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Quantum Mechanics Approach for Risk Aversion, Prudence, and Temperance

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Abstract

The ideas from quantum mechanics (QM) have been used as one of problem-solving methods in the field of economics, especially in game theory and decision theory, starting about "coin flip" and "prisoner's dilemma" and now days "decision paradoxes". In this paper, the concept of QM is applied to prudence and temperance. Classically, risk aversion, prudence, and temperance are characterized by the risk attitude toward losses and its volatility (variance), skewness, and kurtosis as well as by utility theory, where derivatives of the utility are related to risk aversion, prudence, and temperance. Here those are treated as decision paradoxes and in the QM model, probabilities of alternatives are tentatively set as unknown and a person's subjective probabilities toward the alternatives are set as parameterized. Investigating the utility difference before averaging can show the difference among risk aversion, prudence, and temperance. In that sense, a new QM interpretation of risk aversion, prudence, and temperance as opposed to the classical interpretation was founded in the first time.

Keywords

Quantum Mechanics, Risk Aversion, Prudence, Temperance, Utility

1. Introduction

Since the 2000s, ideas from quantum mechanics (QM) have been used as one of problem-solving methods in the field of economics, especially in game theory and decision theory. The QM model has suggested ultra-C solutions for the "coin flip" [1] and "prisoner's dilemma" [2]. Those expand the decision choice alternative. For instance, in the prisoner's dilemma of [2], the choice whether to cooperate or to defect is the matter, and in their QM model calculation of strategy space, the following matrices are set:

$$\begin{pmatrix} e^{i\phi}\cos\left(\frac{\theta}{2}\right) & \sin\left(\frac{\theta}{2}\right) \\ -\sin\left(\frac{\theta}{2}\right) & e^{-i\phi}\cos\left(\frac{\theta}{2}\right) \end{pmatrix},$$

where, classically, strategy cooperate is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, and strategy defect is $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ (See details in [2]).

The new solution is using the concept of entanglement of QM, which allow the model to use above $\sin\left(\frac{\theta}{2}\right)$ or $e^{i\phi}\cos\left(\frac{\theta}{2}\right)$ etc. Classical strategies above are the special case of $(\theta,\phi)=(0,0)$, and $(\theta,\phi)=(\pi,0)$, respectively. QM extended classical concept and find the additional strategies, as also described in section 3, and gave new solutions. Regarding human activity, that research so far thought it useful to solve the paradoxes (For the basic QM theory, see [3] and others).

Generally, the QM model was also used for other game theory studies such as [4] and other decision theory studies comparing Pareto optimum and Nash equilibrium solutions [5] [6] [7]. Conjunction fallacy and disjunction effect research was also conducted by [8], and a study was conducted on the QM model Ellsberg paradox [9] by [10]. This study examines the interpretation of prudence and temperance as well as risk aversion and discusses the model similar to [10]. They used QM concept (Born's rule) of probability and expected payoffs as utilities (See section 3). Their model also introduces mind state, which in this paper is treated as subjective probability. This allows the model to see from another view (projection) and to introduce individual subjective probability and variety of utility calculation.

This paper treats risk aversion, prudence and temperance tendencies are paradoxes and examines in QM way. Typically, these tendencies are described through the utility characteristics, so far. However, they also are paradoxically summarized as decision-making issues (see [11]). Still, classically, the payoff distribution's variance, skewness, and kurtosis of decision-making choice examples are examined. The motivation of this paper is that the decision-making examples could be treated as paradoxes and those characteristics could be clarified by QM framework, which might have more opportunity to minutely modeling human behaviors.

The rest of the paper is structured as follows. Section 2 summarizes the definition and characteristics of prudence and temperance with description as paradoxes. The idea of QM model, mathematical preparation, basic model explanations, and the concept behind are discussed in Section 3 along with the QM model results and implications. Section 4 provides the summary and future challenges.

