orientations (spin parameters). This results in bonding and the formation of crystalline structures. So the m_i/r_i^2 formulation does not hold within such bodies, and the response of the body to external fabric is shared by the crystalline structure. In the extreme near field of coincident particles, the effect becomes one of reactive ends of different particles being co-located (at least at one reactive end each) and synchronising their emissions. The smallest r can be is the size of a single particle A, which in this theory still has a span and occupies volume due to its internal structure [83].

4.2.2. Fabric Effects on Frequency

The Lorentz transformation has been derived using the cordus theory [2], and the results show that frequency and velocity depend also fabric density, which the conventional general relativity does not include. Therefore the fabric density is termed a *covert variable*. The fabric density affects the ability of a particle to complete its own emissions, with greater fabric density retarding the emission [2], hence changing the frequency, velocity and time of the particle. Frequency determines the rate at which the particle can interact with other particles, hence affecting rates of nuclear, chemical & physiological reactions, and this corresponds to time as experienced by the particle [96].

4.2.3. Migration: Motion into Situations of Different Fabric Density

Changes to fabric density, or movement of the particle into situations of different fabric density, affect the rate of time of the particle, hence also time dilation [2]. Consider test particle B with velocity v_{B1} and frequency f_{B1} in situation 1 where the fabric density is ϕ_1 . Subsequently B moves to a new situation of ϕ_2 . Per [2] its intrinsic velocity changes to v_{B2} and frequency to f_{B2} as viewed by Observer A remaining in ϕ_1 .

Observation from originating station

In what follows, $\rightarrow A1$ is used in ambiguous cases to indicate the situation of observation:

$$f_{B2 \rightarrow A1} \phi_2 = f_{B1} \phi_1 \quad (10)$$

and

$$v_{B2 \rightarrow A1} \phi_2 = v_{B1} \phi_1 \quad (11)$$

Hence also

$$v_{B2 \rightarrow A1} = v_{B1} \frac{\phi_1}{\phi_2}$$

Thus if $\phi_2 < \phi_1$ then the velocity of B increases. Examples of where these intrinsic changes would occur are where a star in a galaxy moves distally, or out the plane of the galaxy, or outside of the bar/spiral arm: all these changes are towards reduced fabric density.

From the perspective of an observer A positioned in situation 1 with ϕ_1 the changes to B are detectable. That observer would see a change in the frequency of emission of radiation from B, and the velocity of B against a backdrop of markers of known position.

Co-moving Observation

The changes are not detectable by a co-moving observer since the particles making up the observer will also change in frequency and velocity. However an observer co-moving with B would perceive the rest of the universe as having changed by the inverse relationships.

These changes relate to difference in fabric density, and are separate to relativistic considerations. We refer to these as *intrinsic* changes as they occur via

conservation mechanisms internal to the particle [4] [84], in contrast to *extrinsic* changes such as contact forces or electro-magneto-gravitational fields. Unconventionally, the theory predicts the changes do not require extrinsic energy supply. They are the response of the energisation mechanism of the particle to the changed external constraints imposed by the fabric. The rate of time in this theory is not a dimension, nor is it universally constant, rather it is a property of the fabric density, hence of the distribution of mass.

Observation from a third situation

Relative to observer E in a different situation with ϕ_3 , possibly in a different galaxy at a later epoch of time, the relative local passage of time, measured by frequency f_3 is affected by the fabric density per Equation (10), hence all else being equal $f_3\phi_3 = f_2\phi_2$. If $\phi_3 < \phi_2$ then time passes faster for E, hence E perceives B to have slower frequency $f_{B2 \rightarrow E3}$ and velocity $v_{B2 \rightarrow E3}$. Thus:

$$v_{B2 \rightarrow E3} = v_{B2} \frac{\phi_3}{\phi_2} \quad (12)$$

And if B migrates from ϕ_1 to ϕ_2 then

$$v_{B2 \rightarrow E3} = v_{B2 \rightarrow A1} \frac{\phi_3}{\phi_1} \frac{\phi_3}{\phi_2} = v_{B1} \frac{\phi_1}{\phi_2} \frac{\phi_3}{\phi_1} \frac{\phi_3}{\phi_2} = v_{B1} \left(\frac{\phi_3}{\phi_2} \right)^2 \quad (13)$$

