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Confirmation of 24 h 50 min Lunar Periodicity, Apparently Inexplicable by Classical Factors, in Precession of Allais Pendulum

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Abstract

During 36 days the motion of two pendulums, which were restarted every hour, was continuously recorded. 869 "runs" were thus made, providing each time the precession during the run, as well as other parameters of the motion. Spectral analysis of precession and ellipticity revealed a lunar component of 24 h 50 min, which can only result from an astral action, through mechanisms yet to be discovered. Indeed, an analysis was carried out of the influence of all classical perturbing factors: direct or indirect action of classical gravity, temperature, Earth's magnetic field, etc.… None of them can explain this component, given its amplitude and phase. Its amplitude excludes also an explanation by general relativity. This is consistent with a major result that Allais claimed to have obtained during each one of the six continuous one-month-long experiments he carried out from 1954 to 1960. The numerous and very precise data provided by an automatic alidade give additional information to those gathered by Allais. All that confirms all the scientific interest that there would be to resume long-duration pendulum observations on a much more important scale: continuous observations for at least 2 years, and if possible more.

Keywords

Allais Effect, Pendulum, 24 h 50 min Period, Lunisolar Influence

1. Introduction

1.1. Reminder of the Experimental Results of Maurice Allais

- Maurice Allais (1911-2010), who had studied mathematics and physics at a very high level and received the Nobel Prize in economics in 1988, did not stop, throughout his life, being interested in physics. In particular, he is the first one to have really exploited as an investigative tool the analysis of the precession of a pendulum, which makes it possible to highlight phenomena totally different from those seen by static devices such as gravimeters (and from those seen by the analysis of the period of a pendulum, which can also be used as a gravimeter). Hence, from 1954 to 1960, he organised in his laboratory in Saint-Germain six one-month duration experiments carried out day and night without interruption (the pendulum being restarted every 20 minutes). The 1958 experiment was particularly significant, since a second pendulum has been set up in the very stable environment of an underground quarry[1].

In the evolution of the azimuth of the plane of oscillation of his pendulum, Allais highlighted indeed several periodic components which, taking their amplitude into account, could absolutely not be explained by classical gravitation. Furthermore, an in-depth analysis has shown that at least one of them, whose period was 24 h 50 min, could not be quantitatively explained by any known phenomenon.

Incidentally, while his first experiment was underway, Allais also noticed a strikingly abnormal behavior of the oscillation azimuth during the total solar eclipse of June 30, 1954. He again observed a similar phenomenon, although less pronounced, during the solar eclipse of October 2, 1959.

• From 1957 to 1959, these observations were the subject of publications in several journals, including in particular the "*Comptes rendus de l'Académie des Sciences*" [1]-[9]. They even earned Allais two scientific prizes in 1958[2]. A publication in "*Aerospace Engineering*" [10] [11], at the request of Wernher von Braun, director of NASA, did much to make them known. In a summary book published in 1997 ([12] or [13]), Allais presented all his research in experimental physics together with the analysis that he made of it afterward.

In the late 1950s, Allais' publications and conferences struck a certain chord and were discussed in numerous in-depth debates at a very high level, none of which enabled his findings to be invalidated[3].

[1]This second pendulum was set up in an abandonned underground chalk quarry in Bougival, overlaid with 57 meters of clay and chalk. The horizontal distance from the open surface was about 800 m. It has purposefully been built for that occasion, using the same plans as those of the Saint-Germain pendulum, which has remained in a fixed location from 1954 to 1960. The two pendulums were about 6.5 km apart.

[2]The 1959 Galabert Prize from the *Société Française d'Astronautique* and the 1959 Gravity Research Foundation Prize.

[3]Witness, for instance, the following extract from a letter written by General Bergeron to Wernher von Braun in May 1959: "Before writing to you I thought it necessary to visit both of Professor Allais's laboratories (one of which is located 60 m underground) in the company of eminent specialists-including two professors at the *École Polytechnique*. In the course of a discussion which lasted several hours, it was not possible to locate any significant source of error or any attempted explanation which resisted analysis. I think I ought also to inform you that in the course of these last two years, more than ten members of the *Académie des Sciences* and more than thirty eminent personalities, gravitation specialists of various kinds, have come to visit either his Saint-Germain laboratory or his underground laboratory at Bougival. Detailed discussion took place, not only on these occasions, but also several times in various scientific milieux, notably at the *Académie des Sciences* and the *Centre National de la Recherche Scientifique*. None of them enabled any explanation whatsoever to be brought forward in the context of currently recognized theories."

To the authors' knowledge, no published article has so far disputed the reality of the 24 h 50 min component, and there has been only one article [14], in 1958, to purport conventional explanations (variations in the elasticity of the suspension due to temperature variations, vibrations of the building due to the wind), which the double experiment of 1958 (surface and underground quarry) made it possible to completely eliminate.

In a 2003 article by Van Flandern and Yang "Allais gravity and pendulum effects during solar eclipses explained" [15], it is alledged without proof that anomalies in the precession of the pendulum during solar eclipses would be due to the existence of air currents at ground level. In fact, when you read the article, and contrary to what the title indicates, the so-called explanation is far from a certainty. It is a mere hypothesis, which is furthermore very little substantiated[4]. Anyway, it is very unlikely that in 1958 there were significant air currents at the end of the 800-meter shaft of the Bougival quarry, and that this could explain the observed 24 h 50 min component.

• With a 30 days observation, it is impossible, near the period of 24 h, to utterly discriminate 2 components separated by less than 1/30 day, which is 0.8 hours, or 48 minutes. Hence, a periodic component of 24 h 50 min may result in fact from the composition of the rotation of the Earth in 24 h and from any phenomenon whose period is near 1 month. As it is nearly the period of the wave M1 in tidal theory and the average duration between 2 successive passages of the Moon at meridian (in fact the exact value is 24.84 h, that is 24 h 50 min 24 s; hence, in this experiment, we have considered the 24.84 h component), it is generally considered to be a signature of the influence of the Moon, and this is how Allais interpreted it in most of his publications. In [10] [11], however, he pointed out that it could also come from the Sun rotation on its axis (hence, has a possible connection with the sunspots). The average period of this rotation is 26.5 days (it varies with solar latitude, since the Sun is not a rigid body). Allais chose one-month-long experiments in order to be able to efficiently discriminate such a component from the group of components very close to 24 h found in all known geophysical factors.

The precession of a pendulum (**Appendix B**) is the sum of a precession resulting directly from the action of the various perturbing causes and of a precession resulting indirectly from their action on the ovalization of the trajectory (onset of elliptic orbits). This ellipticity (axis ratio) causes in turn a nonlinear effect, the so-called Airy precession. This precession is all the more important as the pendulum is short and the angular amplitude important. Allais had deduced from preliminary experiments that the unknown perturbing actions he was looking for most probably acted mainly through ovalization. So, he decided to make them more evident by using a pendulum less than 1 m long (equivalent

[4]Van Flandern and Yang indicate that there are, due to temperature variations following an eclipse, large movements of air masses in the upper atmosphere, and that the sharp variation of pressure "can certainly create local air mass movement at ground level, for example, into or out of a building". In fact no more detailed analysis is provided.

simple-pendulum length of 0.83 m) launched at the angular amplitude of 0.11 rd. Airy precession overwhelmed Foucault's precession after a few minutes, so that he could claim that he was essentially measuring ellipticity variations through the precession variations.

• An overall analysis of the 6 experiments carried out with the Saint-Germain pendulum from 1954 to 1960, published only in 1997, in the "Anisotropy of space" ([12] or [13], Chap. V.A and V.B), showed important variations of the amplitude of the 24.84 h component, as well as besides of the average azimuth of the plan of oscillation of the pendulum. The fact that their values are almost exactly the same for the two pendulums of the 1958 experiment shows that these variations could only come from external factors, not from variations in the characteristics of the Saint-Germain pendulum from 1954 to 1960. One of them being apparently in a very stable environment, no classical external phenomenon can explain such long-term variations, in which Maurice Allais found a 5.9-year component. He made the hypothesis that these variations resulted from the action of the solar system as a whole (sunspots, planets[5], …).

• A new opportunity of investigating the eclipse effect occurred with the great total solar eclipse that crossed the northern hemisphere on August 11, 1999. On that occasion, NASA supervised coordination between different experimenters spread over the path of that eclipse. It is then that the phenomenon discovered by Allais during the eclipse of June 30, 1954 was named "Allais effect".

A number of experimenters have since endeavoured to research this eclipse effect, using pendulums and other devices [16]. It emerged from a number of observations, and in particular from observations carried out with several devices [17] [18] [19] [20], that marked anomalies did occur during certain eclipses. A number of other observers did not notice anything (for example [21]). These differences could result from the fact that each eclipse is a particular phenomenon.

Some experimenters observed a variation of the period on the occasion of certain eclipses (for example [22]). In fact, what has been observed was probably that the eclipse created a "linear anisotropy" (see **Appendix B.2**), with the consequence that the period of oscillation along the major axis, which is that which is measured, varied during the observation.

1.2. Objective of This Experiment: To Focus on the 24.84 h Periodic Component

Despite the interest of the regularities highlighted by Allais, which, contrary to the eclipse effect, seemed to be systematically observed, only one experiment tried to find them: the one carried out in 2006-2007 with an automated pendulum by a German private institute (Institut für Gravitationsforschung). The conclusion of the author was positive, but the method used for analyzing the data was quite questionable. The results were not published.

[5]5.9 years is close to half of Jupiter's sidereal revolution period and sunspot period.

All that finally led to the present experiment, which was centered on the search and analysis of the 24.84 h component. Two pendulums of approximately 6.4 m in length were used, with a large enough initial amplitude (0.07 rad), so that Airy precession remains significant. In order to favour the search for this component in a noisy environment, it seemed preferable to do as much as possible the same thing on both pendulums. Hence they both have been relaunched from the same azimuth during the whole experiment, contrary to Allais' pendulums, where the azimuth of each new launch was the final azimuth of the previous one.

A fundamental new element was the use of an automated alidade allowing, for every oscillation cycle, the measurement with great accuracy of all the parameters of the described ellipse, and in particular of its ellipticity. Therefore it was possible to separate the Airy precession from the total precession and, more generally, to make a more elaborated analysis of the motion of the pendulum.

Although the theoretical elements relating to the precession of a Foucault pendulum have been available for more than a century, they are very little known, the vast majority of scientists being only interested in the period of the pendulum, not in its precession. Therefore, as far as this work is concerned, the essential points have been summarized in **Appendix B**.

2. Observations Made at Horodnic

2.1. The Experimental Device

a) The pendulums and their installation (see **Appendix A** for more details).

For the purpose of this research, a special housing tower, which has been named Pendularium, has been erected as an annex to an existing building located in Horodnic de Jos, Romania (47.857˚N, 25.8267˚E). It consisted of a sort of one-storey bunker with concrete floor, cement-block walls, and with two brick chimneys over the roof apertures. On top of the bunker, the roof and the chimneys are protected by a two-storey wooden envelope. The two pendulums were used simultaneously. Their lengths were slightly different[6], so that their periods were not identical, in order to avoid resonances. The suspension was a ball suspension, the ball rolling on a plate, the horizontality of which had been checked to within 2".

A classic flaw of pendulums is indeed that, the elasticity of the support not being the same in all directions, the restoring force is also not the same, and that results in a tendency of the plane of oscillation to be called back toward the direction for which the restoring force is smaller. This ball suspension was adopted because it had been verified experimentally to be weakly anisotropic (see **Appendix B**). One problem with this type of suspension is, however, that, as the ball and the plate are not perfect, this may introduce noise into the motion of the pendulum and the measurements made.

[6]The simple-pendulum equivalent lengths were respectively 6.4162 m and 6.3888 m for Pendulum A and Pendulum B.

The bob was shaped like an assembly of two cemented spherical caps, in order to minimize the aerodynamic effects. It was connected to the suspension by a 1 mm stainless steel wire. The pendulum could be launched from any azimuth. An automatic alidade controlled by a PC made it possible, for each oscillation cycle, to determine the parameters of the described ellipse: major axis a, minor axis b, variation of the azimuth of the major axis since launch, coordinates of the center of the ellipse, oscillation period along the major axis. These parameters were calculated in delayed time from the interrupt timings of four light barriers by an axial pointer (lower stem) under the bob. The manipulation of the pendulum itself remained exclusively manual: positioning of the pendulum in the appropriate azimuth, switching the automatic alidade on, launching the pendulum by the burned-thread method, stopping and stabilizing the pendulum.

b) The environmental data recorded.

