Journal of Modern Physics, 2022, 13, 1147-1156
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

# Including Space-Time in the Extended Group $Cl_3^*$ of Relativistic Form-Invariance

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**How to cite this paper:** Daviau, C. and Bertrand, J. (2022) Including Space-Time in the Extended Group $Cl_3^*$ of Relativistic Form-Invariance. *Journal of Modern Physics*, **13**, 1147-1156.
https://doi.org/10.4236/jmp.2022.138066

**Received:** July 16, 2022
**Accepted:** August 12, 2022
**Published:** August 15, 2022



## Abstract

The inclusion of space-time in the extended group of relativistic form-invariance, $Cl_3^*$, is specified as the inclusion of the whole space-time manifold in this multiplicative Lie group. First physical results presented here are: the geometric origin of the time arrow, a better understanding of the non-simultaneity in optics and a mainly geometric origin for the universe expansion, and its recent acceleration.

## Keywords

Space-Time Manifold, Invariance Group, Standard Model, Acceleration of Expansion

## 1. Introduction

The inclusion of space-time in the group of form-invariance of the relativistic quantum theory of the electron results from our previous works:

1) We have expressed the Dirac theory of the electron in $Cl_3$, Clifford algebra of the 3-dimensional space [1] [2] [3].

2) We have extended the form-invariance of the Dirac theory from $SL(2,\mathbb{C})$ to $GL(2,\mathbb{C}) = Cl_3^*$, where $Cl_3^*$ is the multiplicative group of the invertible elements in $Cl_3$ [4].

3) The value of the quantum wave has been extended to $\mathrm{End}(Cl_3)$, the Lie group of invertible linear applications in $Cl_3$, with its subgroup $Cl_3^*$ as group of relativistic form-invariance, and the $U(1)\times SU(2)\times SU(3)$ group of the Standard Model as group of gauge invariance [5]-[15].

### Nearly a Century Ago

Early quantum physics, as soon as 1927, wrote in the framework of the Pauli's wave equation:

$$\vec{\mathrm{x}} = \begin{pmatrix} \mathrm{x}^3 & \mathrm{x}^1 - i\mathrm{x}^2 \\ \mathrm{x}^1 + i\mathrm{x}^2 & -\mathrm{x}^3 \end{pmatrix} = \mathrm{x}^1\sigma_1 + \mathrm{x}^2\sigma_2 + \mathrm{x}^3\sigma_3. \tag{1}$$

where the three $\sigma_j$ matrices are the well-known Pauli matrices. The set $M_2(\mathbb{C})$ of the 2 × 2 complex matrices is isomorphic to the Clifford algebra $Cl_3$ of the 3-dimensional space (see for instance [14] A.3 for more details). The center of this real algebra is isomorphic to $\mathbb{C}$, this allows quantum physics to identify $Cl_3$ and $M_2(\mathbb{C})$, the center being identified to the set of scalar matrices.

Starting from the Pauli equation, P.A.M. Dirac wrote a relativistic wave equation [16] [17]. Since this equation uses time at the same level as space coordinates, the relativistic invariance needs an extension of the previous inclusion in $M_2(\mathbb{C})$:

$$\mathrm{x} := \mathrm{x}^\mu\sigma_\mu = \mathrm{x}^0 + \vec{\mathrm{x}} = \begin{pmatrix} \mathrm{x}^0 + \mathrm{x}^3 & \mathrm{x}^1 - i\mathrm{x}^2 \\ \mathrm{x}^1 + i\mathrm{x}^2 & \mathrm{x}^0 - \mathrm{x}^3 \end{pmatrix}; \mathrm{x}^0 := ct. \tag{2}$$