In the case where E is an observer on Earth (within the Milky Way), looking back at a denser prior epoch of the universe, and B is a star in the outer reaches of a distant galaxy, then Equation (12) implies observed galaxy rotation curves would not be reliable indicators of actual velocities. The correction is complex because ϕ_2 depends on orbital radius, size and morphology of the galaxy. Equation (12) applies for stars formed in place, whereas Equation (13) is for migratory stars.

4.2.4. Intrinsic Changes in Velocity

The above changes in velocity occur due to the particle experiencing changed fabric density. This occurs without the application of a direct force, at least not an extrinsic force acting on the particle at the time. It is unhelp to consider this an acceleration in the conventional sense. Rather the body natural speeds up or slows down depending on fabric density encountered. Elsewhere in the universe some force must previously have been applied to rearrange matter to cause the fabric density to change, but there need be no extrinsic force on B itself. Hence there is an intrinsic change in velocity Δv given by:

$$\Delta v = v_{B2} - v_{B1} = v_{B1} \left(\frac{\phi_1}{\phi_2} - 1 \right) \quad (14)$$

If a sudden boundary in ϕ were encountered, then the change in velocity would be immediate, *i.e.* the acceleration would be infinite. However this is not a realistic case because there is always some minimum time involved, which is the energisation period of the particle. The time t_ϕ taken for transition from situation 1 to 2 is expected to be large when dealing with cosmological scales and

weak gradients in fabric density, and $\frac{\phi_1}{\phi_2}$ will be close to unity at the scale of a solar system, so the acceleration $a_\emptyset = \Delta v/t_\phi$ is expected to be miniscule. As the acceleration does not require a direct extrinsic force, it is necessary to modify Newton's equation of motion to:

$$a_{B1} = \frac{F_{iB}}{m_{B1}} + a_\emptyset = \frac{F_{iB}}{m_{B1}} + \frac{v_{B1}}{t_\phi} \left(\frac{\phi_1}{\phi_2} - 1 \right) \quad (15)$$

where a_{B1} is acceleration as observed from situation 1, and F_{iB} is the inertial force applied. The first component is the conventional extrinsic part, and the second is the intrinsic part due to the fabric. The latter goes to zero when there is no difference in fabric density, hence recovering Newton's inertial equation. This completes the derivation of how the fabric density parameter affects frequency (hence time) and velocity.

4.3. Inertial Mass

Having established the fabric density effect, it is now necessary to consider the implications for inertial mass.

Intrinsic Changes in Inertial Mass

We assume mass is proportional to the energisation frequency f_B of the particle. (It may instead be a squared or other function—this is not settled within the cordus theory). When the frequency of B increases, it emits discrete forces at a higher rate. Hence there are more discrete gravitational interactions with another body, and the gravitational force increases in strength.

When particle B of mass m_{B1} moves from a situation with ϕ_1 into a new situation of ϕ_2 its frequency changes, and hence the mass changes to m_{B2} as viewed by an Observer A remaining in ϕ_1 :

$$m_{B2 \rightarrow A1} = m_{B1} \frac{\phi_1}{\phi_2} \quad (16)$$

This equation describes a case where a mass moves from ϕ_1 to ϕ_2 . This might be a satellite that leaves the Solar system, or a star that migrates through a galaxy. If $\phi_2 < \phi_1$, *i.e.* when the mass moves into a region of lower fabric density, then the inertial mass increases. This is an intrinsic increase, since mass is intricately linked to energisation frequency in this theory. Hence this theory predicts that mass is not strictly constant, but depends on fabric density.

