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Quantum State Transfer between a Mechanical Oscillator and a Distant Moving Atom

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Abstract

We propose a scheme for high fidelity quantum state transfer from a mechanical oscillator to a distant moving atom. In the scheme, two optical cavities connected by an optical fiber are interacted effectively through adiabatically eliminating fiber mode under large detuning limit. The quantum state transfer fidelity can be raised asymptotically to 100% by optimizing the Gaussian pulse $G(t)$, the maximum atom-cavity coupling strength Ω_{max}, and the atomic velocity *v*. We also show that the affect of dissipation can be obviously depressed by synchronously increasing Ω_{max} and *v*.

Keywords

Quantum State Transfer, Mechanical Oscillator, Cavity QED

1. Introduction

It is well known that designing high fidelity quantum state transfer (QST) between spatially separated hybrid quantum systems plays a key role in quantum information process such as long range quantum communication [1] and distributed quantum computation [2]. By using shaped pulses method, QST can be implemented through quantum interface and map the state of a qubit onto another physically far apart [3]. Systems consist of optical cavity and mechanical oscillator [4], which can use photons to detect mechanical movement with high sensitivity in optical detecting process, is regarded as one of the important candidates for hybrid quantum systems to implement QST and has drawn a lot of research interest both theoretically and experimentally and may induce deep considerations for basic quantum problems [5]. A variety of fascinating schemes have been put forward to discuss QST from a stationary mechanical oscillator to another [6] [7] [8] or a cavity mode [9] [10]. It is shown that high transfer efficiency can be achieved by using adjustable cavity quantum electrodynamics (QED) parameters. For example, in the work presented by Sete, a quantum state can be efficiently transferred from an optical cavity to a distant mechanical oscillator by adjusting cavity damping rates and destructive interference [9]. However, previous approaches to optomechanical QST mostly deal with state transfer between mechanical oscillators and cavity modes. Motivated by the fact that atoms are preferred as suitable candidate for entangled state transfer [11], universal quantum gate [2], and even for multiplexed quantum memory [12], it is reasonable to discuss QST between a mechanical oscillator and an atom. In the present paper, we propose a scheme for QST from a mechanical oscillator to a moving atom based on fiber-mediated cavity QED approach under real-time cavity QED condition. The advantage of the scheme is it works in a robust way since high fidelity can be reached through optimizing coupling parameters to against the dissipation of quantum channel.

2. Theoretical Model

We consider a theoretical model consisted of a mechanical oscillator, two identical optical cavities, and a moving atom, as is shown in **Figure 1**. Optical cavity 1 is coupled to mechanical oscillator. Optical cavity 2 interacts with the moving atom. Two cavities are connected by an optical fiber.

We start by analyzing the subsystem cavity-fiber-cavity and write the Hamiltonian $H_{c,f}$ as [13]

$$H_{c,f} = \omega_1 a_1^+ a_1 + \omega_2 a_2^+ a_2 + \omega_f c^+ c + \nu\left(a_1^+ c + a_2^+ c + a_1 c^+ + a_2 c^+\right) \quad 1)$$

where $a_i\left(a_i^+\right)$ and $c\left(c^+\right)$ are the annihilation (creation) operators of cavity $i\left(i=1,2\right)$ and fiber, respectively. ω_i is the cavity frequency of cavity i (for convenience, we let $\omega_1 = \omega_2 = \omega_c$ in the following discussions), ω_f is the fiber frequency. ν is the coupling strength. The subscriptc c and f indicate cavity and fiber modes. Under the rotation frame transformation $H'_{c,f} = UH_{c,f}U^+$, where $U = e^{\frac{\nu}{\Delta}\left(a_1^+ c + a_2^+ c - a_1 c^+ - a_2 c^+\right)}$, $\Delta = \omega_c - \omega_f$, we can adiabatically eliminate the transition between cavity and fiber in the large detuning limit $\Delta \gg \nu$ and obtain the effective Hamiltonian of subsystem cavity-fiber-cavity as

