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Extended Relativistic Invariance, Quantization of the Kinetic Momentum

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Abstract

The aim of this research is a better understanding of the quantization in physics. The true origin of the quantization is the existence of the quantized kinetic momentum of electrons, neutrinos, protons and neutrons with the $\hbar/2$ value. It is a consequence of the extended relativistic invariance of the wave of fundamental particles with spin 1/2. This logical link is due to properties of the quantum waves of fermions, which are functions of space-time with value into the $Cl_3^* = \mathrm{End}(\mathbb{C}^2)$ and $\mathrm{End}(Cl_3)$ Lie groups. Space-time is a manifold forming the auto-adjoint part of Cl_3^*. The Lagrangian densities are the real parts of the waves. The equivalence between the invariant form and the Dirac form of the wave equation takes the form of Lagrange's equations. The momentum-energy tensor linked by Noether's theorem to the invariance under space-time translations has components which are directly linked to the electromagnetic tensor. The invariance under Cl_3^* of the kinetic momentum tensor gives eight vectors. One of these vectors has a time component with value $\hbar/2$. Resulting aspects of the standard model of quantum physics and of the relativistic theory of gravitation are discussed.

Keywords

Geometry, Invariance Group, Dirac Equation, Electromagnetism, Weak Interactions, Strong Interactions, Clifford Algebras, Gravitation, Quantization

1. Extended Relativistic Invariance

When classical mechanics was replaced by quantum mechanics many changes were necessary. The new theory was built in conformity with Special Relativity by de Broglie [1] and Dirac [2]. The main change brought by relativistic quantum mechanics is the replacement made by Pauli and Dirac of the group of rotations and of the Lorentz group by the multiplicative $SU(2)$ and $SL(2,\mathbb{C})$ Lie groups, respectively. This was made twice: in 1927 space was first put into the Pauli algebra:

$$\vec{\mathrm{x}} = (\mathrm{x}^1,\ \mathrm{x}^2,\ \mathrm{x}^3) = \begin{pmatrix} \mathrm{x}^3 & \mathrm{x}^1 - i\mathrm{x}^2 \\ \mathrm{x}^1 + i\mathrm{x}^2 & -\mathrm{x}^3 \end{pmatrix}. \tag{1}$$

In the following year, the entire space-time was expressed in the Pauli algebra:

$$\mathrm{x} = (\mathrm{x}^0,\ \mathrm{x}^1,\ \mathrm{x}^2,\ \mathrm{x}^3) = \begin{pmatrix} \mathrm{x}^0 + \mathrm{x}^3 & \mathrm{x}^1 - i\mathrm{x}^2 \\ \mathrm{x}^1 + i\mathrm{x}^2 & \mathrm{x}^0 - \mathrm{x}^3 \end{pmatrix}. \tag{2}$$

The full significance of this change was not truly seen. Dirac, for his wave equation of the electron, doubled the number of components of the Pauli spinors and used 4×4 complex matrices. During the next forty years only the $M_4(\mathbb{C})$ algebra was used. The Pauli algebra was only an auxiliary algebra allowing us the study of the Dirac matrices. Next D. Hestenes revisited the Dirac theory, using the Clifford algebra $Cl_{1,3}$ (space-time algebra) built on space-time with signature $+,-,-,-$. This algebra is 16-dimensional on the real field, while $M_4(\mathbb{C})$ is 16-dimensional on the complex field, therefore 32-dimensional on the real field. The Dirac wave, in Hestenes's view, is a function of space-time with value into $Cl_{1,3}^+$, the even sub-algebra of $Cl_{1,3}$, 8-dimensional on $\mathbb{R}$, like the Pauli algebra [3–7].

The departure of our work is the following observation: Equation (2) and values of the Dirac wave use only the eight parameters of the Pauli algebra. It should then be possible to write the entirety of the Dirac theory using only the Pauli algebra. This was studied in [8–37]. Not only is the linear Dirac equation completely described in the Cl_3 algebra (Pauli algebra), another major change is the simplification operated by the replacement of the linear Dirac equation by our improved wave equation. In its completely invariant form, the improved equation of the electron [9, 10, 28] reads in Cl_3:

$$0 = \overline{\phi}(\nabla\widehat{\phi})\sigma_{21} + q\overline{\phi}A\widehat{\phi} + m\rho, \tag{3}$$

$$\phi = \sqrt{2}\begin{pmatrix} \xi_1^1 & -\bar{\eta}_2^1 \\ \xi_2^1 & \bar{\eta}_1^1 \end{pmatrix};\ \widehat{\phi} = \sqrt{2}\begin{pmatrix} \eta_1^1 & -\bar{\xi}_2^1 \\ \eta_2^1 & \bar{\xi}_1^1 \end{pmatrix};\ \overline{\phi} = \widehat{\phi}^\dagger, \tag{4}$$

with:

$$\nabla = \sigma^\mu\partial_\mu;\ A = \sigma^\mu A_\mu;\ \sigma^0 = \sigma_0 = 1;\ \det(\phi) = 2(\xi_1^1\bar{\eta}_1^1 + \xi_2^1\bar{\eta}_2^1), \tag{5}$$

$$\sigma^j = -\sigma_j,\ j = 1,2,3;\ \sigma_{21} = \sigma_2\sigma_1;\ \rho = |\det(\phi)|, \tag{6}$$

$$m = \frac{m_0 c}{\hbar};\ q = \frac{e}{\hbar c}, \tag{7}$$

where m_0 is the proper mass of the electron, e is its charge, A is the electromagnetic potential (a space-time vector), the ξ^1 column is the right spinor wave of the electron and η^1 is the left spinor. This wave equation is invariant under the transformation R defined from any $M \in SL(2,\mathbb{C})$ by:

$$\mathrm{x} \mapsto \mathrm{x}' = R(\mathrm{x}) = M\mathrm{x}M^\dagger;\ \phi \mapsto \phi' = M\phi, \tag{8}$$

$$\xi^1 \mapsto \xi^{1\prime} = M\xi^1;\ \eta^1 \mapsto \eta^{1\prime} = \widehat{M}\eta^1. \tag{9}$$

