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Research on Advertising Volume, Pricing and Promotion Strategies of the Online Video Platform

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Abstract

Considering that the promotion strategies adopted by the platform can increase consumer expectations, the paper constructs decision models for the platform profits maximisation problem and analyse the optimal advertising volume, optimal pricing and optimal promotion strategies of the platform. The results show that the platform is able to influence advertising volume, pricing, or market demand through its promotion strategies, thereby increasing the video profits. Before the video is launched in the market, there exists an optimal promotion period and an optimal allocation proportion of promotion investment. The optimal promotion period before the video is launched in the market is the same under different business models. The optimal promotion investment before video launched is the largest in paid model, the second largest in mixed model, and the smallest in free model.

Keywords

Advertising Volume Strategy, Pricing Strategy, Promotion Strategies, Promotion Period, Promotion Investment

1. Introduction

In recent years, with the development of Internet technology and the popularity of mobile phones, tablets and smart TVs and other terminal devices, online video industry has grown rapidly worldwide, such as Netflix, HBO NOW, YouTube, Amazon Prime, Hulu, IQIYI, etc. In 2018, the number of video subscribers in the US reached 228.8 million. By 2022, the number of video subscribers in the US is expected to reach 248.9 million. In the US, the most popular video platform is YouTube with over 126 million users [1]. In China, online video industry has experienced more than a decade of development and the number of video users has achieved rapid growth. According to the 52st Statistical Report on the Development of the Internet in China released by the China Internet Network Information Centre, the scale of Internet video users in China reached 1.044 billion as of June 2023, an increase of 13.80 million from December 2022, accounting for 96.8% of Internet users as a whole. Watching online videos has become an important leisure and entertainment activity for the majority of Internet users [2].

Against this backdrop, online video platforms pay attention not only to the broadcast model of their programs but also to their promotional strategies in order to attract more consumers to watch the videos they broadcast. Typically, there are three business models for video platforms: free, paid and mixed [3] [4] [5] [6]. In practice, online video platforms invest a certain budget in trailers, commercials and other promotional activities for newly launched popular programs, and start promotional campaigns before the video programs are launched in the market. It follows that under different business models, it becomes increasingly important for the video platforms to maximize revenue for a newly launched video programs by using effective promotional campaigns to successfully attract as many viewers as possible. For example, given budget constraints, how do the video platforms decide on the promotional period before the launch of the video programs, how do they decide on the intensity of investment during the promotional period, and how do they allocate the proportion of promotional investment before and after the launch of the video programs, etc.

Online video programs are media products and their promotion strategies differs from that of ordinary products. Online video has the characteristic of being promoted before it is launched in the market [7]-[12]. Therefore, the general product promotion strategies for advertising are not fully applicable to online video industry. The purpose of this research paper is to provide decision support and guidance for the scientific development of promotional strategies for online video companies. We aim to study the following two questions: how do promotion strategies affect advertising volume and pricing of the video? Under different business models, how do platforms develop their promotion strategies for video programs, *i.e.* how long is the promotion period before the programs is launched in the market, the intensity of the promotion investment during the promotion period, how to allocate the proportion of the promotion investment before and after the programs is launched in the market, etc.

In this paper we consider a monopolistic market consisting of a video platform and consumers. We assume that consumers are heterogeneous. We have developed decision models for the profits maximization problem of video platform under free model, paid model and mixed model respectively. In free model, we obtained the optimal advertising volume and optimal promotional investment intensity of the video platform; in paid model, we obtained the optimal pricing and optimal promotional investment intensity of the video platform; and in mixed model, we obtained the optimal pricing, optimal advertising volume and optimal promotional investment intensity of the video platform. We also compare the impact of promotion effects on optimal advertising volume and optimal pricing. We then compared the profits of the video platform under three business models to select the optimal business model for the video platform. Under different business models, we obtained the optimal promotional period for video platform before the launch of the video programs in the market, and the optimal proportion of promotional investment before and after the launch of the video programs in the market. In addition, we also verified the relevant findings obtained in this paper through numerical analysis.

The research contribution of this paper is mainly reflected in the following two aspects. One, we give the optimal business model choice for the video platform, and under the optimal business model, we get the optimal promotion investment intensity of the platform before and after the video is launched in the market, and analyze the influence characteristics of the promotion strategy on the optimal price, optimal amount of advertisement, market demand and profits of the platform. Second, under the optimal business model, we obtain the optimal promotion period of the platform before the video is launched in the market and the optimal allocation proportion of the promotion investment before and after the video is launched in the market. In addition, we compare the relationship between the above optimal promotion decisions under the optimal business model that satisfies different scenarios.

This paper is organized as follows. In Section 2, we review the related literature. In Section 3, we give the problem descriptions and basic assumptions. In Section 4, we establish the decision models for the profits maximization problem of the video platform in free model, paid model, and mixed model, respectively, and the optimal decisions and related analyses are given. In Section 5, we give the optimal business model choices of the video platform. In Section 6, we give the video platform's optimal promotion period strategy and optimal promotion investment allocation strategy under different business models. In Section 7, we validate the relevant results obtained in this paper through numerical analysis. In Section 8, we summarize the results and managerial insights, and give directions for future research. The proofs are all provided in the Appendix.

2. Related Literature

At present, the research on the market strategies of online video platforms has attracted extensive attention from many scholars, but the research on the promotion strategies of online video platforms has not yet been seen. There are two types of literature related to the problem studied in this paper: one is the research on the business model selection of online video platforms; the other is the research on media-type platforms regarding advertising and promotion.

The business model of online video platforms relies mainly on advertising and paid content. Since the business model is the core of the enterprise to determine the profits model and the source of profits, therefore, studying the business model can help online video platforms to find the most suitable business model for themselves and improve their profitability. Scholars focused that the optimal pricing of paid model of online platform video, the optimal amount of commercial advertisement in free model, the optimal business model choice of online video platform, and the influential factors affecting the business model decision of online video platform. For example, Cheng *et al.* [4] analyzed that the optimal pricing decision of online video platforms by considering consumer's choice behavior. It was found that the greater the sensitivity of consumers to advertisements, the lower the optimal price the platform charges advertisers. Xu *et al.* [5] studied that monopoly online video platform adopts the optimal price decision of paid model, the optimal advertising volume decision of free model, and the joint decision of optimal price and advertising volume of mixed model by considering bandwidth cost, and further analyzed that the optimal business model choice of the platform. Li *et al.* [13] studied the cooperation strategies of duo-oligopoly online video platforms and explored that the pricing decisions and optimal profits of online video platforms in three cases, namely, single-single, multi-single and multi-multi. The results show that when advertisers choose single-homing, the anchor platform can lead to higher profits; when advertisers choose multi-homing, the greater the cost of retransmission rights of videos, the higher the profits of the anchor platform. In addition, video platforms should increase their profits by establishing strong cross-side network effects with multi-attribution advertisers. Rong *et al.* [14] explored that determinants affected consumer stickiness of online video platforms through platform theory. The study found that proprietary resources are crucial for consumer stickiness regardless of the platform's business model, but the price factor does not have a significant effect on consumer stickiness. Chiang *et al.* [15] investigated that the optimal strategy for program content producers to sell their copyrights. They show that content producers sell program rights to online video platforms better than to traditional TV platforms because online platforms are able to leverage online convenience to expand the subscriber base of consumers. Alaei *et al.* [16] examined that there are two types of revenue distribution rules used by online video platforms and video content providers- pro-rata and consumer-centered distribution. The studied shows that both online video platforms and video content providers tend to use the pro-rata payment mechanism. Kim and Mo [17] compared that video distributors' profits with and without commercialization of channels in the online video supply chain. The study shows that video distributors prefer to adopt a free model on direct channels if they offer low quality videos. In addition, video distributors can choose not to offer content to downstream platforms as a paid service. Fan *et al.* [18] analyzed that the optimal business model for online media platforms. The studied pointed that when the quality of content is high and the cost of web access is low, the platform should adopt paid model; when the cost of web access is high, the platform can adopt free strategy. Xue *et al.* [19] analyzed that the pricing strategy of online short video platforms. The results show that when the nuisance cost of consumers is low and the intensity of cross-network externalities is high, the platform should choose the pricing strategy of the advertising model; otherwise, it should choose the pricing strategy of mixed model.