- From an external weather station: outdoor temperature, pressure and wind.

- From a set of sensors installed near Pendulum B: the humidity and temperature in the lower room, where the operator is located, and the temperatures at 5 points of the chimney in which the pendulum is installed (at 60 cm from the top, and at four intermediate points; to simplify the presentation, the values of these four intermediate points have been replaced by their average value). An identical set of sensors had been installed near Pendulum A, but it broke down during a thunderstorm. However, it has been possible to verify before that, at a given height, the temperatures of the two sets of sensors were very close.

2.2. Course of the Observations

They were carried out continuously from July 28, 2019 at 4 h UT to September 2, 2019 at 8 h UT. Every hour, the two pendulums were re-launched. This operation being manual, the two "simultaneous" launches were in fact slightly shifted (2 minutes). 869 launches per pendulum were made. The pendulums were both launched always from the same azimuth $\theta_L = 135^\circ$ (the azimuths are counted counter-clockwise from the north) and stopped after about 50 minutes. The initial amplitude of the oscillations was about 435 mm.

2.3. Need to Relaunch the Pendulum Frequently

Once the pendulum is launched, due to the pendulum's anisotropy (**Appendix B**), the ellipticity of the trajectory increases with time (hence a risk of leaving the domain of validity of the formulas used), and the amplitude decreases (hence a risk of leaving the operating domain of the alidade).

3. Data Processing

In this paper, we shall call a "run" every continuous observation of the pendulum from its launch to its stop.

3.1. There Are 2 Steps of Data Processing

a) The run.

For every pendulum oscillation cycle (1 cycle: about 5 s), parameters other than the above cited ones are calculated, in particular the ellipticity and the Airy precession rate. We therefore have, for each of these quantities, its evolution during the run, with the corresponding curves, as well as summary data for the entire run (average, standard deviation, …).

An example of curve is given by the graph in **Figure 1**.

b) The experiment considered as a whole: global use of the summary data of the different runs.

For a given quantity, there are therefore 869 values spaced 1 hour apart. For various reasons, some runs have proved unusable (11 runs for Pendulum A, 19 for Pendulum B). The corresponding summary data have been replaced by interpolated data, in order to maintain the regularity of the sampling for spectral analysis purposes.

3.2. The Quantities Studied Were

a) The value $\Delta\theta$ of the precession angle at the end of the run. In fact, we preferred to consider the average precession rate over the run $\overline{\theta'} = \Delta\theta/\Delta t$, Δt being the duration of the run. $\Delta\theta$ is directly given by the alidade measurements.

b) The average rate of Airy precession (see **Appendix B.1**) over the run, that is $\overline{\theta'}_{Airy}$. It corresponds to the part of the precession which results from the ellipticity. The rate of Airy precession is deduced from a and b according to the Airy formula; a and b are directly given by the alidade measurements.

c) The average rate of direct precession $\Delta\theta_{dir}$ over the run, that is $\overline{\theta'}_{dir}$. We have $\overline{\theta'}_{dir} = \overline{\theta'} - \overline{\theta'}_{Airy}$.

By subtracting the rate of Foucault precession (which is 0.003098 deg/s in Horodnic) we obtain the average rate of the "residual direct precession" (rdp), that is $\overline{rdp'}$.

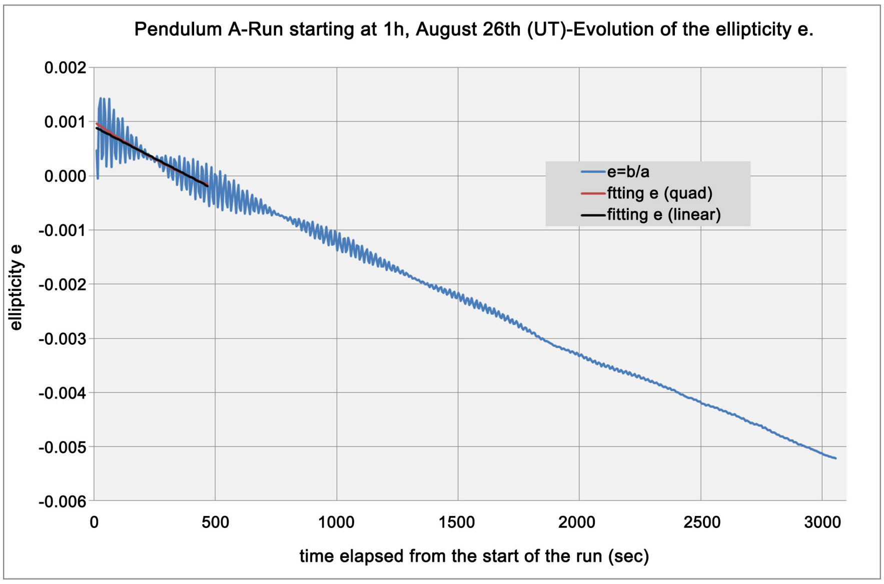

Figure 1. Example of detailed run analysis.

d) The average value over the run of ellipticity $e = a/b$, that is $\overline{e}$.

e) The average value over the run of the derivative of the ellipticity $\mathrm{d}e/\mathrm{d}t$, that is $\overline{e'}$.

This eliminates for the most part the influence of the initial ellipticity, that has an important perturbing action (see **Table 1**). When the thread is burnt, the pendulum is never exactly still: it is always oscillating a little, perpendicularly to the thread. Therefore there is always a little initial ellipticity of the trajectory of the pendulum (see for example **Figure 1**), which causes an Airy precession, that remains approximatively constant during the run (value θ'_{inite}).

3.3. The Quantity $\overline{e'}$ Is the Most Representative of the Action on the Precession of the Unknown Perturbing Actions We Are Looking For

Table 1 shows the standard deviations[7], over the 869 runs, of the average rates of the total precession and its two components, direct precession and Airy precession. Added to that is the standard deviation of the average rate of Airy precession resulting from the initial ellipticity. The latter was estimated, for every run, from fittings to the start of the curve of $e = b/a$ as a function of time[8] (see **Figure 1**).

Table 1 shows, first of all, that the variations of the precession rate come essentially from the variations of the rate of the Airy precession, that is to say from the variations of the ellipticity. It is therefore on the Airy precession or on the ellipticity that we must first focus to explain the variations of the precession. We verify that it is the same to study the variations of the rate of the Airy precession and those of the ellipticity: the correlation between $\overline{\theta'}_{Airy}$ and $\overline{e}$ is very high (Pendulum A: 0.9979; Pendulum B: 0.9981).

Table 1. Standard deviation of the average speed of total precession and of its components (deg/s).

Precession type	Symbol	Pendulum A	Pendulum B
Total precession	$\overline{\theta'}$	0.00017	0.00026
Airy precession	$\overline{\theta'}_{Airy}$	0.00016	0.00025
Direct precession	$\overline{rdp'}$	0.00003	0.00003
Airy precession from initial e	θ'_{inite}	0.00015	0.00018

[7]Since the objective of this study is spectral analysis, only the standard deviations are to be taken into account: the averages do not intervene. As an indication, the average value of $\overline{\theta'}$ over all the runs is close to the speed of Foucault's precession (0.003098 deg/s). The order of magnitude of the differences that we have to study is therefore 1/100th of the latter.

[8]The estimate taken into account is the average of the fittings by a linear function of time and a quadratic function of time.

The last line also shows the importance of the noise introduced on the ellipticity by the initial ellipticity, noise which is eliminated for the most part when we consider the derivative of the ellipticity.

The above justifies that, in order to search for possible unknown phenomena in the evolution of the precession of the Horodnic pendulums, we are primarily interested in the evolution of $\overline{e'}$.

4. Spectral Analysis

4.1. Special Interest of 24 h and 24.84 h Component

- They can only be the result of an astral action.

The spectral analysis of all geophysical factors reveals a line at 24 h, or more generally a group of lines very close to 24 h. These lines result either directly from Earth's rotation relative to the rest of the Universe in a sidereal day (23.98 h), or from the composition of this rotation with astral long term phenomena, the main one of them being the annual revolution of the Earth around the Sun (which gives the 24 h line), with possibly its first harmonics: 6 months, 4 months… An influence of the position of celestial bodies other than the Sun and the Moon, if it exists, affects only this group of lines: their angular velocities in an equatorial system being nil, or very small, only a range of few minutes around the 23.98 h sidereal period is concerned.

We also find, most of the time, close to 24.84 h, which is the average time interval between two successive passages of the Moon at the meridian, a group of lines corresponding to the composition of the rotation of the Earth with monthly astral phenomena. The only known monthly phenomena are the revolution of the Moon around the Earth in one synodic month and the rotation of the Sun.

It does not seem that, in a range of few hours around 24 h, what is the case of a 24 h and a 24.84 h line, there is any known phenomenon which would be a phenomenon only terrestrial, that is to say which would have nothing to do with the rotation of the Earth relative to the rest of the Universe: such phenomenon can only be the result of an astral action.

- The 36-day duration of the experiment makes it possible to discriminate unambiguously between the 2 groups of lines: around 24 h, the resolution is then about 40 minutes. It obviously does not make it possible to distinguish the lines within each group.

As it will be seen in Section 4.3 below, there is both a component of about 24 h and a component of about 24.84 h in $\overline{e'}$, as well in the locally recorded environmental factors. To investigate (Section 5.1) whether the component of about 24.84 h of $\overline{e'}$ can result from one of these factors, we compare the phases of this component and the amplitude ratios of the 24 h and 24.84 h components. It is absolutely essential that the comparisons be made at exactly the same frequencies.

For all the phenomena analyzed, we therefore sought the amplitude and the phase:

- Of the component of exactly 24 h, which we consider to be the one that takes into account all the lines of the first group.

- Of the component of exactly 24.84 h, which we consider to be the one that takes into account all the lines of the second group.

4.2. Use of a Band-Pass Filter

Since the discrete Fourier transform does not allow one to choose the frequencies to be analyzed, the spectral analysis was performed using a band pass filter. This filter is described in **Appendix C**.

The spectrum was scanned from the 5 h period to the 30 h period in 2 minutes increments. The closest period to 24.84 h is 24.8333 h, which was taken into account instead of 24.84 h.

4.3. Results

We saw in Section 3.3 that $\overline{e'}$ is the quantity whose study is the most relevant.

Figure 2 provides the amplitude spectrum of $\overline{e'}$.

We note that, for the two pendulums, there are indeed two very distinct groups of lines, one approximatively around 24 h, and the other approximatively around 24.84 h.

For information, **Appendix D** shows the amplitude spectra of $\overline{\theta'}$ and $\overline{\theta'}_{Airy}$. We find again the two groups of lines, but, at least for Pendulum A, they are less clearly detached from the background.

Figure 3 provides the amplitude spectra of the internal temperatures of the building. Note this time the large preponderance of the 24 h line.

The amplitude spectra of other environmental factors recorded by the sensors are found in **Appendix D**.

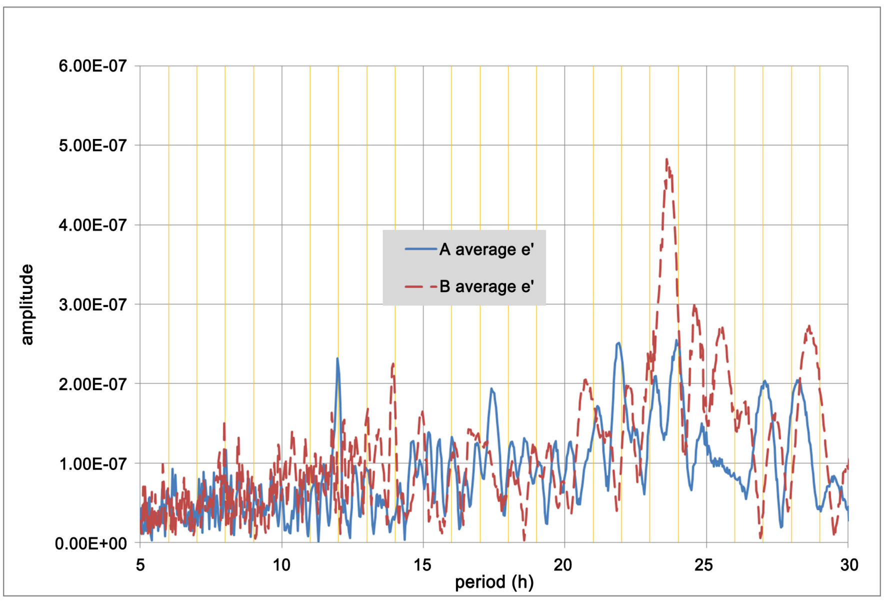

Figure 2. Spectrum of the average derivative of the ellipticity.