2. Risk Aversion, Prudence, and Temperance

In this section, classical understanding of risk aversion, prudence and temperance are summarized and empirically described as paradoxes.

2.1. Definitions

The following definitions and interpretations of prudence and temperance, as well as risk aversion, are based on [11]:

Prudence: This concept was originally introduced in the precautionary savings literature (see [12] [13] [14]); it reflects the desire to increase savings in the face of income risk. According to utility theory, it is considered prudent if $u'''(x) > 0$ and imprudent if $u'''(x) < 0$.

Temperance: This concept was introduced by [15] in his study on the effect of labor income risk on the proportion of savings devoted to risky investment, and is defined as moderation in accepting independent risks. In utility theory, it is considered frugal if $u''''(x) < 0$ and not frugal if $u''''(x) > 0$.

Risk aversion: Originally introduced by [16], this refers to an affinity or aversion to risk. For utility function u, a person is considered risk averse if $u''(x) < 0$ and a risk lover if $u''(x) > 0$.

[17]'s interpretation of utility function defines prudence as "downside (loss) risk aversion" (if you avoid risks for both upward and downward movement, you are more concerned about downside risk). They defined temperance as tail risk avoidance, in which the frequency of losses changes as the losses increases.

2.2. Empirical Research

According to [11], risk aversion, prudence, temperance are described in the empirical research below.

Risk aversion:

Consider an experiment where you compare two options, Choice 1 and Choice 2, and you must find the better one. In Choice 1, you do not gain or lose anything (you can say that you gain 0). For comparability, the situation is set as follows:

Choice 1: Two possibilities of a_1 and b_1.

a_1. There is a 50% probability that you will not gain or lose anything (you can say that you will gain 0).

b_1. There is a 50% probability that you will not gain or lose anything (you can say that you will gain 0).

The redundancy is intentional.

Choice 2: Two possibilities of a_2 and b_2.

a_2. There is a 50% probability that you will not gain or lose anything (you can say that you will gain 0).

b_2. There is a 50% probability that you will obtain the number ε that is normally distributed with mean 0.

When Choice 1 is preferred over Choice 2, this is a state of being risk averse.

Prudence:

Consider an experiment in which you compare two options, Choice 3 and Choice 4, and you must choose the better one.

Choice 3: Two possibilities of a_3 and b_3.

a_3. There is a 50% probability that you will gain $-k$ (k is a constant positive number; if k is 10, you will lose 10 or gain -10).

b_3. There is a 50% probability that you will obtain the number ε that is normally distributed with a mean of 0.

Choice 4: Two possibilities a_4 and b_4.

a_4. There is a 50% probability that you will not gain or lose anything (you can also say that you will gain 0).

b_4. There is a 50% probability that you will obtain the number ε that is normally distributed with a mean of $-k$.

When Choice 3 is preferred over Choice 4, this is the state of prudence.

Temperance:

Next, consider an experiment in which you compare two options, Choice 5 and Choice 6, and you must choose the better one.

Choice 5: Two possibilities of a_5 and b_5.

a_5. There is a 50% probability that you will obtain a number that is normally distributed ($\varepsilon 1$) with a mean of 0.

b_5. There is a 50% probability that you will obtain a number that is independent of the above and has a normal distribution ($\varepsilon 2$) with a mean of 0.

Choice 6: Two possibilities of a_6 and b_6.

a_6. There is a 50% probability that you will not gain or lose anything.

b_6. There is a 50% probability that you will get a normal distribution with a mean of 0 or a normal distribution with an amplitude twice that of ($\varepsilon 1$ + $\varepsilon 2$).

When Choice 5 is preferred over Choice 6, this is the state of temperance.

The preference between Choice 1 and Choice 2 is paradox because the expected payoffs are the same. So are between Choice 3 and Choice 4, and between Choice 5 and Choice 6. Next section solves these paradoxes using QM model.