And then space-time is identified with the auto-adjoint subset of the Pauli algebra $Cl_3$, which is the part of the $M$ elements satisfying $M = M^\dagger$. We note $\overline{\mathrm{x}}$ the co-matrix:

$$\overline{\mathrm{x}} := \mathrm{x}^0 - \vec{\mathrm{x}} = \begin{pmatrix} \mathrm{x}^0 - \mathrm{x}^3 & -\mathrm{x}^1 + i\mathrm{x}^2 \\ -\mathrm{x}^1 - i\mathrm{x}^2 & \mathrm{x}^0 + \mathrm{x}^3 \end{pmatrix}. \tag{3}$$

Thus the space-time metric satisfies:

$$\mathrm{x}\overline{\mathrm{x}} = \overline{\mathrm{x}}\mathrm{x} = \det(\mathrm{x}) = (\mathrm{x}^0)^2 - (\mathrm{x}^1)^2 - (\mathrm{x}^2)^2 - (\mathrm{x}^3)^2. \tag{4}$$

## 2. Form-Invariance of the Dirac Equation

Let $M$ be any nonzero element in $Cl_3$ (that means any fixed nonzero Pauli matrix) and let $R$ be the transformation of space-time into itself such that for any $\mathrm{x}$ is associated $\mathrm{x}'$ given by

$$\mathrm{x}' = \mathrm{x}'^0 + \vec{\mathrm{x}}' = R(\mathrm{x}) = M\mathrm{x}M^\dagger. \tag{5}$$

We note, if $\det(M) \neq 0$:

$$\det(M) = re^{i\theta},\ r = |\det(M)|. \tag{6}$$

Then $r$ is the modulus and $\theta$ is an argument of the determinant of $M$. We get:

$$\begin{aligned}(\mathrm{x}'^0)^2 - (\mathrm{x}'^1)^2 - (\mathrm{x}'^2)^2 - (\mathrm{x}'^3)^2 &= \det(\mathrm{x}') = \det(M\mathrm{x}M^\dagger) \\ &= re^{i\theta}\det(\mathrm{x})re^{-i\theta} = r^2\left[(\mathrm{x}^0)^2 - (\mathrm{x}^1)^2 - (\mathrm{x}^2)^2 - (\mathrm{x}^3)^2\right].\end{aligned} \tag{7}$$

Therefore $R$ multiplies any space-time distance by $r$ and we name this transformation "similitude with ratio $r$". We name $M$ the "dilator" of the similitude $R$, and we define the $R^\mu_\nu$ matrix of this similitude as follows:

$$\mathrm{x}'^{\mu} = R^{\mu}_{\nu}\mathrm{x}^{\nu}. \tag{8}$$

For any dilator $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \neq 0$:

$$2R^0_0 = |a|^2 + |b|^2 + |c|^2 + |d|^2 > 0, \tag{9}$$

Thus $\mathrm{x}'^0$ has the same sign as $\mathrm{x}^0$ at the origin: the similitude $R$ conserves the time arrow. Moreover, for any dilator $M$ in $Cl_3$, we have (proof in [14] A.4.5):

$$\det\left(R^{\mu}_{\nu}\right) = r^4. \tag{10}$$

Hence if $r$ is nonzero $r^4 > 0$: $\det(R) > 0$. Thus $R$ conserves the orientation of space-time and since the transformation conserves the orientation of time, $R$ conserves also the orientation of space. Using only $Cl_3$ for the Dirac theory (see [14] 1.3) the linear Dirac equation is expressed as:

$$\begin{aligned} &0 = \nabla\hat{\phi}\sigma_{21} + qA\hat{\phi} + m\phi; \hat{\phi} := \overline{\phi}^{\dagger}; \sigma_{21} := \sigma_2\sigma_1 = -i\sigma_3; \nabla := \sigma^{\mu}\partial_{\mu}, \\ &\sigma^0 := \sigma_0 = 1; \sigma^j := -\sigma_j, j = 1,2,3. \end{aligned} \tag{11}$$