If we do not restrict M to satisfy $\det(M) = 1$ this condition defining $SL(2,\mathbb{C})$ becomes

$$\det(M) = M\overline{M} = \overline{M}M = re^{i\theta} \neq 0. \tag{10}$$

The transformation R is then a similitude conserving both the orientation of space and the time arrow (see [37] 1.1.2 and A.3):

$$\overline{\phi}'(\nabla'\widehat{\phi}') = \overline{\phi}\,\overline{M}\nabla'\widehat{M}\widehat{\phi} = \overline{\phi}\nabla\widehat{\phi}, \tag{11}$$

$$\overline{\phi}'q'A'\widehat{\phi}') = \overline{\phi}\,\overline{M}q'A'\widehat{M}\widehat{\phi} = \overline{\phi}qA\widehat{\phi}, \tag{12}$$

$$\rho' = |\det(\phi')| = |\det(M)\det(\phi)| = r|\det(\phi)| = r\rho. \tag{13}$$

The main hypothesis of this section is [37]: Suppressing $\det(M) = 1$ the group $SL(2,\mathbb{C})$ is extended to the $GL(2,\mathbb{C}) = Cl_3^*$ Lie group, which may be considered as the true invariance group of any law in physics. The extended relativistic invariance of the wave Equation (3) results from (proof in [37] A.4.4)

$$\nabla = \overline{M}\nabla'\widehat{M};\nabla' = \sigma^\mu\partial'_\mu. \tag{14}$$

The electric gauge invariance means:

$$qA = \overline{M}q'A'\widehat{M}; A' = \sigma^\mu A'_\mu \tag{15}$$

Then the wave equation is invariant if and only if the mass term satisfies:

$$m'\rho' = m'r\rho = m\rho;\ m = m'r. \tag{16}$$

$\phi(\mathrm{x})$ being invertible everywhere in all interesting solutions (plane waves, the hydrogen atom), the invariant wave equation is then equivalent to the improved equation:

$$0 = \nabla\widehat{\phi}\sigma_{21} + qA\widehat{\phi} + me^{-i\beta}\phi;\ \rho e^{i\beta} = \det(\phi), \tag{17}$$

where β is the Yvon-Takabayasi angle. The relativistic invariants m and ρ were already long known in the Dirac theory. The Yvon-Takabayasi angle was enigmatic until G. Lochak used the chiral gauge (with angle β) to build his wave equation of a leptonic magnetic monopole [38–45]. Since ρ is similar to r, which is a scale factor (as ratio of an homothety), the interpretation of ρ as a statistical parameter was wrong. The occupation number of any fermion wave being necessarily 0 or 1, any statistics from such a lone object is necessary irrelevant. We shall encounter as soon as in the next paragraph the true probability density.

Now in all cases where β is null or very small, the improved Equation (17) is reduced to the linear Dirac equation, which reads in Cl_3 (the equivalence with the usual form of the Dirac equation is detailed in [37] 1.3):

$$0 = \nabla\widehat{\phi}\sigma_{21} + qA\widehat{\phi} + m\phi. \tag{18}$$

The resolution of the improved Equation (17) in the case of the hydrogen atom uses a method of separation of variables found by H. Krüger [46]. This gives new solutions to the wave equation, with the same set of quantum numbers and the same energy levels [37]. The wave is then with value everywhere in Cl_3^*. Therefore ρ is like r a ratio of homothety and since $\rho \neq 0$ the two forms of the wave equation, the invariant one (3) and the usual one (17), are equivalent, each may be deduced from the other. The path from the usual form (17) to the invariant form simply uses a multiplication by $\overline{\phi}$ on the left side. Moreover the Lagrangian density is simply the real part, which is one of the eight numeric equations equivalent to (17) (this is detailed in [37] B.1.4):

$$\mathcal{L} = 0 = -w_3 + qA \cdot \mathrm{D}_0 + m\rho =< \overline{\phi}(\nabla\widehat{\phi})\sigma_{21} + q\overline{\phi}A\widehat{\phi} + m\rho >, \quad (19)$$

$$0 = \frac{1}{2}\nabla \cdot \mathrm{D}_2 + qA \cdot \mathrm{D}_1, \quad (20)$$

$$0 = -\frac{1}{2}\nabla \cdot \mathrm{D}_1 + qA \cdot \mathrm{D}_2, \quad (21)$$

$$0 = w_0 + qA \cdot \mathrm{D}_3, \quad (22)$$

$$0 = \frac{1}{2}\nabla \cdot D_3, \quad (23)$$

$$0 = -w_2, \quad (24)$$

$$0 = w_1, \quad (25)$$

$$0 = \frac{1}{2}\nabla \cdot \mathrm{D}_0. \quad (26)$$

where $(\mathrm{D}_0,\ \mathrm{D}_1,\ \mathrm{D}_2,\ \mathrm{D}_3)$ is a mobile orthogonal basis in space-time and w satisfies:

$$\mathrm{D}_\mu = \phi\sigma_\mu\widetilde{\phi};\ \widetilde{\phi} = \phi^\dagger;\ \overline{\phi}(\nabla\widehat{\phi}) - (\overline{\phi}\nabla)\widehat{\phi} = 2iw = 2iw_\mu\sigma^\mu. \quad (27)$$

As it is the real part of an invariant equation, the Lagrangian density is also invariant under Cl_3^*. The current $J = \mathrm{D}_0$ is the probability current linked by Noether's theorem to the electric gauge invariance. The probability density of quantum theory is in the case of the Dirac wave the time component $J^0 = \mathrm{D}_0^0$ of the probability current. The form of the implication from the Lagrangian density (19) into the (17) wave equation is the Lagrange equations as described in 2.3.4 of [37]. Therefore the Lagrangian mechanism does not come as a principle; it is a mere consequence of the non commutative multiplication in Cl_3^*. Consequently the Lagrangian mechanism is necessary for any solution of the wave equation and the momentum-energy is the Tetrode tensor linked to the invariance of the Lagrangian density under space-time translations.