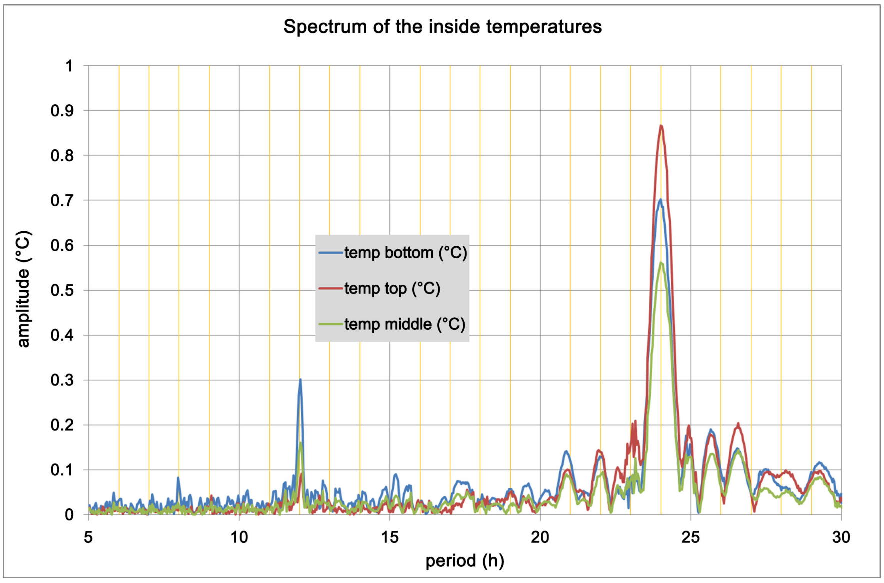

Figure 3. Spectrum of the indoor temperatures.

Table 2 provides, for $\overline{e'}$ and for the other quantities linked with the precession of the pendulum, as well as for the environmental data recorded by the sensors, the results of the spectral analysis for the 24.84 h component and the ratio of the amplitudes of the 24 h and 24.84 h components.

5. As Far as We Can Judge, No Classical Phenomenon Can Explain the 24.84 h Component, Given Its Amplitude and Phase

Remark: We can note also that the corrections to classical mechanics made by general relativity (order of magnitude 10^{-9} in relative value) are on the Earth much too small to be able to explain the amplitudes observed.

5.1. Temperature, Hygrometry, Pressure and Wind

They were recorded by the local sensors.

• **Preliminary remark:** If one of these factors was the main cause of the 24.84 h component of $\overline{e'}$, on the one hand the ratios of the amplitudes of the 24 h and 24.84 h component would be the same as for $\overline{e'}$, and on the other hand there would be phase concordance.

Indeed, the pendulum launched in a given azimuth is a measuring instrument which provides every hour, for every run, from the only data acquired on this run (the instrument is reset at every run), a measure of $\overline{\theta'}$, of $\overline{e'}$, etc.… For sinusoidal inputs of 24 h and 24.84 h, its transfer function is essentially the same.

Even if, what would be quite extraordinary, the assembly constituted by the pendulum and the building had a resonance period exactly at 24.84 h, this would have had absolutely no effect, since each measurement only takes into account data acquired on a little fraction of this period, and since the pendulum is restarted at every run.

• As shown in Table 2, for the derivative of ellipticity $\overline{e'}$, the amplitudes of the 24 h and 24.84 h components remain close (24 h/24.84h ratio = 1.67 for the Pendulum A, and 1.15 for the Pendulum B). Table 2 eliminates temperature[9], wind and humidity as possible causes of the 24.84 h component. This ratio is indeed significantly higher than 4 or more for temperature and wind, and is 3.53 for hygrometry. As regards the pressure (ratio 1.94), the phases do not match at all, which also eliminates it.

• Remark: It results from spectral analysis of pressure and wind speed that the 24.84 h component cannot be explained by air currents resulting from changes in atmospheric pressure at ground level, which was the explanation (in fact very highly purported in the case of the closed Allais' laboratory: see Section 1.1), of the eclipse effect given by Van Flandern and Yang [15].

Table 2. Analysis of the 24.84 h component.

Quantity	Unit	Amplitude	"Lunar hour" of the maximum value (see note)	Sun/Moon (ratio of the amplitudes 24 h/24.84h)
A rate of precession $\overline{\theta'}$	deg/s	2.00×10^{-5}	1.39	1.33
B rate of precession $\overline{\theta'}$	deg/s	4.11×10^{-5}	20.38	1.05
A rate of Airy precession $\overline{\theta'}_{Airy}$	deg/s	1.37×10^{-5}	0.45	2.10
B rate of Airy precession $\overline{\theta'}_{Airy}$	deg/s	4.01×10^{-5}	20.45	0.90
A ellipticity $\overline{e}$		1.78×10^{-4}	0.67	2.05
B ellipticity $\overline{e}$		4.89×10^{-4}	20.63	0.92
A derivative of ellipticity $\overline{e'}$		1.50×10^{-7}	2.00	1.67
B derivative of ellipticity $\overline{e'}$		2.49×10^{-7}	22.28	1.15
hygrometry	%	0.18	9.05	3.53
bottom temperature	˚C	0.18	20.78	3.99
top temperature	˚C	0.16	0.18	5.34
average middle temperature	˚C	0.13	22.49	4.19
outdoor temperature	˚C	1.12	15.48	6.01
wind speed	mph	0.67	12.15	4.07
pressure	mb	0.19	10.76	1.94

Note: "lunar hour" 0 corresponds to the average time of Moon's passage at the antimeridian.

[9]It is not surprizing. The temperature acts in the first order on the length of the pendulum, and therefore on the period of oscillation, but this has no effect in the first order on the precession. If it has a significant action on the precession, it can only be a very indirect action (deformation of the suspension resulting in a modification of its anisotropy, for example…).

5.2. Direct or Indirect Gravitational Action of the Celestial Bodies

a) **Direct gravitational action.**

The forces of attraction of the celestial bodies at a point on the Earth, often called "tidal forces", are very well known theoretically for a long time, and the consistency with what is observed by means of static devices such as gravimeters is excellent. Only the action of the Sun and Moon is not negligible. Calculating the influence of the lunisolar forces on the precession of a pendulum is therefore the first question that arose for Allais, who was the first one to have exploited its analysis as an investigative tool. He did this calculation in 1957, in the *Comptes rendus de l'Academie des Sciences* [9], and reminded it in 1958, in an article of *Aerospace Engeneering* [10] [11]. It is also reminded, with much more details, in [12] or [13] (Chap. IB2): see **Appendix B.5.c**. The result is that the influence on the precession, which is about six orders of magnitude smaller than the amplitude of the observed periodic component, can absolutely not explain them. In fact, this influence is completely undetectable over the duration of a run, and thus cannot, moreover, explain an eclipse effect.

Those conclusions remain utterly valid for the Horodnic pendulums, as it is verified in **Appendix B.5**, which shows also that the action on the pendulum of the tidal forces exerted by a celestial body is a "linear anisotropy", as this concept has been defined in **Appendix B.2**.

b) **Indirect gravitational actions: general information.**

These actions are those that result from the displacement of elements of the Earth under the effect of tidal forces: ocean tides, atmospheric tides, Earth tides.

• **Preliminary remark:** Influence on the precession of external accelerations acting on the bob, when they are very small compared to the acceleration of gravity *g*.

- The influence on the precession of a vertical component can be neglected.

It results in a change in the apparent value of *g*. As regards precession, when there is ellipticity, this results in a very small change (it is proportional to the ratio vertical acceleration/*g*) in the speed of Airy precession: see Formula (3)).

- Hence, it can be considered that only horizontal accelerations may act. They always deviate slightly the apparent vertical line given by the pendulum (tilt). This tilt, which modifies the angle between the apparent vertical given by the pendulum and the axis of rotation of the Earth, modifies the velocity of Foucault precession in the 1st order. This velocity is indeed given by the formula $\theta'_F = \Omega_T \sin(\lambda + \Delta\lambda)$, where Ω_T is the rotational speed of the Earth, λ the latitude, and $\Delta\lambda$ the value of the tilt, expressed in rd.

Hence, $\Delta\theta'_F = \Omega_T \cos(\lambda)\Delta\lambda$.

With $\lambda \approx 45$ deg, $\Delta\theta'_F \approx 2.95\times10^{-3}\Delta\lambda$ deg/s.

With for example $|\Delta\lambda| \le 10^{-4}$ rd, $|\Delta\theta'_F| \le 2.95\times10^{-7}$ deg/s, which is very small (and 10^2 times smaller than the amplitudes of the observed 24.84 h components: see **Table 2**).

Anyway, the Foucault precession, which is a direct precession, does not act on the ellipticity e, and therefore on its derivative e', which is the physical quantity we have preferred to study, instead of the precession itself.

The tilt has also an action in the 2nd order, which can only be negligible, on the apparent gravity exerted on the pendulum, and therefore on its pulsation, which intervenes in the Airy precession (Formula (3) in **Appendix B**).

- A constant horizontal acceleration has no other effect on the precession, and no effect on e and e' (the action on one half cycle is exactly cancelled by the effect on the next one).

- Only variations in tilt during the run might therefore have an effect. The calculation confirms[10] that, when the tilt, which is very small, moreover varies slowly (which is the case for diurnal variations), the effects on the precession and the ellipticity can only be absolutely negligible: it was verified that it is the case for a diurnal tilt the amplitude of which is $<10^{-4}$ rd.

• **Whether the attractive mass is a celestial body (see Appendix B.5.c) or an element linked to the Earth[11], the gravitational action acts by 2 ways on the precession of the pendulum:**

1) By the tilt due to direct attraction of the attractive mass: see above "Preliminary remark".

The main consequence is a variation of the velocity of Foucault precession proportional to the tilt, which has no effect on the ellipticity. The value of the tilt is:

$$Tilt\left(\mathrm{rd}\right)=\frac{GM\sin z}{gd^{2}} \quad (1)$$

where G is the gravitational constant, g the acceleration of gravity, d the distance of the attractive body from the point of rest of the pendulum, M its mass, and z its zenith distance.

We can summarize by saying that a diurnal tilt whose amplitude is $\leq 10^{-4}$ rd could not explain the observed precession, and anyway would have no perceptible effect on $\overline{e'}$. We can note that daily variations of the tilt of 10^{-4} rd, which are quite enormous[12], would be easily detectable by accelerometers.

2) By the variation in the restoring force of the pendulum towards its equilibrium position, due to variations in the force of attraction in the space swept by the pendulum at each oscillation. This perturbation acts in resonance with the pendulum oscillations.

If the lines of force of the gravitationnal field are parallel in this space, which

[10]The variation of the horizontal acceleration during the run results in a deviation of SG, G being the center of gravity of the pendulum and S its suspension point, and therefore in a rotation of the almost horizontal axis around which the pendulum swings. Hence a gyroscopic couple, etc.… Finally the analysis shows that it acts on the ellipticity, and therefore on the precession, which thus can be calculated knowing the law of variation of the horizontal acceleration. In fact only acts the component of this acceleration perpendicular to major axis of the ellipse. The calculation shows that only the second derivative of acceleration begins to act (in average over one cycle the acceleration and its primary derivative have no effect at all).

[11]The calculation for a celestial body was made by Maurice Allais, and reminded in **Appendix B.5.c**. For an attractive body linked to the Earth the formulas are not exactly the same.