The form-invariance of the Dirac equation results from:

$$\phi' = M\phi; \mathrm{x}' = M\mathrm{x}M^{\dagger}; \nabla = \overline{M}\nabla'\hat{M}; \nabla' = \sigma^{\mu}\frac{\partial}{\partial \mathrm{x}'^{\mu}} \tag{12}$$

Which gives:

$$0 = \nabla'\hat{\phi}'\sigma_{21} + q'A'\hat{\phi}' + m'\phi', \tag{13}$$

$$qA = \overline{M}q'A'\hat{M}; m = r\mathrm{e}^{i\theta}m'. \tag{14}$$

We then have a double inclusion: space-time of special relativity is included in $Cl_3$ and the $SU(2)$ group of invariance of non-relativistic quantum theory is a subgroup of $SL(2,\mathbb{C})$, itself a subgroup of $GL(2,\mathbb{C}) = Cl_3^*$, where $Cl_3^*$ is the multiplicative Lie group of the invertible elements in $M_2(\mathbb{C}) = Cl_3$, itself Lie algebra of $Cl_3^*$. Moreover $Cl_3^*$ is a subgroup of $\mathrm{End}(Cl_3)$, which is a group containing the $U(1)\times SU(2)\times SU(3)$ group of the Standard Model (see [14] Chapter 2 and Chapter 3).

The only difficulty of (14), the $\mathrm{e}^{i\theta}$ factor, is solved with the simplification of the Dirac Lagrangian containing $\overline{\psi}\psi = \rho\cos(\beta) = \Re[\det(\phi)]$ where we have suppressed the cosine (see [14] 1.5). This gives our improved nonlinear wave equation:

$$0 = \overline{\phi}\left(\nabla\hat{\phi}\right)\sigma_{21} + \overline{\phi}qA\hat{\phi} + m\rho, \tag{15}$$

where the form-invariance of the wave equation results now from:

$$0 = \overline{\phi}'\left(\nabla'\hat{\phi}'\right)\sigma_{21} + \overline{\phi}'q'A'\hat{\phi}' + m'\rho', \tag{16}$$

$$\phi' = M\phi; qA = \overline{M}q'A'\hat{M}; m = rm'. \tag{17}$$

And since each interesting solution of the Dirac equation has values in $Cl_3^*$ (see [14] 1.5.3 and 1.5.7), we may suppose the inclusion of the space-time manifold itself in $Cl_3^*$. This hypothesis also relies on the experimental building of geometry, by telescopes which are turned before each observation: the invariance under rotation is always assumed [18] and since quantum mechanics replaces the invariance under rotation by the invariance under $SU(2)$, the invariance under a subgroup of $Cl_3^*$ is necessarily at the center of the geometry of the universe. Moreover, space-time is a 4-dimensional manifold, thus $Cl_3$, which is 8-dimensional, is large enough to host the space-time manifold.

## 3. Space-Time Manifold in $Cl_3^*$

### 3.1. Local and Global Structure of Space-Time

Since any measurement of length is always a measurement of the ratio between two lengths, we let

$$\mathbf{x} := \frac{\mathrm{x}}{l_a}; \mathbf{x} \in Cl_3, \tag{18}$$

where $l_a = \sqrt{\alpha} l_P$ is an absolute length, linked to the fine structure constant $\alpha$ and to the Planck length $l_P$ [15]. The first difference with classical geometry is that the origin of the measure of time and space is at $\mathbf{x} = 1$ (neutral element of the Lie group), not 0 which is the neutral element of the Lie algebra. Second, $Cl_3$ is the Lie algebra of the $Cl_3^*$ multiplicative group. This means that the vicinity of any point $O$ is isomorphic to $Cl_3$. This set is a linear space which contains two subsets: $Cl_3^*$, which is the set of $\mathbf{x}$ satisfying $\det(\mathbf{x}) \neq 0$, and the light cone, which is the set of $\mathbf{x}$ satisfying $\det(\mathbf{x}) = 0$. Third, these conditions exclude themselves, therefore the light cone is included in each (local) Lie algebra, not in the (global) Lie group $Cl_3^*$. Fourth, the only link between each Lie algebra and the whole Lie group is the exponential function, which we calculate as follows:

$$\mathbf{x} = a + b\mathbf{u}; \mathbf{u} = x^1\sigma_1 + x^2\sigma_2 + x^3\sigma_3; (x^1)^2 + (x^2)^2 + (x^3)^2 = 1,$$

$$\mathbf{x}^n = \frac{1}{2}\left[(a+b)^n(1+\mathbf{u}) + (a-b)^n(1-\mathbf{u})\right] \tag{19}$$

$$\exp(\mathbf{x}) = \sum_{n=0}^{\infty}\frac{\mathbf{x}^n}{n!} = \frac{1}{2}\left[\mathrm{e}^{a+b}(1+\mathbf{u}) + \mathrm{e}^{a-b}(1-\mathbf{u})\right] = \mathrm{e}^a\left[\cosh(b) + \sinh(b)\mathbf{u}\right]. \tag{20}$$

Thus the same unitary vector $\mathbf{u}$ ($\mathbf{u}^2 = 1$) is used for $\mathbf{x}$ and for $\exp(\mathbf{x})$. Moreover, we have:

$$\det\left[\exp(\mathbf{x})\right] = \exp\left[\mathrm{tr}(\mathbf{x})\right] = \mathrm{e}^{2a}. \tag{21}$$

Thus, with $\exp(\mathbf{x}) = A + B\mathbf{u} = A + B(x^1\sigma_1 + x^2\sigma_2 + x^3\sigma_3)$ we obtain:

$$\mathrm{e}^{2a} = \det\left[\exp(\mathbf{x})\right] = (A + B\mathbf{u})(A - B\mathbf{u}) = A^2 - B^2 \tag{22}$$

This implies that the light cone ($A^2 = B^2$) is the boundary of the space-time manifold and that nothing exists outside this boundary, since $\mathrm{e}^{2a} > 0$. From this sign we may see the purely local character of the classification of events in five categories.[1] We obtain:

$$\mathrm{e}^{a}=\sqrt{A^2-B^2};\cosh(b)+\sinh(b)\mathbf{u}=\frac{A+B\mathbf{u}}{\sqrt{A^2-B^2}}.$$

$$a=\ln\left(\sqrt{A^2-B^2}\right)=\frac{1}{2}\left[\ln(A+B)+\ln(A-B)\right], \tag{23}$$

$$b=\sinh^{-1}\left[\frac{B}{\sqrt{A^2-B^2}}\right]=\frac{1}{2}\left[\ln(A+B)-\ln(A-B)\right],$$

$$a+b=\ln(A+B);\ A+B=\mathrm{e}^{a+b}. \tag{24}$$

## 3.2. The EPR Paradox

Two photons are emitted at the point-event $O$. We suppose, simplifying the calculation, that they are emitted in two orthogonal directions, $\sigma_1$ and $\sigma_2$. They are absorbed at the same time $y>0$, also to simplify the calculation. The photon emitted in the direction $\sigma_1$ is absorbed at the point-event:

$$\mathbf{x}_1=a_1+b_1\mathbf{u}_1=(a+y)+\left(bx^1+y\right)\sigma_1+b\left(x^2\sigma_2+x^3\sigma_3\right),$$

$$a_1=a+y;\mathbf{u}_1=x_1^1\sigma_1+x_1^2\sigma_2+x_1^3\sigma_3;\left(x_1^1\right)^2+\left(x_1^2\right)^2+\left(x_1^3\right)^2=1,$$

$$\left(x^1+y/b\right)^2+\left(x^2\right)^2+\left(x^3\right)^2=1+2x^1y/b+(y/b)^2, \tag{25}$$

$$b_1=b\sqrt{1+2x^1y/b+(y/b)^2};\mathbf{u}_1=\frac{\left(x^1+y/b\right)\sigma_1+x^2\sigma_2+x^3\sigma_3}{\sqrt{1+2x^1y/b+(y/b)^2}}.$$