3. Model

This chapter discusses the idea of QM model, mathematical preparation, basic model explanations, and the concept behind.

3.1. Preparation and Concept of QM

Paradoxes are often described as to choose specific urn and pick up ball of black or white. Supposing the utility U is an expected payoff, classically, U (chose #1 urn, Pick black ball) is the below:

U(#1 urn, Pick black ball) (≡U(#1, black), #1 urn has been chosen.)

= (Probability of pick a black ball from #1 urn) × Payoff

In QM model, the expected payoff is calculated as follows:

$$\begin{aligned} U(\#1,\text{black}) &= E\Bigg[\left(\alpha_{\text{black}}\begin{pmatrix}1 & 0\end{pmatrix}+\alpha_{\text{white}}\begin{pmatrix}0 & 1\end{pmatrix}\right)\times\left(\begin{pmatrix}\text{Payoff}_{\text{black}} & 0\\ 0 & 0\end{pmatrix}\right. \\ &\left.+\begin{pmatrix}0 & 0\\ 0 & \text{Payoff}_{\text{black}}\end{pmatrix}\right)\times\left(\alpha_{\text{black}}\begin{pmatrix}1\\0\end{pmatrix}+\alpha_{\text{white}}\begin{pmatrix}0\\1\end{pmatrix}\right)\Bigg] \\ &= \langle\Psi|A|\Psi\rangle\ \ (\text{where } \Psi=\text{state},\ A=\text{payoff matrix}) \end{aligned}$$

Probability is defined as above description and in the case 50% and 50% of black and white indicate $\alpha_{\text{black}} = \alpha_{\text{white}} = \frac{1}{\sqrt{2}}$. $\text{Payoff}_{\text{black}}$ is just in case the payoff of not picking black ball. The last line describe QM's Dirac bra-ket notation. This is one of the characteristics of QM model and allows the model to extend parameter flexibility (as later shown as phase θ).

This allows more mathematical extension and flexibility. Where this comes from? QM holds that physical quantities, such as the state of a wave, can be determined only by measurement. Here, the wave function ψ is defined by considering the physical quantities of matter as a wave. The Dirac bra-ket notation is introduced as $|\psi\rangle$, which is called a ket, and $\langle\psi|$, which is called bra. These sets are then discussed as components of the Hilbert space. Mathematically, these denote vectors in a complex vector space, and physically they represent states of (wave) systems.

Next, Born's rule is introduced. In measuring physical quantities, assuming that the system state $|\psi\rangle$ is expanded using the eigenstate $|\omega_i\rangle$ ($i = 1, 2, \cdots, N$), we have

$$|\Psi\rangle = c_1|\omega_1\rangle + c_2|\omega_2\rangle + \cdots + c_N|\omega_N\rangle.$$

However, the probability of obtaining the measured value ω_j by measuring the physical quantity before measurement is

$$\langle\omega_j|\Psi\rangle^2 = |c_j|^2.$$

This refers to the interpretation given by Born's rule.

Moreover, when considering the two states $|\psi\rangle$ and $|\phi\rangle$, there can be an overlap of intermediate states before measurement. However, by measurement, we can never find a system in an intermediate state between $|\psi\rangle$ and $|\phi\rangle$ but can always find a system in either $|\psi\rangle$ or $|\phi\rangle$. Whether it is found in state $|\psi\rangle$ or $|\phi\rangle$ depends on the measurement. Furthermore, the projection principle (contraction of state due to measurement) refers to the idea that the state before measurement changes from the state of superposition to a specific state by measurement (called projective measurement). Furthermore, entangled states (quantum entanglement), which are important in resolving the prisoner's dilemma, are generally the key to solving various decision paradoxes using the concept of QM. The key to this study is the related concept of redundant gauge degrees of freedom. Using this QM mathematical characteristic, the model introduces subjective probability. This allows the model to introduce individual subjective probability (see the below and discussions here after).

