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Proposal of a Model of a Three-Dimensional Spherical Universe

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Abstract

Despite the existence of several exact solutions to the general theory of relativity, it is still difficult to explain the entire structure of the universe. In this paper, we propose a novel three-dimensional spherical (S^3) universe model. According to this model, the universe had a powerful gravity source at its origin, and a part of the gravitational source formed a bubble of spacetime in which the universe was born. The universe expanded explosively immediately after the Big Bang. The energy obtained from the gravity source at the birth of the universe is constant, that is, it remains constant from the birth of the universe to the end. The expansion and contraction of the universe is determined by the passage of coordinate time. The S^3 universe observed from outside can be considered to be a two-dimensional spherical surface by projecting it onto a three-dimensional space. On the other hand, the visible universe viewed from inside is observed as a three-dimensional sphere with an arbitrary observation point. The origin of the S^3 universe can be seen to be evenly spread in the outer shell of the visible universe. We define *visible longitude* as the difference between an observation point and its farthest light source. At an initial phase of the expansion of the universe, few stars can be seen, because the visible longitude is small. However, when the expansion of the universe progresses, the number of visible stars also increases, since visible longitude increases. In our S^3 universe model, the redshifts observed and reported so far in literature appear to indicate that the expansion of the universe is accelerating. Our S^3 universe model can thus lead to a novel exact solution to general relativity.

Keywords

Redshift, S^3 Universe, Expansion, General Relativity, Big Bang

1. Introduction

Current exact solutions to general relativity can correctly explain the deformation of spacetime under specific conditions [1] [2] [3] [4] [5], but Freedman's solution, which explains the chronological changes of the entire universe, cannot explain the accelerated expansion of the universe [6] [7] [8]. We present in this paper a three-dimensional spherical (S^3) universe model. This model can show the entire structure of the universe as seen from outside of the universe from its birth to its end, as well as the chronological changes of the universe as observed from inside. The S^3 universe has a strong source of gravity in its center, and a part or all of it became a spacetime bubble where the universe was born [9]. The S^3 universe was explosively expanded by the Big Bang shortly after its birth, just like the current universe [9], eventually becoming a great sphere after the rate of expansion had slowed. Soon after, the S^3 universe started to contract gradually; the speed of the contraction increased until it finally contracted to the original origin (**Figure 1**) [9] [10].

When the S^3 universe is viewed from outside, it is possible to define the chronological changes of the entire structure using coordinate time, which can express the expansion and contraction of the S^3 universe. On the other hand, when viewed from inside, the S^3 universe appears, similar to the current universe, as a celestial sphere that spreads evenly in all directions [11] [12]. Soon after the birth of the S^3 universe, light reaching an arbitrary observation point is only the nearest light of the Big Bang. After the birth of stars, the light from neighboring stars reaches sequentially, and when the expansion speed of the S^3 universe slows

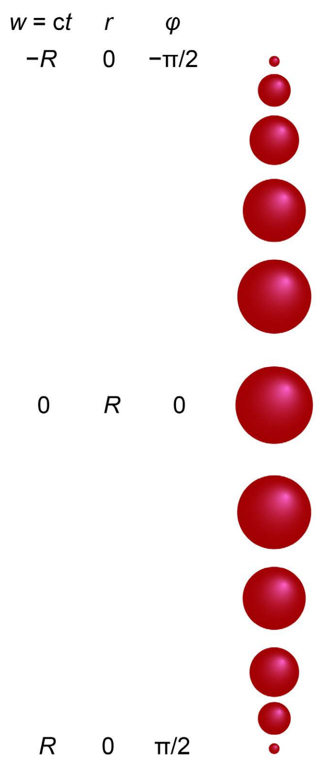

Figure 1. Life of the S^3 universe. The red spheres show the expansion and contraction of the S^3 universe with the passage of coordinate time from an external view. r represents the radius at coordinate time $w = ct$, and φ represents the latitude of the S^3 universe.

down, light from a long distance away starts to arrive sequentially. The longitude difference between the farthest light source from which light can be emitted and the observation point is defined as *visible longitude*. Stars visible in the entire celestial sphere are sparse at the beginning of the expansion since the visible longitude is small then, but the number of visible stars increases as the universe expands as a result of the increase in visible longitude. When the universe reaches its maximum expansion, the light from the observation point reaches the visible longitude of π/2. The visible longitude increases even during the contraction phase, as does the number of visible stars. Finally, the universe contracts to the origin and ends when the contraction speed increases and cosmic microwave background (CMB) reaches the longitude difference of π [9]. From the standpoint of an internal view of the universe, the gravity source, which is the origin of the universe, is in the outer shell of the celestial sphere, and the entire universe can be interpreted to be the bubble of spacetime floating in the gravity source at the origin [9].

