Journal of Modern Physics, 2022, 13, 1403-1410
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

An SU(3) Electroweak Unified Model Using Generalized Yang-Mills Theory

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How to cite this paper: Wang, L., Zhou, X.Y., Wang, D.F., Huang, S.Z. and Guo, Y.Q. (2022) An SU(3) Electroweak Unified Model Using Generalized Yang-Mills Theory. *Journal of Modern Physics*, **13**, 1403-1410.
https://doi.org/10.4236/jmp.2022.1311087

Received: October 11, 2022
Accepted: November 13, 2022
Published: November 16, 2022

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Abstract

Generalized Yang-Mills theory has a covariant derivative which contains both vector and scalar gauge bosons. Based on this theory, we construct an $SU(3)$ unified model of electromagnetic and weak interactions to simplify the Weinberg-Salam model. By using the Nambu-Jona-Lasinio mechanism, the symmetry breaking can be realized dynamically. The masses of $W^{\pm}$, Z^0 are obtained and interactions between various particles are the same as that of the Weinberg-Salam model. At the same time, $\sin^2\theta_w = 1/4$ can be given.

Keywords

Yang-Mills Theory, Dynamical Symmetry Breaking, Nambu-Jona-Lasinio Mechanism

1. Introduction

Up to now, many experimental results have proved that the Weinberg-Salam (WS) model [1] [2] is correct in the current energy range, but as Weinberg himself pointed out, the WS model still has some unsatisfactory points [3]. The WS model describes the weak and electromagnetic interactions in the energy range ≤ 10^2 GeV with two different coupling constants g and g' for the gauge groups $SU(2)$ and $U(1)$, respectively. Thus, there is no real explanation of the different strengths displayed by the two interactions. For example, the experimentally determined Weinberg angle is approximately equal to 30˚, which cannot be directly obtained by WS model itself. On the other hand, although the 125 GeV Higgs boson has been discovered in 2012 [4] [5], there is still no evidence that Higgs particles are basic or compound particles and the number of Higgs particles is without theoretical guidance. Therefore, the improvement of the WS model is still necessary.

There have been different types of ideas to improve these situations. The most widely accepted one by far has been to use a large group of which $SU(2)\otimes U(1)$ is just a small subgroup. The original work is an old idea proposed by Fairlie [6] and Ne'eman [7], of using supergroup $SU(1/2)$ as the unification group and putting the Higgs fields in the adjoint along with the vector fields. But it increases the dimensions of space-time; meanwhile, the number of the Higgs bosons increases, which is not expected to be seen in theory. In Ref. [8], the authors have constructed an $SU(3)$ unified model of electroweak interaction; by using different realizations of $SU(3)$ algebra, the correct quantum numbers of the leptons and the Weinberg angle can be given. However, since $SU(3)$ group has eight generators, there are four more vector gauge fields $V^{\pm}$ and $U^{\pm}$ than the WS model, as well as some heavy fermions and scalar particles in the model.

Then, is there a more natural way for us to introduce the Higgs fields to physical theories? Many scholars have taken efforts to solve this problem. In Ref. [9], the authors have constructed a unified $U(3)$ model of electroweak interaction using a generalized Dirac covariant derivative, that contained both vector and pseudo-scalar fields. However, since $U(3)$ group has nine generators, it will have a extra field that does not interact with other particles than the WS model. And what's more, there is no the Higgs potential $V(\varphi)$ in the model; thus the spontaneous symmetry breaking cannot be applied, and the particles in the model cannot obtain masses. Recently, some authors have attempted to construct the so-called generalized Yang-Mills theory (GYMT) [10] [11] [12] [13] [14], which the generalized Dirac covariant derivative D is besides the vector part A_μ; it can also contain a scalar part φ, a pseudo-scalar part P, an axial-vector part V_μ and a tensor part $T_{\mu\nu}$. In Ref. [12], by using a covariant derivative with both vector and scalar gauge fields, the authors have constructed a generalized Yang-Mills model, which is invariant under local gauge transformations of a Lie group. Since the GYMT does not involve the potential energy term about the scalar fields, it is difficult to realize the Higgs mechanism [15] directly. It is shown, in terms of the Nambu-Jona-Lasinio (NJL) mechanism [16], that the gauge symmetry breaking can be realized dynamically.