Research on the advertising and promotional aspects of media based programs has focused on areas such as film and television. For instance, Julia *et al.* [8] and Lee *et al.* [9] argued that pre-launch promotion of experiential products such as movies is crucial, and that advance promotion is a key driver influencing the success of a movie's premiere at the box office. Franses [7] proposed that a revenue model for movie box office, the results of the study showed that consumers based on intrinsic motivation are attracted by trailers, advertisements before the release of the movie resulting in the peak of the movie box office tends to occur in the first week and then the new revenues slowly disappear. Ehrenberg and Andrew [20], through their study, found that spending the largest share of the advertising and promotional budget on a movie to be conducted prior to its release is a key factor in influencing the box office during the premiere stage of a movie. Thomas [21] studied that trailer promotion is the most effective film promotion medium and has the greatest impact on consumer behavior as compared to other forms of film promotion such as websites and posters. Similarly, Salma and Lidia [11] analyzed that the effectiveness of trailer campaigns and found that trailer ads help consumers form expectations about the future success of a movie, and that earlier placement of trailers positively affects the movie's box office. Finsterwalder *et al.* [22] also explored that factors such as the style of the trailer, the content of the movie exposed, etc. have an impact on consumer attention and movie box office. Trehan *et al.* [12] explored that the role of advance publicity for television programs and found that advance promotion stimulates consumer's interest in the programs thus achieving consumer engagement. Furthermore, Rennhoff and Adam [23] examined that the effectiveness of advertising promotion after the release of movie and found that exhibitors who extend the film's run period along with advertising promotion can increase box office.

Reviewing the above literature, existing studies have made important contributions to the market strategies of online video platforms and have drawn some valuable conclusions and managerial insights. However, the literature that examines the promotion strategies of online video platforms from a theoretical models perspective has not yet been seen. In practice, promotion strategies are important means for online video platforms to increase revenue, and there is a saying in the industry that "no promotion, no launch". It should be noted that there are some differences between the promotion strategies of online video platforms and traditional media platforms, such as movies, because movies usually only use paid business model, while online video platforms have free, paid, mixed business models. Therefore, the findings of promotion strategies in industries such as movies are not entirely applicable to online video platforms. Thus, it is necessary to further study the promotion strategies of online video platforms under different business models to make up for the shortcomings or limitations of existing studies.

In summary, considering the limited cost of promotion, the promotion strategies of the online video platform usually include the strategy of the promotion period before the video is launched in the market, the strategy of the allocation of the promotion investment before and after the video is launched in the market, and the strategy of the intensity of the continuous investment during the promotion period, and so on. Since this study is a monopoly market problem and all of the above strategies are time-dependent, and optimal control theory is limited to studying monopoly problems, we believe that to apply optimal control theory to study the above problem is the most appropriate. To this end, by establishing an optimal control model for the online video platform in this paper, we first analyse the optimal advertising volume, optimal pricing and optimal business model of the platform, and then study the promotion strategies under different business models.

3. Problem Descriptions and Basic Assumptions

In this study, we focus on a representative video program as a case study (referred to as "the video"). A monopolistic online video platform (hereinafter referred to as "the platform") has acquired the exclusive right to broadcast the video, and consumers can only watch the video on this platform and not on other platforms.

The platform has three business models available for the video: free model, paid model and mixed model [16] [17] [24] [25]. When the platform is free, consumers have only one option, *i.e.* they can only watch the video embedded with a certain number of commercials for free, when the platform's revenue comes only from charging the companies that place the ads. When the platform adopt paid model, consumers also have only one option, *i.e.* they pay a fee to the platform to watch the video without ads, at which point the platform's revenue comes from consumers paying to watch. When the platform adopt mixed model, consumers have two options: either consumers watch the video embedded with a certain number of ads for free, or consumers pay to watch the video without ads. As a result, the platform generates revenue from both advertisers and consumers. In practice, consumers pay to watch the video featuring platforms that provide HD quality and better sound effects for the videos, and no ads to distract consumers, so they get a better experience by paying to watch videos. The feature of free video viewing for consumers is that they don't have to pay for it, but they have to bear the distraction of ads inserted in the program while watching the video, so the experience they get from free video viewing is lower than the experience they get from paid video viewing.

For ease of description, the text uses $i = n$ to denote the platform's choice of free model, $i = p$ to denote the platform's choice of paid model and $i = m$ to denote the platform's choice of mixed model. The following basic assumptions are made prior to the research question:

1) We assume that the quality of paid video is q_p and the quality of free video is q_n. Since the quality of paid video is higher than the quality of free video, thus $q_p > q_n$;

2) Since media platforms usually promote and advertise programs before they are launched in the market and during the broadcast period [8] [9] [23], we assume that the platform promotes the video both before and after it is launched in the market, and that the pre-launch promotion period of the video is Δt and the broadcast period of the video is $[0, T]$. The above period segmentation is shown in **Figure 1**;

Following classical literature such as Dylan *et al.* [26], Jianghua *et al.* [27], the impact effect of market promotion has a cumulative character, but it also has a decaying character over time. Based on this, video programs as a typical experience product, we can think that the promotional effect should also have the above characteristics. Therefore, we assume that the intensity of promotional investment in the program at moment τ is $k(\tau)$, and the decay of the impact effect at the future moment *t* is $e^{-\delta(t-\tau)}k(\tau)$. Where $\delta > 0$ is the decay factor, and the cumulative effect of the intensity of promotional investment prior to moment *t* is $A(t) = \int_{-\Delta t}^{t} e^{-\delta(t-\tau)}k(\tau)\,d\tau$, $t \in [-\Delta t, T]$;

3) We assume that the total budget invested by the platform in the promotion of the video is *C*, where the cost of promotion used for the program before it is launched in the market is χC, and the cost of promotion used for the program after it is launched in the market is $(1-\chi)C$, $\chi \in [0,1]$. Following Filippo and Demetrios [28], Liu [29], Thomas and Michael [30], where investment intensity and investment cost are characterized by a quadratic function, we hypothesize that the relationship between promotional investment intensity and investment cost is $\int_{-\Delta t}^{0} k^2(\tau)\,d\tau = \chi C$, $\int_{0}^{T} k^2(t)\,dt = (1-\chi)C$;

4) Based on the commercials embedded in the video, the industry usually adopts CPM (cost per impression) billing, which means that advertisers are charged based on the number of users whose ads are displayed [31] [32]. Therefore, in this paper, using CPM billing, we assume that the platform charges advertisers an advertising rate of κ per unit of free consumers;

5) As different consumers have heterogeneity for the same video, *i.e.*, different types of consumers have differences in the quality of the same video. Therefore, we assume that the consumer type is θ. For the video with quality *q*, the actual quality of the experience is θq. Following Cheng *et al.* [4], Xue *et al.* [19] and Xu *et al.* [5], we assume that at any moment *t*, the market size is 1 and that consumers obey a uniform distribution on $[0,1]$.

