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On Lorentz Transformations and the Theory of Relativity

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Abstract

In the course of a research study on Lorentz transformations and the theory of relativity, the fundamentals of the relativistic concepts of space and time, the relations of those concepts to Lorentz transformations, and equivalence of mass and energy were studied. Many important references on the said subjects were reviewed. This paper draws attention to some critical questions that have risen in the course of that research study on the concepts of expansion of time and unbounded increase of a particle's mass with velocity.

Keywords

Lorentz Transformations, Relativity Theory, Mass, Energy

1. Introduction

Maxwell's equations have been used as the fundamental form of electromagnetic equations for more than a century. These equations in point form are:

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{1}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} \tag{2}$$

$$\nabla \cdot B = 0 \tag{3}$$

$$\nabla \cdot D = \rho \tag{4}$$

where E is the electric field intensity, B is the magnetic flux density, H is the magnetic field intensity, D is the electric flux density, ρ is the volume charge density and J is the current density.

The above equations are accompanied by:

$$B = \mu H \tag{5}$$

$$D = \epsilon E \tag{6}$$

$$J = \sigma E \tag{7}$$

$$J = \rho v \tag{8}$$

where ϵ is permittivity, μ is permeability, σ is conductivity, v is the velocity of charge, with Equation (8) defining the convection current density. The above set of Maxwell's equations is complemented by the Lorentz force equation as:

$$F = \rho\left(E + v \times B\right) \tag{9}$$

where F is force per unit volume.

Lorentz transformations [1] [2] were introduced to enable physicists to apply the same equations in different coordinate systems with relative constant motion with respect to one another and also to explain the failure of the Michelson-Morley experiment [3]. Lorentz transformations were used as the basis of Einstein's special theory of relativity [4] which in turn was used later as the basis of general theory of relativity [5]. Lorentz transformations and the theory of relativity have been the subject of numerous studies to this date (e.g., [6] [7]). At the same time, the relativity theory and quantum mechanics have had significant contradictions which have been the subject of many debates and various new theories have been devised to bridge the gap between the two major theories. The purpose of this research was to get an in depth understanding of Lorentz transformations and the theory of relativity. This paper draws attention to some basic questions on these subjects that may need clarification.

2. Lorentz Transformations and the Relativistic Concepts of Space and Time

Considering two cartesian coordinate systems, one assumed as stationary (x, y, z, t), and the other in relative constant motion along the x direction with respect to the other at a constant velocity, v, (x', y', z', t'), Lorentz provided the following transformations so the physical equations would remain unchanged in the two coordinate systems:

$$x' = \frac{x - v \cdot t}{\sqrt{1 - \frac{v^2}{C^2}}} \tag{10}$$

$$y' = y \tag{11}$$

$$z' = z \tag{12}$$

$$t' = \frac{t - \frac{v \cdot x}{C^2}}{\sqrt{1 - \frac{v^2}{C^2}}} \tag{13}$$

where C is the speed of light in free space considered as constant in all coordinate systems. In the well-known Michelson-Morley's experiment, in which a single source of light, a glass plate, and two mirrors were used, it was hypothesized that the earth velocity could be measured by measuring the difference in the light travel time to and from the mirrors. That difference, however, turned out to be zero and the experiment did not produce any result. Lorentz transformations (Equations (10)-(13)) as stated above were used to describe the reason for the failure of the experiment. Focusing on Lorentz Equation (10) and applying that equation to two points x_1', and x_2' in the moving coordinate system yields:

$$x_1' = \frac{x_1 - v \cdot t_1}{\sqrt{1 - \frac{v^2}{C^2}}} \tag{14}$$

$$x_2' = \frac{x_2 - v \cdot t_2}{\sqrt{1 - \frac{v^2}{C^2}}} \tag{15}$$

where x_1' and x_2' are the positions measured along the x' axis in the moving system, x_1 and x_2 are the positions along the x axis in the stationary system, and t_1 and t_2 are the corresponding measured times in the stationary system. Subtracting Equation (14) from Equation (15) yields:

$$\Delta x' = \frac{\Delta x - v \cdot \Delta t}{\sqrt{1 - \frac{v^2}{C^2}}} \tag{16}$$

where $\Delta x = x_2 - x_1$, $\Delta x' = x_2' - x_1'$ and $\Delta t = t_2 - t_1$. Rearranging Equation (16) yields:

$$\Delta x = \Delta x' \sqrt{1 - \frac{v^2}{C^2}} + v \cdot \Delta t \tag{17}$$

The first term on the right side of Equation (17) shows that the distance between the two points in the moving system as viewed by an observer in the stationary system is multiplied by the factor γ:

$$\gamma = \sqrt{1 - \frac{v^2}{C^2}}$$

Therefore, this distance appears as contracted by the stationary observer as v increases. This phenomenon as predicted by Lorentz transformations is referred to as contraction of space and could explain the null results of Michelson-Morley's experiment. Later, it was noted by a number of scientists that since the Michelson-Morley's measurement system was moving with the earth and was not stationary, the experiment could not produce the intended result. In relation to the same experiment, the concept of expansion of time was proposed as follows:

$$\tau = \frac{\tau'}{\sqrt{1 - \frac{v^2}{C^2}}} \tag{18}$$

where τ' is the travel time measured in the moving system and τ is the corresponding duration of time as observed in the stationary system. Focusing on Equation (13) of Lorentz transformations above and applying it to two points in time in the moving system results as:

$$t_1' = \frac{t_1 - \frac{v \cdot x_1}{C^2}}{\sqrt{1 - \frac{v^2}{C^2}}} \tag{19}$$

$$t_2' = \frac{t_2 - \frac{v \cdot x_2}{C^2}}{\sqrt{1 - \frac{v^2}{C^2}}} \tag{20}$$