1.1. Chiral Parts of the Wave

The $\phi = \phi^1$ wave of the electron is the sum of a left part and a right part:

$$R^1 = \phi^1(1+\sigma_3)/2 = \sqrt{2}(\xi^1 \quad 0);\ \xi^1 = \begin{pmatrix}\xi_1^1 \\ \xi_2^1\end{pmatrix}, \quad (28)$$

$$\widehat{L}^1 = \widehat{\phi}^1(1+\sigma_3)/2 = \sqrt{2}(\eta^1 \quad 0);\ \eta^1 = \begin{pmatrix}\eta_1^1 \\ \eta_2^1\end{pmatrix}. \quad (29)$$

All tensorial densities of the Dirac wave are built from these left and right parts. The improved wave equation is equivalent to the system (see [37] 1.5.1):

$$\begin{aligned} 0 &= -i\nabla\widehat{L}^1 + qA\widehat{L}^1 + me^{-i\beta}R^1, \\ 0 &= -i\widehat{\nabla}R^1 + q\widehat{A}R^1 + me^{i\beta}\widehat{L}^1. \end{aligned} \quad (30)$$

And the probability current satisfies:

$$\begin{aligned} \mathrm{J} = \phi\widetilde{\phi} = \mathrm{D}_R + \mathrm{D}_L;\ \mathrm{D}_R = R^1\widetilde{R}^1;\ \mathrm{D}_L = L^1\widetilde{L}^1, \\ \rho e^{-i\beta} = \widehat{\phi}\widetilde{\phi} = \widehat{L}^1\widetilde{R}^1 + \widehat{R}^1\widetilde{L}^1;\ \rho e^{i\beta} = \phi\overline{\phi} = L^1\overline{R}^1 + R^1\overline{L}^1. \end{aligned} \quad (31)$$

The local reduced velocity v is so defined:

$$\mathrm{v} = \frac{1}{\rho}\mathrm{J};\ \mathrm{v}\widehat{\mathrm{v}} = 1;\ \mathrm{v}^{-1} = \widehat{\mathrm{v}}. \tag{32}$$

This gives:

$$\begin{aligned}
\mathrm{v}\widehat{L}^1 &= \frac{1}{\rho}(R^1\widetilde{R}^1 + L^1\widetilde{L}^1)\widehat{L}^1 = \frac{1}{\rho}R^1\widetilde{R}^1\widehat{L}^1,\\
\widetilde{R}^1\widehat{L}^1 &= 2\begin{pmatrix}\bar{\xi}_1 & \bar{\xi}_2\\ 0 & 0\end{pmatrix}\begin{pmatrix}\eta_1 & 0\\ \eta_2 & 0\end{pmatrix} = \begin{pmatrix}\bar{a}_1 & 0\\ 0 & 0\end{pmatrix};\ a_1 = \rho e^{i\beta},\\
R^1\widetilde{R}^1\widehat{L}^1 &= R^1\bar{a}_1\frac{1+\sigma_3}{2} = \bar{a}_1 R^1,\\
me^{-i\beta}R^1 &= \frac{m}{\rho}\bar{a}_1 R^1 = \frac{m}{\rho}R^1\widetilde{R}^1\widehat{L}^1 = m\mathrm{v}\widehat{L}^1. \qquad (33)\\
\mathrm{v}\widehat{R}^1 &= \frac{1}{\rho}(R^1\widetilde{R}^1 + L^1\widetilde{L}^1)\widehat{R}^1 = \frac{1}{\rho}L^1\widetilde{L}^1\widehat{R}^1 \qquad (34)\\
&= \frac{1}{\rho}L^1\overline{\widetilde{R}^1\widehat{L}^1} = \frac{1}{\rho}L^1\bar{a}_1\frac{1-\sigma_3}{2} = \frac{\bar{a}_1}{\rho}L^1 = e^{-i\beta}L^1.
\end{aligned}$$

Then the system (30) is equivalent to the system seeming uncrossed:

$$\begin{aligned}
0 &= (-i\nabla + qA + m\mathrm{v})\widehat{L}^1,\\
0 &= (-i\widehat{\nabla} + q\widehat{A} + m\widehat{\mathrm{v}})R^1,
\end{aligned} \tag{35}$$

equivalent to:

$$\begin{aligned}
0 &= (-i\nabla + qA + m\mathrm{v})\eta^1,\\
0 &= (-i\widehat{\nabla} + q\widehat{A} + m\widehat{\mathrm{v}})\xi^1.
\end{aligned} \tag{36}$$

These systems are not truly uncrossed, since v contains both right and left currents. The Lagrangian density satisfies:

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_L + \mathcal{L}_R, \qquad (37)\\
\mathcal{L}_L &= \Re\big[\eta^{1\dagger}[(-i\nabla + qA)\eta^1 + me^{-i\beta}\xi^1]\big],\\
\mathcal{L}_R &= \Re\big[\xi^{1\dagger}[(-i\widehat{\nabla} + q\widehat{A})\xi^1 + me^{i\beta}\eta^1]\big],
\end{aligned}$$

because right and left waves satisfy:

$$\begin{aligned}
&\eta^{1\dagger}me^{-i\beta}\xi^1 + \xi^{1\dagger}me^{i\beta}\eta^1 = me^{-i\beta}\eta^{1\dagger}\xi^1 + me^{i\beta}\xi^{1\dagger}\eta^1\\
&= me^{-i\beta}\frac{1}{2}\rho e^{i\beta} + me^{i\beta}\frac{1}{2}\rho e^{-i\beta} = m\rho.
\end{aligned} \tag{38}$$

With the covariant derivative

$$d_\mu = \partial_\mu + iqA_\mu, \tag{39}$$

the Lagrangian density reads (see [37] 1.9):

$$\mathcal{L} = \Re\big[-i(\eta^{1\dagger}\sigma^\mu d_\mu\eta^1 + \xi^{1\dagger}\widehat{\sigma}^\mu d_\mu\xi^1)\big] + m\rho. \tag{40}$$

1.2. Normalization of the Wave

Since the wave equation is homogeneous the Lagrangian density is null for any solution of the wave equation. The momentum-energy tensor issued from the invariance of the Lagrangian density under space-time translations (Tétrode's tensorial densities) reads:

$$\begin{aligned}
T^\mu_\nu &= \Re[i\eta^{1\dagger}\sigma^\mu d_\nu\eta^1 + i\xi^{1\dagger}\widehat{\sigma}^\mu d_\nu\xi^1)] - \delta^\mu_\nu\mathcal{L}\\
&= \Re[i\eta^{1\dagger}\sigma^\mu d_\nu\eta^1 + i\xi^{1\dagger}\widehat{\sigma}^\mu d_\nu\xi^1)].
\end{aligned} \tag{41}$$

For a wave with an energy E satisfying:

$$-id_0\eta^1 = \frac{E}{\hbar c}\eta^1; \ -id_0\xi^1 = \frac{E}{\hbar c}\xi^1, \tag{42}$$

the momentum-energy tensor satisfies:

$$T_0^0 = \Re[i(\eta^{1\dagger}d_0\eta^1 + i\xi^{1\dagger}d_0\xi^1)] = -\frac{E}{\hbar c}(\eta^{1\dagger}\eta^1 + \xi^{1\dagger}\xi^1) = -E\frac{J^0}{\hbar c}. \tag{43}$$