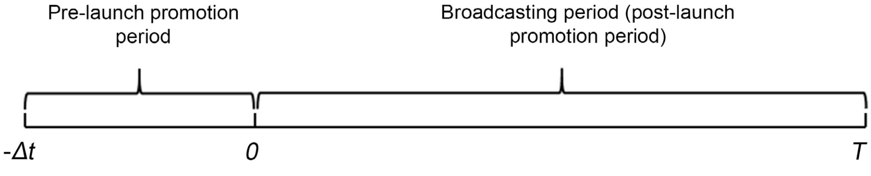

Figure 1. Period segmentation.

4. Model

In this section, the broadcast period of the video is considered to be $[0,T]$, without prejudice to the conclusions of the analysis, we assume that the required costs other than the promotional costs of the video are zero. We construct decision models for the profits maximization problem of the platform in free model, paid model and mixed model respectively. In free model, we give the optimal advertising volume and the optimal promotional intensity of the platform; in paid model, we give the optimal price of the platform and the optimal promotional intensity of the platform; and in mixed model, we give the optimal price of the platform, the optimal advertising volume and the optimal promotional intensity of the platform.

4.1. Free Model

In case of the platform with free model, consumers can only watch the video through free means.

Based on reality, and also according to literature such as Gavious and Lowengart [33], He *et al.* [34], we assume that the platform's promotion of the video to the market can increases consumers' expected utility of the video, and that the greater the cumulative effect of the promotion, the higher consumers' expectations of the video. Following Cheng *et al.* [4], Meng and He [10], Xue *et al.* [19], at any time *t*, we assume that if the consumer chooses free model to watch the video, the expected utility gained by the consumer is $E_n(t)=\theta q_n-\alpha_n n(t)+\eta A(t)$, where $n(t)$ is the number of ads inserted by the platform in the video at moment *t*, $\alpha_n>0$ is the coefficient of consumer sensitivity to the number of advertising in the video, the term "advertising sensitivity" refers to the negative utility per unit of advertising viewed by the consumer. $\eta>0$ is the sensitivity coefficient of consumer to the cumulative effect of promotion, the term "promotion sensitivity" refers to the positive utility to the consumer per unit of cumulative effect of the promotion. If $E_n(t)\geq 0$, the consumer chooses to watch the video for free. Otherwise, the consumer chooses not to watch the video. Then at time *t*, the consumers' market demand is

$$D_n(t)=\int_{\frac{\alpha_n n(t)-\eta A(t)}{q_n}}^{1}\mathrm{d}\theta=1-\frac{\alpha_n n(t)-\eta A(t)}{q_n} \tag{1}$$

As the problem is a two-stage problem, the inverse order solution method is used in this paper.

At stage $[0,T]$, the platform decides the number of ads to be inserted in the video and the intensity of the promotional investment to maximize profits during the broadcast period. The platform's decision objective is

$$\begin{cases}\max\limits_{n(t),k(t)}\pi_{n2}=\int_0^T\left(\kappa n(t)D_n(t)-k^2(t)\right)\mathrm{d}t\\ \text{s.t. } x_n(0)=0,\dot{x}_n(t)=D_n(t),\dot{\xi}_n(t)=k(t),\int_0^T k^2(t)\mathrm{d}t=(1-\chi)C\end{cases} \tag{2}$$

The Hamiltonian function for the above profits maximization problem is

$$H_{n2}=\kappa n(t)D_n(t)-k^2(t)+\lambda_n(t)D_n(t)+\mu_{n\xi}(t)\dot{\xi}_n(t)$$

According to the optimality first order condition, $n(t)$ and $k(t)$ should satisfy the following equations

$$\begin{cases}\dfrac{\partial H_{n2}}{\partial n(t)}=\kappa D_n(t)-\dfrac{\kappa\alpha_n n(t)}{q_n}-\dfrac{\alpha_n\lambda_n(t)}{q_n}=0\\ \dfrac{\partial H_{n2}}{\partial k(t)}=\mu_{n\xi}(t)-2k(t)=0\end{cases} \tag{3}$$

The co-state variables $\lambda_n(t)$ and $\mu_{n\xi}(t)$ should satisfy the following equations

$$\begin{cases}\dot{\lambda}_n(t)=-\dfrac{\partial H_{2n}}{\partial x_n(t)},\lambda_n(T)=0\\ \dot{\mu}_{n\xi}(t)=-\dfrac{\partial H_{2n}}{\partial \xi_n(t)}\end{cases} \tag{4}$$

Solving Equations (3) and (4) together gives

$$\begin{cases}n^*(t)=\dfrac{q_n+\eta A(t)}{2\alpha_n}\\ k_n^*(t)=\sqrt{\dfrac{(1-\chi)C}{T}}\end{cases} \tag{5}$$

At stage $[-\Delta t,0]$, the platform decides the intensity of the promotional investment $k(\tau)$ before the video is launched in the market, so that the platform maximises its total profits at stage $[-\Delta t,T]$. The platform's decision objective is

$$\begin{cases}\max\limits_{k(\tau)}\pi_n=\int_0^T\left(\kappa n^*(t)D_n(t)-k_n^{*2}(t)\right)\mathrm{d}t-\int_{-\Delta t}^0 k^2(\tau)\mathrm{d}\tau\\ \text{s.t.}\ \dot{\zeta}_n(\tau)=k(\tau),\int_{-\Delta t}^0 k^2(\tau)\mathrm{d}\tau=\chi C\end{cases} \tag{6}$$

The Hamiltonian function corresponding to the objective function (6) is

$$H_n=\mu_{n\zeta}(\tau)\dot{\zeta}_n(\tau)-k^2(\tau)$$

According to the optimality first order condition, $k(\tau)$ should satisfy the following equation

$$\frac{\partial H_n}{\partial k(\tau)}=\mu_{n\zeta}(\tau)-2k(\tau)=0 \tag{7}$$

The co-state variable $\mu_{n\zeta}(\tau)$ should satisfy the following equation

$$\dot{\mu}_{n\zeta}(\tau)=-\frac{\partial H_n}{\partial \zeta_n(\tau)} \tag{8}$$

Solving for Equations (7) and (8), we get

$$k_n^*(\tau)=\sqrt{\frac{\chi C}{\Delta t}} \tag{9}$$

From Equations (5) and (9), we are able to obtain the following Lemma 1.