[12]We find very well with accelerometers the tilt resulting directly from the tidal forces, which is at most 10^{-7} rd.

is the case with a sufficiently distant attractive body, one finds, by calculating the restoring force, that this creates a "linear anisotropy". This time, there is an action both on the precession and $\overline{e'}$.

In the end, the tilt being, as we will see, too small to have a non-negligible action, the variation of the restoring force in the space swept by the pendulum remains the only way of action of gravitational forces to be considered.

The direction of anisotropy, which is the direction of the anisotropy vector, is the direction of the attractive body, and we find that the coefficient of anisotropy, which is its modulous, is given by the formula:

$$\eta = \frac{GMl \sin^2 z}{2gd^3} \tag{2}$$

where l is the length of the pendulum (considered as a simple pendulum).

The important point is that η, which is inversely proportional to the cube of *d*, decreases very rapidly with *d*.

The order of magnitude of the measured anisotropies is 10^{-6} (see **Appendix B.5**): an anisotropy variation vector whose modulus η would be $<10^{-9}$ would have no noticeable effect.

c) **Gravitational action of ocean and atmospheric tides.**

Horodnic is more than 300 km away from the sea (Mediterranean sea). It follows from Formula (2) that, to obtain an anisotropy whose coefficient is 10^{-9}, it would be necessary to place at 300 km a mass of 1.33×10^{15} tons, which corresponds to 1.33×10^{15} m^3 of water. With a tide height of 2 m, this corresponds to an area of 6.65×10^8 km^2, what is more than the surface of the Earth: we are absolutely not in the necessary orders of magnitude…

The same is true of the influence of atmospheric density variations resulting from atmospheric tides[13].

d) **Gravitational action of Earth tides.**

• There results from lunisolar forces not only ocean tides and atmospheric tides, but diurnal deformations of the Earth's itself ("Earth tides"), which are not negligible at all (several tens of centimeters per day at the surface of the Earth). In principle, these deformations are vertical, which has effects on gravimeters, but no effect on the precession. Nevertheless it cannot be ruled out it might occur locally horizontal motions. For a horizontal deformation of 1 m peak to peak of period 24 h, the maximum variation of tilt would remain $<2.7 \times 10^{-10}$ rad, which can only have a negligible influence.

• However, we cannot definitively conclude with regard to the action of the Earth tides. For there to be a detectable action, it is necessary:

- That the horizontal attraction of the whole of the Earth on the pendulum is not zero, that is to say that there is an anisotropy in the environment of the place of observation. Hence a tilt, which can be measured, as well as a linear anisotro-

[13]In their analysis of the effect on the motion of a pendulum of the displacement of large air masses in the upper atmosphere, Van Flandern and Yang [15] also calculated the influence of the gravitational attraction of these masses to be totally negligible.

py of the pendulum, the coefficient of which can be calculated by Formula (2).

- That, moreover, due to tidal forces (deformation of the solid part of the Earth or variations in the density of the magma), this anisotropy varies sufficiently (module of the vector variation of anisotropy at least $>10^{-9}$).

Horodnic not being in a mountainous area, this seems unlikely, but due to lack of data no calculation could be made.

It should also be noted that a priori variations under the effect of tidal forces in the azimuth of the anisotropy of the pendulum environment cannot be large. This is not incompatible with the results of Horodnic, where the pendulums were always started in the same azimuth. But this could not explain the results of Allais ([12] or [13]), whose procedure (pendulum started from the final azimuth of the previous run) made it possible to follow the direction of the anisotropy. Over one month, the change could be greater than 90˚.

5.3. Variations of the Earth's Magnetic Field

- **Effect of these variations on the pendulum suspension wire (which is made of steel).**

The order of magnitude ΔB of the diurnal variations of the Earth's magnetic field is 200 nT peak to peak[14]. To have a very rough order of magnitude of the effect, we consider that the wire is a rigid magnet whose magnetic moment, which is vertical, is M. Hence a torque $M\Delta B$ relative to the point of suspension of the pendulum, which corresponds to an horizontal force $M\Delta B/L$ applied to the bob (L = 6.4 m is the length of the pendulum). If m is the mass of the bob (m = 12 kg), the maximum tilt over a day is $M\Delta B/(Lmg) = 2.7\times10^{-10}$ rad. Even if the value of M was 10^5 A/m^2, the influence on the precession would remain negligible (see Section 5.2.c, "Preliminary remark").

In addition, it was verified that the stainless steel from which the wire is made could only be very little magnetic: there is no perceptible attraction by a strong magnet. The influence of a possible magnetic action on the precession can only be negligible.

- **Effect of these variations on the eddy currents induced in the disc by the movement of the pendulum.**

This action is null, these currents changing direction with every half-oscillation.

5.4. Variations of the Electric Field

The contact between the mobile part, which is entirely metal, and the suspension, which is also metal, allows the mobile part to be grounded. It was besides verified experimentally that grounding had no detectable influence on the motion of the pendulum.

5.5. Eigenfrequencies of the Pendulum

Their periods remain much less than one hour[15]. The composition of the pen-

[14]We can find this order of magnitude by considering that the diurnal variation of the magnetic declination is about 0.2 degrees and that the value of the magnetic field is about 50 µT.

dulum's natural frequencies with a frequency of 24 h cannot create frequencies close to 24 h.

5.6. Earthquakes of 24.84 h Period

There might be earthquakes of near 24 h period: human activities have a 24 h period, and there is a possible link between lunisolar forces and seismic activity. But this has not been found in the individual analysis of each run (see Section 3.1.a). In such an analysis, the great sensitivity of the alidade makes it easy to identify possible earthquakes.

5.7. Other Conventional Perturbing Factors

There are a lot of other perturbing factors: the noise of the measuring device, the pendulum is never launched in exactly the same way, possible mechanical perturbations: (episodic earthquakes, natural or resulting from human activities, short-duration air currents due to motion of the experimenter…).

These factors are either specific to the run (the pendulum is restarted every hour in the same azimuth, which resets the measuring instrument it constitutes to zero), or short duration, and therefore uncorrelated, or very weakly correlated, from one run to the next (there are 10 minutes of rest between them). We can therefore consider that the overall influence of all these factors is uncorrelated from one run to the next one. Hence, for a given quantity, a random uncorrelated noise on the 869 values of its summary data.

Although such a noise has no periodic component, its spectral analysis at a given frequency always reveals a component, whose amplitude depends statistically on the importance of the noise. The probability that a fake 24.84 h component would result from that noise is calculated for $\overline{e'}$ in **Appendix C**.

The hypothesis that the 869 values of $\overline{e'}$ would be only noise is really very pessimistic. Indeed, if it was so, the spectrum would be flat. That is absolutely not the case, as shown in **Figure 4**. There is in particular a very marked diurnal influence (with even its Harmonics 2 and 3).

Nevertheless, with this hypothesis, this probability remains small for each of the two pendulums: 5.2×10^{-3} for Pendulum A, 1.4×10^{-2} for Pendulum B.

The true probability is certainly much lower. Thus, with the probably still pessimistic hypothesis that the noise resulting from other disturbing factors corresponds to 2/3 of the standard deviation of the 869 values of $\overline{e'}$, the values found are 7.8×10^{-6} for Pendulum A and 6×10^{-5} for the Pendulum B.

6. Does the 24.84 h Component Result from a Lunar Action or from an Action of the Rotation of the Sun in about 1 Month?

This question was asked by Section 4.1.

[15]In their analysis of the effect on the motion of a pendulum of the displacement of large air masses in the upper atmosphere , Van Flandern and Yang [15] also calculated the influence of the gravitational attraction of these masses to be totally negligible.

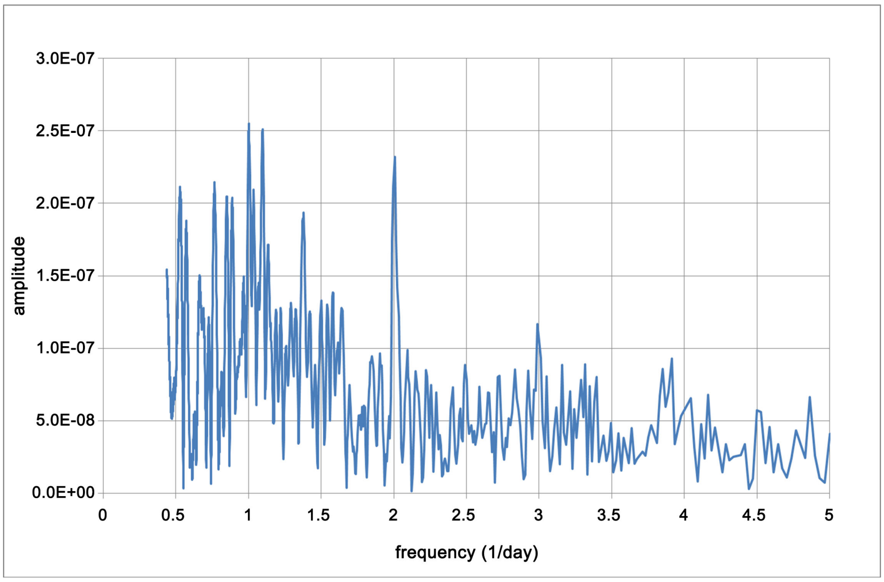

Figure 4. Pendulum A: spectrum of the average derivative of the ellipticity.

Table 2 shows that, for $\overline{e'}$, the average on the 2 pendulums of the lunar hours of the maximum is 0.14 h, which is very close to the time of the passage of the Moon at the antimeridian of Horodnic. Assuredly, this goes in the direction of an action of the Moon. Nevertheless, other observations would be necessary to conclude with certainty.

7. Comments on the Previous Results

7.1. About the Differences between the Two Pendulums

The amplitudes of the 24 h and 24.84 h components are much smaller for Pendulum A. This does not call into question the above results concerning the 24.84 h component of $\overline{e'}$, since they were established for each pendulum considered alone.

The explanation of these differences is therefore to be found in the fact that the pendulums are not identical:

- They each have their own noise, which can have a significant impact on both amplitude and phase.

- As explained in **Appendix B**, $\overline{e'}$ depends mainly on the total anisotropy of each pendulum, which is the vector composition of its intrinsic anisotropy, a priori fixed, and specific to it, and of an external variable anisotropy, a priori the same for both pendulums. All these anisotropies are very small (the measured anisotropy coefficients are in the range of 10^{-6}), and the external anisotropy, which is the one we are interested in, is certainly not large compared to the intrinsic anisotropies. The external anisotropy can therefore act in quite a different way on the two pendulums.

7.2. About Other Periodic Components

They are not at all involved in the analysis carried out in Section 5. Nevertheless the detailed study of the entire spectrum could certainly provide some interesting additional information. But this study, which moreover cannot be simple, would deviate from the main objective of this article.

We will simply note, at period 25.67 h, the existence of a peak for Pendulum B, to which corresponds a not very clear, but nevertheless quite probable, maximum for Pendulum A. This period is close to 25.82 h, period which results from the composition of the rotation of the Earth in 24 h with the harmonic 2 of the synodic month, the 24.84 h period resulting from the composition with the synodic month (29.53 day). We can therefore think that the 25.67 h component originates also from the composition of the Earth's rotation with a monthly phenomenon, as the 24.84 h component. The fact that the amplitude is also lower for Pendulum A also goes in this direction.

7.3. New Phenomena That Would Be Only Minor Modifications of Classical Gravitation (Such as the Existence of a "Screening Effect") Could Not Explain the Observed Precession Anomalies

We have seen (see Section 5.2) that the gravitational attraction of a celestial body could not have any detectable action on the precession of a Pendulum. A "screening effect" (attenuation of gravity when a body interposes itself more or less completely between 2 bodies[16]), which would be only a very minor adjustment of the classical laws of gravitation, therefore could not have any detectable action (although, if it exists, it could explain what gravimeters observed during certain eclipses).

7.4. What Is the Mechanism by Which the Unknown Action Acts on the Precession of the Pendulum?

a) As we saw above, it acts mainly through an action on the ellipticity (as in the case of Allais's pendulum).

b) It can be explained in a major part by the hypothesis of a restoring force towards the equilibrium point of the pendulum varying with the azimuth of the plane of oscillation.