The condition of normalization of the wave function:

$$\iiint dv \frac{J^0}{\hbar c} = 1, \tag{44}$$

is then equivalent to

$$0 = E + \iiint dv T_0^0. \tag{45}$$

The left term of this sum is the total energy E of the electron, linked to the gravitational proper mass (gravitational redshift) while the right term is the sum of the local density of energy of the wave. This local density is linked to inertia in the form of the Lorentz force (see [37] 1.9):

$$0 = \partial_\mu T^\mu + F_{\mu\nu}\mathrm{j}^\mu\sigma^\nu; \ T^\mu = T^\mu_\nu\sigma^\mu. \tag{46}$$

where F is the electromagnetic tensor. With

$$F = \nabla\widehat{A} = \vec{E} + i\vec{H}; \ \mathrm{j} = q\mathrm{J} = \rho_e + \vec{\mathrm{j}}; \ \mathrm{f} = \mathrm{f}_0 + \vec{\mathrm{f}}, \tag{47}$$

where $\vec{E}$ is the electric field, $\vec{H}$ is the magnetic field, ρ_e is the density of charge, $\vec{\mathrm{j}}$ is the density of current and $\vec{\mathrm{f}}$ is the density of force, (46) gives:

$$\vec{\mathrm{f}} = \rho_e\vec{E} + \vec{\mathrm{j}} \times \vec{H}; \ \mathrm{f}_0 = \vec{E} \cdot \vec{\mathrm{j}}. \tag{48}$$

The normalization of the electron wave is then exactly equivalent to the equality between gravitational mass and inertial mass, the principle at the basis of general relativity. This normalization is then a necessary consequence of physical laws.

1.3. Electromagnetic Field

If the A potential is not exterior to the quantum wave, but linked to the fermion wave by:

$$A = \mathrm{D}_R - \mathrm{D}_L, \tag{49}$$

the electromagnetic field:

$$F = \nabla\widehat{A} = \vec{E} + i\vec{H}; \ \vec{E} = E^j\sigma_j; \ \vec{H} = H^j\sigma_j, \tag{50}$$

satisfies the following relations with the Tetrode momentum energy tensor (see [37] 1.10):

$$\begin{aligned} E^1 &= 2(T_2^3 - T_3^2); \ H^1 = 2(T_0^1 + T_1^0), \\ E^2 &= 2(T_3^1 - T_1^3); \ H^2 = 2(T_0^2 + T_2^0), \\ E^3 &= 2(T_1^2 - T_2^1); \ H^3 = 2(T_0^3 + T_3^0). \end{aligned} \tag{51}$$

This implies that the electromagnetic field has components that are directly linear combinations of the densities of impulse-energy which are components of the Tetrode tensor. These equalities relieve quantum field theory of the infinities coming from the electric field created by electric charges. The energy of a charged fermion is the sum on all space of the local energy of the Tetrode tensor, linked to the electric and magnetic field, not to the square of these fields. These equalities also mean that the components of the electromagnetic field are mere densities of impulse-energy. This joins the beginning of quantum physics, when Einstein discovered that light contained grains of impulse-energy, nowadays called photons.

1.4. Quantized Kinetic Momentum

Since the Lagrangian density is a sum of two chiral parts, a second Lagrangian density exists as difference of these parts. Then a second momentum-energy tensor, first noted by O. Costa de Beauregard [47], satisfies:

$$\mathcal{L}^- = \mathcal{L}_L - \mathcal{L}_R, \tag{52}$$

$$V^\mu_\lambda = \Re\big[-i[\eta^{1\dagger}\sigma^\mu d_\lambda \eta^1 - \xi^{1\dagger}\widehat{\sigma}^\mu d_\lambda \xi^1]\big]. \tag{53}$$

The invariance of these densities under the Cl_3^* group (inducing rotations, boosts and similitudes) introduces the general transformation [48]:

$$M = 1 + \frac{1}{2}(\delta\omega^0 + \delta\omega^j\sigma_j + \delta\omega^{3+j}i\sigma_j + \delta\omega^7 i) \tag{54}$$

where the eight $\delta\omega^n$ are infinitely small. We have:

$$M^\dagger = 1 + \frac{1}{2}(\delta\omega^0 + \delta\omega^j\sigma_j - \delta\omega^{3+j}i\sigma_j - \delta\omega^7 i)$$

$$\mathrm{x}' = \mathrm{x}'^\mu\sigma_\mu = M\mathrm{x}M^\dagger = \mathrm{x} + \delta\mathrm{x}^\mu\sigma_\mu;\ \delta\mathrm{x}^\mu = X^\mu_i\delta\omega^i \tag{55}$$

This gives:

$$\begin{aligned}
\delta\mathrm{x}^0 &= \mathrm{x}^0\delta\omega^0 + \mathrm{x}^1\delta\omega^1 + \mathrm{x}^2\delta\omega^2 + \mathrm{x}^3\delta\omega^3,\\
\delta\mathrm{x}^1 &= \mathrm{x}^0\delta\omega^1 + \mathrm{x}^1\delta\omega^0 + \mathrm{x}^2\delta\omega^6 - \mathrm{x}^3\delta\omega^5,\\
\delta\mathrm{x}^2 &= \mathrm{x}^0\delta\omega^2 - \mathrm{x}^1\delta\omega^6 + \mathrm{x}^2\delta\omega^0 + \mathrm{x}^3\delta\omega^4,\\
\delta\mathrm{x}^3 &= \mathrm{x}^0\delta\omega^3 + \mathrm{x}^1\delta\omega^5 - \mathrm{x}^2\delta\omega^4 + \mathrm{x}^3\delta\omega^0.
\end{aligned} \tag{56}$$

The only non-null X^μ_i are then:

$$\begin{aligned}
&X^0_0 = \mathrm{x}^0;\ X^0_1 = \mathrm{x}^1;\ X^0_2 = \mathrm{x}^2;\ X^0_3 = \mathrm{x}^3,\\
&X^1_0 = \mathrm{x}^1;\ X^1_1 = \mathrm{x}^0;\ X^1_5 = -\mathrm{x}^3;\ X^1_6 = \mathrm{x}^2,\\
&X^2_0 = \mathrm{x}^2;\ X^2_2 = \mathrm{x}^0;\ X^2_6 = -\mathrm{x}^1;\ X^2_4 = \mathrm{x}^3,\\
&X^3_0 = \mathrm{x}^3;\ X^3_3 = \mathrm{x}^0;\ X^3_4 = -\mathrm{x}^2;\ X^3_5 = \mathrm{x}^1,
\end{aligned} \tag{57}$$