Lemma 1. In free model, the platform's optimal promotional investment intensity strategy $k_n^*(\tau)$ before the video launch, the optimal advertising volume strategy $n^*(t)$ after the video launch, and the optimal promotional investment intensity strategy $k_n^*(t)$ are given by the following equations

$$k_n^*(\tau)=\sqrt{\frac{\chi C}{\Delta t}},\quad \tau\in[-\Delta t,0] \tag{10}$$

$$n^*(t)=\frac{q_n+\eta A^*(t)}{2\alpha_n},\quad t\in[0,T] \tag{11}$$

$$k_n^*(t)=\sqrt{\frac{(1-\chi)C}{T}},\quad t\in[0,T] \tag{12}$$

Lemma 1 shows that the optimal advertising volume strategy is determined by factors such as the sensitivity of consumers to the volume of advertising and the sensitivity of consumers to the cumulative effect of promotion, and the strategy of the intensity of promotional investment before the video is launched in the market is determined by factors such as the cost of promotional investment and the period of promotion, and the strategy of the intensity of promotional investment after the video has been launched in the market is determined by factors such as the cost of promotional investment and the duration of the broadcasting period.

Substituting Equations (10)-(12) into Equations (1) and (6), we can obtain the market demand and the profits of the platform as the following equations, respectively, *i.e.*

$$D_n^*(t)=\frac{q_n+\eta A^*(t)}{2q_n},\quad t\in[0,T] \tag{13}$$

$$\pi_n^*(t)=\frac{\kappa}{4\alpha_n q_n}\left(q_n+\eta A^*(t)\right)^2-k_n^{*2}(t) \tag{14}$$

According to Lemma 1 and Equations (13-14), we are able to obtain Corollary 1.

Corollary 1. (i) $\frac{\partial n^*(t)}{\partial A^*(t)}>0$; (ii) $\frac{\partial D_n^*(t)}{\partial A^*(t)}>0$; (iii) $\frac{\partial \pi_n^*(t)}{\partial A^*(t)}>0$; (iv) $\frac{\partial k_n^*(\tau)}{\partial \Delta t}<0$, $\frac{\partial k_n^*(t)}{\partial T}<0$, $\frac{\partial k_n^*(\tau)}{\partial \tau}=0$, $\frac{\partial k_n^*(t)}{\partial t}=0$.

Proof. See Appendix 1.

Corollary 1(i) shows that the amount of adverts in the video is positively correlated with the cumulative effect of promotion. Therefore, the platform should take measures to increase the cumulative effect of promotion so that the platform can place more adverts, thus increasing the platform's profits.

Corollary 1(ii) shows that the market demand for free video is positively related to the cumulative effect of promotion. Therefore, the platform can increase the market demand for the video through the strategy of increasing the cumulative effect of promotion.

Corollary 1(iii) shows that the cumulative effect of promotion has a positive impact on total platform profits. Combined with Corollary 1(i-ii), it can be seen that the cumulative effect of promotion enhances the platform's profits by influencing the number of advertisements implanted in the video and the market demand.

Corollary 1(iv) suggests that the promotion intensity of the video before launch is negatively related to the promotion period, and the promotion intensity of the video after launch is negatively related to the broadcast period, and the promotion intensity is independent of the time. Obviously, the platform should decide the intensity of promotion investment before and after the launch of the video according to the length of the promotion period and broadcasting period, and during the promotion period, the platform should evenly distribute the cost of promotion investment at each moment.

4.2. Paid Model

When the platform adopt paid model, consumers can only watch the video by paying for it.

Similarly, and similar to the assumptions in Section 4.1, the expected utility function that a consumer obtains by paid viewing of the video under paid model is $E_p(t) = \theta q_p - \alpha_p p(t) + \eta A(t)$, where $p(t)$ is the price charged for the video, $\alpha_p > 0$ is the consumer's sensitivity coefficient to the price charged, the term "price sensitivity" refers to the negative utility per unit of price to the consumer. If the consumer expected utility function satisfies $E_p(t) \geq 0$, the consumer will choose to pay to watch the video; otherwise, the consumer chooses not to watch the video. Then, at time *t*, the consumers' market demand is

$$D_p(t) = \int_{\frac{\alpha_p p(t) - \eta A(t)}{q_p}}^{1} \mathrm{d}\theta = 1 - \frac{\alpha_p p(t) - \eta A(t)}{q_p} \tag{15}$$

At stage $[0, T]$, the platform decides on the price of the video and the intensity of the promotional investment so that the platform maximises its profits during the broadcast period, with an objective functions of the following form, *i.e.*

$$\begin{cases} \max\limits_{p(t),k(t)} \pi_{p2} = \int_0^T \left(p(t) D_p(t) - k^2(t) \right) \mathrm{d}t \\ \text{s.t. } x_p(0) = 0, D_p(t) = \dot{x}_p(t), \dot{\xi}_p(t) = k(t), \int_0^T k^2(t)\mathrm{d}t = (1-\chi)C \end{cases} \tag{16}$$

The Hamiltonian function corresponding to the objective function (16) is

$$H_{p2} = p(t) D_p(t) - k^2(t) + \lambda_p(t) D_p(t) + \mu_{p\xi}(t) \dot{\xi}_p(t)$$

According to the optimality first order condition, $p(t)$ and $k(t)$ should satisfy the following equations

$$\begin{cases} \dfrac{\partial H_{p2}}{\partial p(t)} = D_p(t) - \dfrac{\alpha_p p(t)}{q_p} - \dfrac{\alpha_p \lambda_p(t)}{q_p} = 0 \\ \dfrac{\partial H_{p2}}{\partial k(t)} = \mu_{p\xi}(t) - 2k(t) = 0 \end{cases} \tag{17}$$

The co-state variables $\lambda_p(t)$ and $\mu_{p\xi}(t)$ should satisfy the following equations

$$\begin{cases}\dot{\lambda}_p(t) = -\dfrac{\partial H_{p2}}{\partial x_p(t)}, \lambda_p(T) = 0 \\ \dot{\mu}_{p\xi}(t) = -\dfrac{\partial H_{p2}}{\partial \xi_p(t)}\end{cases} \tag{18}$$

Solving Equations (17) and (18) together gives

$$\begin{cases}p^*(t) = \dfrac{q_p + \eta A(t)}{2\alpha_p} \\ k_p^*(t) = \sqrt{\dfrac{(1-\chi)C}{T}}\end{cases} \tag{19}$$

At stage $[-\Delta t, 0]$, the platform decides the intensity of the promotional investment $k(\tau)$ before the video is launched in the market, so that the platform maximises its total profits at stage $[-\Delta t, T]$. The platform's decision objective is

$$\begin{cases}\max\limits_{k(\tau)} \pi_p = \int_0^T \left(p^*(t) D_p(t) - k_p^{*2}(t)\right) \mathrm{d}t - \int_{-\Delta t}^0 k^2(\tau) \mathrm{d}\tau \\ \text{s.t.}\ \dot{\zeta}_n(\tau) = k(\tau), \int_{-\Delta t}^0 k^2(\tau) \mathrm{d}\tau = \chi C\end{cases} \tag{20}$$

The Hamiltonian function corresponding to the objective function (20) is

$$H_p = \mu_{p\zeta}(\tau) \dot{\zeta}_p(\tau) - k^2(\tau)$$

According to the optimality first order condition, $k(\tau)$ should satisfy the following equation

$$\frac{\partial H_p}{\partial k(\tau)} = \mu_{p\zeta}(\tau) - 2k(\tau) = 0 \tag{21}$$

The co-state variable $\mu_{p\zeta}(\tau)$ should satisfy the following equation

$$\dot{\mu}_{p\zeta}(\tau) = -\frac{\partial H_p}{\partial \zeta_p(\tau)} \tag{22}$$

Solving for Equations (21) and (22), we get

$$k_p^*(\tau) = \sqrt{\frac{\chi C}{\Delta t}} \tag{23}$$

From Equations (19) and (23), we are able to obtain the following Lemma 2.