The analysis of the total ellipticity (**Appendix B.5.b**) shows indeed that it can be explained in a major part, once the initial ellipticity has been deduced, by a restoring force varying with the azimuth of the plane of oscillation as a sinusoidal function whose period is 180˚ ("linear anisotropy": see **Appendix B.2**). It can reasonably be assumed that this is also true for the 24.84 h component.

Hence, the above analysis provides indications that may be very useful in the search for the nature of the unknown action. For example, if the hypothesis is that it is a force field, this field mainly intervenes through its variations in the

[16]A screening effect is one of the explanations of the Allais effect that have been considered (see for example [15], Section I).

[17]Its average value in this space has no other effect than very slightly deviating the apparent vertical line given by the pendulum at rest (tilt), which can only have negligible effects on the precession: see Section 5.2.b) Indirect gravitational action, Preliminary remarks.

space swept by the pendulum[17], which results in a restoring force varying with the azimuth of the plane of oscillation. The comparison with the experimental results of the effects calculated from the characteristics of the field then makes it possible to check the validity of that hypothesis.

Mathematically, the anisotropy of the restoring force can also be explained by an anisotropy of the medium in which the pendulum oscillates. The unknown action then acts on the pendulum not directly, by a force, but indirectly, by creating this anisotropy. That is the hypothesis proposed by Allais[18].

7.5. About the Ratio of the Amplitudes of the 24 h and 24.84 h Components

Allais pointed out [9] that, for all known geophysical phenomena (except for the tidal forces which, as we saw, only have a negligible effect on the precession of the pendulum), the amplitude of the component of about 24.84 h is significantly smaller than that of the 24 h component: the fact that these amplitudes are close in all his observations ([12] or [13] p. 92) is therefore in itself remarkable. We also find in Horodnic this peculiarity. The average value of the ratio 24.84 h/24h for the two pendulums is 0.71. Over Allais' six experiments, this ratio remains between 0.54 and 2.71, its average value being 1.39.

8. Conclusions

• The observation of the precession of two pendulums carried out in Horodnic continuously from 28 July 2019 to 2 September 2019 revealed, in addition to a 24 h line, a 24 h 50 min line. Over one month, we can distinguish these two lines without ambiguity, but we cannot go any further. In fact, they are to be considered as grouping all the lines very close to each of these two periods.

These periods can only result from an astral action, by the composition of the rotation of the Earth relative to the Universe with, as regards to the 24 h 50 min period, astral phenomena whose period is about 1 month, and, as regards to the 24 h period, with astral phenomena whose period is much longer (annual, semi-annual, …). It does not seem, indeed, that there is any known natural phenomenon whose period is in the range of 24 h - 25 h, and which would have nothing to do with the rotation of the Earth.

In the case of the 24 h 50 min line, there are only two known phenomena whose period is about 1 month: the revolution of the Moon around the Earth in one synodic month, and the rotation of the Sun. The fact that the maximum is close to the passage of the Moon to the antimeridian points to an action of the Moon.

• The astral action which results in the 24 h 50 min line (whether it is an action of the Moon or any other celestial body) apparently cannot be done through

[18]See Allais ([12] or [13], Section I.F.3). Mathematically, everything happens as if the mass of inertia varied with the azimuth of the plane of oscillation of the pendulum. Obviously, this does not mean that it is indeed the mass of inertia of the pendulum that has been changed. However, this explains the hypothesis of an "anisotropy of inertial space" proposed by Allais.

a classical mechanism. Indeed we have excluded that it could result:

a) Of classical gravitational action, direct (direct action of tidal forces) or indirect, i.e. through the displacement of elements of the Earth resulting from tidal forces: ocean tides, atmospheric tides and Earth tides.

Indeed, the precession of the pendulum being, unlike gravimeters and inclinometers, very insensitive to small and slowly varying accelerations (especially with the procedure used, in which the pendulum is frequently stopped and restarted), the calculation shows that it can only result in effects at least 100 000 times smaller than the observed ones.

Note, however, that, although it looks quite unlikely, it is impossible to conclude definitively with regard to the action of the Earth's tides, because we know little about what is happening inside the Earth: this does not allow calculations to be made.

b) Of astral influence on known environmental factors (temperature, pressure, hygrometry, influence of the wind on the structure, terrestrial magnetic and electric fields). Indeed, either their influence on the pendulum can only be negligible (magnetic and electric fields), or they cannot explain the amplitude and/or phase of the 24 h 50 min component observed (analysis of data recorded by local sensors).

If there had been earthquakes linked to the diurnal rhythm of human activity and of tidal forces, this would have appeared in the detailed analysis of each run.

• A probability calculation showed that the 24 h 50 min component could not result from random disturbances affecting each run: noise from the measuring devices, microearthquakes of natural or human origin not linked to diurnal activities and tidal forces, the pendulum is never launched exactly in the same way…

• With regard to non-classical mechanisms, we can note:

- That the corrections to classical mechanics resulting from general relativity, which are on Earth in relative value $<10^{-9}$, are far too small to explain the observed lines.

- Nor can these lines be explained (as the "eclipse effect" on the precession of a pendulum) by a gravitational screen phenomenon (since precession is very insensitive to tidal forces).

• With regard to the mode of action of the unknown action on the pendulum, the automatic alidade provided much more precise and numerous information than those obtained by Allais, which may be very useful in finding its nature.

This made it possible to confirm with certainty that the unknown action was, at least, for the most part, the result of an action on the ellipticity. Therefore it was preferred to study in depth the ellipticity, and more precisely the derivative of ellipticity, rather than precession itself.

It also showed, as Allais had hypothesized, that this action on the ellipticity was mainly due to the fact that the restoring force to the equilibrium position varied slightly, under the effect of the unknown action, with the azimuth of the plane of oscillation. This is mathematically compatible with the hypothesis of the existence of anisotropy of inertial space suggested by Allais.

• Today, there is therefore no explanation of the experimental results of Horodnic within the framework of the current theories, and the same is still true of those of Allais. All that confirms all the scientific interest that there would be to resume long-duration experiments on a very much more important scale than what has been done in August 2019:

a) This requires complete automation of the pendulum, while keeping the absolutely essential requirement that it be frequently and regularly relaunched.

b) It should be aimed that the observations can be spread out over many years. As it was reminded in the introduction, what the analysis of the observations of Maurice Allais, which were spread over 6 years, tells us that, very possibly, there is in the variations of the precession of the pendulum an important influence of the solar system as a whole (planets, sunspots…). A spread of the observations over two years is a minimum. This should already yield very interesting data.

c) It is obvious that it would be better to have several pendulums.

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Authors' Contribution

Thomas Goodey: Co-instigator of the project, main organizer of experiments, co-designer of the laboratory enclosing the pendulums, co-designer of the pendulums, co-designer of alidades, co-developer of interpretation software, co-responsible for data acquisition and contributor to the writing of the article.

Dimitrie Olenici: Co-instigator of the project, co-organizer of experiments, the main designer of the laboratory, co-designer of pendulums, co-designer of alidades and co-responsible for data acquisition.

Jean-Bernard Deloly: Co-instigator of the project, co-organizer of experiments, co-developer of interpretation software, responsible for the writing of the article, responsible for spectral analysis and the overall exploitation of data.

René Verreault: Co-designer of alidades, co-developer of interpretation software, contributor to data analysis and editing reviewer.

Data Accessibility

Experimental data and specific software can be accessed via the link: https://borealisdata.ca/dataverse/reneverreault.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendices

Appendice A: The Pendulums

A dedicated 6.1-meter wide, 2.3-meter deep and 8-meter high rectangular tower, incidentally called Pendularium, has been erected to host the experiment. A central cross-section of the building is shown in Figure A1.

Figure A2 shows some construction details of each of the two pendulum implementations. The bob is made out of brass. The diameter of the stainless steel wire is 1 mm.

Figure A2(a): Oblique view of the paraconical suspension. The suspension mechanism consists of a ring-shaped stirrup solidary of a 6-mm diameter hardened-steel ball that rolls on a horizontal optically polished hardened-steel flat plate. The horizontality of this support plate has been checked to ±2" of arc.

Figure A2(b): Bob-stem assembly. Lateral view of the Olenici-style lenticular bob with its upper and lower stems. The rationale behind that shape aims at minimizing aerodynamic drag for a given mass/volume ratio. The sharp edge appears to be the best compromise for to and fro motion along any direction. It has been verified by test runs that the Q-factor of that pendulum type at 0.07 rd amplitude

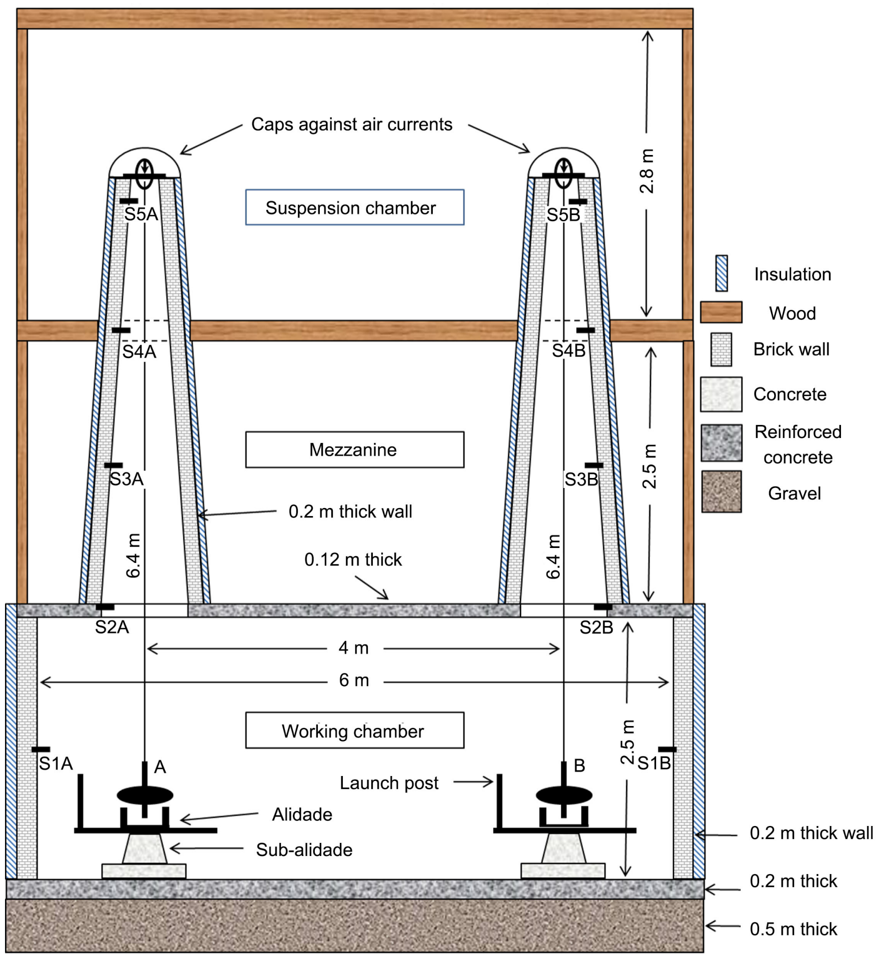

Figure A1. Central section through the longest building dimension.