2. Definition of the S^3 Space Model

The S^3 universe is represented by four dimensions: x, y, and z-coordinates for space, and the w-coordinate that indicates space expansion and contraction. The w coordinate has a dimension of distance determined by multiplying time t by the speed of light c, where t is defined as the coordinate time of the S^3 universe. When the radius of the great sphere of S^3 at coordinate time zero is R, the relationship $x^2 + y^2 + z^2 + w^2 = R^2$ holds. When the radius of the S^3 universe at coordinate time t ($w = \mathrm{c}t$) is r, the latitude of the S^3 is defined as φ as shown in **Figure 1** and **Figure 2**. Latitude φ is zero on the great sphere ($w = 0$) and $\varphi = \arcsin(\mathrm{c}t/R)$ at arbitrary coordinate time t ($w = \mathrm{c}t$), where $-\pi/2 \le \varphi \le \pi/2$. The rate of increase of the radius r at coordinate time t is defined as the expansion speed of the S^3 universe, which is expressed by v. They are related by the expressions $r = R\cos(\varphi)$ and $v = \mathrm{d}r/\mathrm{d}t = -\mathrm{c}\tan(\varphi)$. As shown by the green line in **Figure 2**, the radius r of the S^3 universe becomes zero at a start point ($w = -R$), R at the great sphere ($w = 0$), and zero again at an end point ($w = R$). On the other hand, as shown by the blue line in the figure, the expansion speed v of the radius r is maximum immediately after the start point, zero on the great spherical surface, and negative maximum just before the end point.

In this paper, without considering local spacetime distortions, we believe that the expansion and contraction of the S^3 universe occur evenly in all directions of the space coordinates, and that the celestial bodies in the S^3 universe are stationary at each respective position. Therefore, even if the radius of the S^3 universe changes, the relative positions of the celestial bodies in the space coordinates do not change. As shown in **Figure 3**, the angle β formed by the directions of the expansion velocity vectors of the two celestial bodies represents the longitude difference between the two celestial bodies, and this longitude difference is always constant regardless of the expansion and contraction of the S^3 universe.

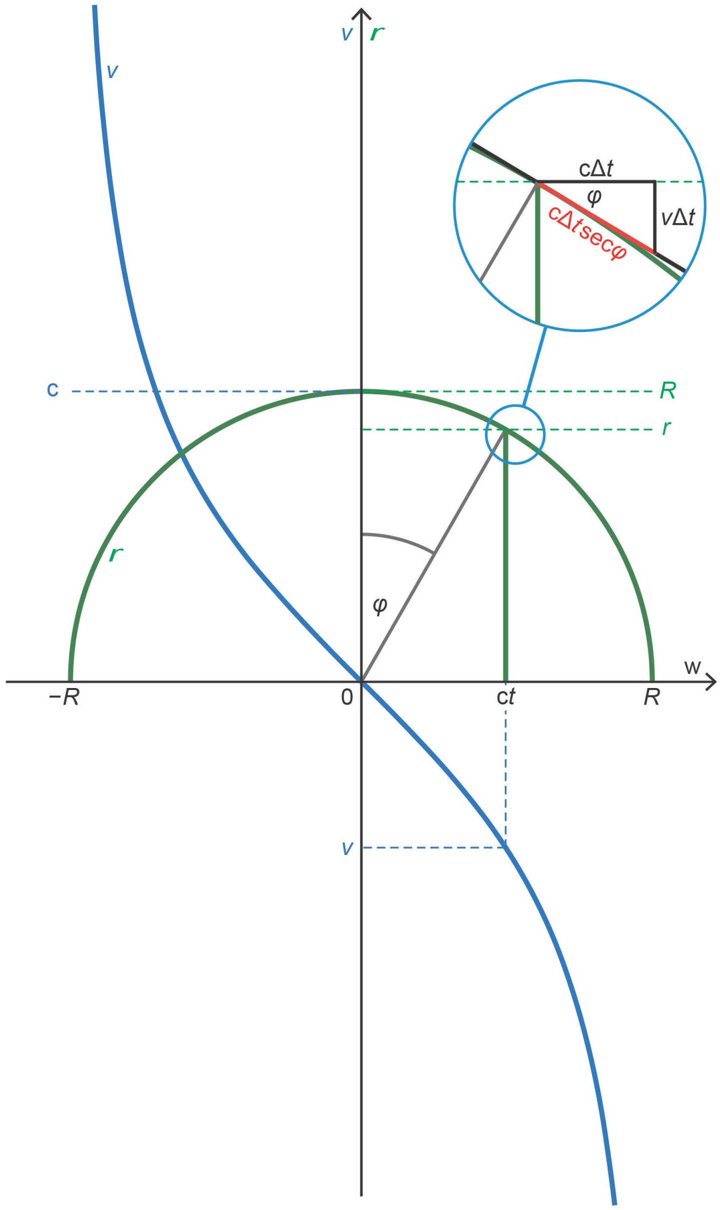

Figure 2. Radius of the S^3 universe and its expansion velocity. The green semicircle shows the change in the radius r of the S^3 universe with the passage of $w = ct$ at the coordinate time. φ is the S^3 latitude of the radius r at $w = ct$ with respect to the great sphere at $w = 0$. The blue curve depicts the expansion speed v of the radius r. In the enlarged circled inset, the ratio of the elongation $v\Delta t$ of the radius r to the advancement $c\Delta t$ of coordinate time is shown to be the same as the expansion speed ratio $v/c = -\tan(\varphi)$ with respect to the speed of light at latitude φ. Since this figure shows a contraction process, v is negative.