The changes in mass (Equation (17)) and velocity (Equation (11)) are in the same direction (both increase or decrease). Hence the cordus theory predicts that the conservation of momentum equation must also be modified to include the fabric density term:

$$m_{B1} v_{B1} \phi_1^2 = m_{B2} v_{B2} \phi_2^2 \quad (17)$$

Another way of looking at this is to state that the particle changes to a more energised state, hence also becomes heavier, when the fabric density decreases.

Having examined the effect of fabric density on inertial mass, we next address gravitational mass. As identified above, for reasons of self-consistency of discrete force identity, it is assumed that the inertial and gravitational masses are the same. However this does not necessarily mean that their relationship with fabric density is trivial.

4.4. Gravitation

The operation of the gravitational field has previously been developed in the cordus theory [85], and relevant parts are briefly introduced here. Consider a central mass A comprising one or more particles, with a satellite mass B. We are only concerned here with the gravitational effect on B. In this theory the gravitational field of a particle arises from torsion in its emitted flux tube [85]. This torsion arises from the sequence (and hence handedness) of emission of its three discrete forces. For the particle B receiving and responding to a gravitational field, the interaction arises as a constraint on displacement during the energisation cycle of the particle [4] [85]. The particle needs to emit its own discrete forces, detects the gradient in the fabric, and attempts to move in a direction that maximises its opportunity to emit its own discrete forces [4]. Hence the reactive ends of the unconstrained particle move along the field gradient to maximise the mutual compatibility. Compatibly here relates to the handedness of emissions sequence. In a three dimensional spatial system with three discrete forces, there are only two ways this energisation may be accomplished, hence two hands, dexter and sinister, and these are attributed to the matter-antimatter species respectively [97]. The interaction is not continuous but rather follows a sinusoidal function over time, with the reactive end undertaking a torsional displacement during its de-energised phase [84] (see **Figure 2** above). Typically the remote particle B is of the same species as body A, *i.e.* both are matter. Then B finds it easier to make its own emissions if they are synchronised with those it receives from A. This increases the compatibility, and hence B moves up the field gradient, *i.e.* matter-matter gravitation is attractive. Matter-antimatter is predicted to be gravitationally repulsive, and antimatter-antimatter attractive [85]. This concept of gravitation being underpinned by a torsional mechanism is consistent with other developments in the field, such as Einstein-Sciama-Kibble torsion gravity developments [110], but originates from a different line of reasoning.

4.4.1. Gravitational Dependency on Fabric

The strength of the gravitational interaction depends proportionally on the mass of B and the gravitational field intensity, hence the effect is proposed to be multiplicative:

$$F_{BG} \approx \left| \overline{\phi}_2 \right| m_B \quad (18)$$

The gravitational mass of B (m_B) is the product of quantity of discrete forces it emits determined by particle identity [58], total number of particles, and frequency (which is moderated by the fabric density ϕ_2).

For example, a satellite body B in a situation of lower fabric density ϕ_2 has faster re-energisation of its reactive ends than A, *i.e.* time passes faster [96], compared to a reference situation with ϕ_1 . Its frequency is per Equation (10)

$$f_{B2 \rightarrow A1} = f_{B1} \frac{\phi_1}{\phi_2}. \text{ Consequently B has greater receptivity to gravitational interaction,}$$

hence it experiences a stronger gravitational force than does A from B, by a receptivity enhancement of the ratio of fabric densities.

Note that this frequency effect applies irrespective of whether or not B originated in the same location as A, *i.e.* this is not a migration effect per se, but rather a difference in the fabric densities. This can also be viewed as a time dilation effect [96]—the perspectives are complementary. Time dilation exists in the presence of a gravitational field, and hence by inference the two bodies cannot experience the same temporal summation of force. This asymmetry is unconventional. This finding is not accessible to conventional relativity which assumes forces are continuous parameters, nor to conventional quantum mechanics which assumes forces arise from bilateral exchange of infinitely compact bosons. The cordus theory proposes that forces have a sinusoidal nature, and it is this characteristic that makes the difference.