Bailin [48] notes the following relation for the different fields φ_a and their variations

$$\delta\varphi_a = \phi^a_i\delta\omega^i. \tag{58}$$

Since we may use the adjoint term to obtain the real part we can consider only two spinor fields:

$$\varphi_1 = \eta^1;\ \varphi_2 = \xi^1. \tag{59}$$

And we have:

$$\eta^1+\delta\eta^1=\widehat{M}\eta^1;\ \xi^1+\delta\xi^1=M\xi^1,$$
$$\widehat{M}=1+\frac{1}{2}(\delta\omega^0-\delta\omega^j\sigma_j+\delta\omega^{3+j}i\sigma_j-\delta\omega^7 i). \tag{60}$$

This gives:

$$2\delta\xi^1=\delta\omega^0\xi^1+\delta\omega^1\sigma_1\xi^1+\delta\omega^2\sigma_2\xi^1+\delta\omega^3\sigma_3\xi^1 \tag{61}$$
$$+\delta\omega^4 i\sigma_1\xi^1+\delta\omega^5 i\sigma_2\xi^1+\delta\omega^6 i\sigma_3\xi^1+\delta\omega^7 i\xi^1,$$

$$2\delta\eta^1=\delta\omega^0\eta^1-\delta\omega^1\sigma_1\eta^1-\delta\omega^2\sigma_2\eta^1-\delta\omega^3\sigma_3\eta^1 \tag{62}$$
$$+\delta\omega^4 i\sigma_1\eta^1+\delta\omega^5 i\sigma_2\eta^1+\delta\omega^6 i\sigma_3\eta^1-\delta\omega^7 i\eta^1.$$

Through the relations (59) we arrive at

$$\phi_0^1=\frac{\eta^1}{2};\ \phi_1^1=-\sigma_1\frac{\eta^1}{2};\ \phi_2^1=-\sigma_2\frac{\eta^1}{2};\ \phi_3^1=-\sigma_3\frac{\eta^1}{2},$$
$$\phi_4^1=i\sigma_1\frac{\eta^1}{2};\ \phi_5^1=i\sigma_2\frac{\eta^1}{2};\ \phi_6^1=i\sigma_3\frac{\eta^1}{2};\ \phi_7^1=-i\frac{\eta^1}{2}, \tag{63}$$

$$\phi_0^2=\frac{\xi^1}{2};\ \phi_1^2=\sigma_1\frac{\xi^1}{2};\ \phi_2^2=\sigma_2\frac{\xi^1}{2};\ \phi_3^2=\sigma_3\frac{\xi^1}{2},$$
$$\phi_4^2=i\sigma_1\frac{\xi^1}{2};\ \phi_5^2=i\sigma_2\frac{\xi^1}{2};\ \phi_6^2=i\sigma_3\frac{\xi^1}{2};\ \phi_7^2=i\frac{\xi^1}{2}, \tag{64}$$

Noether's theorem associates with each of the eight parameters ω^n a conservative current:

$$j_i^\mu=\Big(\frac{\partial\mathcal{L}^-}{\partial(\partial_\mu\varphi_a)}(\partial_\nu\varphi_a)\Big)X_i^\nu-\frac{\partial\mathcal{L}^-}{\partial(\partial_\mu\varphi_a)}\phi_i^a. \tag{65}$$

The j_7 current is particular because i belongs to the center of Cl_3^*. Quantum theory previously used only the quantities j_1^μ to j_6^μ. These six space-time vectors now join two other vectors and it is precisely one of these new vectors, j_7, that we will presently use. We have:

$$j_7^\mu=\Big(\frac{\partial\mathcal{L}^-}{\partial(\partial_\mu\varphi_a)}(\partial_\nu\varphi_a)\Big)X_7^\nu-\frac{\partial\mathcal{L}^-}{\partial(\partial_\mu\varphi_a)}\phi_7^a. \tag{66}$$

Now the only X_i^ν that are not null are listed in (57) and this list contains none of X_7^ν. This results from the following property: the generator i of the chiral gauge $U(1)$ belongs to the kernel of the homomorphism $f: M\mapsto R$ from Cl_3^* into the D^* group of similitudes. We then have:

$$j_7^\mu=-\frac{\partial\mathcal{L}^-}{\partial(\partial_\mu\varphi_a)}\phi_7^a. \tag{67}$$

Using the equalities (63) and since the adjoint of a real is the real itself we obtain:

$$j_7^\mu=2\frac{i}{2}\eta^{1\dagger}\sigma^\mu(-i)\frac{\eta^1}{2}-2\frac{i}{2}\xi^{1\dagger}\sigma^\mu(+i)\frac{\xi^1}{2}. \tag{68}$$

This implies:

$$j_7=\frac{1}{2}(\mathrm{D}_L^1+\mathrm{D}_R^1)=\frac{1}{2}\mathrm{J}, \tag{69}$$

$$\iiint dv\frac{1}{c}j_7^0=\frac{1}{2c}\iiint dv\mathrm{J}^0=\frac{\hbar}{2}. \tag{70}$$

Therefore the quantization of the kinetic momentum of the electron with the value $\hbar/2$ is a necessary consequence of the equality between gravitational and inertial mass and of the enlarged relativistic invariance under Cl_3^*.

1.5. Pauli's Principle

The Bohr atomic model fits Mendeleev's classification trough two hypotheses: the quantification of the action and the Pauli principle. A true theory of fields must account for this principle from the properties of the field. The resolution of the improved wave equation in the case of the hydrogen atom gives a set of quantum states which are not only normalized, but orthogonal. This orthogonality was proved as early as 1934 by L. de Broglie [49] for Darwin's solutions. This orthogonality, first obtained for the Hermitian scalar product of the Dirac theory, is equivalent to the orthogonality for the Euclidean product of the real Clifford algebra [8]. This result remains for the new solutions of the Dirac equation with a Yvon-Takabayasi angle everywhere defined and small [8, 37]. The Pauli principle may then be stated as: Two electrons in an atom must have orthonormal states. This implies that the chiral currents, the J current, and the electric current, are additive [37]. The origin of the orthonormalization is double: the necessity of the normalization comes from the principle of equivalence between gravitational and inertial mass. The use of two sets of Dirac matrices, one for non-relativistic approximation, the other for chiral waves and weak interactions, induces the existence of two families of solutions. The normalization in one family is equivalent to the orthogonality in the other family. If ϕ_1 and ϕ_2 are two solutions then $\phi_1 + \phi_2$ and $\phi_1 - \phi_2$ are approximations of the solutions corresponding to the wave of two electrons. The wave of an electron in a system of electrons is also a function of space-time with value in Cl_3^*. Therefore the Dirac wave does not need the configuration space of non-relativistic quantum mechanics. Moreover the additivity of the chiral currents implies both the additivity of the charges and of the mass-energy.