3. A Light Path of the S^3 Universe

For a stationary light source and observation point, the light path (distance of geodesic) that runs in the S^3 universe is elongated when compared to the light path that runs in a stationary space, because the radius r of the S^3 universe changes during the time it takes for the light emitted from the light source to reach the observation point. As shown in **Figure 2** (circled), the light path of a

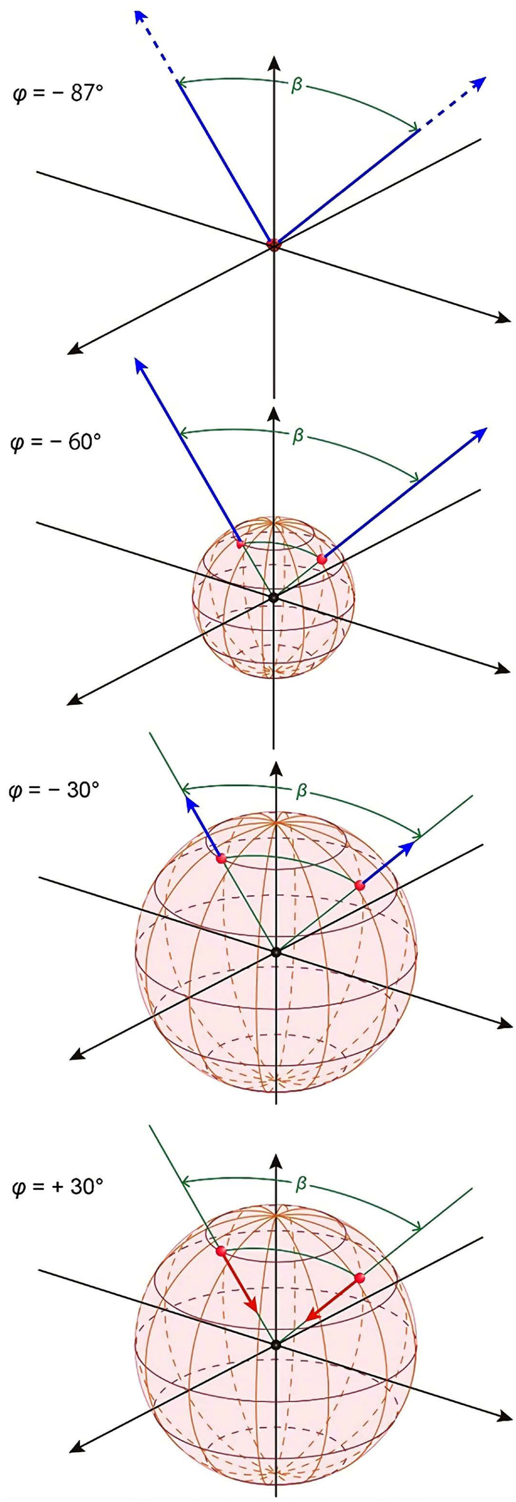

Figure 3. Longitude difference between two celestial bodies. This represents the S^3 universe at each latitude φ as seen from the outside. The small red spheres represent both stationary celestial bodies, and the arrows extending from the celestial bodies represent positive expansion velocity vectors in blue and negative expansion velocity vectors in red, respectively. The angle β formed by the expansion velocity vectors of the two celestial bodies is the longitude difference between their spatial coordinates, and it is always constant regardless of the latitude φ of the S^3 universe.

minute distance is elongated to the distance of hypotenuse $c\Delta t\sec(\varphi)$ compared to the distance $c\Delta t$, when light runs in a stationary space. This elongation of the light path occurs due to the principle of constant light speed. The reason for this is that the proper time needed for the light to travel this minute distance is greater than the coordinate time required to travel the same distance. When the proper time of the S^3 universe at latitude φ is τ, the light path of the minute distance, $c\Delta\tau \approx c\Delta t\sec(\varphi)$, and the elongation of the proper time relative to the coordinate time can be expressed as $d\tau/dt = \sec(\varphi)$, as shown in **Figure 4** (blue line). When the ratio $\sec(\varphi)$ is integrated so that the proper time begins from zero and the integration constant is $\pi/2$, the following equation that shows the elongation of the proper time with respect to coordinate time is obtained as shown in **Figure 4** (green line).