4.4.2. Gravitation Formalism

Per Equation (9) the gravitational field vector experienced by B at location 2 is

$$\overline{\phi}_2 = \sum_B \frac{m_i}{r_i^2} \text{ where the computation is B-centric. Hence combining the above}$$

Eqns, the gravitational force experienced by B at location 2 is:

$$F_{BG \rightarrow 2} = \left| \overline{\phi}_2 \right| m_B \frac{\phi_1}{\phi_2} k_G = \left| \sum_B \frac{m_i}{r_i^2} \right| m_B \frac{\phi_1}{\phi_2} k_G \quad (19)$$

where k_G is a factor to account for the conversion of the torsional effect of the discrete forces into gravitational force. Hence the conventional gravitational constant G comprises:

$$G = \frac{\phi_1}{\phi_2} k_G \quad (19a)$$

This factor is not explored further here.

Another way to look at this is that gravitation is an emergent property of the fabric, rather than an invariant attribute of space-time. This idea of gravitation being emergent is somewhat similar to the position taken by entropic gravity, though that approaches it from the different direction of assuming dark matter arises from dark energy effects at the particle level.

4.5. The Inference of Equivalence

This theory expects that a body will behave in internally consistent ways in both inertial and gravitational cases, because the underlying mechanisms of mass are related to a common cause in the emission of discrete forces. Superficially this

means that the cordus theory supports the equivalence of inertial and gravitational mass. Now, having established the fabric dependencies, the question of what the equivalence looks like from a formulaic perspective may be addressed.

Assume a circular gravitational orbit. Assume that the centripetal acceleration a_{B2} applies in the radial direction per Equation (7) ($a_{B2} = \frac{v_B^2}{r_B} = \frac{F_{rB}}{m_B}$). For radial force F_{rB} substitute F_{BG} from Equation (19) as the gravitational force is radially directed for a circular orbit. Hence:

$$\frac{v_B^2}{r_B} = \frac{F_{BG}}{m_B} = \left| \sum_B \frac{m_i}{r_i^2} \right| \frac{\phi_1}{\phi_2} k_G \quad (20)$$

For a gravitational field dominated by a single body A (*i.e.* not a multi-body problem), the gravitational field $|\vec{\mathcal{O}}| = \nabla \mathcal{O}$ which is determined simply by m_A and the orbital radius:

$$\left| \sum_B \frac{m_i}{r_i^2} \right| = \frac{m_A}{r_B^2} \quad (21)$$

Hence:

$$v_B^2 = \frac{m_A}{r_B} \frac{\phi_1}{\phi_2} k_G$$

Thus

$$v_B = \sqrt{\frac{m_A}{r_B} \frac{\phi_1}{\phi_2} k_G} \propto \frac{1}{\sqrt{r_B}} \quad (22)$$

Thus the ratio of fabric densities appears within the equivalence. If the fabric densities are assumed to be the same, which is what Newtonian gravitation assumes, then the conventional equivalence is recovered.

5. Discussion

5.1. Interpretation

5.1.1. Disjointed Equivalence

The surprise in the equivalence formulation of Equation (22) is the covert term $\frac{\phi_1}{\phi_2}$. Our interpretation is that the equivalence holds in any one case under

examination, in terms of the relationship between m_A, r_B, v_B , but a different equivalence holds when the fabric densities change. We refer to this as a *disjointed* equivalence. In terms of Newtonian gravitation this corresponds to a variable gravitational constant. This implies the need to abandon the concept of a gravitational constant that is constant in time and place. This is unconventional, but is consistent with the observation that G has been difficult to determine with the precision expected of fundamental constants.

5.1.2. Composition of the ϕ Term

In general the \mathcal{O} terms may be simplified into several summed components:

1) The fabric density of the background universe ϕ_0 at that location in space and temporal epoch. Assuming an expanding universe, the fabric density was greater at early epochs. The universe term may also include the super-structures of galaxies, and the galaxy in which the star is located. The latter is a complex relationship of galaxy morphology, galaxy mass, and orbital radius of the star. The morphology is especially complex with disk galaxies with their cores, disks and arms.