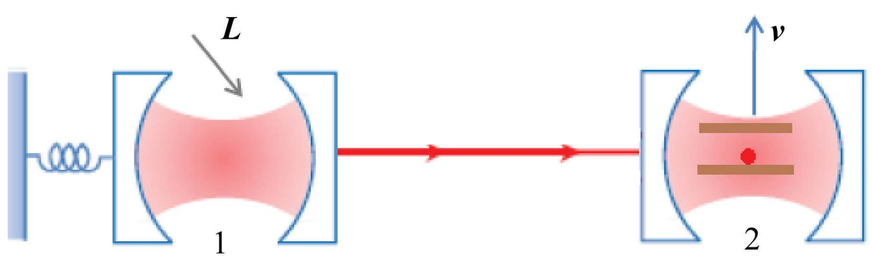


Figure 1. Schematic of proposed model. Optical cavity 1 is driven by a laser field *L*. A two-level atom moves along a direction perpendicular to cavity 2 mode with velocity *v*. Cavity waist $w = 5\ \mu m$.

$$H'_{c,f} = \left(\omega_c + \frac{2\nu^2}{\Delta}\right)a_1^+ a_1 + \left(\omega_c + \frac{2\nu^2}{\Delta}\right)a_2^+ a_2 + \left(\omega_f - \frac{4\nu^2}{\Delta}\right)c^+ c + \frac{2\nu^2}{\Delta}\left(a_1^+ a_2 + a_1 a_2^+\right) \tag{2}$$

Now we include the optomechanical subsystem and cavity-atom subsystem. The Hamiltonian of the global system without dissipation can be written as

$$H = \omega_m b^+ b + \omega_a \sigma^z - \left(\delta - \frac{2\nu^2}{\Delta}\right)a_1^+ a_1 + \left(\omega_c + \frac{2\nu^2}{\Delta}\right)a_2^+ a_2 + \frac{2\nu^2}{\Delta}\left(a_1^+ a_2 + a_1 a_2^+\right) - g_0 a_1^+ a_1\left(b + b^+\right) + \Omega\left(a_2 \sigma^+ + a_2^+ \sigma^-\right) \tag{3}$$

where $b\left(b^+\right)$ is the mechanical annihilation(creation) operator, σ_z and $\sigma^+\left(\sigma^-\right)$ are atomic spin and raising (lowering) operators. ω_m is the mechanical frequency, ω_a is the frequency of atomic internal transition. $\delta = \omega_L - \omega_c$ is the detuning of cavity 1 and the driving field. g_0 is the vacuum optomechanical coupling strength [14] [15]. Ω is the coupling strength of cavity to atom. For an atom moving along x scale perpendicular to the cavity mode with velocity v, the cavity-atom coupling strength can be represented by $\Omega(t) = \Omega_{max} e^{\frac{-(x_0+vt)}{w^2}}$, where $\Omega_{max} = \Omega_0 \cos(kz)$ is the maximum atom-cavity coupling strength, Ω_0 is Rabi frequency, x_0 is atomic initial position. The "c^+c" terms that does not influence the systematic transition is neglected.

3. Quantum State Transfer Protocol

The task of QST between two two-state ($|a\rangle_{1,2}$ and $|b\rangle_{1,2}$) systems is to accomplish the implementation $|\Psi_{in}\rangle = \left(\alpha|a\rangle_1 + \beta|b\rangle_1\right)\otimes|b\rangle_2 \to |\Psi_{out}\rangle = |b\rangle_1 \otimes \left(\alpha|a\rangle_2 + \beta|b\rangle_2\right)$ deterministically [11], where $|\Psi_{in}\rangle$ and $|\Psi_{out}\rangle$ are inputting initial state and outputting target state, α and β are normalized coefficients. The efficiency of QST can be illustrated by fidelity defined as $F = \left|\langle\Psi_{out}|\Psi(t)\rangle\right|^2$. In our model, we assume that only the mechanical oscillator is initially excited, with initial system state $|\Psi_{in}\rangle = |1\rangle_m|0\rangle_1|0\rangle_2|0\rangle_a$ and target state $|\Psi_{out}\rangle = |0\rangle_m|0\rangle_1|0\rangle_2|1\rangle_a$. The system state $|\Psi(t)\rangle = \sum_i C_i(t)|\phi_i\rangle$ $(i = 1,2,3,4)$ is restricted within the Hilbert space spanned by basis vectors $|\phi_1\rangle = |1\rangle_m|0\rangle_1|0\rangle_2|0\rangle_a$, $|\phi_2\rangle = |0\rangle_m|1\rangle_1|0\rangle_2|0\rangle_a$, $|\phi_3\rangle = |0\rangle_m|0\rangle_1|1\rangle_2|0\rangle_a$, $|\phi_4\rangle = |0\rangle_m|0\rangle_1|0\rangle_2|1\rangle_a$, and is governed by Schrödinger equation $\frac{\partial|\Psi(t)\rangle}{\partial t} = -iH|\Psi(t)\rangle$, where $C_i(t)$ are normalized coefficients.