Based on the GYMT given in Ref. [12], the work of the present paper is to construct an $SU(3)$ gauge-invariant unified model of electroweak interaction and that it naturally assigns the correct quantum numbers to the leptons and Higgs bosons. By using the GYMT, we introduce vector fields and scalar fields as the gauge fields into the model by the requirement of localization gauge invariance. We will show that, in terms of the NJL mechanism, the symmetry breaking can be realized dynamically and the masses of $W^{\pm}$ and Z^0 particles are obtained. Meanwhile, interactions between various particles are the same as that of the WS model.

2. Generalized Yang-Mills Theory

The main idea of the GYMT in Ref. [12] is as follows: Corresponding to each generator of the Lie group there is a gauge field, it does not matter whether vector fields or scalar fields. The generalized Dirac covariant derivative D can be constructed by taking each of the N generators and multiplying it by one of its associated gauge fields and summing them together

$$D = \gamma_\mu \partial_\mu - i\gamma_\mu A_\mu + \varphi, \tag{1}$$

where

$$A_\mu = gA_\mu^a T_a, \varphi = g\varphi^b T_b, \tag{2}$$

with the subscript a varies from 1 to N_A, b varies from $N_A + 1$ to N. Define the transformation for the gauge fields as

$$-i\gamma_\mu A_\mu + \varphi \to U\left(-i\gamma_\mu A_\mu + \varphi\right)U^{-1} - \left(\gamma_\mu \partial_\mu U\right)U^{-1}, \tag{3}$$

from which we can obtain that the covariant derivative must transform as $D \to UDU^{-1}$. When the covariant derivative acts on the matter field ψ, its gauge fields A_μ and φ will acquire certain coefficients called the charges Q_A and Q_φ of each gauge field with respect to ψ with the result

$$D_\psi = \gamma_\mu \partial_\mu - iQ_A \gamma_\mu A_\mu + Q_\varphi \varphi. \tag{4}$$

From our knowledge of the standard model we can only conclude that $Q_A = 1$. As for Q_φ, it is only known that parameter gQ_φ is related to the mass of the matter field. If $Q_A = Q_\varphi = 1$, the expressions of covariant derivatives D and D_ψ will be the same form, which is a considerable question that will be discussed in the next part.

The Lagrangian density of the GYMT contains only covariant derivatives and matter fields, and that it possesses both gauge and Lorentz invariance:

$$L = -\bar{\psi} D_\psi \psi + \frac{1}{2g^2} \widetilde{Tr}\left(\frac{1}{8}\left(TrD^2\right)^2 - \frac{1}{2} TrD^4\right), \tag{5}$$

in which the trace with the tilde is over the matrices of the Lie group and the one without tilde is over the matrices of the spinorial representation of Lorentz group.

3. The *SU*(3) Unified Model of Electroweak Interaction

In this section, in terms of the above GYMT, we will construct an *SU*(3) unified model of electromagnetic and weak interactions of electron-type leptons. By considering an *SU*(3) gauge invariant GYMT, it will be naturally assign the correct isospin T_3 and hypercharge Y quantum numbers to the neutrino and the electron, so long as we place them in *SU*(3) fundamental representation

$$\psi = \begin{pmatrix} \nu_L \\ e_L \\ e_R \end{pmatrix}. \tag{6}$$

In the Lagrangian density (5), the covariant derivative D will be of the form Equation (1) where $A_\mu = gA_\mu^a T_a \left(a = 1,2,3,8\right)$ is the vector gauge field, $\varphi = g\varphi^b T_b \left(b = 4,5,6,7\right)$ is the scalar gauge field, and g is the coupling constant, we have

$$D = \gamma_\mu \partial_\mu - i\gamma_\mu g A_\mu^a T_a + g\varphi^b T_b, \tag{7}$$

in which $T_i = (1/2)\lambda_i$ are the generators of $SU(3)$ group in three-dimensional representation, the λ_i are 3×3 traceless Hermitian matrices, which can be chosen to have the form

$$\begin{aligned}
&\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
&\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \\
&\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\
&\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}.
\end{aligned} \tag{8}$$

One can see that the matrices from λ_1 to λ_7 are still the usual Gell-Mann matrices. Meanwhile the last one λ_8 takes the minus sign. Correspondingly, the components containing indices 8 in the structure constants f_{ijk} and d_{ijk} of $SU(3)$ group will take the minus sign too.