2) The contribution of massy body A to the fabric density, ϕ_A . Depending on the context, body A may refer to a satellite orbiting a star (which is a relatively simple case of m_A/r^2), or a multibody galactic core. At the centre of solid body A (a star) there is a self-contribution to the fabric determined as:

$$\phi_A = \int_0^{R_A} \frac{4\pi r^2 \rho dr}{r^2} = 4\pi \rho R_A = \frac{3m_A}{R_A^2} \quad (23)$$

where this assumes a homogenous spherical body of outer radius R_A and mass m_A and density ρ . This shows that the fabric density is primarily a mass density property, and no singularity arises at the centre of a massy body (or cluster of masses). Separately A also contributes to the gradient via the $\frac{m_i}{r_i^2}$ term.

3) In addition body B makes a fabric contribution which we ignore as small. However in some cases, such as binaries, this will not be a safe assumption.

Hence the fabric density at location A is:

$$\phi_1 = \phi_0 + \frac{3m_A}{R_A^2} \quad (24a)$$

And for location B:

$$\phi_2 = \phi_0 + m_A/r_B^2 \quad (24b)$$

Note that R_A is the outer radius of the mass surface of A, whereas r_B is the orbital radius of body B.

5.2. Gravitation Special Cases

There are several cases with specific assumptions relating to the epoch of the universe. Fabric density varies with the temporal epoch of the universe, being denser in the past. Hence more distant galaxies are predicted in general to have greater background fabric density. Fabric density also varies with galaxy size, with larger galaxies having greater fabric density. In addition, fabric density varies across any one galaxy, being denser and even uniform in the centre (because of many-many mass interactions), and lighter at the periphery. Furthermore this is complicated by the shape of the galaxy, being different in- and out- of the galactic plane. Hence the theory implies that gravitation over cosmological scales is a much more complex interaction than experienced in Earth environs. Several specific cases are enumerated below.

1) Earth location and epoch: Newtonian gravitation

As the universe evolves over time t , its mass becomes spatially distributed, and a

fabric density $\phi_0(t)$ develops. For observations in the vicinity of Earth, cocooned among other stars and galaxies, ϕ_0 is assumed to be large, and approximately homogeneous and static. Hence $\phi_0 \gg \frac{3m_A}{R_A^2}$ where A refers to the Sun, thus per Equation (24), $\phi_1 \cong \phi_0$ and likewise $\phi_2 \cong \phi_0$ which means the receptivity enhancement tends to unity, and the gravitational field intensity is dominated by body A (the Sun), giving for Equation (19):

$$F_{BG \rightarrow 2} = \frac{m_A}{r_B^2} m_B k_G \tag{25}$$

which recovers the form of Newtonian gravitation. Hence we propose that the gravitational constant is identified with $G = k_G$ but only in this special case. More generally $G = \phi_2 / \phi_2 k_G$ hence is not constant but rather has a covert fabric dependency. The equivalence for this special case becomes, per Equation (20):

$$\frac{v_B^2}{r_B} = \frac{m_A}{r_B^2} k_G \tag{26a}$$

Hence:

$$v_B \propto \sqrt{\frac{1}{r_B}} \tag{26b}$$

This recovers the conventional Newtonian formulation, where orbital velocity decreases with separation.

2) Galaxy rotation curves

Fabric density will be reduced for a star in a more distal part of a galaxy, or out the plane of the galaxy, or outside of the bar/spiral arm. We assume an approximately spherical galaxy shape, with star B at radius r_B from the galactic centre, though we acknowledge that disk galaxies have a more complex distribution of mass. At the centre of galaxy A, $\phi_1 = \phi_0 + \frac{3m_A}{R_A^2}$, where R_A is approximated as the outer edge of the galaxy, and m_A is the mass of the galaxy. Assume the fabric density of the background university is some fraction $\frac{1}{a}$ of the galaxy,

$$\phi_0 = \frac{1}{a} \frac{3m_A}{R_A^2} \text{ hence}$$

$$\phi_1 = \frac{3m_A}{R_A^2} \left(\frac{1}{a} + 1 \right) \tag{27a}$$

$$\phi_2 = \phi_0 + \frac{m_A}{r_B^2} = m_A \left(\frac{3}{aR_A^2} + \frac{1}{r_B^2} \right) \tag{27b}$$

Thus:

$$\frac{\phi_1}{\phi_2} = \frac{3 \left(\frac{1}{a} + 1 \right)}{\frac{3}{a} + \frac{R_A^2}{r_B^2}} \cong a + 1 \tag{27c}$$

For example, with $a = 1$, *i.e.* the fabric density of the galaxy at its core is the same strength as the universe at large, $\frac{\phi_1}{\phi_2} = 2$.