The photon emitted in the direction $\sigma_2$ is absorbed at the point-event:

$$\mathbf{x}_2=a_2+b_2\mathbf{u}_2=(a+y)+bx^1\sigma_1+\left(bx^2+y\right)\sigma_2+bx^3\sigma_3. \tag{26}$$

And we also have:

$$a_2=a+y;\mathbf{u}_2=x_2^1\sigma_1+x_2^2\sigma_2+x_2^3\sigma_3;\left(x_2^1\right)^2+\left(x_2^2\right)^2+\left(x_2^3\right)^2=1,$$

$$\left(x^1\right)^2+\left(x^2+y/b\right)^2+\left(x^3\right)^2=1+2x^2y/b+(y/b)^2, \tag{27}$$

$$b_2=b\sqrt{1+2x^2y/b+(y/b)^2};\mathbf{u}_2=\frac{x^1\sigma_1+\left(x^2+y/b\right)\sigma_2+x^3\sigma_3}{\sqrt{1+2x^2y/b+(y/b)^2}}.$$

On the space-time manifold, the emission is at $O=\mathbf{x}/l_a=A+B\mathbf{u}$ while the photon emitted in the direction $\sigma_1$ is absorbed at the point-event $M=\mathbf{x}_1/l_a=\exp(\mathrm{x}_1)$. The photon emitted in the direction $\sigma_2$ is absorbed at the point-event $P=\mathbf{x}_2/l_a=\exp(\mathrm{x}_2)$. The position of the point event $P$, seen from 1, is:

$$\mathbf{x}_2^0=\left[\exp(\mathbf{x})\right]^{-1/2}\exp(\mathrm{x}_2)\left[\exp(\mathbf{x})\right]^{-1/2}. \tag{28}$$

The position of the point event $P$, seen from $M$, is:

[1]E being a given event, the five categories are: events on the future light cone of E; events on the past light cone of E; events inside the future light cone of E; events inside the past light cone of E; elsewhere: all other events.

$$\mathbf{x}_2^1 = \left[\exp(\mathbf{x}_1)\right]^{1/2}\left[\exp(\mathbf{x})\right]^{-1/2}\exp(\mathrm{x}_2)\left[\exp(\mathbf{x})\right]^{-1/2}\left[\exp(\mathrm{x}_1)\right]^{1/2}. \tag{29}$$

The position of the point event *M*, seen from 1, is:

$$\mathbf{x}_1^0 = \left[\exp(\mathbf{x})\right]^{-1/2}\exp(\mathrm{x}_1)\left[\exp(\mathbf{x})\right]^{-1/2}. \tag{30}$$

The position of the point event *M*, seen from *P*, is:

$$\mathbf{x}_1^2 = \left[\exp(\mathbf{x}_2)\right]^{1/2}\left[\exp(\mathbf{x})\right]^{-1/2}\exp(\mathrm{x}_1)\left[\exp(\mathbf{x})\right]^{-1/2}\left[\exp(\mathrm{x}_2)\right]^{1/2}. \tag{31}$$

And we have, since the determinant of a product is the product of the determinants:

$$\det\left(\mathbf{x}_2^1\right) = \mathrm{e}^{a+y}\mathrm{e}^{-a}\mathrm{e}^{2(a+y)}\mathrm{e}^{-a}\mathrm{e}^{a+y} = \mathrm{e}^{2(a+y+y)},$$
$$\det\left(\mathbf{x}_1^2\right) = \mathrm{e}^{a+y}\mathrm{e}^{-a}\mathrm{e}^{2(a+y)}\mathrm{e}^{-a}\mathrm{e}^{a+y} = \mathrm{e}^{2(a+y+y)}. \tag{32}$$