Next, we will give the specific form of the covariant derivative D_ψ in Equation (5). Following from Ref. [8], in order to obtain the correct hypercharge of e_R, we choose one of the four realizations of $SU(3)$ algebra. Then the above eight generators T_i can be divided into two groups $T_n\,(n = 2,4,6)$ and $T_s\,(s = 1,3,5,7,8)$ respectively. Define $T_i^{(5)}\,(i = 1,\cdots,8)$ as

$$T_n^{(5)} = T_n, T_s^{(5)} = T_s\gamma_5. \tag{9}$$

It can be easily proved that $T_i^{(5)}$ satisfy the same commutation rules as T_i

$$\left[T_i^{(5)}, T_j^{(5)}\right] = i f_{ijk} T_k^{(5)}. \tag{10}$$

Here, $T_i^{(5)}$ is the one of the four realizations of $SU(3)$ algebra.

Following the above discussion we can give the covariant derivative D_ψ as

$$D_\psi = \gamma_\mu \partial_\mu - i\gamma_\mu A_\mu^{(5)} + Q_\varphi \varphi^{(5)}, \tag{11}$$

where $A_\mu^{(5)} = g A_\mu^a T_a^{(5)}\,(a = 1,2,3,8), \varphi^{(5)} = g\varphi^b T_b^{(5)}\,(b = 4,5,6,7)$. By using $\gamma_5 L = L$, and $\gamma_5 R = -R$, we can obtain

$$\begin{aligned}
i\bar{\psi}\gamma_\mu A_\mu^{(5)}\psi &= ig\bar{\psi}\gamma_\mu\left[\gamma_5\left(A_\mu^1 T_1 + A_\mu^3 T_3 + A_\mu^8 T_8\right) + A_\mu^2 T_2\right]\psi \\
&= ig\bar{\psi}\gamma_\mu\left[A_\mu^1 T_1 + A_\mu^2 T_2 + A_\mu^3 T_3 + A_\mu^8 T_8'\right]\psi,
\end{aligned} \tag{12}$$

in which $T_8' = 1/\left(2\sqrt{3}\right) diag(-1,-1,-2)$. This means that the hypercharges of

ν_L, e_L and e_R are $-1,-1,-2$, respectively.

Substituting Equation (12), Equation (11) and Equation (7) into Equation (5), the Lagrangian density changes to be

$$L = -\bar{\psi}\gamma_\mu \partial_\mu \psi + i\bar{\psi}\gamma_\mu A_\mu^{(5)}\psi - Q_\varphi \bar{\psi}\varphi^{(5)}\psi \\ -\frac{1}{2g^2}\widetilde{Tr}\left(\partial_\mu A_\nu - \partial_\nu A_\mu - i\left[A_\mu, A_\nu\right]\right)^2 - \frac{1}{g^2}\widetilde{Tr}\left(\partial_\mu \varphi - i\left\{A_\mu, \varphi\right\}\right)^2. \tag{13}$$

By using the Pauli matrices $\sigma_{a'}\left(a'=1,2,3\right)$, Equation (6) and Equation (12), one can give the expansion of the Lagrangian density (13) as

$$L = -\bar{\theta}_L \gamma_\mu \left(\partial_\mu - \frac{1}{2}igA_\mu^{a'}\sigma_{a'} + \frac{1}{2}ig'A_\mu^8\right)\theta_L - \bar{e}_R\gamma_\mu\left(\partial_\mu + ig'A_\mu^8\right)e_R \\ -\frac{G_F}{\sqrt{2}}\bar{e}_R\phi^{(5)+}\theta_L - \frac{G_F}{\sqrt{2}}\bar{\theta}_L\phi^{(5)}e_R - \frac{1}{4}F_{\mu\nu}^{a'}F_{\mu\nu}^{a'} - \frac{1}{4}F_{\mu\nu}^8 F_{\mu\nu}^8 \\ -\left|\left(\partial_\mu - \frac{1}{2}igA_\mu^{a'}\sigma_{a'} - \frac{1}{2}ig'A_\mu^8\right)\phi\right|^2, \tag{14}$$