Lemma 2. In paid model, the platform's optimal promotional investment intensity strategy $k_p^*(\tau)$ before the video launch, the optimal price optimal strategy $p^*(t)$ after the video launch, and the optimal promotional investment intensity strategy $k_p^*(t)$ are given by the following equations

$$k_p^*(\tau) = \sqrt{\frac{\chi C}{\Delta t}}, \quad \tau \in [-\Delta t, 0] \tag{24}$$

$$p^*(t) = \frac{q_p + \eta A^*(t)}{2\alpha_p}, \quad t \in [0, T] \tag{25}$$

$$k_p^*(t)=\sqrt{\frac{(1-\chi)C}{T}},\quad t\in[0,T] \tag{26}$$

Substituting Equations (24-26) into Equations (15) and (20), we can obtain the market demand and platform profits, respectively, *i.e.*,

$$D_p^*(t)=\frac{q_p+\eta A^*(t)}{2q_p} \tag{27}$$

$$\pi_p^*(t)=\frac{1}{4\alpha_p q_p}\left(q_p+\eta A^*(t)\right)^2-k_p^{*2}(t) \tag{28}$$

From Lemma 2 and Equations (27-28), we are able to obtain the following corollary 2.

Corollary 2. (i) $\frac{\partial p^*(t)}{\partial A^*(t)}>0$; (ii) $\frac{\partial D_p^*(t)}{\partial A^*(t)}>0$; (iii) $\frac{\partial \pi_p^*(t)}{\partial A^*(t)}>0$; (iv) $\frac{\partial k_p^*(\tau)}{\partial \Delta t}<0$, $\frac{\partial k_p^*(t)}{\partial T}<0$, $\frac{\partial k_p^*(\tau)}{\partial \tau}=0$, $\frac{\partial k_p^*(t)}{\partial t}=0$.

Proof. See Appendix 2.

Corollary 2 leads to similar conclusions as Corollary 1, *i.e.*, under paid model, the platform should take measures to increase the cumulative effect of promotion, thereby increasing the profits of the video. In addition, the platform should determine the intensity of the promotional investment based on the length of the promotional period and the broadcast period. The intensity of the promotional investment should be the same at every moment during the promotional period.

4.3. Mixed Model

When the platform adopt mixed model, consumers have two choices: to watch the video with commercials for free and to pay to watch the video without ads.

Similarly to the assumptions in Sections 4.1 and 4.2, a consumer who pays to watch the video has an expected utility function of $E_{mp}(t)=\theta q_p-\alpha_p p(t)+\eta A(t)$ and a consumer who watches the video for free has an expected utility function of $E_{mn}(t)=\theta q_n-\alpha_n n(t)+\eta A(t)$. In mixed model, where the relevant parameters have the same meaning as those in sections 4.1 and 4.2. If $E_{mp}(t)>E_{mn}(t)$ and $E_{mp}(t)\geq 0$, consumers will choose to pay to watch the video; if $E_{mn}(t)>E_{mp}(t)$ and $E_{mn}(t)\geq 0$, consumers will choose to watch the video for free; if $E_{mn}(t)>E_{mp}(t)$ and $E_{mn}(t)<0$, consumers will choose not to watch the video. When $E_{mp}(t)=E_{mn}(t)$, the point of no difference in utility between consumers choosing to pay to watch the video and choosing to watch the video for free can be obtained as $\theta_1(t)=\frac{\alpha_p p(t)-\alpha_n n(t)}{q_p-q_n}$; when $E_{mn}(t)=0$, the point of no difference in utility between consumers choosing to free to watch the video and choosing not to watch the video can be obtained as $\theta_2(t)=\frac{\alpha_n n(t)-\eta A(t)}{q_n}$. In order to ensure non-negative demand from paid and free consumers, we assume that $\theta_1(t)\geq\theta_2(t)$. Then at time *t*, the market demand for paid and free consumers is respectively, *i.e.*

$$\begin{cases} D_{mp}(t) = \int_{\theta_1}^{1} \mathrm{d}\theta = 1 - \dfrac{\alpha_p p(t) - \alpha_n n(t)}{q_p - q_n} \\ D_{mn}(t) = \int_{\theta_2}^{\theta_1} \mathrm{d}\theta = \dfrac{\alpha_p p(t) - \alpha_n n(t)}{q_p - q_n} - \dfrac{\alpha_n n(t) - \eta A(t)}{q_n} \end{cases} \tag{29}$$

Similarly to Sections 4.1 and 4.2, we use the inverse order solution method to calculate the optimal strategy for the platform at each stage.

At stage $[0,T]$, the platform determine the pricing of the video fees, the amount of free ads placed and the intensity of promotional investment to maximize its profits during the broadcast period. The objective function of the platform is as follows, *i.e.*

$$\begin{cases} \max\limits_{p(t),n(t),k(t)} \pi_{m2} = \int_0^T \left(p(t) D_{mp}(t) + \kappa n(t) D_{mn}(t) - k^2(t) \right) \mathrm{d}t \\ \text{s.t. } x_{mi}(0) = 0, D_{mi}(t) = \dot{x}_{mi}(t), \dot{\xi}_m(t) = k(t), \int_0^T k^2(t)\mathrm{d}t = (1-\chi)C, i = p, n \end{cases} \tag{30}$$

The Hamiltonian function for the above profits maximization problem is

$$H_m = p(t) D_{mp}(t) + \kappa n(t) D_{mn}(t) - k^2(t) + \sum\nolimits_{i=p}^{n} \lambda_{mi}(t) \dot{x}_{mi}(t) + \mu_{m\xi}(t) \dot{\xi}_m(t)$$

According to the optimality first order condition, $p(t)$, $n(t)$ and $k(t)$ should satisfy the following equations, *i.e.*

$$\begin{cases} \dfrac{\partial H_m}{\partial p(t)} = 1 - \dfrac{\alpha_p p(t) - \alpha_n n(t)}{q_p - q_n} - \dfrac{\alpha_p p(t)}{q_p - q_n} + \dfrac{\kappa \alpha_p n(t)}{q_p - q_n} = 0 \\ \dfrac{\partial H_m}{\partial n(t)} = \dfrac{\alpha_n p(t)}{q_p - q_n} + \kappa D_{mn}(t) - \kappa n(t) \left(\dfrac{\alpha_n}{q_p - q_n} - \dfrac{\alpha_n}{q_p - q_n} \right) = 0 \\ \dfrac{\partial H_m}{\partial k(t)} = \mu_{m\xi}(t) - 2k(t) = 0 \end{cases} \tag{31}$$

The co-state variables $\lambda_{mi}(t)$ and $\mu_{m\xi}(t)$ should satisfy the following equations

$$\begin{cases} \dot{\lambda}_{mi}(t) = -\dfrac{\partial H_{m2}}{\partial x_{mi}(t)}, \lambda_{mi}(T) = 0 \\ \dot{\mu}_{m\xi}(t) = -\dfrac{\partial H_{m2}}{\partial \xi_m(t)} \end{cases} \tag{32}$$