2. Gravitation

The full meaning of (1) is the inclusion of the space-time manifold in Cl_3^*, which is a Lie group, then also a 8-dimensional manifold. The quantum wave of the electron allows us the definition of a mobile orthogonal (but not orthonormal) basis $(\mathrm{D}_0, \mathrm{D}_1, \mathrm{D}_2, \mathrm{D}_3)$. Using:

$$S_k = \phi\sigma_k\overline{\phi};\ S_k^\dagger = \widehat{\phi}\sigma_k\phi^\dagger, \tag{71}$$

$$\mathcal{S}_{(k)} + i\mathcal{S}'_{(k)} = \frac{\nabla S_k^\dagger}{\det(\phi^\dagger)}, \tag{72}$$

$$\mathcal{A}_{(k)} + i\mathcal{A}'_{(k)} = \frac{AS_k^\dagger}{\det(\phi^\dagger)}, \tag{73}$$

$$\tau = \frac{1}{2}[(\nabla\widehat{\phi})\phi^\dagger - \sigma^\mu\widehat{\phi}\partial_\mu\phi^\dagger], \tag{74}$$

$$\mathcal{T} + i\mathcal{T}' = \frac{\tau}{\det(\phi^\dagger)}. \tag{75}$$

The τ tensor is the density of spin of Durand [10, 47]. With the improved wave equation of the electron, this gives the following connection (see [37] D.4) :

$$\Gamma^0_{1\mu} = D_\mu \cdot [\mathcal{S}_{(1)} - 2q\mathcal{A}_{(2)}] + 2m\rho\delta^2_\mu, \tag{76}$$
$$\Gamma^0_{2\mu} = D_\mu \cdot [\mathcal{S}_{(2)} + 2q\mathcal{A}_{(1)}] - 2m\rho\delta^1_\mu, \tag{77}$$
$$\Gamma^0_{3\mu} = D_\mu \cdot \mathcal{S}_{(3)}, \tag{78}$$
$$\Gamma^2_{3\mu} = -D_\mu \cdot [\mathcal{S}'_{(1)} - 2q\mathcal{A}'_{(2)}], \tag{79}$$
$$\Gamma^3_{1\mu} = -D_\mu \cdot [\mathcal{S}'_{(2)} + 2q\mathcal{A}'_{(1)}], \tag{80}$$
$$\Gamma^1_{2\mu} = -D_\mu \cdot [\mathcal{S}'_{(3)} + 2qA] - 2m\rho\delta^0_\mu, \tag{81}$$
$$\Gamma^0_{0\mu} = D_\mu \cdot [-2\mathcal{T} + 2q\mathcal{A}'_{(3)}], \tag{82}$$

with $\delta^0_0 = 1$, $\delta^j_j = -1, j = 1,2,3$ and $\delta^\nu_\mu = 0, \mu \neq \nu$. Cl_3^* is not only a manifold including space-time, it is also the invariance group of the wave equations. Using:

$$M = 1 + \mathrm{dx}^\mu(a^0_\mu + a^j_\mu\sigma_j + a^{3+j}_\mu i\sigma_j + a^7_\mu i), \tag{83}$$

where the a^n_μ, for $\mu = 1,2,3,4$ and $n = 0,1,\dots,7$ are 32 numeric functions of x sufficiently smooth and the dx^μ are the increments of x at this point-event in the local basis. The similitude R defined by M which changes x into x′, such as $\mathrm{x}' = R(\mathrm{x}) + \mathrm{a} = M\mathrm{x}M^\dagger + \mathrm{a}$ where a is the vector $\mathrm{a} = \mathrm{a}^\mu\sigma_\mu$ of a translation, gives:

$$\begin{aligned}
\mathrm{x}'^0 &= \mathrm{x}^0 + \mathrm{dx}^0 + 2(a^0_\mu\mathrm{x}^0 + a^1_\mu\mathrm{x}^1 + a^2_\mu\mathrm{x}^2 + a^3_\mu\mathrm{x}^3)\mathrm{dx}^\mu,\\
\mathrm{x}'^1 &= \mathrm{x}^1 + \mathrm{dx}^1 + 2(a^1_\mu\mathrm{x}^0 + a^0_\mu\mathrm{x}^1 + a^6_\mu\mathrm{x}^2 - a^5_\mu\mathrm{x}^3)\mathrm{dx}^\mu,\\
\mathrm{x}'^2 &= \mathrm{x}^2 + \mathrm{dx}^2 + 2(a^2_\mu\mathrm{x}^0 - a^6_\mu\mathrm{x}^1 + a^0_\mu\mathrm{x}^2 + a^4_\mu\mathrm{x}^3)\mathrm{dx}^\mu,\\
\mathrm{x}'^3 &= \mathrm{x}^3 + \mathrm{dx}^3 + 2(a^3_\mu\mathrm{x}^0 + a^5_\mu\mathrm{x}^1 - a^4_\mu\mathrm{x}^2 + a^0_\mu\mathrm{x}^3)\mathrm{dx}^\mu.
\end{aligned} \tag{84}$$

Then the Christoffel symbols $\mathbf{\Gamma}^\alpha_{\beta\gamma}$ satisfy:

$$\mathbf{\Gamma}^0_{0\mu} = \mathbf{\Gamma}^1_{1\mu} = \mathbf{\Gamma}^2_{2\mu} = \mathbf{\Gamma}^3_{3\mu} = 2a^0_\mu, \tag{85}$$
$$\mathbf{\Gamma}^1_{0\mu} = \mathbf{\Gamma}^0_{1\mu} = 2a^1_\mu\ ;\ \ \mathbf{\Gamma}^2_{0\mu} = \mathbf{\Gamma}^0_{2\mu} = 2a^2_\mu, \tag{86}$$
$$\mathbf{\Gamma}^3_{0\mu} = \mathbf{\Gamma}^0_{3\mu} = 2a^3_\mu;\ \mathbf{\Gamma}^2_{3\mu} = -\mathbf{\Gamma}^3_{2\mu} = 2a^0_\mu, \tag{87}$$
$$\mathbf{\Gamma}^3_{1\mu} = -\mathbf{\Gamma}^1_{3\mu} = 2a^5_\mu\ ;\ \ \mathbf{\Gamma}^1_{2\mu} = -\mathbf{\Gamma}^2_{1\mu} = 2a^6_\mu. \tag{88}$$