Thus the gravitational formulation in Equation (19) becomes:

$$F_{BG \rightarrow 2} = \frac{m_A}{r_B^2} m_B (a+1) k_G \quad (28)$$

The equivalence for this special case becomes, per Equation (20):

$$\frac{v_B^2}{r_B} = \frac{F_{BG \rightarrow 2}}{m_B} = \frac{m_A}{r_B^2} (a+1) k_G \quad (29a)$$

Hence:

$$v_B \propto \sqrt{\frac{a+1}{r_B}} \quad (29b)$$

This predicts that more peripheral stars will orbit faster than the Newtonian prediction, because they experience a stronger gravitation. This is consistent with known observations, though the consistency is conceptual rather than quantified at this stage, because we cannot yet suggest a way for parameter a to be determined. The orthodox cosmological interpretation attributes dark matter as the cause of anomalous galaxy rotation curves, though no evidence of such matter has yet been found. The MOND family of theories [69, 70] are successful at modelling the galaxy rotation curves, and hence obviate the need for dark matter, by assuming that gravitation get stronger with distance. While MOND is quantitatively accurate, it has no underlying ontological explanation of why gravitation should increase in strength with distance. The cordus theory offers such an explanation, and further proposes that the effect is not solely distance, but also fabric density. Increased gravitational force is also provisioned in the non-geodesic theories, *i.e.* an additional force is involved [111], involving both vector and scalar fields [112]. The cordus theory also involves vector and scalar fields in gravitation. Hence there is a possibility that the cordus theory might provide an underlying rationale based on physical realism for the MOND and vector-scalar field theories. There is a conceptual consistency, though additional work would be necessary to check this.

Galaxy rotation velocities are determined from Doppler shift in spectral lines. Our interpretation is that these frequencies would need to be corrected for differences in fabric density those velocities can be determined. Given the difficulty of determining fabric density for remote locations, it is currently unclear how this may be done.

3) Genesis epoch for the universe

Expansion of the universe

Evidence of the accelerated expansion of the universe has been accumulated from multiple studies, e.g. based on redshifts [113], or supernovae [114], considering also anisotropies [115], and these results generally support the idea of a cosmological constant [116]. This constant may represent a vacuum energy den-

sity [117], which is commonly believed to be created by a dark energy. The conventional interpretation of the expansion of the universe is that the metric changes scale, *i.e.* that the spatial dimensions themselves expand. This is premised on there being nothing for space to expand into, *i.e.* nothing outside the universe. There are numerous models for the expansion, and theories for its mechanisms. The dominant conventional model is Lambda cold dark matter (Λ CDM), which assumes the existence of dark energy to drive the cosmological expansion, and of cold dark matter to explain the galaxy rotation curves. The model shows good quantitative agreement with empirical measurements [118]. However the identities of neither dark energy nor dark matter are known. Alternative theories for the expansion include modifications to general relativity, modifications to gravitation such as MOND [119] and similar such as entropic gravity, and biometric gravity.