Therefore at each point-event, when a photon is absorbed at the local time $a+y$, each observer sees the absorption of his photon as preceding, with the same length of time *y*, the arrival of the photon for the other observer: the absorption of the other photon is in the future of each observer, not at the moment of arrival. This strange result seems very similar to the fact that each observer sees any length shorter for a moving object: an observer in the moving object also sees the other observer as moving, thus with shorter length. The paradox is that a measurement made on either of the particles apparently collapses the state of the entire entangled system and does so *instantaneously*, before any information about the measurement result could have been communicated to the other particle. Our previous calculation shows the key of the paradox: the instantaneous character of the measurement is simply false, the "collapse" of the quantum wave only results from the supposition (without any mathematical proof) that this situation may be described by a tensor product of Hilbert spaces. Attention! We don't deny quantum entanglement; we say that the paradox is only in the interpretation of this situation by a non-relativistic Hermitian theory, whereas physics must account for this: each "fixed" observer is journeying in time on the space-time manifold, even if he does not travel in space.

The understanding of the true geometry of space-time simply requires the use of the space-time manifold itself, not merely the use of a flat tangent space-time at the particular point-event *O*. The main difference between the flat space-time of restricted relativity and the space-time manifold as part of the $Cl_3^*$ Lie group is the fact that the light cone is not included in the manifold: it is the single boundary of the manifold, included only in the Lie algebra $Cl_3$. This was difficult to detect, because the only indication to see this inclusion was the two-valued representation of rotations in quantum mechanics.

Einstein, Podolsky and Rosen said [19]: "From this follows that either: 1) the quantum-mechanical description of reality given by the wave function is not complete or 2) when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality. For if both of them had simultaneous reality—and thus definite values—these values would enter into the complete description, according to the condition of completeness."

Experiments with the polarization of two photons simultaneously emitted can neither prove (1) nor (2) because the absorption of these photons cannot be simultaneous at the points where each absorption is effective. The quantum wave used in [14], with value in $\mathrm{End}(Cl_3)$, not only with value in $\mathbb{C}$, is enough to prove that (1) was true in 1935, independently of what we now think about (2). More generally no contradiction can exist between general relativity and quantum mechanics. Any apparent contradiction results from bad approximations of relativistic laws.

### 3.3. The Time Arrow and the Expansion of the Universe

Any point of the space-time manifold is at a position:

$$X = l_a \exp(a + b\mathbf{u}) = l_a (A + B\mathbf{u});\ A = \mathrm{e}^a \cosh(b);\ B = \mathrm{e}^a \sinh(b). \tag{33}$$

Then the time position $l_a \mathrm{e}^a \cosh(b)$ is the product of positive real numbers: time is an oriented quantity, **the time arrow has a geometric root**. The $A$ variable goes from 0 to $+\infty$.

Now we consider a photon received at this position $X$, coming from a distant galaxy, for instance with the $\sigma_1$ direction. It was emitted at the position:

$$l_a \exp\left[a - y + \left(bx^1 - y\right)\sigma_1 + b\left(x^2\sigma_2 + x^3\sigma_3\right)\right] = l_a \exp(a_1 + b_1\mathbf{u}_1), \tag{34}$$

with[2]

$$a_1 = a - y;\ \mathbf{u}_1 = x_1^1\sigma_1 + x_1^2\sigma_2 + x_1^3\sigma_3;\ \left(x_1^1\right)^2 + \left(x_1^2\right)^2 + \left(x_1^3\right)^2 = 1,$$

$$\left(x^1 - y/b\right)^2 + \left(x^2\right)^2 + \left(x^3\right)^2 = 1 - 2x^1 y/b + \left(y/b\right)^{22}, \tag{35}$$

$$b_1 = b\sqrt{1 - 2x^1 y/b + \left(y/b\right)^2};\ \mathbf{u}_1 = \frac{\left(x^1 - y/b\right)\sigma_1 + x^2\sigma_2 + x^3\sigma_3}{\sqrt{1 - 2x^1 y/b + \left(y/b\right)^2}}.$$