$$\alpha = \frac{c}{R}\int \sec(\varphi)\,dt = \frac{c}{R}\int\left[\arcsin\left(\frac{ct}{R}\right)\right]dt = \arcsin\left(\frac{ct}{R}\right) + \frac{\pi}{2} = \varphi + \frac{\pi}{2} \tag{1}$$

The result of this integration shows the advancement of longitude (angular distance), in which light runs in the expanding or contracting space coordinate from the origin, while coordinate time advances in the direction of the w-coordinate

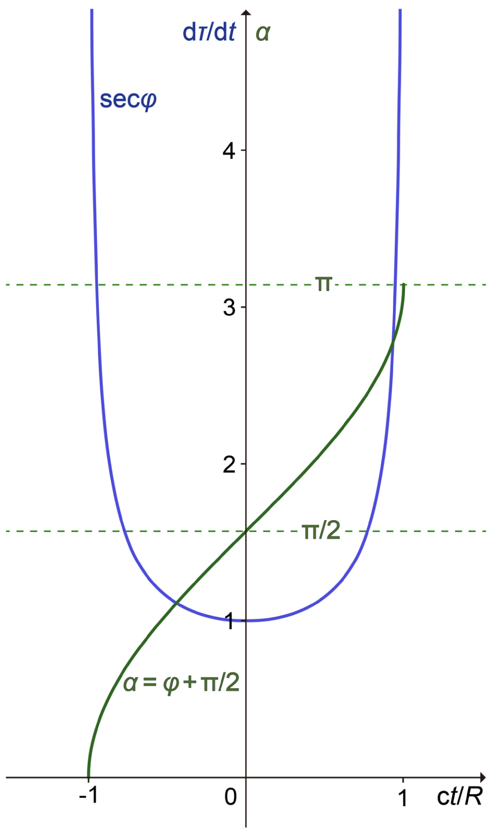

Figure 4. Progression of proper time and visible longitude of the S^3 universe. The horizontal axis represents the ratio of the w-coordinate to the maximum radius R. The blue line shows the progression of the proper time with respect to the coordinate time $d\tau/dt$. The green line depicts the angular distance α traveled by the light of the Big Bang, which is the visible longitude of the observable universe.

from $w = -R$ to ct. Since this is what we call visible longitude α, the equation of $\alpha = \varphi + \pi/2$ holds true. The path of light in the S^3 universe from origin to latitude φ is shown by a green arc of radius $R/2$ in Figure 5(d), and the distance of the light path is the same as the arc of the great circle $R\alpha$ of radius R and angle α depicted in Figure 5. This $R\alpha$ represents the optical path length of the Big Bang light, namely the CMB observed at an arbitrary latitude φ, and $R\alpha$ is hereinafter referred to as an optical radius. In Figure 5, the longitude difference between the observation point G and the light sources G_0, G_1, G_2, and G_3 on the optical path is always constant. Therefore, the longitude difference β between the observation point and each light source is equal to the difference between latitude φ when the light is observed at the observation point and latitude φ_s when the light is emitted from the light source. In other words, $\beta = \varphi - \varphi_s$ holds.

4. Structure of the S^3 Universe as Revealed by an Internal View

We consider a single path of light from the Big Bang that ran soon after the birth of the S^3 universe. When this light travels on an S^3 circumference with varying radius r and arrives at an observation point, it appears to an observer to be travelling in a one-dimensional straight line. This one-dimensional straight line is a line segment of a certain length for an observer standing in the center. The distance from the observer to the end of the line segment is optical radius a of the light from the Big Bang. The observer observes the light simultaneously with light emitted on the same line by a star. Here we assume a three-dimensional space in which all lights appear straight, and we use Cartesian coordinates x', y', z' with an observation point G as an origin. Two paths of light are emitted at the Big Bang and reach the observation point by running right and left on the circumference of the S^3 universe becoming one-dimensional segments starting from an origin G and extending right and left on the x'-axis of these coordinates, as shown in Figure 6(a). When this line is rotated 180 degrees (π) around the z'-axis, it becomes an inside-filled circle of radius $R\alpha$ in a two-dimensional x-y plane, as shown in Figure 6(b). Lights coming from all directions to an observer in the center include not only the lights from the Big Bang, but also lights originating in stars and traveling along the light paths. Furthermore, when the two-dimensional circle is rotated 180 degrees (π) around the y-axis, it generates an inside-filled three-dimensional sphere of optical radius $R\alpha$ at the time of observation, as shown in Figure 6(c). This sphere represents the visible universe at latitude φ, and the origin of the S^3 universe appears evenly spread in the outer shell of the visible universe at that time. We show the changes in latitude φ and optical radius $R\alpha$ of the visible universe with the passage of coordinate time in Figure 7(a) and Figure 7(b). The rate of change of the optical radius of the visible universe is shown by the blue line in Figure 4 and is the same as the change of $d\tau/dt$; it decreases during an expansion phase and increases during a contraction phase. When the farthest light (electromagnetic wave) from visible longitude π reaches the observation point, the entire universe converges to one point and ends.