where, $F_{\mu\nu}^{a'} = \partial_\mu A_\nu^{a'} - \partial_\nu A_\mu^{a'} + gf_{a'b'c'}A_\mu^{b'}A_\nu^{c'}\left(a',b',c'=1,2,3\right)$, $F_{\mu\nu}^8 = \partial_\mu A_\nu^8 - \partial_\nu A_\mu^8$, $G_F = gQ_\varphi$, $\phi = \left(\varphi^4 - i\varphi^5, \varphi^6 - i\varphi^7\right)^{\mathrm{T}}\big/\sqrt{2}$, $\phi^{(5)} = \left(\varphi^4 - i\gamma_5\varphi^5, \varphi^6 - i\gamma_5\varphi^7\right)^{\mathrm{T}}\big/\sqrt{2}$, $\theta_L = \left(\nu_L, e_L\right)^{\mathrm{T}}$ and $g' = g\big/\sqrt{3}$. By considering $g' = g\big/\sqrt{3}$, one can conclude that $\sin^2\theta_w = 1/4$ easily. Seeing from Equation (14), one can obtain the correct hypercharge $Y=1$ of the scalar field (the Higgs field) as in the WS model.

In Equation (14), the factor $G_F\big/\sqrt{2} = gQ_\varphi\big/\sqrt{2}$ is the Yukawa coupling constant for the coupling between the scalar gauge field and the fermion field. The fermion masses are proportional to $gQ_\varphi\big/\sqrt{2}$ after electroweak symmetry breaking. And the different masses of all elementary fermions are determined by the different values of Q_φ. If $Q_\varphi = 1$, as we have mentioned above, is there any possibilities that the elementary fermion mass spectrum can be obtained as usual? In Ref. [17], the authors have discussed this problem and proposed a possible way to solve it.

4. Dynamical Breaking of *SU*(3) Gauge Symmetry

As is known to us, Equation (14) is almost the Lagrangian density of the WS model, except that no the Higgs potential $V(\varphi)$, which means that the spontaneous symmetry breaking mechanism cannot be utilized directly. In this section, we will show that by using the NJL mechanism, the *SU*(3) gauge symmetry breaking can be realized dynamically.

Substituting Equation (13) into Euler equation, one can obtain the equations of motion for the fermion field ψ, the scalar gauge fields φ^b, and the vector gauge fields $A_\mu^{a'}$

$$\gamma_\mu\left(\partial_\mu - igA_\mu^a T_a^{(5)}\right)\psi + G_F\varphi^b T_b^{(5)}\psi = 0, \tag{15}$$

$$\left(\partial_\mu^2 - g^2 dA_\mu^a A_\mu^a\right)\varphi^b - G_F\bar{\psi}T_b^{(5)}\psi = 0, \tag{16}$$

$$\left(\partial_\mu F_{\mu\nu}^{a'} + gf_{a'b'c'}A_\mu^{b'}F_{\mu\nu}^{c'}\right) + g^2 d\left(\varphi^b\right)^2 A_\nu^{a'} - ig\bar{\psi}\gamma_\nu T_{a'}^{(5)}\psi = 0. \tag{17}$$

With $d = d_{abc}d_{abc}\left(a = 1,2,3,8, b = 4,5,6,7\right)$. Multiplying $A_{\nu}^{a'}$ on both sides of Equation (17), we obtain

$$\left[\left(\partial_{\mu}F_{\mu\nu}^{a'} + gf_{a'b'c'}A_{\mu}^{b'}F_{\mu\nu}^{c'}\right) + g^2 d\left(\varphi^b\right)^2 A_{\nu}^{a'} - ig\bar{\psi}\gamma_{\nu}T_{a'}^{(5)}\psi\right]A_{\nu}^{a'} = 0, \tag{18}$$

after taking the vacuum expectation value of Equation (18), to the lowest-order approximation in $\hbar$, we obtain a simple formula

$$f\left\langle A_{\mu}^{a'}A_{\mu}^{a'}\right\rangle = d\left\langle\left(\varphi^b\right)^2\right\rangle, \tag{19}$$

where $f = f_{a'b'c'}f_{a'b'c'}\left(a', b', c' = 1,2,3\right)$. We can see that in the ground state, Equation (19) gives an important relationship between the vector gauge fields and the scalar gauge fields. One can denote the vacuum expectation of the scalar fields as

$$\left\langle\varphi^b\right\rangle = \left\langle\varphi^6\right\rangle = \upsilon \neq 0, \tag{20}$$

which means

$$\left\langle\phi\right\rangle = \left\langle\phi^{(5)}\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \upsilon\end{pmatrix}. \tag{21}$$