In contrast the cordus theory supports a different interpretation. For a start, time is not a dimension in the cordus theory, but an emergent property of matter, and is communicated through the fabric [96]. The theory also proposes the primacy of the spatial dimensions, so that there is something for the universe to expand into. Consequently there is also a frontier of expansion, which is the cosmological boundary [93]. At genesis the baryogenesis is proposed to have occurred via pair production [104] and the remanufacture of the anti-electron into the proton [58], resulting also in domain warfare between competing matter and antimatter pathways. The matter and antimatter bodies are expected to have comprised coherent neutron-species, and the interaction between them is predicted to be repulsive by the synchronous/strong force [95] for coherent conditions, and also repulsive by gravitational for the decoherent state [85]. Hence this provides a mechanism for a rapid explosion (“inflation”) of the primal massy material, followed by a momentum-driven expansion of the universe. Consequently the cordus theory disfavours the idea of metric expansion of space, and instead proposes it to be a more conventional expansion by movement of matter. Hence when the universe expands, other masses move outwards, their separations increase, and the overall fabric density drops, *i.e.* a change in the r_i

components within $\varnothing = \sum_x \left| \frac{m_i}{r_i^2} \right|$ (Equation (8)). This causes an increase in

intrinsic velocity per Equations (14)-(15). Thus the cordus theory does not conceptually need dark energy, but instead proposes that the expansion of the universe is an intrinsic change in velocity caused by the reduction in fabric density that arises from the expansion itself.

The inertial-gravitational relationship at the genesis epoch is predicted as follows. Assume the universe comprised a single mass A located at 1, and a small remote test body B located at 2. Also assume body A being in a decoherent state at least in its far-field emissions. In this extreme case, with no other mass in the universe the fabric density is entirely created by A, hence $\varnothing_1 = \frac{3m_A}{R_A^2}$, $\varnothing_2 = \frac{m_A}{r_B^2}$,

and with $\left| \sum_B \frac{m_i}{r_i^2} \right| = \frac{m_A}{r_B^2}$ substituting into Equation (19):

$$F_{BG \rightarrow 2} = \left| \sum_B \frac{m_i}{r_i^2} \right| m_B \frac{\phi_1}{\phi_2} k_G = m_B \frac{3m_A}{R_A^2} k_G \quad (30)$$

Note there is no surviving r_B term representing the orbital radius of body B. Hence the theory makes the unusual prediction of a genesis gravitational force that is initially constant with separation. The equivalence for this special case becomes, per Equation (20):

$$\frac{v_B^2}{r_B} = \frac{3m_A}{R_A^2} k_G \quad (31a)$$

Hence

$$v_B = \sqrt{r_B \frac{3m_A}{R_A^2} k_G} \propto \sqrt{r_B} \quad (31b)$$

Hence orbital velocity would increase slowly as separation increased. In contrast for the Newtonian case $v_B \propto \frac{1}{\sqrt{r_B}}$.

The current prediction of stronger gravitation at earlier epochs of the universe is consistent with other findings. First, there is empirical evidence for the accelerating expansion of the universe. This is conventionally attributed to dark energy or a variety of other causes [120]. The dark energy formulation of Chevallier-Polarski-Linder has a linear dependency on redshift z of the form $z/(1+z)$ which predicts an increase in dark matter for greater z . In the present model, greater redshift corresponds to earlier phases when the universe was denser and hence gravitation stronger. Second, the dark matter findings may also be interpreted as consistent with the current model. Per the Navarro, Frenk & White (NFW) model [121], the density of a Lambda cold dark matter (Λ CDM) halo depends on the mean density of the universe at its location, *i.e.* later formed haloes have lower density [122]. Furthermore, the Einasto power law has coefficient α which is not constant but rather increases with redshift and mass [122]. An increase in dark matter corresponds to a stronger gravitational force.

Future evolution of the universe

As the matter of the universe expands further apart in the far future, there is expected to be a reduction in the fabric density of the universe as a whole, so ϕ_0 tends to smallness. For body B orbiting A, the fabric density will be dominated by A, hence $\phi_1 = \frac{3m_A}{R_A^2}$ and $\phi_2 = \frac{m_A}{r_B^2}$ hence $\frac{v_B^2}{r_B} = \frac{3m_A}{R_A^2} k_G$ as per the genesis epoch. Thus the gravitational force is predicted to be stronger at genesis, weaker during middle epochs, and strong again at the end of the universe. We are unsure whether this implies an open or closed universe.