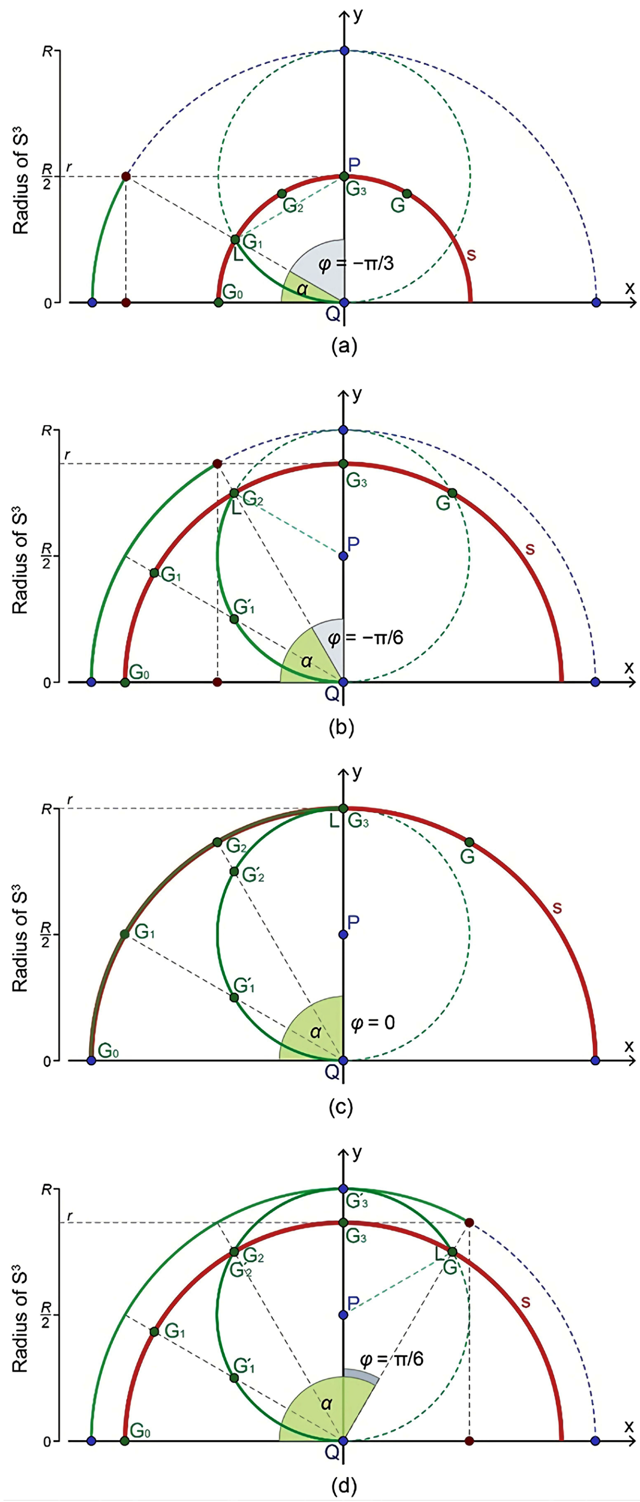

Figure 5. The red semicircles in panels (a), (b), (c) and (d) show x-y cross sections of the S^3 universe at latitude φ, and the green arc QL of radius $R/2$ and center P shows a path of light emitted from origin Q and running the distances of $\alpha = \pi/6$, $\pi/3$, $\pi/2$ and $2\pi/3$, respectively. The light path of arc QL is the same as the length $R\alpha$ of the arc EM, which is a projection of radius R on the great sphere. G_0, G_1, G_2, G_3 and G are celestial bodies on the light path, and longitude differences between them do not change despite the changes in the radius of the S^3 universe.