Solving Equations (31) and (32) together gives

$$\begin{cases} p_m^*(t) = \dfrac{2\kappa \alpha_n q_p (q_p - q_n) + \kappa \eta A(t)(q_p - q_n)(\kappa \alpha_p + \alpha_n)}{M} \\ n_m^*(t) = \dfrac{q_n (\kappa \alpha_p + \alpha_n)(q_p - q_n) + 2\kappa \alpha_p \eta A(t)(q_p - q_n)}{M} \\ k_m^*(t) = \sqrt{\dfrac{(1-\chi)C}{T}} \end{cases} \tag{33}$$

where $M = 4\kappa\alpha_p\alpha_n q_p - q_n\left(\kappa\alpha_p + \alpha_n\right)^2$. Since $p_m^*(t)$ and $n_m^*(t)$ need to satisfy the non-negative condition and the numerator of both $p_m^*(t)$ and $n_m^*(t)$ in Equation (33) is greater than zero, it is important that $M > 0$ is satisfied.

At stage $[-\Delta t, 0]$, the platform decides the intensity of the promotional investment $k(\tau)$ before the video is launched in the market, so that the platform maximises its total profits at stage $[-\Delta t, T]$. The platform's decision objective is

$$\begin{cases} \max\limits_{k(\tau)} \pi_m = \int_0^T \left(p_m^*(t) D_{mp}(t) + \kappa n_m^*(t) D_{mn}(t) - k_m^{*2}(t) \right) \mathrm{d}t - \int_{-\Delta t}^0 k^2(\tau)\,\mathrm{d}\tau \\ \text{s.t.}\ \dot{\zeta}_m(\tau) = k(\tau), \int_{-\Delta t}^0 k^2(\tau)\,\mathrm{d}\tau = \chi C \end{cases} \tag{34}$$

The Hamiltonian function corresponding to the objective function (34) is

$$H_m = \mu_{m\zeta}(\tau)\dot{\zeta}_m(\tau) - k^2(\tau)$$

According to the optimality first order condition, $k(\tau)$ should satisfy the following equation

$$\frac{\partial H_m}{\partial k(\tau)} = \mu_{m\zeta}(\tau) - 2k(\tau) = 0 \tag{35}$$

The co-state variable $\mu_{m\zeta}(\tau)$ should satisfy the following equation

$$\dot{\mu}_{m\zeta}(\tau) = -\frac{\partial H_m}{\partial \zeta_m(\tau)} \tag{36}$$

Solving for Equations (35) and (36), we get

$$k_m^*(\tau) = \sqrt{\frac{\chi C}{\Delta t}} \tag{37}$$

From Equations (33) and (37), we are able to obtain the following Lemma 3.

Lemma 3. In mixed model, the platform's optimal promotional investment intensity strategy $k_m^*(\tau)$ before the video is launched in the market, the platform's optimal pricing strategy $p_m^*(t)$, optimal advertising volume strategy $n_m^*(t)$ and optimal promotional investment intensity strategy $k_m^*(t)$ after the video is launched in the market are given by the following equations

$$k_m^*(\tau) = \sqrt{\frac{\chi C}{\Delta t}}, \quad \tau \in [-\Delta t, 0] \tag{38}$$

$$p_m^*(t) = \frac{\left(q_p - q_n\right)\left[2\kappa\alpha_n q_p + \kappa\eta A^*(t)\left(\kappa\alpha_p + \alpha_n\right)\right]}{M}, \quad t \in [0, T] \tag{39}$$

$$n_m^*(t) = \frac{\left(q_p - q_n\right)\left[q_n\left(\kappa\alpha_p + \alpha_n\right) + 2\kappa\alpha_p\eta A^*(t)\right]}{M}, \quad t \in [0, T] \tag{40}$$

$$k_m^*(t) = \sqrt{\frac{(1-\chi)C}{T}}, \quad t \in [0, T] \tag{41}$$

Substituting Equations (38-41) into Equations (29) and (34), we can obtain the market demand and platform profits, respectively, *i.e.*,

$$\begin{cases} D_{mp}^*(t) = \dfrac{\kappa\alpha_p\left[\alpha_n\left(2q_p - q_n\right) - \kappa\alpha_p q_n\right] + \kappa\alpha_p\eta A^*(t)\left(\alpha_n - \kappa\alpha_p\right)}{M} \\ D_{mn}^*(t) = \dfrac{\alpha_n q_p q_n\left(\kappa\alpha_p - \alpha_n\right) + \alpha_n\eta A^*(t)\left[2\kappa q_p\alpha_p - q_n\left(\kappa\alpha_p + \alpha_n\right)\right]}{q_n M} \end{cases} \quad (42)$$

$$\pi_m^*(t) = p_m^*(t)D_{mp}^*(t) + \kappa n_m^*(t)D_{mn}^*(t) - k_m^{*2}(t) \quad (43)$$

Since the market demand for the video has to satisfy the condition of being greater than zero, according to Equation (42), $D_{mp}^*(t) > 0$ and $D_{mn}^*(t) > 0$ hold only if $r = \frac{\alpha_p}{\alpha_n}$ satisfies the $r > r_1^*$ and $r < r_2^*$ cases. Where $r_1^* = \frac{q_n\left(q_p + \eta A^*(t)\right)}{\kappa\left[q_n\left(q_p - \eta A^*(t)\right) + 2q_p\eta A^*(t)\right]}$ and $r_2^* = \frac{2q_p - q_n + \eta A^*(t)}{\kappa\left(q_n + \eta A^*(t)\right)}$. Therefore, under mixed model, we assume that r satisfies $r \in \left(r_1^*, r_2^*\right)$ and M satisfies $M > 0$.

From Lemma 3 and Equations (42-43), we are able to obtain the following corollary 3.

Corollary 3. (i) $\frac{\partial p_m^*(t)}{\partial A^*(t)} > 0$, $\frac{\partial n_m^*(t)}{\partial A^*(t)} > 0$; (ii) If $\frac{\alpha_p}{\alpha_n} < \frac{1}{\kappa}$, $\frac{\partial D_{mp}^*(t)}{\partial A^*(t)} > 0$, otherwise $\frac{\partial D_{mp}^*(t)}{\partial A^*(t)} < 0$; if $\frac{\alpha_p}{\alpha_n} > \frac{q_n}{\kappa\left(2q_p - q_n\right)}$, $\frac{\partial D_{mn}^*(t)}{\partial A^*(t)} > 0$, otherwise $\frac{\partial D_{mn}^*(t)}{\partial A^*(t)} < 0$; (iii) $\frac{\partial \pi_m^*(t)}{\partial A^*(t)} > 0$; (iv) $\frac{\partial k_m^*(\tau)}{\partial \Delta t} < 0$, $\frac{\partial k_m^*(t)}{\partial T} < 0$, $\frac{\partial k_m^*(\tau)}{\partial \tau} = 0$, $\frac{\partial k_m^*(t)}{\partial t} = 0$.

Proof. See Appendix 3.

It can be seen that the results of Corollary 3(i) and 3(iv) are consistent with Corollary 1(i) and 1(iv), so we omit the elaboration.