Subtracting Equation (19) from Equation (20) yields:

$$t_2' - t_1' = \Delta t' = \frac{\Delta t - \frac{v \cdot \Delta x}{C^2}}{\sqrt{1 - \frac{v^2}{C^2}}} \tag{21}$$

Rearranging Equation (21) yields:

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{C^2}}} - \frac{\frac{v \cdot \Delta x}{C^2}}{\sqrt{1 - \frac{v^2}{C^2}}} \tag{22}$$

This equation does not show the concept of expansion of time as shown in Equation (18). In Chapter 15 of a well-known reference book on this subject [8], an example of a light clock is used to demonstrate the concept of expansion of time in a moving system as observed from a stationary system. However, "time" can be measured in various ways. For example, the unit of time, second, may be defined as about 9 billion oscillations of the cesium atom or similarly in terms of biological procedures. Choosing a time clock in which the distance travelled by light is directly affected by the velocity of the coordinate system may result in different conclusions than by using a biological clock or a clock based on atomic oscillations. Another question that arises is that if the distance in a moving system as viewed by a stationary observer is contracted and the corresponding time is not contracted at the same rate and is expanded at that rate, how will that affect the speed of light which should remain constant in all coordinate systems.

3. Mass and Energy

While physicists were puzzled about the nature of light, Louis De Broglie proposed in 1923 that everything manifested both particle and wave properties, which became known as wave-particle duality. De Broglie came up with the following wave equation:

$$P = \frac{h}{\lambda} \tag{23}$$

where P is the particle's momentum, h is the Planck's constant, and λ is the wavelength. The relation between the energy and frequency of a wave is given by

Planck's equation as:

$$\text{Energy} = h \cdot f \tag{24}$$

where f is the wave frequency. Substituting h from Equation (23) into Equation (24) yields:

$$\text{Energy} = P \cdot \lambda \cdot f \tag{25}$$

Since $f = \dfrac{C}{\lambda}$:

$$\text{Energy} = P \cdot C \tag{26}$$

For a mass m_0 at velocity, v:

$$\text{Energy} = m_0 \cdot v \cdot C \tag{27}$$

If the velocity of the particle, v, is raised to the maximum possible velocity which is the speed of light, C, at which point all the mass of the particle can be said to have been transformed to energy, Equation (27) can be written as:

$$\text{Energy} = m_0 \cdot C^2 \tag{28}$$

Equation (28) whose validity has been verified in many experiments was provided by Einstein. In Chapter 15 of Reference #8, a derivation of Equation (28) as well as the following equation is provided:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{C^2}}} \tag{29}$$

In the said reference, the derivation starts by assuming the validity of Equation (29) and ends up by concluding the validity of the said equation as well as Equation (28) above. There seem to be some issues with this approach. First, one may not begin by assuming the validity of a mathematical equation and use it as correct, in order to conclude the validity of the same equation. Second, Equation (29) which does not seem to have been supported by sufficient evidence to this date, expresses that the mass of a particle tends to become very large and approaches infinity as the velocity of that particle increases. Whether the resulting mass is called the "inertial mass", "the observed mass", or energy, the concept of a physical quantity to approach "infinity" does not seem to be plausible. The experiments performed to this date in which a lot of energy was given to a charged particle, resulted in the additional energy transformed to radiation energy given by the particle and not a measurable increase in mass.

4. Summaries

Relativity and quantum physics are two major theories that have significant contradictions. Some aspects of relativity have been verified experimentally while some others have not. At the same time, quantum physics has been used quite successfully to predict many experimental results, particularly at the atomic and sub-atomic levels. Albert Einstein once stated "… the great initial success of the quantum theory does not make me believe in the fundamental dice game," and his famous quote was "I cannot believe that god plays dice". Einstein might have been quite right that the world does not function based on probabilities. However, quantum physics is based on probabilistic mathematics which is devised for and is quite successful when the amount of data is enormous, maybe partially unknown, and deterministic analysis is practically impossible; these conditions exist in the atomic and subatomic world.

However, as described in this paper, there are some important questions on Lorentz transformations and the theory of relativity that need to be addressed. Notwithstanding that Lorentz transformations are empirical, the concept of contraction of space in a moving coordinate system as viewed by a stationary observer can be derived from Lorentz transformations. The main questions raised in this paper are the following:

1) With regard to the concept of expansion of time in a moving system as seen by a stationary observer, there does not seem to be a clear derivation of that concept based on Lorentz transformations that are themselves empirical in nature, or otherwise, as discussed above. Also, taking the concepts of expansion of time and contraction of space together may present a contradiction with the principle of constancy of the speed of light in all coordinate systems.

2) Another issue is with Equation (29) that describes a particle's mass (or inertial mass) as a function of velocity and predicts that it will increase unbounded to infinity as velocity increases. There does not seem to be sufficient proof of this concept to this date. Furthermore, the use of Equation (29) by some authors to derive Equation (28) that has been verified in many experiments and can be derived from Equations (23)-(27) as shown above, does not seem to be warranted. These issues deserve more attention and may need to be addressed.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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