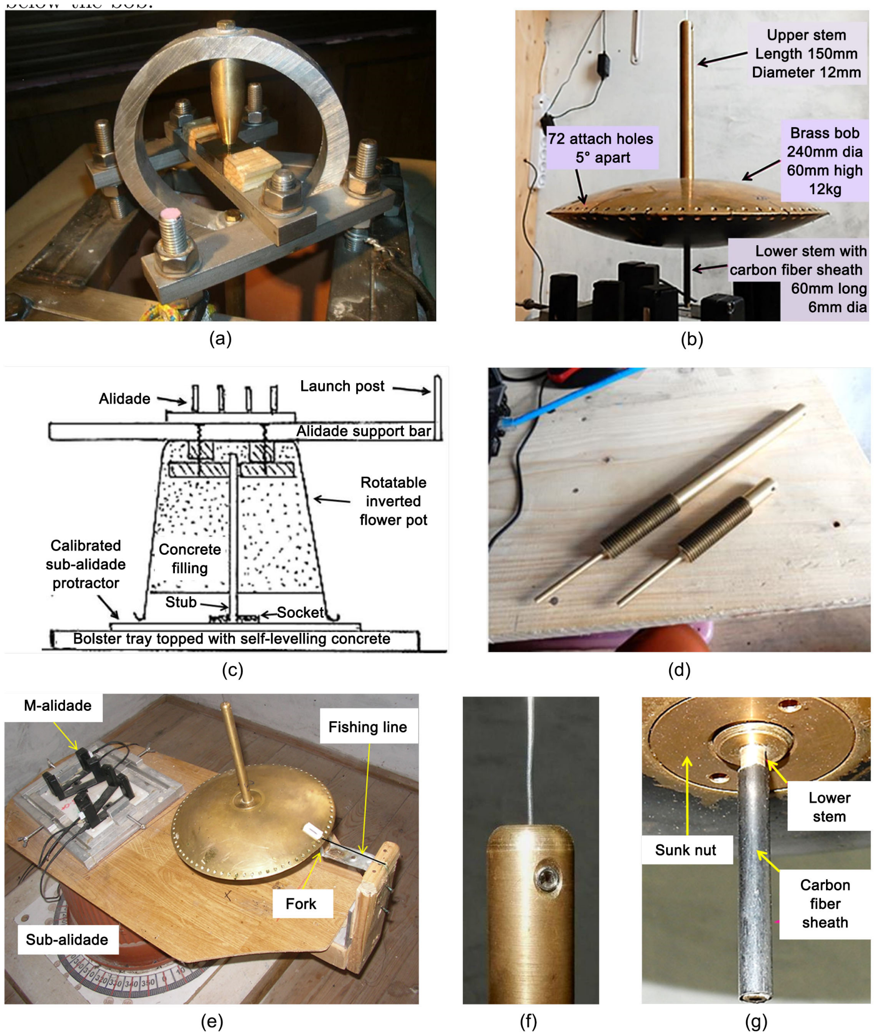

Figure A2. Construction details of a pendulum. (a) Oblique view of the paraconical suspension. (b) Bob-stem assembly. (c) Section of the alidade and sub-alidade system. (d) Typical upper-stem-lower-stem rods. (e) Oblique view of the alidade and sub-alidade showing the bob attached against a stabilizing fork at the launch post. (f) Detail of the stem-wire junction. (g) Detail of lower-stem pointer with flush sunk nut below the bob.

lies around 11,000. The 72 holes along the rim allow the pendulum to be launched from any azimuth with not more than ±2.5 degrees of twist in the suspension line.

Figure A2(c): Section of the alidade and sub-alidade system. In order to be able to launch the pendulum from any prescribed azimuth, the launching post and the alidade need to be rotatable through 360 degrees around the vertical (Z-axis). That role is fulfilled by a heavy and stable sub-alidade which rotates about a pivot anchored into a massive horizontal base. The azimuth of the sub-alidade is measured along a calibrated protractor disk solidary of the horizontal base, which consists of a 100 kg wooden bolster tray that has been topped with self-leveling concrete. Fine centering of the bolster tray under the pendulum's rest point is also enabled.

Figure A2(d), **Figure A2(f)** and **Figure A2(g)**: They illustrate various features of the stem that materializes the bob axis. The upper/lower stem unit is attached to the suspension line with two opposite set screws. It is inserted into an axial bore of the bob and held in place by two countersunk nuts that are tightened flush with the upper and lower bob surfaces, in order to minimize drag during bob motion.

Figure A2(e): Oblique view of the alidade and sub-alidade showing the bob attached at the launch post. The bob is attached against the fork, which on the one hand enables a constant launch amplitude, and on the other hand helps to reduce the lateral oscillations of the pendulum. The thread is a 0.2 mm diameter braided nylon fishing line.

Appendice B: Some Theoretical Elements about Pendulum

If the period of a pendulum has been a classic investigative tool for several centuries, it is not at all the same for its precession which, apart from Foucault's precession, is very little known. In addition, the authors who have studied it theoretically have used different notations, and sometimes different coordinate axes.

We have therefore gathered here a number of essential elements for its analysis, using the same notations and coordinate axes as in the article.

We have also explicitly introduced the notion of "anisotropy" and more particularly of "linear anisotropy", which is a disturbance model making it possible to account for the effective action on the pendulum of a lot of disturbing phenomena.

The formulas used are approximate formulas, which are valid only for very small ellipticities, which is possible only if the pendulum is regularly stopped and restarted.

Appendice B.1: Two Kinds of Precession May Result from a Given Perturbing Action, in the More General Case

They are:

- the "direct" precession, which results from a direct action of the perturbing action on the trajectory,

- the Airy precession, which results from an indirect action: the perturbation acts on the ovalization and, as the pendulum is a spherical pendulum, the ovalization makes it precess.

The rate of that precession is given by Airy's formula [23] [24] [25]:

$$\theta' = \frac{3}{8}\alpha\beta\sqrt{\frac{g}{l}} \tag{3}$$

where g is the acceleration of gravity, l the length of the equivalent simple pendulum, and α and β the angular half major axis[19] and the half minor angular axis of the ellipse (α and $\beta \ll 1$). We can observe that the Airy pre-

[19]Angular half major axis = half major axis/physical length of the pendulum.

cession is all the more important as α is large, and l small.

Whether you have a direct action or an indirect action depends on the phase of the perturbation [25]. For example, if you have a lateral impulsion exactly when the pendulum is at the extremity of its trajectory (or, more generally, in phase with the oscillation), you have only ovalisation. If you have a lateral impulsion exactly when the pendulum is at the middle of its trajectory (or at 90 degrees with the oscillation), you have only direct precession. The Coriolis force induces only a direct precession, which is the well-known Foucault precession.

Formula (3) is only an approximate formula. It was verified, by numerical integration, that for Horodnic pendulums, and an ellipticity of less than 0.01 (which is significantly higher than the measured ellipticities, which are of the order of 0.001), the difference with Formula (3) was only a few per thousand.

Appendice B.2: Anisotropy of a Pendulum; Notion of "Linear Anisotropy"

As a general rule, the behavior of the pendulum is anisotropic: its trajectory depends on the launch azimuth. This results from the fact that the symmetry of the device is never perfectly revolutionary, on the one hand, and on the other hand that many external perturbing actions are directional.

Indeed a number of them make that the restoring force, which calls back the pendulum to its equilibrium position, is not exactly the same in 2 perpendicular directions: hence a disturbance that acts in resonance with the oscillations of the pendulum. In particular this is the case when the elasticity of the suspension varies with the azimuth of the plane of oscillation, which is a classic flaw of the pendulums [26]. It is also the case when the pendulum is in a field of forces which varies in the space swept during each oscillation.

Hence, the interest of studying this kind of anisotropy. It is the simplest model (called "linear anisotropy"[20]) that has been considered:

- For a given azimuth the restoring force is proportional to the distance of the gravity center of the pendulum from the equilibrium point. The restoring coefficient is a function of the azimuth, which is periodic, with a period of 180˚.
- This 180˚ periodic function is a sinusoid.
- We consider that the oscillations remain small, which makes it possible to assimilate the movement of the pendulum to that of a plane oscillator.

The movement of the projection of the center of gravity of the pendulum on a straight line Ox of given azimuth is then a solution of the differential equation $x'' + kx = 0$, k being the restoring coefficient. The solutions are sinusoids of pulsation $\omega = \sqrt{k}$, and therefore of period $T = 2\pi/\omega$.

Considering that the variations in the restoring coefficient are very small in relative value, we have, for the azimuth θ, as regards the period and the restoring coefficient:

$$T = T_0\left[1 + \eta\cos 2\left(\theta - \theta_A\right)\right], \tag{4}$$

[20]This designation results from the fact that an analogy can be drawn between this kind of anisotropy affecting the behaviour of a plane oscillator and the notion of "linear polarization" of a field perpendicular to its propagation direction [26].

$$k = k_0\left[1 - 2\eta\cos 2\left(\theta - \theta_A\right)\right], \tag{5}$$

where $\eta > 0$ and $\ll 1$. The period is maximum and the restoring force minimum in the direction θ_A.

η is called the "coefficient of anisotropy", and θ_A the "direction of anisotropy".

We can demonstrate[21] that it results from Equation (4) or Equation (5), with the additional hypothesis that the ellipticity $e \ll 1$:

a) A direct precession, the rate of which is θ'_d:

$$\theta'_d = 2\eta\omega\frac{\beta}{\alpha}\cos 2\left(\theta - \theta_A\right) = 2\eta\omega e\cos 2\left(\theta - \theta_A\right), \tag{6}$$

with $\omega = 2\frac{\pi}{T} = \sqrt{\frac{g}{l}}$. Hence, the direct action calls back the major axis towards the azimuth $\theta_A + \frac{\pi}{2}$.

b) An ovalization $e = \beta/\alpha$ of the trajectory, which is such that:

$$e' = \mathrm{d}\left(\frac{\beta}{\alpha}\right)\Big/\mathrm{d}t = -\eta\omega\sin 2\left(\theta - \theta_A\right). \tag{7}$$

Hence, we can consider that α remains constant, as the pendulum is frequently relaunched:

$$e(t) = -\eta\omega\int_0^t \sin 2\left(\theta - \theta_A\right)\mathrm{d}t + e_L = -t\eta\omega\overline{\sin 2\left(\theta - \theta_A\right)} + e_L, \tag{8}$$

where $\overline{\sin 2\left(\theta - \theta_A\right)}$ = average value of $\sin 2\left(\theta - \theta_A\right)$ on $[0, t]$, and e_L is the initial ellipticity.

Hence, from Equations (3) and (8), the speed θ'_i of the Airy precession:

$$\theta'_i = \frac{3}{8}\omega\alpha^2 e = -\frac{3}{8}t\eta\omega^2\alpha^2\overline{\sin 2\left(\theta - \theta_A\right)} + \frac{3}{8}\omega\alpha^2 e_L. \tag{9}$$

Therefore, the indirect effect tends to call back the plane of oscillation towards the direction of anisotropy θ_A.

The indirect effect is predominant. The ratio between the direct effect and the indirect effect can be deducted from Equations (6) and (9):

$$\frac{\theta'_i}{\theta'_d} = \frac{3\alpha^2}{16\eta\cos 2\left(\theta - \theta_A\right)} > \frac{3\alpha^2}{16\eta} \tag{10}$$

The value of α to be taken into account is its average value over a run.

Hence, the indirect effect is dominant if the coefficient of anisotropy η remains very small.

In the case of the Horodnic pendulums, $\alpha = 0.058\ \mathrm{rd}$, $\eta < 5\times 10^{-6}$. Hence $\frac{\theta'_i}{\theta'_d} > 126$.

In the case of Allais' pendulums[22], $\alpha = 0.1\ \mathrm{rd}$, η is about 10^{-5}. Hence

[21]See, with different notations, [25] p. 83 (with different coordinate axes), or [24].

[22]The value of η can be calculated from the data provided in [12] or [13], E.3 pp. 176-182

$$\frac{\theta'_i}{\theta'_d} > 180 .$$

Appendice B.3: Composition of Several "Linear Anisotropies"

All these perturbations are very small: we are in the linear area. Thus, if we consider for example the period (Equation (4)), and 2 linear anisotropies, we have:

$$T = T_0\left[1+\eta_1\cos 2\left(\theta-\theta_{A_1}\right)+\eta_2\cos 2\left(\theta-\theta_{A_2}\right)\right] = T_0\left[1+\eta_{tot}\cos 2\left(\theta-\theta_{A_{tot}}\right)\right] \quad (11)$$

Indeed, the sum of two sinusoids of period 180˚ is a sinusoid of period 180˚. The composition of several linear anisotropies is therefore also a linear anisotropy.

If we define an "anisotropy vector", the modulus of which is η and the argument $2\theta_A$, it is easy to verify that the anisotropy vector of the composition of several anisotropies is the sum of the anisotropy vectors.

Appendice B.4: How to Measure the "Linear Anisotropy" of a Pendulum?

The total anisotropy comes on one hand from the pendulum itself (its "intrinsic anisotropy", which is a priori constant) and on the other hand from external perturbing actions (the "external anisotropy", which is always varying). We can only measure the total anisotropy. Two methods can be used:

a) The "round the clock" method: n runs at azimuths spaced by 180 deg/n.