From Corollary 3(ii), it is found that unlike free and paid models, the relationship between the market demand for the video and the cumulative effect of promotion under mixed model is affected by the combined effect of consumer sensitivity to the price charged and sensitivity to the volume of ads. When the ratio of consumer sensitivity to price and sensitivity to advertising volume is less than $\frac{q_n}{\kappa\left(2q_p - q_n\right)}$, the cumulative effect of promotion has positive impact on the market demand for paid video, but negative impact on the market demand for free video; when the ratio of consumer sensitivity to price and sensitivity to advertising volume is greater than $\frac{1}{\kappa}$, the cumulative effect of promotion has positive impact on the market demand for free video, but negative impact on the market demand for paid video; when the ratio of consumer sensitivity to price to sensitivity to advertising volume is between $\frac{q_n}{\kappa\left(2q_p - q_n\right)}$ and $\frac{1}{\kappa}$, the cumulative effect of promotion has a positive impact on the market demand for both paid and free video.

From Corollary 3(iii), the cumulative effect of promotion has different impacts on the market demand for paid and free under different situations where consumer's sensitivity to price and sensitivity to the amount of ads are satisfied, but the cumulative effect of promotion still has a positive impact on the total platform profits, so the platforms can still improve the video profits by increasing the cumulative effect of promotion under mixed model.

5. Optimal Business Model for the Platform

According to Equations (14), (28) and (43), we compare the optimal profits of the platform for free model, paid model and mixed model and obtain the following Lemma 4.

Lemma 4. If $r < r^*$, $\pi_p^* > \max\{\pi_n^*, \pi_m^*\}$; if $r \in (r^*, r_2^*)$, $\pi_m^* > \max\{\pi_p^*, \pi_n^*\}$; if $r > r_2^*$, $\pi_n^* > \max\{\pi_p^*, \pi_m^*\}$.

Proof. See Appendix 4.

In case where the ratio of consumer's sensitivity to video price and sensitivity to advertising volume is low (*i.e.*, $r < r^*$), consumers are more sensitive to the amount of advertising in free model, and therefore, it is the optimal strategy for the platform to adopt paid model; in case where the ratio of consumer's sensitivity to video price and sensitivity to advertising volume is high (*i.e.*, $r > r_2^*$), consumers are more sensitive to the charged price, and therefore, the platform adopt free model as the optimal strategy; in case where the ratio of consumer's sensitivity to video price and sensitivity to advertising volume is moderate (*i.e.*, $r \in (r^*, r_2^*)$), consumers are not particularly sensitive to either the price charged or the volume of advertisements, and therefore, it is optimal for the platform to adopt mixed model as the optimal strategy. In summary, the platform should choose the optimal business model based on the relationship between consumer's sensitivity to the price charged for the video and their sensitivity to the volume of advertisements in order to maximise the profits.

Based on Lemma 4, we explore how the platform maximizes its profits by deciding the promotion period before the video is launched in the market, and the allocation proportion of promotion investment before and after the video is launched in the market, given the above three optimal business models and the unchanged promotion budget.

6. Optimal Promotion Strategies for the Platform

6.1. Free Model

Proposition 1. (i) The optimal promotion period before the video launch is Δt_n^*; (ii) The optimal allocation of the total promotion budget before and after the video launch is $\chi_n^* C$ and $(1-\chi_n^*)C$.

Proof. See Appendix 5.

Proposition 1 suggests that in free model, the platform can increase its profits more effectively by setting the promotion period before the launch of the video and allocating the proportion of promotional investment before and after the launch of the video. In other words, under the premise of constant promotion cost, the platform can maximise the promotion effect by setting the promotion period before the launch of the video and allocating the investment proportion before and after the launch of the video.

In free model, proposition 2 can be analysed as to which scenarios the platform should focus on promotion before the video is launched and which scenarios should focus on promotion after the video is launched.

Proposition 2. If $\delta < \delta_n$, we have $\chi_n^* > \frac{1}{2}$; if $\delta > \delta_n$, we have $\chi_n^* < \frac{1}{2}$.

Proof. See Appendix 6.

Proposition 2 shows that the decay factor of the promotion effect affects the platform's promotion investment before and after the launch of the video. When the decay factor is low (*i.e.*, $\delta < \delta_n$), the promotion investment before the video launch is higher than that after the launch; when the decay factor is high (*i.e.*, $\delta > \delta_n$), the promotion investment after the video launch is higher than that before the video launch. Therefore, the platform should decide the promotion investment before and after the launch of the video in the market based on the degree of forgetting of the impact of the promotion by consumers (*i.e.*, the decay factor). In case of a low decay factor, the platform should focus on the promotional strategy before the video launch; conversely, the platform should focus on the promotional strategy after the video launch.

6.2. Paid Model

Proposition 3. (i) The optimal promotion period before the video launch is Δt_p^*; (ii) The optimal allocation of the total promotion budget before and after the video launch is $\chi_p^* C$ and $\left(1-\chi_p^*\right)C$.

Proof. See Appendix 7.

Proposition 4. If $\delta < \delta_p$, we have $\chi_p^* > \frac{1}{2}$; if $\delta > \delta_p$, we have $\chi_p^* < \frac{1}{2}$.

Proof. See Appendix 8.

The results of Propositions 3-4 are consistent with those of Propositions 1-2, that is, under the condition that paid model and the total cost of promotion remain unchanged, the platform can maximize its profits by formulating the promotion period before the launch of the video as well as allocating the proportion of promotion investment before and after the launch of the video. At the same time, the platform should also decide the focus of the promotion strategies according to the size of the decay factor, when the decay factor is low, the platform should take the promotion of the video in the pre-launching period as the focus; when the decay factor is high, the platform should take the promotion of the video in the post-launching period as the focus of the promotion.

6.3. Mixed Model

Proposition 5. (i) The optimal promotion period before the video launch is Δt_m^*; (ii) The optimal allocation of the total promotion budget before and after the video launch is $\chi_m^* C$ and $\left(1-\chi_m^*\right)C$.

Proof. See Appendix 9.

Proposition 6. If $\delta < \delta_m$, we have $\chi_m^* > \frac{1}{2}$; if $\delta > \delta_m$, we have $\chi_m^* < \frac{1}{2}$.

Proof. See Appendix 10.

As can be seen, the findings in section 6.3 are consistent with those in sections 6.1-6.2. We do not discuss it here.

6.4. Comparative Analysis

On the basis of the propositions obtained in Sections 6.1-6.3, we compare the optimal promotion period before the video is launched in the market, and the optimal proportion of promotional investment before and after the video is launched in the market under free, paid and mixed models, and we obtain the following Proposition 7.

Proposition 7. (i) $\Delta t_n^* = \Delta t_p^* = \Delta t_m^*$; (ii) $\chi_p^* > \chi_m^* > \chi_n^*$.

Proof. See Appendix 11.

Proposition 7(i) shows that the optimal promotion period for the video before it is launched in the market is the same whether the platform adopts free model, paid model or mixed model. This is because the optimal promotion period of the video before it is launched in the market is only related to the degree of consumer forgetting the impact of the promotion (*i.e.*, the decay factor). Therefore, the optimal promotion period before the video launched in the market is the same under different business models. In other words, a change in the platform's business model does not change the optimal promotion period of the video before launch. Therefore, when the platform adjusts the business model of the video, the platform does not need to change the promotion period of the video before launch.