To accomplish high fidelity QST, the coupling strengthes are designed as follows. Initially, only the driving field is turned on. At time t_1, the driving field is turned off and the cavity-cavity interaction is turned on. At time t_2, the cavity-cavity interaction is turned off and the atom enters cavity with velocity v.

In the time $0 \le t \le t_1$, the QST implementation from mechanical oscillator to cavity 1 is only dominated by the Hamiltonian of optomechanical subsystem. By using the standard "linearized approximation" procedure for optomechanics under the substitution $a_1 = \sqrt{\bar{n}_1} + A_1$ (where A_1 is the fluctuation of cavity 1 [15]) and considering dissipation, the subsystem is effectively described by a non-Hermitian Hamiltonian (under rotating wave approximation) as

$$H_{\mathrm{m},1} = \omega_{\mathrm{m}} b^{+} b - \delta A_1^{+} A_1 - G(t)\left(A_1^{+} b + A_1 b^{+}\right) - \frac{i}{2}\gamma_{\mathrm{m}} b^{+} b - \frac{i}{2}\kappa A_1^{+} A_1 \tag{4}$$

with mechanical decay rate γ_{m} and cavity leakage κ.

The pulsed many-photon optomechanical coupling is given by $G(t) = G_0 \mathrm{e}^{-(t-t_0)^2/2s^2}$ [9], where G_0 is the maximum optomechanical coupling strength. It has been experimentally demonstrated that optomechanical coupling is proportional to the mean number of the laser photons $\bar{n}_1$ [14], s represents the width of the Gaussian pulse. The designed coupling strengths sequence is shown in **Figure 2**, where $t_1 = \frac{4}{G_0}$, $t_2 = \frac{8}{G_0}$.

Obviously, in the time $0 \le t \le t_1$, the cavity-cavity coupling and cavity-atom interaction are negligible, and only the transition between mechanical oscillator and cavity 1 is considered. the coefficients C_i satisfy the equations

$$\begin{aligned} \dot{C}_1(t) &= -i\left[\omega_{\mathrm{m}} C_1(t) - G(t) C_2(t)\right] - \frac{\gamma_{\mathrm{m}}}{2} C_1(t) \\ \dot{C}_2(t) &= -i\left[-\delta C_2(t) - G(t) C_1(t)\right] - \frac{\kappa}{2} C_2(t) \end{aligned} \tag{5}$$

Under the condition $\omega_{\mathrm{m}} = -\delta$, and in a frame rotating with mechanical oscillator frequency ω_{m}, the equations can be simplified as

$$\begin{aligned} \dot{C}_1(t) &= iG(t) C_2(t) - \frac{\gamma_{\mathrm{m}}}{2} C_1(t) \\ \dot{C}_2(t) &= iG(t) C_1(t) - \frac{\kappa}{2} C_2(t) \end{aligned} \tag{6}$$

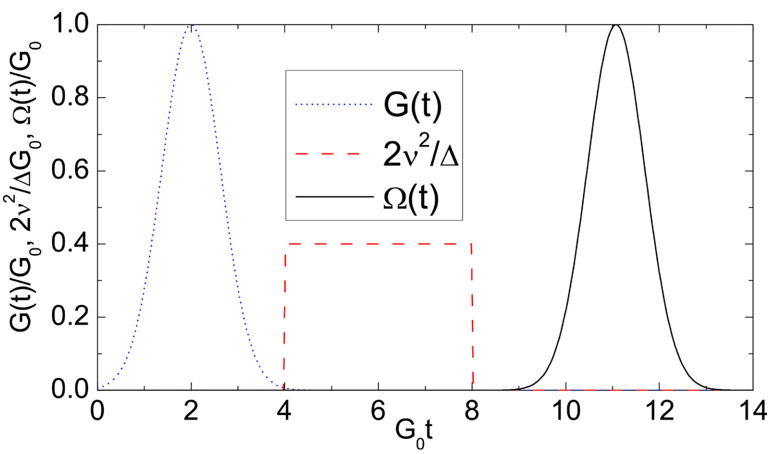


Figure 2. The time profiles of $G(t)$ (blue dotted line), $2v^2/\Delta$ (red dashed line), and $\Omega(t)$ (black solid line) normalized by G_0 as a function of normalized time $G_0 t$. $G_0 = 2\ \mathrm{MHz}$, $\Omega_{\max} = G_0$, $\frac{v}{w} = 1.626\Omega_{\max}$, $v = 2G_0$, $\Delta = 10v$.