For example, we make 18 runs from the azimuths 0, 10, …, 170, what gives the period T_L and the derivative of the ellipticity e'_L at the launching instant t_L (the runs don't need to be long: runs of about 10 min every 15 minutes, for example). Then we can deduce η and θ_A from the period T_L and Formula (4). But the use of T_L is delicate, because it is necessary to take in account that T_L is very sensitive to the temperature. Therefore it is easier to use e'_L and Formula (7), but that supposes that a "linear anisotropy" is the only cause of e'.

b) With the hypothesis that a linear anisotropy is the only cause of e', use of Formula (7) with one long run (about 50 min or more).

During the run the Foucault effect makes the pendulum precess (about 10 degrees during 50 minutes). The fitting of e' as a function of θ by a sinusoid with a period of 180 degrees gives an approximate estimation of η and θ_A. But when the coefficient of anisotropy η is very small ($<5 \times 10^{-6}$, to fix the ideas), the fitting is very much disrupted by the noise affecting e'. Moreover, the quality of the fitting depends on the value of ($\theta-\theta_A$). In fact, when η is very small, the interest of this method is mainly to verify that η remains very small, but not at all to calculate its precise value and the precise value of θ_A during a given run.

Appendice B.5: Case of the Horodnic Pendulums

a) Anisotropy of the pendulums.

For each pendulum a "round the clock" experiment, during which the orientation of the suspension had been modified, had shown that the participation to the intrinsic anisotropy of the pendulum of the building, on which the suspension is fixed, was not detectable.

Since May 2018 the two pendulums have been equipped with the current suspension. It was verified, both with a "round the clock" experiment and the exploitation of several hundred runs with the method b), which the average value of η remained very small: about 2.5×10^{-6}, almost all the values remaining under 5×10^{-6}. In that value of total anisotropy, it was not possible to distinguish between what comes from the intrinsic anisotropy, what comes from the external anisotropy, and what comes from noise. But that means with certainty that the intrinsic anisotropies of the two pendulums were very small.

Nevertheless each pendulum has certainly its own intrinsic anisotropy, with its own intrinsic η and its own intrinsic θ_A. Therefore a same external anisotropy may act differently on both pendulums.

b) A "linear anisotropy" can explain most of the ellipticity, once the initial ellipticity has been deduced.

As seen above, with the hypothesis that the changes in the ellipticity *e* during a run results only from a linear anisotropy, an estimate of η and θ_A can be made for every run. From these values, Formula (8), with taking into account the measure of the initial ellipticity e_L, gives an estimate of the evolution of *e* during the run. The difference between the measured e and its estimate is the part of *e* which cannot be explained by the initial ellipticity and a linear anisotropy. Table A1 shows that this residual ellipticity is small compared to the total ellipticity after deducing the initial ellipticity. This hypothesis is therefore reasonable.

c) Influence of the lunisolar forces on the precession and on the derivative of the ellipticity $\overline{e'}$.

Allais calculated ([12] or [13], Chap. IB2) the action of lunisolar forces on the precession of a pendulum. It emerges from Equation (3) of Table V in THIA Chap. IB2, p. 120, that the gravitational action on the pendulum of a celestial body i is the sum of 2 terms[23]:

Table A1. Analysis of the ellipticity.

	Pendulum A	Pendulum B
average ratio (quadratic average over every run of the residual ellipticity /quadratic average over every run of the ellipticity after deducing the initial ellipticity)	0.27	0.11

[23]Equation (3) gives the expression for the absolute acceleration γ of the center of gravity G of the pendulum as a function of the acceleration of G with respect to the Earth, of the Coriolis acceleration, and of the entrainment acceleration of the Earth's center. An essential condition is that we have:

$$grad_G U_i - grad_T U_i = \left(grad_S U_i - grad_T U_i\right) + \left(grad_G U_i - grad_S U_i\right)$$

The notation $grad_G U_i$ means that the gradient of the gravitational potential of the celestial body i is considered at the point G (S being the point of suspension of the pendulum, and T the center of the Earth).

1) The action of the deviation from the vertical (tilt) resulting from the tidal force created by the celestial body. What Allais writes ("it has no influence") is not completely exact: that tilt, whose maximum value is $<10^{-7}$ rad, has an influence which is extremely small, but can be calculated (see Section 5.2.b, "Preliminary remark"). The main effect of the tilt is a very small change in the velocity of Foucault precession. This velocity is indeed given by the formula $\theta'_F = \Omega_T \sin(\lambda + \Delta\lambda)$, where Ω_T is the rotational speed of the Earth, λ the latitude, and $\Delta\lambda$ the value of the tilt, expressed in rd. Hence $\Delta\theta'_F = \Omega_T \cos(\lambda)\Delta\lambda$.

Hence, with $\lambda \approx 45$ deg and $|\Delta\lambda| < 10^{-7}$, $|\Delta\theta_F| < 2.95\times10^{-10}$ deg/s, which is 10^5 much smaller than the amplitudes of the observed 24.84 h components (see **Table 2** in Section 4.3).

Anyway, the Foucault's precession, which is a direct precession, does not act on the ellipticity e, and therefore on its derivative e', which is the physical quantity we have preferred to study, instead of the precession.

2) The difference between the tidal force created by the celestial body at the center of gravity of the pendulum and at its point of suspension. The action of this difference was calculated by Allais[24], the result being the differential Equations (3) and (4) in Table VI, p. 128 of "The Anisotropy of Space" ([12] or [13]). These equations reveal a restoring force which depends sinusoidally on the azimuth modulo 180 deg: it is therefore a "linear anisotropy"[25]. Formula (7) of Table VI provides, with Allais notations, the derivative of the minor axis:

$$\beta' = \frac{\alpha}{2p} K_i \sin^2(A_i - \Phi) \tag{12}$$

The correspondence with the notations of this article is as follows: $\alpha = \alpha$; $\beta = \beta$; $p = \omega$; $A_i = \theta_{Ai}$ ($\theta_{Ai} = \theta_A$ for celestial body i); $\Phi = \theta$.

According Formula (5) of Table VI,

$$K_i = \frac{3}{2}\sin 2z_i C_i \tag{13}$$

where z_i is the zenith distance of the celestial body i, and C_i is a coefficient associated with it.

Hence $|K_i| \le \frac{3}{2}C_i$

If we consider that α remains constant during the run, we have, from (12):

$$e' = \frac{\beta'}{\alpha} = \frac{K_i}{2\omega}\sin 2(\theta - \theta_{Ai}) \tag{14}$$

Hence,

$$|e'| \le \frac{K_i}{2\omega} \le \frac{3C_i}{4\omega} \tag{15}$$

[24]These calculations have been verified.

[25]More generally one finds, by calculating the restoring force of the pendulum, that a field of forces whose lines of force are parallel in the space swept by the pendulum, which is the case of the gravitational attraction of a sufficiently distant body, creates a "linear anisotropy".

For Horodnic pendulums, $\omega = 1.3$, and in the case of the Moon $C_i = C_M = 0.862 \times 10^{-13}$.

Hence $|e'| \le 5 \times 10^{-14}$, which is over a million times smaller than the amplitude of the 24.84 h component of $\overline{e'}$, which is >10^{-7} ((see **Table 2** in Section 4.3).

Remark: Identifying Equations (7) and (14) makes it easy to calculate the anisotropy coefficient η_L:

$$\eta_L = \frac{K_L}{2\omega^2} < 1.3 \times 10^{-13} \tag{16}$$

Which is over a million times smaller than the measured anisotropies.

Appendice C: Band-Pass Filter Used

Appendice C.1: Description of the Band-Pass Filter Used for the Spectral Analysis

Figure A3 shows an overall diagram of the spectrum analyzer.

Let be a temporal sequence $s_i = s(t_i)$ ($i = 1, \cdots, N$; in the Horodnic experiment, N = 869). The date t_i, which is counted from an initial date Θ, is expressed in days and hours. We gather all the elements of the sequence corresponding to a given hour, and then we calculate their average. Hence 24 values $z_0, z_j, \cdots, z_{23}$ corresponding to hours $0, 1, \cdots, 23$. A band-pass filtering[26] was thus carried out around the period T = 1 day = 24 h. The extraction of harmonic 1 of this new sequence provides amplitude and phase of the 24 h component of the sequence s_i.

To perform filtering around frequency $\frac{1}{T}$, we operate as above, after having expressed t_i in "days T" and in "hours T" (1 "hour T" = T (expressed in hours)/24 hours, and 1 "day T" = 24 "hours T").

Appendice C.2: Estimation of the Probability That a Random Not Auto-Correlated Noise Creates a Fake Periodic Component of Period *T*

We hypothesize that, at the input of the spectrum analysis, s_i is only a non autocorrelated random stationary and ergodic noise: $s_1, \cdots, s_N$ are N independent random variables with the same probability law. The random variable a_T is the amplitude of the component of period T of the stochastic sequence $s_1, \cdots, s_N$.

The experimental result is then one random draw of the $s_1, \cdots, s_N$. Its standard deviation σ_s is also the standard deviation of the random variable s_i.

Its spectrum analysis gives b as the amplitude of the component of period T. The probability that $a_T \ge b$, which is the probability that this component is in fact a fake component, is given by Formula (17):

$$proba(u_T \ge v) = \mathrm{e}^{-v^2 \frac{N^2}{4(N-1)}} \tag{17}$$

where $u_T = \frac{a_T}{\sigma_s}$, $v = \frac{b}{\sigma_s}$ and N = 869.

[26]This is in fact a particular case of the classic Buys-Ballot filter, which allows band-pass filtering when the period studied is a multiple of the sampling period.

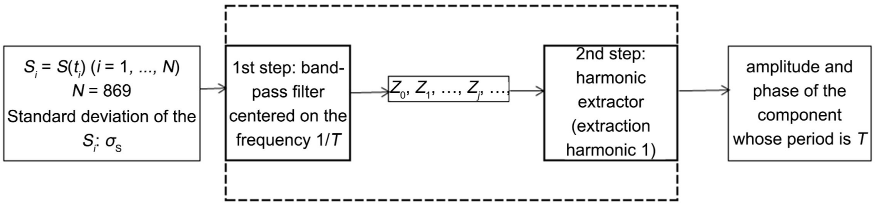

Figure A3. Spectral analysis: search of the component of period T of the sequence $S_i = S(t_i)$.

Table A2. Probability of a fake 24.84 h component if $\overline{e'}$ is only noise.

	A	B
Value of b/σ_s for the average derivative of the ellipticity $\overline{e'}$	0.155	0.141
Probability creation of a fake 24.84 h component	5.2×10^{-3}	1.4×10^{-2}

Table A3. Probability of a fake 24.84 h component if 2/3 sd($\overline{e'}$) come from noise.

	A	B
Value of b/σ_s for the average derivative of the ellipticity $\overline{e'}$	0.232	0.212
Probability creation of a fake 24.84 h component	7.8×10^{-6}	6×10^{-5}

Remark: The above formula is deduced, with different notations, from the test developed by A. Schuster [27], in the case where the random sequence is not autocorrelated. We can also find it, more clearly expressed, in [28] or [29], in which Allais has extended this test to the case where the random sequence is autocorrelated.

Case of the 24.84 h component of Horodnic pendulums:

It results from the perturbing factors considered in Section 5.7 ("Other possible perturbing factors") a random uncorrelated noise on $\overline{e'}$. As indicated in Section 5.7 the hypothesis that the 869 values of $\overline{e'}$ are only noise is very pessimistic. Nevertheless, the probability that the 24.84 h component is a fake one remains small for each of the 2 pendulums (**Table A2**). In **Table A2**, the value of σ_s is the measured standard deviation of the average derivative of the ellipticity $\overline{e'}$ (9.63×10^{-7} for Pendulum A, and 1.77×10^{-6} for Pendulum B), and the value of b for T = 24.84 h, which is the measured average ellipticity $\overline{e'}$, is given by **Table 2** in Section 4.3 (1.50×10^{-7} for Pendulum A, and 2.49×10^{-7} for Pendulum B).