The identity between inertia ($\Gamma^\alpha_{\beta\gamma}$ symbols) and gravitation ($\mathbf{\Gamma}^\alpha_{\beta\gamma}$ symbols) is equivalent to the identification between the two connections:

$$\mathbf{\Gamma}^\alpha_{\beta\mu} = \Gamma^\alpha_{\beta\mu}. \tag{89}$$

There are actually only 28 independent equalities ($28 = 8 \times 7/2$ is the dimension of $SO(8)$). The quantum wave in a gravitational non-null field follows exactly the same invariant wave equations as in a null field, with only the change of the ∇ operator into the invariant derivation $\mathbf{D}$ (see [37] 4.1.2) such as:

$$a^n = \sigma^\mu a^n_\mu, \tag{90}$$
$$\begin{aligned}
\mathbf{D}\widehat{\phi} &= [\nabla - \frac{1}{2}(\nabla\widehat{M}^{-1})\widehat{M}]\widehat{\phi}\\
&= [\nabla - \frac{1}{2}(a^0 - a^j\sigma_j + a^{3+j}i\sigma_j - a^7 i)]\widehat{\phi}.
\end{aligned} \tag{91}$$

Here all 32 functions, including the four ones of a^7 that are not implied in the calculation of the tensors of GR, must be considered.

Einstein thought that something was lacking in the physical theory for the integration of quantum physics into classical physics. The lack was in classical physics.

When the gravitational field is weak the double link between the invariant equation and the Lagrangian density is preserved, conserving the probability current. With:

$$\mathbf{D} = \sigma^\mu \mathbf{D}_\mu = \mathrm{D} - i\frac{\mathrm{b}}{2}, \tag{92}$$
$$\mathrm{D} = \sigma^\mu \mathrm{D}_\mu = \nabla - \frac{a^0}{2} + (\frac{a^j}{2} - i\frac{a^{3+j}}{2})\sigma_j;\ \mathrm{V} = \mathrm{V}_\mu \sigma^\mu.$$

This gives:

$$\begin{aligned}\mathrm{D} + i\mathrm{V} &= \nabla + \frac{1}{2}[-a^0 + (a^j - ia^{3+j})\sigma_j] + i\mathrm{V} \\ &= \begin{pmatrix} \partial_0 - \partial_3 & -\partial_1 + i\partial_2 \\ -\partial_1 - i\partial_2 & \partial_0 + \partial_3 \end{pmatrix} + \frac{1}{2}(X_\mu + iY_\mu)\sigma^\mu + i\mathrm{V}, \\ X_0 &= -a_0^0 - a_1^1 - a_2^2 - a_3^3;\ X_3 = -a_3^0 - a_2^4 + a_1^5 - a_0^3, \\ Y_0 &= a_1^4 + a_2^5 + a_3^6;\ Y_3 = -a_1^2 + a_2^1 + a_0^6, \\ X_1 &= -a_1^0 - a_0^1 - a_3^5 + a_2^6;\ X_2 = -a_2^0 + a_3^4 - a_0^2 - a_1^6, \\ Y_1 &= a_0^4 - a_3^2 + a_2^3;\ Y_2 = a_1^3 + a_0^5 - a_3^1,\end{aligned} \tag{93}$$

where V is a gauge vector proper to each spinor equation. For instance the invariant wave equation of L^1 is equivalent to the system formed by its real part (Lagrangian density) and by its imaginary part:

$$0 = -\frac{i}{2}\eta^{1\dagger}(\nabla\eta^1) + \frac{i}{2}(\nabla\eta^1)^\dagger\eta^1 + (\frac{1}{2}Y_\mu + V_\mu)\mathrm{D}_L^{1\mu}, \tag{94}$$
$$0 = -\frac{i}{2}(\partial_\mu + X_\mu)\mathrm{D}_L^{1\mu}. \tag{95}$$

when $X_\mu = 0$ the probability current is conservative, Lagrange equations operate with conservative momentum-energy tensor and quantized kinetic momentum $\hbar/2$. On the contrary when $X_\mu \neq 0$ (inflation?) the gravitational field is dominated by the curvature of homothety. The probability current is no longer conservative, there are no Lagrange equations and no more conservation of momentum-energy (see [37] 4.3).

The unification between gauge forces and gravitation relies on two following potential vectors:

$$0 = a^7 + \mathrm{b};\ \mathrm{b} = \frac{q}{\sqrt{3}}B, \tag{96}$$

where B is the potential term of the $U(1)$ gauge group. This equality simplifies the equations of the quark sector for electro-weak forces (see [37] 4.2).

The principle of equivalence is also a consequence of the properties of the quantum waves: for instance the L^1 wave has for the Lagrangian density, in the case $X_\mu = 0$:

$$0 = \mathcal{L}^1 = -\mathcal{L}_i^1 + \mathcal{L}_g^1, \tag{97}$$
$$\mathcal{L}_i^1 = \frac{i}{2}\eta^{1\dagger}\nabla\eta^1 - \eta^{1\dagger}(\frac{3}{2}\mathrm{b} + 3\mathrm{w}^3 - m_a\mathrm{v})\eta^1, \tag{98}$$
$$\mathcal{L}_g^1 = \frac{i}{2}(\partial_\mu\eta^{1\dagger})\sigma^\mu\eta^1 + \eta^{1\dagger}(\frac{1}{2}a_\mu^7 + \frac{1}{2}Y_\mu\sigma^\mu + m_b\mathrm{v})\eta_1,$$
$$m = m_b - m_a;\ \mathcal{L}_i^1 = \mathcal{L}_g^1. \tag{99}$$

The mass term of the wave equation is then always a difference between two mass-energy terms. The frequency of the de Broglie wave is then also a difference between the frequency of the considered system including a particle and the frequency of another system without this particle. This also applies to the Mössbauer effect.