$$\begin{pmatrix} y^2 & e^{-i\theta} y\sqrt{1-y^2} \\ e^{i\theta} y\sqrt{1-y^2} & 1-y^2 \end{pmatrix}$$

3.2. Solution to Paradoxes

The proposed way to solve problems in economics using the QM concept is to apply the Born probability and extend the mathematical model to the wave function state with complex numbers [18]. In the probability interpretation of state vectors, as state vectors $|\psi\rangle$ and $|\psi'\rangle = e^{i\theta}|\psi\rangle$ give the same result, we can conclude that both represent the same state. This complex coefficient $e^{i\theta}$ is called a phase factor, while the degree of freedom in selecting phase θ is called the state vector's redundant gauge degree of freedom. When the system is in state $|\psi\rangle$, the probability P of finding the system state in $|\phi\rangle$ by measurement is given by the square of $|\langle\phi|\psi\rangle|$. Here, $\langle\phi|\psi\rangle$ before squaring is called the probability amplitude.

It is generally difficult to understand the meaning of probability amplitudes including complex numbers. This will be explained later, but note that the degree of prudence and degree of temperance are related. This flexibility allows the opportunity to solve the paradoxes.

3.3. Formulation

The basic state $|\omega_i\rangle$ (basic Hilbert space state, $i = a$, b) is such that, for each Choice m, a_m (denoted as a_m) and b_m (denoted as b_m) occur, and we set them as $|a_m\rangle$ and $|b_m\rangle$ (m = 1, 2, 3, 4, 5, 6), respectively. The following expressions define the event probability $P\omega$.

$$Pa_m = |a_m\rangle\langle a_m|$$

$$Pb_m = |b_m\rangle\langle b_m|$$

(Trace(Pa_m+Pb_m) = 1, m = 1, 2, 3, 4, 5, 6)

After standardizing the actions, states (Ψa_1 for a_1., Ψb_1 for b_1. and others) can be expressed as follows, based on each choice in Section 2.

$$\text{In Choice } m,\quad \Psi_m = \frac{1}{\sqrt{2}}\left(\Psi a_m + \Psi b_m\right) \quad (m = 1, 2, 3, 4, 5, 6).$$

that is,

$$\text{In Choice 1},\quad \Psi_1 = \frac{1}{\sqrt{2}}\left(\Psi a_1 + \Psi b_1\right), \text{ and etc.}$$

This can be expressed as Aa and Ab, using the payoff function $u(\ )$ for the payoffs of a. and b., respectively:

$$A_m = u(a_m) \times Pa + u(b_m) \times Pb$$

The expected utilities are described as follows:

$$U(m) = E\left[\langle\Psi_m|A_m|\Psi_m\rangle\right]$$

3.3.1. Risk Aversion

From here on, we use a matrix rather than Braket description to calculate the expected utility. First, for Choices 1 and 2, as the options for each of the two choices will be selected with a 50% probability, the difference in expected utilities will be as follows:

$$U(1)=E\left[\frac{1}{\sqrt{2}}\left[\begin{pmatrix}1 & 0\end{pmatrix}+\begin{pmatrix}0 & 1\end{pmatrix}\right]\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}\frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}\right]\right]=E[0]=0$$

$$U(2)=E\left[\frac{1}{\sqrt{2}}\left[\begin{pmatrix}1 & 0\end{pmatrix}+\begin{pmatrix}0 & 1\end{pmatrix}\right]\begin{pmatrix}0 & 0\\ 0 & \varepsilon\end{pmatrix}\frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}\right]\right]=E[\varepsilon]=0$$

$$U(1)-U(2)=E[-\varepsilon]=0$$

This means that there is no difference between Choice 1 and Choice 2.