Substitute Equation (21) into Equation (14), we can give the masses of the neutrino and the electron

$$m_e = \frac{G_F\upsilon}{2}, m_{\nu} = 0. \tag{22}$$

Let us now take the vacuum expectation value of Equation (16). To the lowest-order approximation in $\hbar$, by using Equation (19), the self-consistency equation can be given as

$$g^2 d^2 f^{-1}\left\langle\varphi^6\right\rangle^3 = -\frac{1}{2}G_F\left\langle\bar{e}e\right\rangle. \tag{23}$$

In Equation (23), with an invariant momentum cut-off at $p^2 = \Lambda$ in the momentum integral, $\left\langle\bar{e}e\right\rangle$ will be finite value as

$$\begin{aligned}\left\langle\bar{e}e\right\rangle &= -TrS_F\left(0\right) = 2G_F\left\langle\varphi^6\right\rangle\int\frac{\mathrm{d}^4 p}{\left(2\pi\right)^4}\frac{i}{p^2 + G_F^2\left\langle\varphi^6\right\rangle^2/4}\\ &= -\frac{G_F\left\langle\varphi^6\right\rangle}{4\pi^2}\left[\Lambda^2 - \frac{G_F^2\left\langle\varphi^6\right\rangle^2}{4}\ln\left(\frac{4\Lambda^2}{G_F^2\left\langle\varphi^6\right\rangle^2} + 1\right)\right].\end{aligned} \tag{24}$$

Substituting Equation (24) into Equation (23), we have

$$\left\langle\varphi^6\right\rangle^2 = \frac{G_F^2 f}{8\pi^2 d^2 g^2}\left[\Lambda^2 - \frac{G_F^2\left\langle\varphi^6\right\rangle^2}{4}\ln\left(\frac{4\Lambda^2}{G_F^2\left\langle\varphi^6\right\rangle^2} + 1\right)\right]. \tag{25}$$

From Equation (25), one can finally obtain the non-vanishing vacuum expectation value $\left\langle\varphi^6\right\rangle$ of the scalar field, which is determined by the self-energy of the fermion field. And then the *SU*(3) gauge symmetry is broken down dynamically.

Substituting the definition

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(A_\mu^1 \mp iA_\mu^2\right), Z_\mu = \frac{1}{2}\left(-\sqrt{3}A_\mu^3 + A_\mu^8\right), B_\mu = \frac{1}{2}\left(A_\mu^3 + \sqrt{3}A_\mu^8\right), \quad (26)$$

and Equation (21) into Equation (14), we can obtain the masses of the vector gauge particles

$$m_W^2 = \frac{1}{4}g^2\upsilon^2, m_Z^2 = \frac{4}{3}m_W^2, m_B^2 = 0. \quad (27)$$

This result is exactly the same as that of the WS model.

5. Summary and Remarks

In this paper, based on the generalized Yang-Mills theory, we have constructed an $SU(3)$ unified model of electromagnetic and weak interactions. By using the NJL mechanism, the $SU(3)$ gauge symmetry breaking can be realized dynamically, although there is no the Higgs potential $V(\varphi)$ in the GYMT. The masses of $W^\pm$ and Z^0 particles are obtained. Interactions and quantum numbers of various particles are the same as that of the WS model. Compared to the WS model, the present model has several advantages. Firstly, since the present model is based on $SU(3)$ gauge group, there is only one coupling constant, and $\sin^2\theta_w = 1/4$ can be obtained directly. Secondly, the scalar fields are considered to be as gauge fields in the present model, then the introduction of the scalar fields becomes natural, and the number of the scalar fields can become certain too.

Acknowledgements

Sincere thanks to the members of JMP for their professional performance.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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