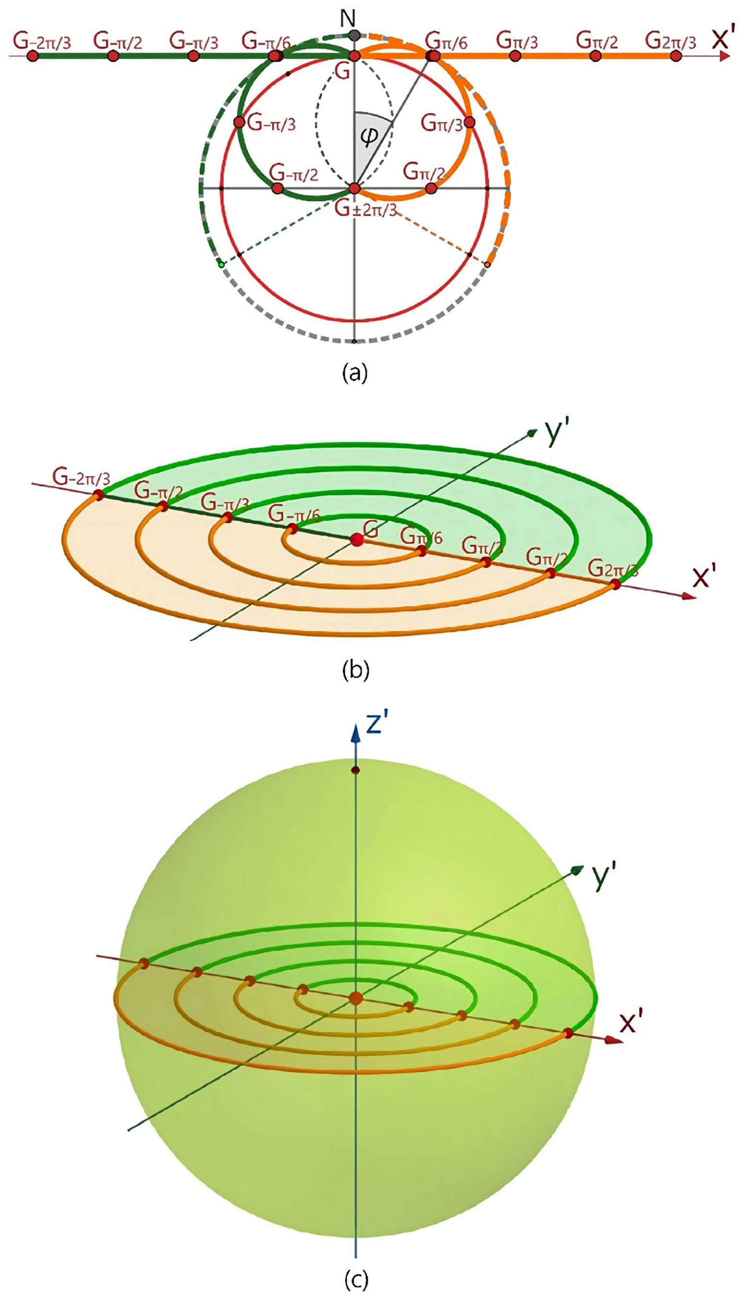

Figure 6. Visible structure of the S^3 universe. (a) The red circle is the circumference s at latitude $\varphi = \pi/6$, and G is the stationary observation point on the circumference. The green and orange arcs indicate optical paths (light trajectories) from the visible longitude $\pm\alpha$ ($\alpha = \varphi + \pi/2 = 2\pi/3$) observed at point G at $\varphi = \pi/6$. Gs with a subscript indicate the optical path positions of a light source (an astronomical object) at the time of emission. Each subscript represents the longitude β from observation point G. The horizontal green and orange lines extending from G to the left and right are line segments on an x'-coordinate axis of three-dimensional Cartesian coordinates (x'-, y'-, z'-axis) with observation point G as the origin. The optical radius ($R\alpha = 2\pi R/3$) is given from the observation point G to the left and right. (b) When this one-dimensional line segment is rotated through 180 degrees (π), it becomes a two-dimensional circle with radius a on the x-y plane. (c) When this two-dimensional circle is rotated 180 degrees (π), it transforms into a three-dimensional sphere with an optical radius $R\alpha$.

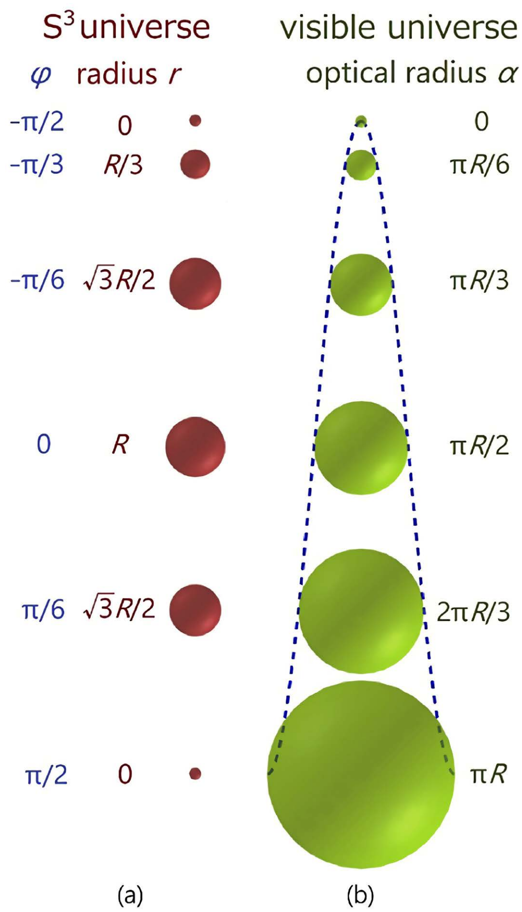

Figure 7. The S^3 universe as seen from external and internal perspectives. (a) The red spheres show the S^3 universe as seen from the outside at some latitude φ. (b) Yellow-green spheres are full three-dimensional spheres that represent the visible universe with an optical radius a that can be observed from any point G inside the S^3 at latitude φ.