Here, ambiguity (no notion that both options will be selected with a 50% probability) and subjective probability ([19] [20] [21] [22] and others) are introduced. These states are described as follows. First, we introduce

$$\Psi_1(x)=x|a_1\rangle+\sqrt{1-x^2}|b_1\rangle$$

$$\Psi_2(x)=x|a_2\rangle+\sqrt{1-x^2}|b_2\rangle$$

as the ambiguity state (including the 50% probability case with $x^2 = 0.5$). This means that in Choices 1 and 2, we do not know whether the exact probability of a_m and b_m (m = 1, 2) is 0.5. Then, we introduce subjective probability. People who choose options have their own subjective probability of a_m (m = 1, 2) and b_m (m = 1, 2), which can be presumed to be y^2 and $(1-y^2)$, respectively, with gage degree of freedom θ, described as Ψ_{people} below.

$$\Psi_{people}(y,\theta)=y|a_1\rangle+e^{i\theta}\sqrt{1-y^2}|b_1\rangle=y|a_2\rangle+e^{i\theta}\sqrt{1-y^2}|b_2\rangle$$

In this case, the following can describe the probabilities of Pa_m and Pb_m (m = 1, 2).

$$P_{people}Pa_mP_{people},$$

$$P_{people}Pb_mP_{people}$$

($P_{people}=\begin{pmatrix}y^2 & e^{-i\theta}y\sqrt{1-y^2}\\ e^{i\theta}y\sqrt{1-y^2} & 1-y^2\end{pmatrix}$ This is the same even for m = 3, 4, 5, 6).

Here, the difference in expected utilities is as follows:

$$\begin{aligned}&U(1)-U(2)\\&=E\left[\begin{pmatrix}x & \sqrt{1-x^2}\end{pmatrix}\begin{pmatrix}y^2 & e^{-i\theta}y\sqrt{1-y^2}\\ e^{i\theta}y\sqrt{1-y^2} & 1-y^2\end{pmatrix}(-\varepsilon)\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}y^2 & e^{-i\theta}y\sqrt{1-y^2}\\ e^{i\theta}y\sqrt{1-y^2} & 1-y^2\end{pmatrix}\begin{pmatrix}x\\ \sqrt{1-x^2}\end{pmatrix}\right]\\&=E[-\varepsilon]\times(1-y^2)\left\{x^2y^2+(1-x^2)(1-y^2)+2xy\sqrt{1-x^2}\sqrt{1-y^2}\cos(\theta)\right\}\end{aligned}$$

In this way, subjective probability is introduced. It might be tentative and at last it would be 50% probability ($y^2=0.5$). However, this zooms up the difference among risk aversion, prudence and temperance. In QM theory, it is match to projection, observing the value from the specific basic state combination view.

3.3.2. Prudence

First, for Choice 3, as each of the two options is selected with a 50% probability, the expected value obtained as a result of investment is

$$U(3)=E\left[\frac{1}{\sqrt{2}}\left[\begin{pmatrix}1 & 0\end{pmatrix}+\begin{pmatrix}0 & 1\end{pmatrix}\right]\begin{pmatrix}-k & 0\\ 0 & \varepsilon\end{pmatrix}\frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}\right]\right]$$
$$=E\left[\frac{1}{2}(-k+\varepsilon)\right]=-\frac{1}{2}k$$

This is the same for Choice 4:

$$U(4)=\frac{1}{\sqrt{2}}\left[\begin{pmatrix}1 & 0\end{pmatrix}+\begin{pmatrix}0 & 1\end{pmatrix}\right]\begin{pmatrix}0 & 0\\ 0 & -k+\varepsilon\end{pmatrix}\frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}\right]$$
$$=E\left[\frac{1}{2}(-k+\varepsilon)\right]=-\frac{1}{2}k$$
$$U(3)-U(4)=E[0]=0$$

Next, when the following is introduced, as discussed in the previous section, the results will be as stated below.