In the time $t_1 \le t \le t_2$, note that at the end of Gaussian pulse in cavity 1, $\bar{n}_1 = 0$, which leads to $a_1 = A_1$, we obtain equations:

$$\begin{aligned}\dot{C}_2(t) &= -i\frac{2v^2}{\Delta}C_3(t) - \frac{\gamma_m}{2}C_2(t)\\ \dot{C}_3(t) &= -i\frac{2v^2}{\Delta}C_2(t) - \frac{\gamma_m}{2}C_3(t)\end{aligned} \tag{7}$$

while in the time $t_2 \le t$, the coefficients satisfy:

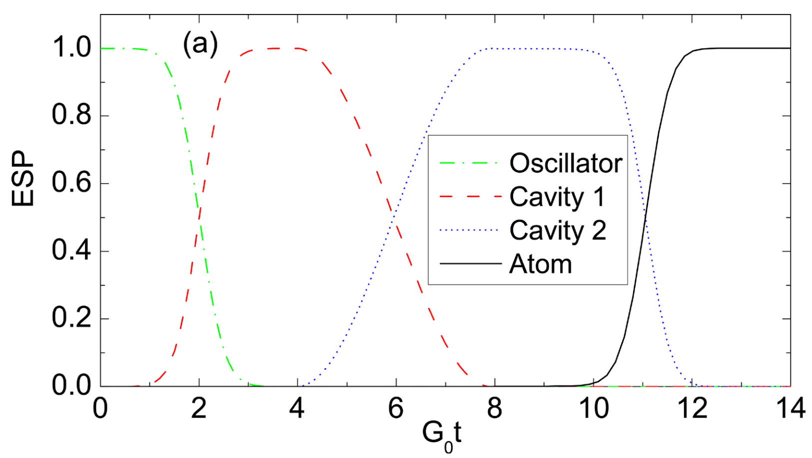
$$\begin{aligned}\dot{C}_3(t) &= -i\Omega(t)C_4(t) - \frac{\gamma_m}{2}C_3(t)\\ \dot{C}_4(t) &= -i\Omega(t)C_3 - \frac{\gamma}{2}C_4(t)\end{aligned} \tag{8}$$

where γ is atomic decay rate (spontaneous emission rate).

The above equations can be solved numerically under the initial condition $C_1(0) = 1$, $C_2(0) = 0$. The excited state populations (ESP) of mechanical oscillator, cavity 1, cavity 2, and atom are represented by $|C_1(t)|^2$, $|C_2(t)|^2$, $|C_3(t)|^2$, and $|C_4(t)|^2$, respectively.

Figure 3(a) shows the ESP under characteristic experimental parameters without dissipation. The fidelity of QST can be calculated through the formula $F = |\langle \Psi_f | \Psi(T+t_2)\rangle|^2$ for atomic transit time $T = \frac{L}{v}$ under the condition of atomic transit distance $L \gg w$. It can be proved that the fidelity turns out to be $|C_4(T+t_2)|^2$. Numerical results show that the quantum state initially encoded on mechanical oscillator can be transferred to atom with a fidelity 100% if the influence of dissipation is excluded. It is known that the system dissipation inevitably decreases the fidelity. However, the mechanical decay is not a big factor for maximum fidelity, which is 99.9% at $\frac{\gamma_m}{G_0} = 5\times10^{-4}$ (this result is obtained to demonstrate the affect of mechanical decay, all other decays are excluded).

Further more, Figure 3(b) shows the ESP in presence of system dissipation with specified decay rates [9] [14]. The maximum fidelity is 90.1%. One can see that the maximum fidelity strongly relies on cavity leakage and atom decay.