The photon was emitted at:

$$x_e = l_a \mathrm{e}^{a_1}\left[\cosh(b_1) + \sinh(b_1)\mathbf{u}_1\right]. \tag{36}$$

At this point-event the local time was $t_e = l_a \mathrm{e}^{a_1}\cosh(b_1) \approx l_a \mathrm{e}^{a_1 + b_1}/2$. The same photon is absorbed at the point-event $X$, then at the local time $t_a = l_a \mathrm{e}^a \cosh(b) \approx l_a \mathrm{e}^{a+b}/2$. The only constant object of this geometry is the Lie algebra: each local tangent space, in each point of the manifold, is isomorphic to the Lie algebra of the group. We will then suppose that:

$$\mathrm{d}(a_1 + b_1) = \mathrm{d}(a + b);\ \frac{\mathrm{d}t_e}{t_e} = \frac{\mathrm{d}t_a}{t_a} \tag{37}$$

And we have:

[2]Since we now look at past, $a_1 < a$.

$$\frac{\nu_a}{\nu_e} = \frac{\mathrm{d}t_e}{\mathrm{d}t_a}. \tag{38}$$

In first approximation, $b_1 \approx b$, we obtain:

$$\frac{1}{1+z} = \frac{\nu_a}{\nu_e} = \frac{\mathrm{d}t_e}{\mathrm{d}t_a} = \frac{\mathrm{d}\left[l_a \mathrm{e}^{a_1}\cosh(b_1)\right]}{\mathrm{d}\left[l_a \mathrm{e}^{a}\cosh(b)\right]} \approx \frac{l_a \mathrm{d}a \mathrm{e}^{a-y}\cosh(b)}{l_a \mathrm{d}a \mathrm{e}^{a}\cosh(b)} = \frac{1}{\mathrm{e}^y} \approx \frac{1}{1+y}. \tag{39}$$

This means that the redshift, previously interpreted as a Doppler effect, due to the expansion of the universe, is a direct effect of the geometry of space-time, and the $z$ parameter, defined as $(\nu_e - \nu_a)/\nu_a$, is almost equal to $y$. But this is true only as a crude approximation, or as a false velocity. When $y$ is small this redshift seems proportional to $y$. The Hubble parameter (73.3 ± 1.4 km/s/Mpc) gives for the distance 1 Mpc the value $z = 0.0002443$, thus giving $R = l_a \mathrm{e}^{a+b}/2 \approx 6.3\times 10^{25}\ \mathrm{m}$.

Using the geometric condition (37), which results from the Lie algebra as the only fixed framework, independent from the space-time position on the manifold, we may calculate more precisely the ratio $\mathrm{d}t_e/\mathrm{d}t_a$ in the case where $y$ is small. We have:

$$\frac{\mathrm{d}\left[l_a \mathrm{e}^{a_1}\cosh(b_1)\right]}{\mathrm{d}\left[l_a \mathrm{e}^{a}\cosh(b)\right]} = \frac{\mathrm{d}\left[\mathrm{e}^{a-y}\cosh(b_1)\right]}{\mathrm{d}\left[\mathrm{e}^{a}\cosh(b)\right]} = \frac{\mathrm{e}^{-y}\cosh(b_1)}{\cosh(b)} = \frac{1}{f(y)} \tag{40}$$

$$f(y) := \mathrm{e}^y \frac{\cosh(b)}{\cosh(b_1)} \approx f(0) + yf'(0) + y^2\frac{f''(0)}{2} + \cdots \tag{41}$$