5.3. Critique

With inductive reasoning the conclusions are not logically certain. This is because there remains an element of doubt about the starting principles, and not all the empirical evidence (in this case from cosmology and particle physics) has been considered. Nonetheless the inductive approach has the benefit of amplifying a topic into a broader set of principles, *i.e.* the theory is further developed and new principles discovered.

In the case of the present paper those new principles are the prediction of a disjointed equivalence conditional on fabric densities, a variable gravitational constant G , and a gravitational formulation that changes with fabric density and the evolution of the universe. These findings are consistent with empirical observations of galaxy rotation curves.

Furthermore the present findings are logically consistent with the other published work on the cordus theory. This is a useful quality test for conceptual development of a candidate new theory of physics like this.

5.3.1. Falsifiable Predictions

Falsifiable feature of the theory are:

- 1) A unidirectional causality of the gravitational forces exists at the fundamentally level.
- 2) In the current epoch and location, the fabric density contributed by the observable universe is comparatively larger than that contributed by local gravitational effects (e.g. Sun, Earth).
- 3) A body will have faster velocity than predicted by gravitation alone, when it moves into situations of lower fabric density.

5.3.2. Limitations and Future Research

The cordus theory developed here is a conceptual and logical work, built on a starting conjecture for the structure of matter. The current paper has provided a theoretical formalism for mass equivalence, but has not tested this against empirical results. Left for future work is the task of analysing galaxy rotation curves to determine how well this theory fits those observations. This would seem to be the most direct route to test the theory, since the fabric density—which is otherwise invisible—can perhaps be computed from mass and morphology of galaxies. We do not underestimate the potential difficulty in determining absolute values of fabric densities for various astronomical objects. If this cordus theory is true, determining these densities would be a necessity to understand the evolution of the universe. This would seem to require new mathematical methods and empirical approaches, hence a large potential future area of cosmology research.

6. Conclusions

The principles of the cordus non-local hidden-variable theory have been extra-

polated by inductive reasoning to examine the identity of mass, and explore the inertial-gravitational equivalence. Key findings of this theory are as follows. A disjointed equivalence is predicted, whereby inertial and gravitational masses are equivalent in any one situation, but a different equivalence holds when the fabric densities change. This is interesting, because it means that the equivalence holds in any one situation, but the nature of the equivalence changes with location. Consequently this theory predicts that the gravitational constant G varies with fabric density, and hence would be different across the universe and across time. Other theories also propose a variation in gravitation, such as the Brans-Dicke theory [123] where the gravitational constant is replaced with a variable scalar field (also called ϕ), though the approach and formulation differ from the present theory. Similarly MOND [69] [70] proposes that the gravitational parameters are non-constant.

In this cordus theory, not only is the gravitational constant non-constant, but the formulation of gravitation changes with fabric density. Specifically, the theory predicts gravity is stronger at genesis (and the end of the universe) such that orbit velocity $v_B \propto \sqrt{r_B}$ (where r_B is orbit radius), compared to weaker gravitation at middle life epochs such that $v_B \propto \frac{1}{\sqrt{r_B}}$. The current Earth location and epoch

correspond to the latter case, *i.e.* Newtonian gravitation is recovered by a special-case simplification of the cordus theory. This is an original finding.

Furthermore, a novel explanation is provided for the accelerating expansion of the universe. The theory proposes that the expansion of the universe is an intrinsic change in velocity caused by the reduction in fabric density that arises from the expansion itself.

If true, this has implications for understanding the evolution & expansion of the universe, and the interpretation of galaxy rotation curves. The findings disfavour the existence of both dark energy and dark matter, and instead attribute these effects to differences in the fabric density.

In summary the original contribution of this work is the elucidation of fabric density as a covert variable in the mass equivalence formulations, the identification that mass equivalence is disjointed—inertial and gravitational masses are equivalent in any one situation, but a different equivalence holds when the fabric densities change, and the identification that the gravitation formulation itself changes with relative fabric densities.

Conflicts of Interest

The author declares that there are no financial conflicts of interest regarding this work.

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