$$\Psi_m(x)=x|a_m\rangle+\sqrt{1-x^2}\,|b_m\rangle$$
$$P_{people}Pa_mP_{people},$$
$$P_{people}Pb_mP_{people}\quad (m=3,4)$$

$$U(3)=E\left[\begin{pmatrix}x & \sqrt{1-x^2}\end{pmatrix}\begin{pmatrix}y^2 & \mathrm{e}^{-i\theta}y\sqrt{1-y^2}\\ \mathrm{e}^{i\theta}y\sqrt{1-y^2} & 1-y^2\end{pmatrix}\begin{pmatrix}-k & 0\\ 0 & \varepsilon\end{pmatrix}\begin{pmatrix}y^2 & \mathrm{e}^{-i\theta}y\sqrt{1-y^2}\\ \mathrm{e}^{i\theta}y\sqrt{1-y^2} & 1-y^2\end{pmatrix}\begin{pmatrix}x\\ \sqrt{1-x^2}\end{pmatrix}\right]$$

$$U(4)=E\left[\begin{pmatrix}x & \sqrt{1-x^2}\end{pmatrix}\begin{pmatrix}y^2 & \mathrm{e}^{-i\theta}y\sqrt{1-y^2}\\ \mathrm{e}^{i\theta}y\sqrt{1-y^2} & 1-y^2\end{pmatrix}\begin{pmatrix}0 & 0\\ 0 & -k+\varepsilon\end{pmatrix}\begin{pmatrix}y^2 & \mathrm{e}^{-i\theta}y\sqrt{1-y^2}\\ \mathrm{e}^{i\theta}y\sqrt{1-y^2} & 1-y^2\end{pmatrix}\begin{pmatrix}x\\ \sqrt{1-x^2}\end{pmatrix}\right]$$

Those like $\begin{pmatrix}-k & 0\\ 0 & \varepsilon\end{pmatrix}$ and $\begin{pmatrix}0 & 0\\ 0 & -k+\varepsilon\end{pmatrix}$ are different as form $k\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$,

$$\begin{aligned}&U(3)-U(4)\\&=E\left[\begin{pmatrix}x & \sqrt{1-x^2}\end{pmatrix}\begin{pmatrix}y^2 & \mathrm{e}^{-i\theta}y\sqrt{1-y^2}\\ \mathrm{e}^{i\theta}y\sqrt{1-y^2} & 1-y^2\end{pmatrix}k\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}y^2 & \mathrm{e}^{-i\theta}y\sqrt{1-y^2}\\ \mathrm{e}^{i\theta}y\sqrt{1-y^2} & 1-y^2\end{pmatrix}\begin{pmatrix}x\\ \sqrt{1-x^2}\end{pmatrix}\right]\\&=E\left[k\times(1-2y^2)\left\{x^2y^2+(1-x^2)(1-y^2)+2xy\sqrt{1-x^2}\sqrt{1-y^2}\cos(\theta)\right\}\right]\\&=k\times(1-2y^2)\left\{x^2y^2+(1-x^2)(1-y^2)+2xy\sqrt{1-x^2}\sqrt{1-y^2}\cos(\theta)\right\}\end{aligned}$$

This means as follows:

- If the person's probabilities of options for a_4 and b_4 are 1/2 (y^2 is 1/2), the difference will be 0.
- Unless x^2, y^2, and θ have a specific relationship, the difference will remain as a scalar quantity that does not disappear even when the expected value is taken.

3.4. Temperance

We present below the same discussion for Choices 5 and 6.

$$U(5) = E\left[\frac{1}{\sqrt{2}}\left[(1\quad 0)+(0\quad 1)\right]\begin{pmatrix}\varepsilon 1 & 0\\ 0 & \varepsilon 2\end{pmatrix}\frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}\right]\right]$$
$$= E\left[\frac{1}{2}(\varepsilon 1+\varepsilon 2)\right] = 0$$

$$U(6) = \frac{1}{\sqrt{2}}\left[(1\quad 0)+(0\quad 1)\right]\begin{pmatrix}0 & 0\\ 0 & \varepsilon 1+\varepsilon 2\end{pmatrix}\frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}\right]$$
$$= E\left[\frac{1}{2}(\varepsilon 1+\varepsilon 2)\right] = 0$$