5. Redshift Observed in the S^3 Universe

The observation that the current universe appears to be expanding in an accelerated way [13] [14] [15] [16] is also confirmed in our S^3 universe model. The reason for this observation is that we are looking at past light emitted by a star when the velocity of expansion was larger than that at the time of observation. According to the principle of constant light velocity, we believe that light in a flat spacetime travels at velocity c with respect to the coordinate time t, and light in an expanding or contracting S^3 universe travels at velocity of c with respect to the proper time τ of a passing point. Consequently, the expansion velocity of the radius r against the light velocity $v/c = -\tan(\varphi)$ can be regarded as elongation of proper time τ at latitude φ against coordinate time t. During the expansion phase of the universe, the rate of expansion decreases depending on the light path, namely the difference in longitude, during the period of light emission to the observation point with the passage of time (the area of positive v in **Figure 2**). Therefore, redshift increases, as the passage of proper time at the time of observation is delayed compared to that at light emission, and the wave length of the observed light increases. Since the energy of light at the time of emission reflects the frequency determined by the proper time, and the increase of proper time compared to coordinate time at the time of emission at latitude φ is $\sec(\varphi)$ (blue line in **Figure 4**), the frequency ν_τ of light L at the emission time in terms of proper time becomes $\sec(\varphi)$-fold of the frequency ν_t of coordinate time. In other words, $\nu_\tau/\nu_t = \sec(\varphi)$, and the ratio of wave lengths becomes the inverse of this equation, namely, $\lambda_\tau/\lambda_t = \cos(\varphi)$.

When the observation point G is on the S^3 universe of latitude φ, we consider the light that has travelled a distance of $R\beta$ from the light source G_s to an observation point G with longitude difference β. If we assume that a light source G_s at the time of light emission is of latitude φ_s, the wave length of the light in terms of proper time at the time of emission is λ_s, the latitude of the observation point G at the time of observation is φ and the wave length in terms of proper time is λ. We obtain $\varphi_s = \varphi - \beta$, because the longitude difference β between the observation point and the light source is the same as the latitude difference $\varphi - \varphi_s$ between the times of light emission and observation. If the elongation of the wave lengths of a light source and an observation point, in terms of proper time are respectively $\lambda_s/\lambda_t = \cos(\varphi_s)$ and $\lambda/\lambda_t = \cos(\varphi)$, the elongation of the wave length at the time of observation against that of emission is expressed as follows.

$$\frac{\lambda}{\lambda_s} = \frac{\lambda/\lambda_t}{\lambda_s/\lambda_t} = \frac{\cos(\varphi)}{\cos(\varphi_s)} = \frac{\cos(\varphi)}{\cos(\varphi-\beta)} \tag{2}$$

An uppercase letter Z is used to denote the redshift to distinguish it from the letter z for coordinate. When the above equation is inserted in the redshift equation $Z = \lambda/\lambda_s - 1$, the following equation that shows redshift in the S^3 universe is obtained.

$$Z = \frac{\cos(\varphi)}{\cos(\varphi-\beta)} - 1 \tag{3}$$

Figure 8 shows a relationship between the longitude difference β from the observation point to the light source and redshift by selecting latitudes of the observation points of the S^3 universe at regular intervals. It can be understood from this figure that the redshift increases in an accelerated fashion as the longitude difference β between the light source and the observation point increases, and the curve of increase becomes milder with the increase in φ of the observation point. On the other hand, during the contraction phase ($\varphi > 0$), blueshift starts to appear, with minus Z beginning from the light of small β and the light source to large β in this order. Upon the increase in latitude φ, the area of the blueshift expands. At the observation point during the contraction phase, blueshift increases when the longitude difference is $\beta < \varphi$, since Z becomes minus; blueshift becomes maximum at $\beta = \varphi$, and the next blueshift decreases at $\varphi < \beta < 2\varphi$, and finally $Z = 0$ at $\beta = 2\varphi$. Furthermore, we found that the redshift Z increases in an accelerated fashion even during the contraction phase, depending on the increase in longitude difference β, when the longitude difference β exceeds