3. Second Quantization and the Standard Model

The first sections used the Dirac wave with its two chiral parts. The perspective of second quantization considers a wave with value on a set of linear operators (creation and annihilation operators), conserving all results of the first quantization (for instance the results for the hydrogen atom). Moreover the occupation number of the quantum wave, in the case of a fermion, can be only 0 or 1. So we now let us suppose that the set of the values of the fermion wave is enlarged from Cl_3^* into End(Cl_3). This large multiplicative group contains Cl_3^* since the Pauli algebra contains diagonal matrices (this allows us to conserve the results of the first quantization). Moreover a function with value in a Lie group of endomorphisms may be considered as acting on itself, like creation and annihilation operators. The Ψ wave of second quantization is then supposed to be a function of space-time with value into End(Cl_3):

$$\Psi : \phi \mapsto \phi_e,\ \phi_e = \Psi(\phi); \Psi \in \mathrm{End}(Cl_3);\ \mathrm{x} = \phi_e \mathrm{y} \widetilde{\phi}_e. \tag{100}$$

where y belongs to the auto-adjoint part of Cl_3^*. The set of the y (numeric space-time) is included into the Cl_3^* Lie group, which is also an 8-dimensional manifold. Since End$(Cl_3) = Cl_{3,3} = M_8(\mathbb{R})$, Ψ reads:

$$\Psi = \Psi(\mathrm{x}) = \begin{pmatrix} \Psi_l + i\Psi_b & \Psi_r + \Psi_g \\ \Psi_r - \Psi_g & \Psi_l - i\Psi_b \end{pmatrix}, \tag{101}$$

$$\Psi_l = \mathcal{P}_1 - i\mathcal{I}_1;\ \mathcal{P}_1 = \begin{pmatrix} \phi_e & 0 \\ 0 & \widehat{\phi}_e \end{pmatrix};\ \mathcal{I}_1 = \begin{pmatrix} 0 & \phi_n^\dagger \\ \overline{\phi}_n & 0 \end{pmatrix},$$

$$\Psi_r = -i\mathcal{P}_2 + \mathcal{I}_2;\ \mathcal{P}_2 = \begin{pmatrix} \phi_{dr} & 0 \\ 0 & \widehat{\phi}_{dr} \end{pmatrix};\ \mathcal{I}_2 = \begin{pmatrix} 0 & \phi_{ur}^\dagger \\ \overline{\phi}_{ur} & 0 \end{pmatrix},$$

$$\Psi_g = -i\mathcal{P}_3 + \mathcal{I}_3;\ \mathcal{P}_3 = \begin{pmatrix} \phi_{dg} & 0 \\ 0 & \widehat{\phi}_{dg} \end{pmatrix};\ \mathcal{I}_3 = \begin{pmatrix} 0 & \phi_{ug}^\dagger \\ \overline{\phi}_{ug} & 0 \end{pmatrix},$$

$$\Psi_b = -i\mathcal{P}_4 + \mathcal{I}_4;\ \mathcal{P}_4 = \begin{pmatrix} \phi_{db} & 0 \\ 0 & \widehat{\phi}_{db} \end{pmatrix};\ \mathcal{I}_4 = \begin{pmatrix} 0 & \phi_{ub}^\dagger \\ \overline{\phi}_{ub} & 0 \end{pmatrix}. \tag{102}$$

The Ψ term is then composed of two different kinds of terms: Ψ_l which is alone, Ψ_r, Ψ_g and Ψ_b, which are three similar terms, different from Ψ_l. This means that the difference between a lepton part Ψ_l and a quark part $(\Psi_r,\ \Psi_g,\ \Psi_b)$ directly proceeds from the definition of the whole quantum wave. The lepton part contains a diagonal part which is the wave of the electron, with its left and right parts and the wave of the neutrino. This neutrino wave is equally the sum of a right and a left part. The standard model previously used only the left part, while the full wave describes the magnetic monopole of Lochak's theory [38–45]. The quark wave appears naturally in three parts named in the standard model r, g, b. This implies the conservation of the baryonic quantum number. The lepton part of the wave is studied in [20, 22, 28, 31] [36, 37]. The full wave is made of 16 chiral parts, eight left waves η^n and eight right waves ξ^n. Each wave follows an equation similar to the previous equations of the electron:

$$i\nabla\eta^n = p_1^n\eta^n; i\widehat{\nabla}\xi^n = \widehat{p}_2^n\xi^n, \tag{103}$$

where the p_1^n and p_2^n are space-time vectors containing gauge terms and mass terms. These equations are form-invariant under the enlarged relativistic group Cl_3^*. The lepton part of the wave is gauge-invariant under the $U(1)\times SU(2)$ gauge group of the electro-weak theory, with a Weinberg-Salam angle equal to 30° [24,36,37]. The value of the electric charges are calculated. The gauge invariance is both compatible with the form invariance and with the presence of mass terms in the wave equations. The full wave is gauge invariant under the $U(1) \times SU(2) \times SU(3)$ group of the standard model, the $SU(3)$ part of the gauge group acts only on the quark part of the full wave, explaining the fact that leptons do not see strong interactions. The double link between the wave equations and the Lagrangian density is similar to the electron case. Therefore the existence of two kinds of tensorial densities for the momentum energy, T for the sum and V for the difference between left and right parts of the waves, is completely similar to the electron case previously described. The extremal principle of Lagrangian physics is then not a meta-physical principle; it is only a consequence of the form of each wave equation and of the non commutative structure of the $GL(2, \mathbb{C})$ group.

As in the electron case, the J current is the sum of all chiral currents, ρ^2 is the scalar product $\mathrm{J}\cdot\mathrm{J}$. The mass term of each Equation (103) contains the unitary space-time vector $\mathrm{v} = \mathrm{J}/\rho$. The wave Equation (103) are then only partially uncrossed, v depending on all chiral waves. The normalization of the wave and the quantization of the kinetic momentum are obtained with the same method previously described in the first section (electron case). This quantization is obtained only for the whole wave. This explains the confinement of the quarks in the proton or in the neutron since the quarks do not have a quantized kinetic momentum $\hbar/2$. Only the whole proton or the whole neutron have a quantized kinetic momentum $\hbar/2$.

4. Concluding Remarks

The wave with value in $\mathrm{End}(Cl_3)$ accounts for all particles of the first generation. The wave equation contains a σ_{21} product that is only one of six similar products: σ_{21}, σ_{32}, σ_{13}, σ_{12}, σ_{23} and σ_{31}. Charge conjugation is the $C = PT$ symmetry corresponding simply to the change of σ_{jk} by σ_{kj}. The passing from σ_{21} to σ_{32} and σ_{13} may account for the two other generations.

The wave equations of the gauge fields are obtained by using the iterative process giving second order, third order, etc. wave equations.

The maximal violation of the parity in weak interactions, meaning a preference for the left waves, is also a consequence of the properties of material waves, (see [37] 3.8).

More generally, the laws and even the principles governing these laws (principle of exclusion, extremal principle, principle of equivalence, maximal violation of parity in weak interactions) are themselves consequences of the properties of the material waves.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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