When the following is introduced, the results will be as stated below.

$$\Psi_m(x) = x|a_m\rangle + \sqrt{1-x^2}\,|b_m\rangle$$

$$P_{people}Pa_mP_{people},$$

$$P_{people}Pb_mP_{people}$$

(In this case, m = 5, 6).

$$U(5) = E\left[\left(x\quad \sqrt{1-x^2}\right)\begin{pmatrix}y^2 & e^{-i\theta}y\sqrt{1-y^2}\\ e^{i\theta}y\sqrt{1-y^2} & 1-y^2\end{pmatrix}\begin{pmatrix}\varepsilon 1 & 0\\ 0 & \varepsilon 2\end{pmatrix}\begin{pmatrix}y^2 & e^{-i\theta}y\sqrt{1-y^2}\\ e^{i\theta}y\sqrt{1-y^2} & 1-y^2\end{pmatrix}\begin{pmatrix}x\\ \sqrt{1-x^2}\end{pmatrix}\right]$$

$$U(6) = E\left[\left(x\quad \sqrt{1-x^2}\right)\begin{pmatrix}y^2 & e^{-i\theta}y\sqrt{1-y^2}\\ e^{i\theta}y\sqrt{1-y^2} & 1-y^2\end{pmatrix}\begin{pmatrix}0 & 0\\ 0 & \varepsilon 1+\varepsilon 2\end{pmatrix}\begin{pmatrix}y^2 & e^{-i\theta}y\sqrt{1-y^2}\\ e^{i\theta}y\sqrt{1-y^2} & 1-y^2\end{pmatrix}\begin{pmatrix}x\\ \sqrt{1-x^2}\end{pmatrix}\right]$$

Those like $\begin{pmatrix}\varepsilon 1 & 0\\ 0 & \varepsilon 2\end{pmatrix}$ and $\begin{pmatrix}\varepsilon 1 & 0\\ 0 & \varepsilon 1+\varepsilon 2\end{pmatrix}$ are different as form $-\varepsilon 2\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$,

$$U(5)-U(6)$$
$$= E\left[(-\varepsilon 2)\right]\times\left(1-2y^2\right)\left\{x^2y^2+\left(1-x^2\right)\left(1-y^2\right)+2xy\sqrt{1-x^2}\sqrt{1-y^2}\cos(\theta)\right\}$$

Temperance thus differs from prudence in that its formula has a fixed value k or variable $\varepsilon 1$.

3.5. Classical and QM Interpretation

Here, we discuss the classical interpretation of risk aversion, prudence, and temperance in addition to their QM interpretations are summarized in **Table 1** and **Table 2**.

Classically, as shown in **Table 1**, to find why people opt for Choice 1 rather than Choice 2 in Section 2, we calculate the expected utility (payoff) and the difference. Even when the averages are both zero, risk aversion would be an issue if the averages and their volatility (variance, or standard deviation of utility difference) is different between them. For prudence and temperance, to examine why people selected Choice 3 rather than Choice 4 and Choice 5 rather than Choice 6, we first calculated thevolatility, but found this to be the same. However, there should be other indicators. If the figures of the skew of each utility are different, prudence would be an issue. Even when the skew of the utility is the same,

Table 1. Classical analysis for risk aversion, prudence, and temperance.