With the hypothesis, probably still pessimistic, that the noise resulting from the random disturbing factors corresponds to 2/3 of the standard deviation of $\overline{e'}$, the probabilities are given by **Table A3**.

Appendice D: Spectral Analysis

This appendix gathers, for the 2 pendulums, the amplitude spectra of:

- The average rate of the total precession $\overline{\theta'}$ (**Figure A4**).

- The average rate of Airy precession $\overline{\theta'}_{Airy}$ (**Figure A5**).
- The outdoor and indoor temperatures (**Figure A6**).
- The atmospheric pressure (**Figure A7**).
-The hygrometry (**Figure A8**).
- The wind speed (**Figure A9**).

Figure A10 further shows that, in the case of Pendulum A, the importance of the 23 h line in $\overline{\theta'}$ and $\overline{\theta'}_{Airy}$ is the result of the noise introduced by the initial *e*. The graph in **Figure 2** of Section 4.3 shows that this influence has indeed been eliminated very largely in the amplitude spectrum of $\overline{e'}$.

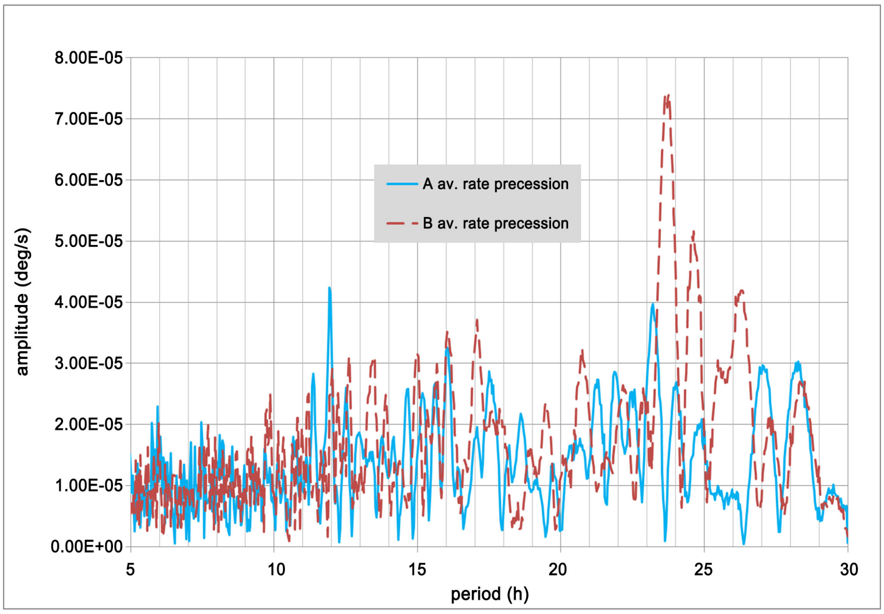

Figure A4. Spectrum of the average rate of the total precession.

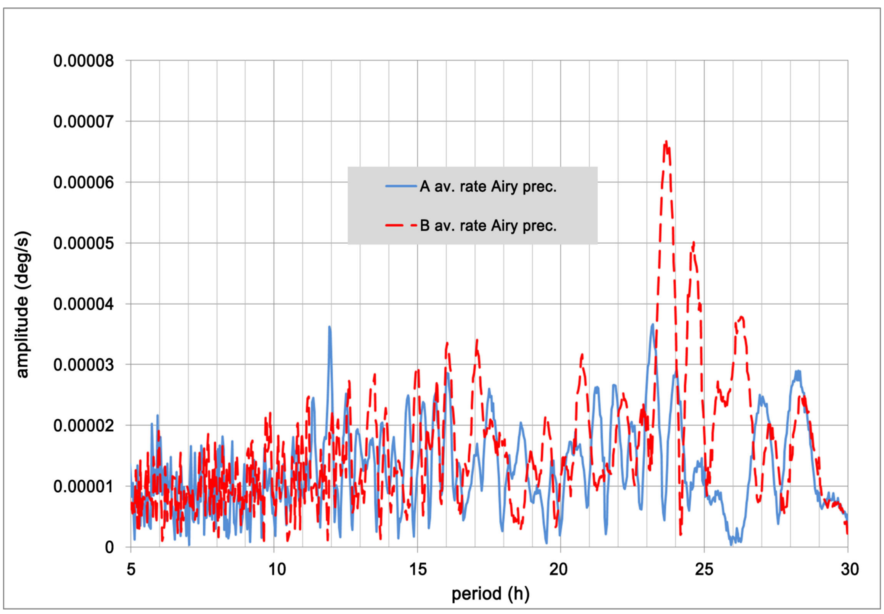

Figure A5. Spectrum of the average rate of Airy precession.

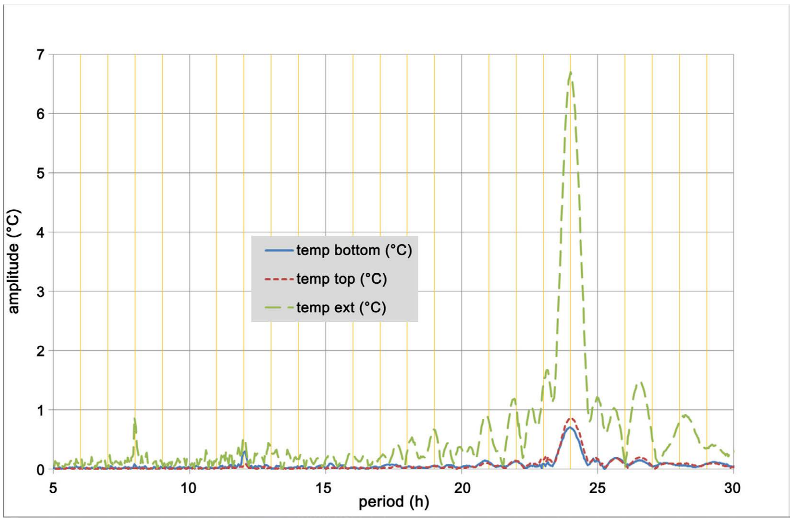

Figure A6. Outdoor and indoor temperatures.

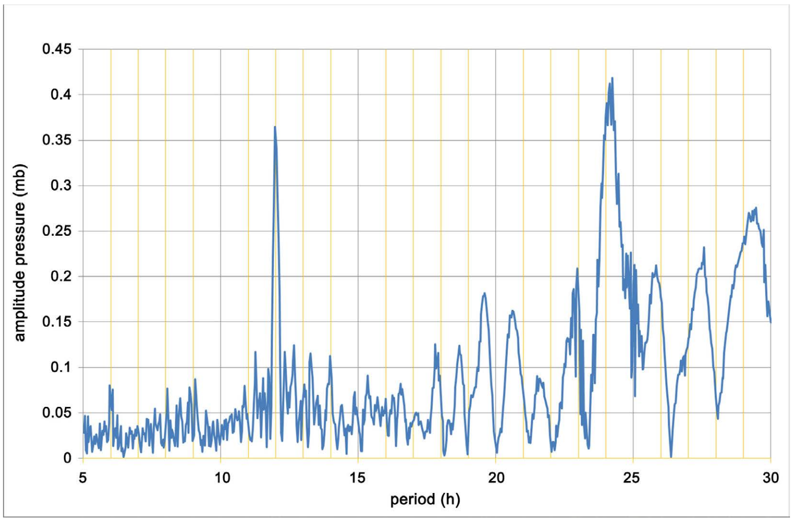

Figure A7. Spectrum of atmospheric pressure.

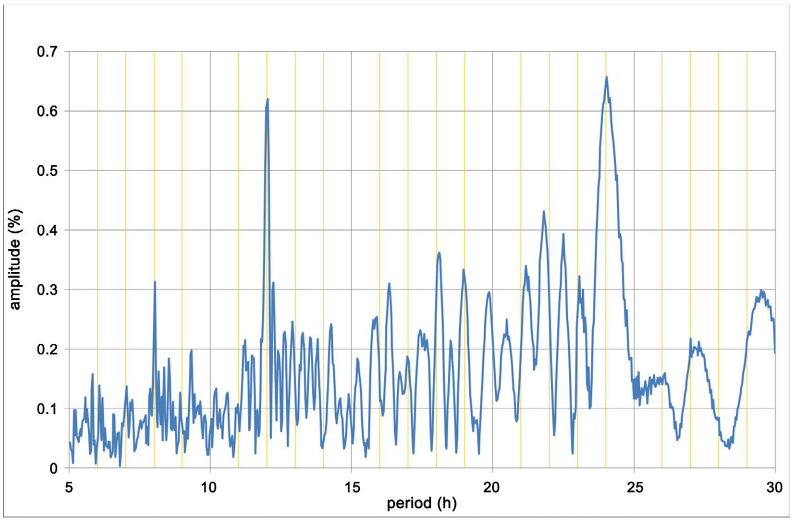

Figure A8. Hygrometry spectrum.

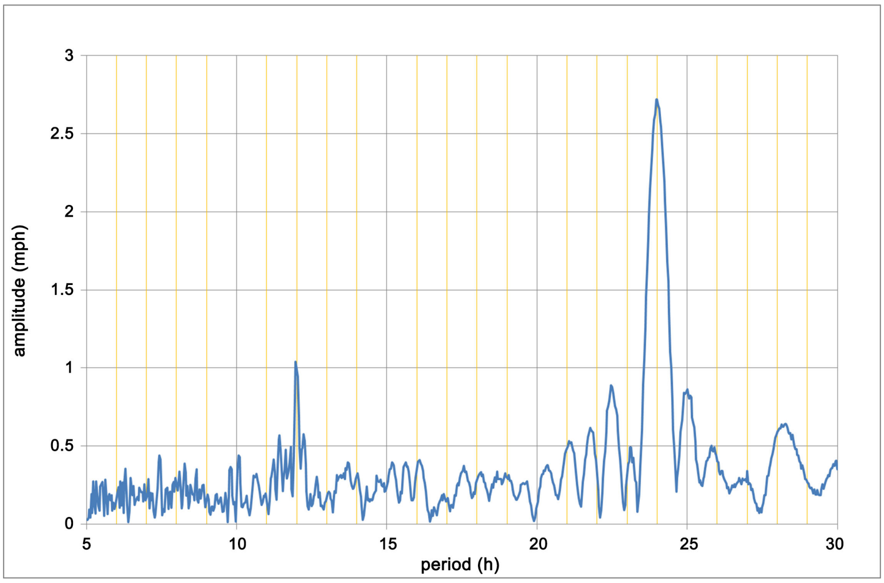

Figure A9. Wind speed spectrum.

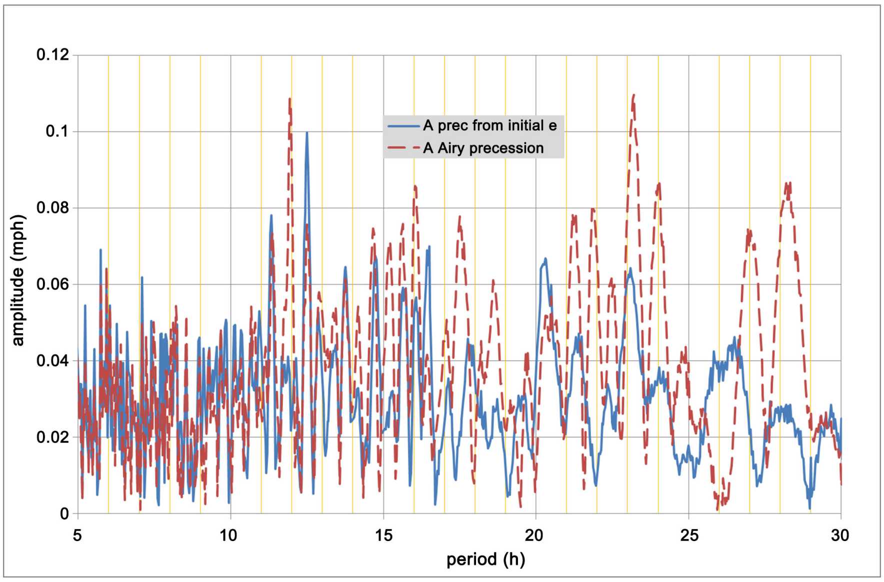

Figure A10. Pendulum A: total Airy precession and Airy precession from initial *e*.