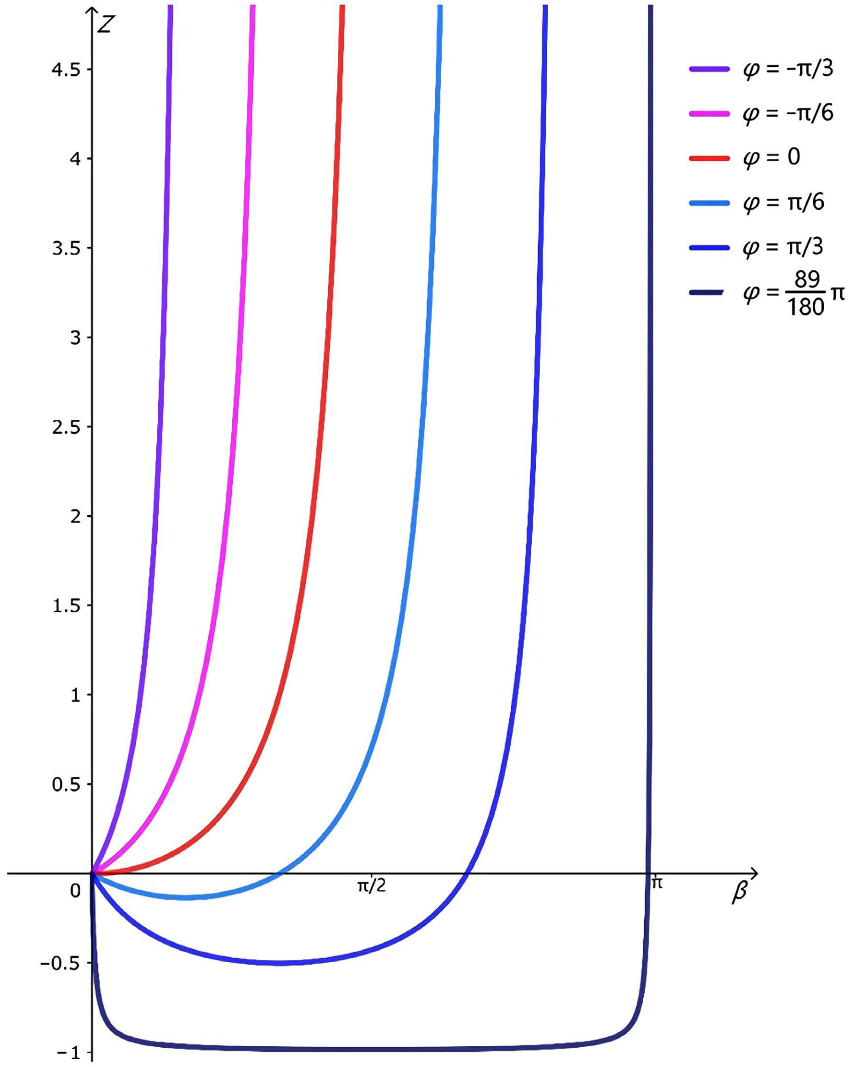

Figure 8. Redshift in the S^3 universe. This figure shows redshift of light observed at latitudes $\varphi = -\pi/3$, $-\pi/6$, 0, $\pi/6$, $\pi/3$ and $(89/180)\pi$. The vertical axis represents redshift Z, and the horizontal axis depicts the difference in longitude β between the observation point and the light source.

2φ. We also discovered that the redshift Z at the observation point of latitude φ is determined by the light path $R\beta$ up to the light source, because the horizontal axis β of **Figure 8**, namely the difference β, is proportional to the length of the light path $R\beta$.

6. Conclusion

In our S^3 universe model, the visible universe is seen from the internal viewpoint as a three-dimensional sphere centered on an arbitrary observation point G, and the origin of the S^3 universe appears to spread evenly over the entire celestial body, which is the outer shell of the visible universe. The longitude difference between stationary celestial bodies in the S^3 universe is always constant, and the optical path length of light emitted from the celestial body of the light source and reaching the celestial body of the observation point is always constant, regardless of the change in the dynamic diameter of the S^3 universe. However, as shown in **Figure 8**, the redshift observed in the S^3 universe is determined by the longitude difference β between the light source and the observation point, as well as the latitude φ of the S^3 universe at the time of observation. From the internal viewpoint of the S^3 universe, when the universe expands and reaches the large spherical surface and then contracts, the blueshift is observed in the order from the celestial body with the smallest longitude difference (**Figure 8**, $\varphi = \pi/6$ blue curve, and $\pi/3$ dark blue curves). On the other hand, blueshifts have been observed in some of the closest galaxies in the present universe, which in the S^3 universe model coincides with the early stage of contraction immediately after the large sphere. Furthermore, from the observation of type Ia supernovae, it seems that the current universe began to accelerate and expand again, which is consistent with the increase in negative expansion rate in the early stages of the contraction of the S^3 universe. This result suggests that the S^3 universe model is one of the leading candidates for the temporal change in the geometry of the present universe.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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