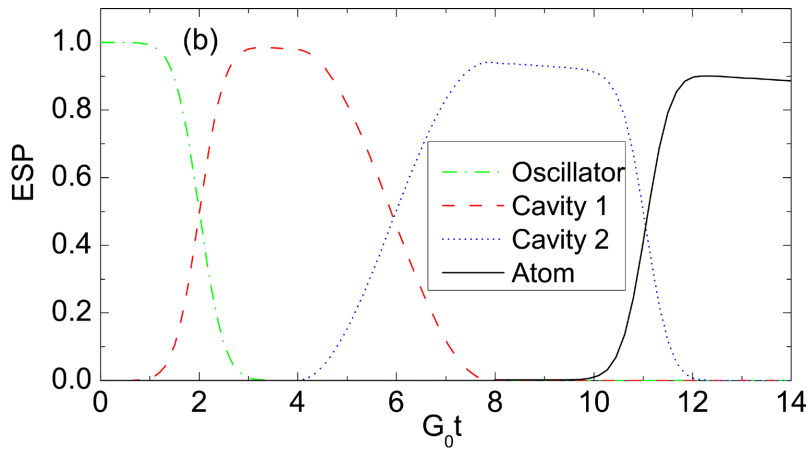


Figure 3. Excited state populations (ESP) of oscillator (green dash-dotted line), cavity 1 (red dashed line), cavity 2 (blue dotted line), atom (black solid line) versus G_0t. The quantum state initially encoded on oscillator is transferred to cavity 1, then to cavity 2 far apart via fiber, finally received by moving atom. The profiles of populations is modulated by parameters in Figure 2 but for (a) without dissipation, (b) with dissipation, and the oscillator, cavity, and atom decay rates are characterized as $\gamma_m = 5\times10^{-4}G_0$, $\kappa = \gamma = 0.01G_0$, respectively [9] [14].

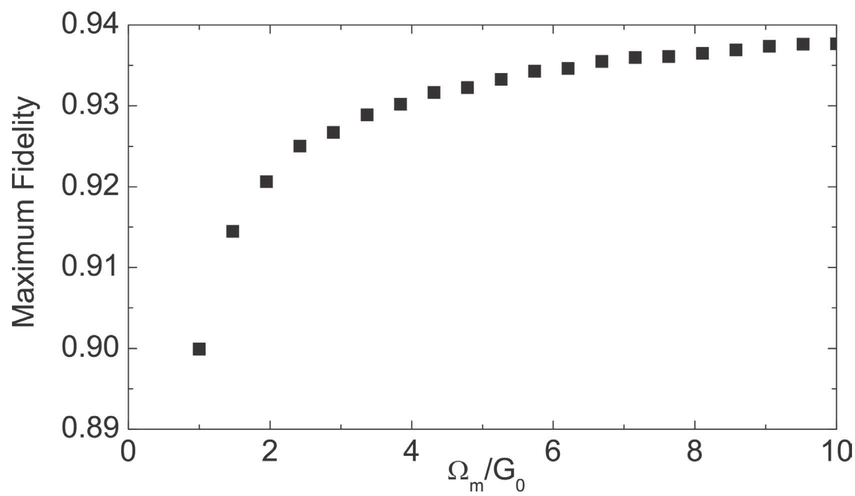


Figure 4. The maximum fidelity versus the maximum cavity-atom coupling strength Ω_m, where $v = 1.626\Omega_{max}w$.

Nevertheless, the fidelity can be obviously improved by optimal parameters. It is shown in Figure 4 that synchronously increasing the maximum atom-cavity coupling strength Ω_{max} and atom velocity v increases maximum fidelity from 90.1% to 93.8%.

4. Conclusion

To summarize, we have analyzed a scheme to implement a quantum state transfer between a mechanical oscillator and a distant moving two-level atom mediated by cavity-fiber-cavity channel. By designing appropriate coupling strengths sequence within the experimental parameters range, a QST process is accomplished with fidelity 100%. We showed the scheme works in a robust way since the oscillator decay rate has very little impact on the fidelity and the affect of quantum channel has been reduced by adiabatically eliminating fiber mode. By synchronously increasing maximum atom-cavity coupling strength and atomic velocity, although the cavity leakage and atomic decay decrease the fidelity of QST, the maximum fidelity can be obviously raised to 93.8% from 90.1%. Furthermore, in the regime of "good cavity limits" with $\Omega_{max} \gg \kappa, \gamma$ [16], the internal interaction of cavity-atom is much large than the dissipation that results in the decoherence of atom-cavity state, the affect of cavity leakage and atomic decay on QST can be effectively suppressed. Given the very successfully realized experimental real-time cavity QED technology [13] and strong-coupling optomechanical system [14], our scheme may be feasible and realizable.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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