Utility (Payoff) (options are with 50% probability)	Risk Aversion	Prudence	Temperance
Expected Utility	The same	The same	The same
Volatility	Different	The same	The same
Skew		Different	The same
Kurtosis			Different

Table 2. QM model analysis for risk aversion, prudence, and temperance

Utility Difference before Expectation Calculation	Risk Aversion	Prudence	Temperance
Probability of options is x^2 (ambiguity), and probability of people's mind is y^2 (subjective probability)	Variable $-\varepsilon\times(1-y^2)\{x^2y^2 + (1-x^2)(1-y^2) + 2x\sqrt{1-x^2} \times y\sqrt{1-y^2}\cos(\theta)\}$	Constant $k\times(1-2y^2)\{x^2y^2 + (1-x^2)(1-y^2) + 2x\sqrt{1-x^2} \times y\sqrt{1-y^2}\cos(\theta)\}$	Variable $-\varepsilon 2\times(1-2y^2)\{x^2y^2 + (1-x^2)(1-y^2) + 2x\sqrt{1-x^2} \times y\sqrt{1-y^2}\cos(\theta)\}$

temperance can be an issue if kurtosis is different among them and temperance is the reason for the choices.

Characteristics of risk aversion, prudence, and temperance outcomes are summarized in regard to differences between choices of section 2.2 and their volatility (standard deviation), skew and kurtosis.

However, for QM models, as shown in Table 2, we use only the calculation of "utility difference before expectation calculation" with ambiguity and people's subjective probability to outline the reason behind the decision as follows:

First, input $y^2 = 0.5$; then, risk aversion case will not have a 50% probability of options, even though a 50% probability of people's minds. Here risk aversion is $-\varepsilon\times\left\{\frac{1}{2}+\frac{1}{2}\cos(\theta)\right\}$, both prudence and temperance would be 0. This means that because of risk aversion, Choice 1 is preferred to Choice 2 and risk aversion makes no difference between Choice 3 and Choice 4 or between Choice 5 and Choice 6.

To find the difference between the Choice 3 and Choice 4 pair and Choice 5 and Choice 6 pair, we use probability option of x^2 (ambiguity), and probability of people's mind of y^2 (subjective probability), and average the utility difference. The expected utility difference in the former pair is not zero, but that in the latter pair is zero. This shows that guides the choice between Choice 3 and Choice 4, as prudence. Temperance matters when the volatility of the utility difference is not zero (though the expected utility difference is zero).

After probabilities of alternatives are tentatively set as unknown and a person's subjective probabilities toward the alternatives are set, the analysis for risk aversion, prudence, and temperance performed. Those utility difference before expectation calculation could be variable or constant.

Considering rational and implication of above, human behavior of not fully believing the situation or setting could affect the paradoxical choices and QM setting including subjective probability reveals the characteristics or magnitude of the risk in case the situation or setting is not as told (fair etc.).

4. Discussion, Summary, and Future Challenges

The study applied the QM model to risk aversion, prudence, and temperance utilizing the QM concept in decision making theory and game theory. The study first developed the flipping coin paradox and prisoner's dilemma paradox solutions, giving the solution more flexibility and more strategy opportunities. QM also gave solutions for several paradoxes of decision-making. Empirically, risk aversion, prudence, and temperance are explained "choosing urns and picking a color ball" paradoxes. This paper applied QM mode to them. When calculating probability using the born probability interpretation in Hilbert space, we found a new QM interpretation of risk aversion, prudence, and temperance as opposed to the classical interpretation.

To detect risk aversion, prudence, and temperance status difference, probabilities of alternatives are tentatively set as unknown and a person's subjective probabilities toward the alternatives are set. This method fit for QM model and this might mean that even people are told this is fair (say 50% probability), to some extent people daut that. This leads to and reveals the difference among risk aversion, prudence, and temperance. A new QM interpretation of risk aversion, prudence, and temperance as opposed to the classical interpretation was founded in the first time.

As a future challenge (in QM as well), the physical quantity observed should be real, and not simulated, as the imaginary part of $e^{i\theta}$ and $e^{-i\theta}$ are canceled out. However, the probability amplitude (its square is probability) is complex and the phase state is important [23]. The phase difference is related to the observation/measurement stability [24]. Amplitude and phase state are future issues for decision theory research.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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