We use:

$$b_1 := bg(y) = \sqrt{b^2 - 2x^1 by + y^2};\ g(y) = \sqrt{1 - 2\frac{x^1}{b}y + \left(\frac{y}{b}\right)^2},$$

$$g(y) \approx 1 - \frac{x^1}{b}y + \frac{1-\left(x^1\right)^2}{2b^2}y^2 + \frac{x^1\left[1-\left(x^1\right)^2\right]}{2b^3}y^3 + \cdots. \tag{42}$$

And we obtain:

$$f(y) \approx \mathrm{e}^y \frac{\mathrm{e}^b}{\mathrm{e}^{b_1}} = \mathrm{e}^{a(y)} \tag{43}$$

$$a(y) = y + b - b_1 \approx \left(1+x^1\right)y - \frac{1-\left(x^1\right)^2}{2b}y^2 - \frac{x^1\left[1-\left(x^1\right)^2\right]}{2b^2}y^3 + \cdots,$$

$$f'(y) \approx a'(y)\mathrm{e}^{a(y)} = \left(1+x^1\right)\left[1 - \frac{1-x^1}{b}y - \frac{3x^1\left(1-x^1\right)}{2b^2}y^2 + \cdots\right]\mathrm{e}^{a(y)}. \tag{44}$$

From values of the Hubble parameter and of $l_a$ we obtain $a+b \approx 142$. We only know that $a > b > 0$. The ratio $a/b$ is unknown. If our position in the manifold is anywhere, for instance is $(a+b)/a \approx a/b$, we could have $a \approx 88$ and $b \approx 54$. This should give a ratio $B/A$ very close to 1. We now look at the acceleration or deceleration of the expansion.

### 3.4. Beginning of the Acceleration

Defining $h$ such that $h(y) := f(y)/y$ the redshift seems accelerated if and only if $h$ is increasing, hence if $h'(y) > 0$. We obtain:

$$\begin{aligned} y^2 h'(y) &= yf'(y) - f(y) \approx \left[ya'(y) - 1\right] \mathrm{e}^{a(y)} \\ &= \left[-1 + \left(1 + x^1\right) y - \frac{1 - \left(x^1\right)^2}{b} y^2 - \frac{3x^1 \left[1 - \left(x^1\right)^2\right]}{2b^2} y^3 + \cdots\right] \mathrm{e}^{a(y)}. \end{aligned} \tag{45}$$

For instance if $b = 40$ and $x^1 = 0.6$ we have:

$$y^2 h'(y) \approx \left[-1 + 1.6y - 0.016y^2 - 0.00036y^3 + \cdots\right] \mathrm{e}^{a(y)} \tag{46}$$

Thus, in this case, $h'(y) > 0$ if and only if

$$y > y_0,\ y_0 \approx 0.63. \tag{47}$$

Moreover the sign of the coefficient of $y^3$ indicates a sign change for large $y$, but the method of calculation used here does not give the value of this new change of sign.

Hence the acceleration of the expansion seems to begin near $y_0$, with possible differences depending on the directions of observation. And the expansion seems to decelerate for very large $z$. Thus there is no need for either black matter (but the movement of stars in galaxies and the movement of galaxies in galaxy clusters is another question) or repulsive gravity to explain all modern observations of cosmological redshifts.

## 4. Conclusions

The expansion of the Universe, as resulting of the geometry of the whole space-time, is much more satisfying than cosmological models issued from the hypothesis of a homogeneous and uniform density of matter at a large scale:

1) We obtain, without any other hypothesis than (37), an acceleration of the expansion with a beginning of this acceleration.

2) Since the main part of the expansion is not linked to the density of matter, the cosmic microwave background is necessarily uniform, its non uniformity coming only from the non uniformity of the density of matter. This non-uniformity is automatic since gravitation is highly nonlinear.

3) The hypothesis of uniformity of the matter density at a very large scale is contrary to all observations of modern astronomers [20].

4) We know that there is not enough ordinary matter to satisfy an expansion ruled by gravitation, and no satisfying supplementary matter has been found.

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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