Proposition 7(ii) shows that the optimal allocation of the platform's promotion investment in the video before its launch is different under different business models. The platform's promotion investment before the video launch is the largest when the platform adopt paid model, the second largest when the platform adopt mixed model, and the lowest when the platform adopt free model. This is because the optimal promotional investment before the video is launched in the market is related to the ratio of consumer sensitivity to the price charged and consumer sensitivity to the volume of ads (*i.e.*, $r = \frac{\alpha_p}{\alpha_n}$). The proof of Proposition 7 shows that the lower r is (*i.e.*, lower α_p or higher α_n), the larger $\chi_j^*, j = p,n,m$ is. Therefore, the platform should develop the strategy for allocating promotion investment before and after the launch of the video in the market according to the business model it adopts. If the platform adopts paid model, the platform's promotion investment before the video launched in the market should be the highest relative to other business models; if the platform adopts mixed model, the platform's promotion investment before the video launched in the market should be in the middle relative to other business models; if the platform adopts free model, the platform's promotion investment before the video launched in the market should be the lowest relative to other business models.

7. Numerical Analysis

In this section, we validate the main results obtained in this paper through a visual analysis of numerical simulations, while providing additional management insights into the platform through optimal decision making. According to the basic conditions to be satisfied by Lemmas 1-3 in this paper, we set the following basic parameters as $q_p = 1$, $q_n = 0.8$, $\eta = 0.1$, $C = 5$, $T = 2$, $\delta = 0.5$, $\kappa = 1$, respectively. In this section, when examining a factor, we assume that the other non-examined factors are fixed.

We first examine the impact of the cumulative effect of promotion on the platform's market demand, the video price, the volume of advertising and the profits, as well as the characterization of the promotional investment intensity as a function of the promotional cycle and the time within the promotional period. Since the results of Corollary 1-3 are similar and the results of Corollaries 1-2 are more intuitive, we only validate the conclusion of Corollary 3, as shown in Figures 2-5, respectively. Then we examine the effect of r (*i.e.*, the ratio of consumer's sensitivity to price and sensitivity to advertisement volume) on the platform's profits, as shown in Figure 6; finally, we examine the effect of the promotion period before the video is launched in the market, the proportion of the promotional investment before and after the video is launched in the market on the platform's profits and further examine the relationship between the above optimal decisions under different business models, as shown in Figures 7-9, respectively.

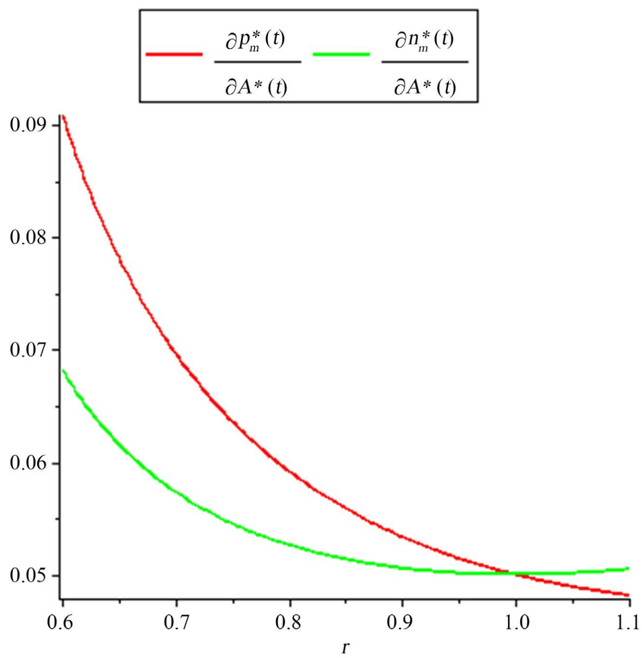

Figure 2. Effect of $A^*(t)$ on $p_m^*(t)$ and $n_m^*(t)$.

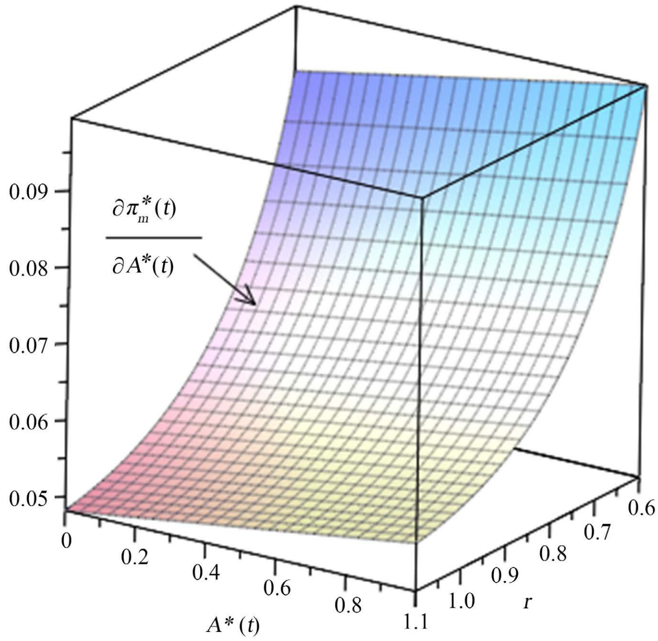

Figure 3. Effect of $A^*(t)$ on $\pi_m^*(t)$.

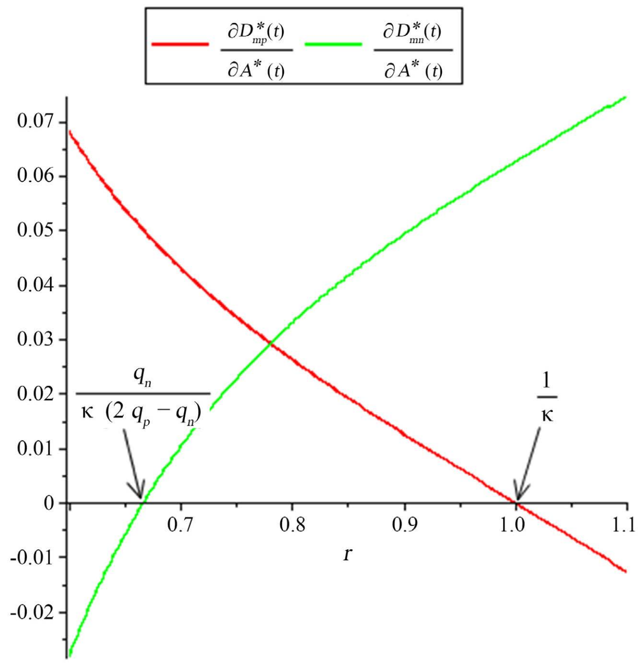

Figure 4. Effect of $A^*(t)$ on $D_{mp}^*(t)$ and $D_{mn}^*(t)$.

Figure 2 and **Figure 3** show that in mixed model, for any given ratio *r* of price sensitivity and advertising volume sensitivity, the cumulative effect of promotion has a positive impact on the platform's charged price, the amount of free ads, and the platform's profits. Thus the platform is able to increase the video profits through the promotion strategy. The above results validate the results of Corollary 3(i) and (iii).

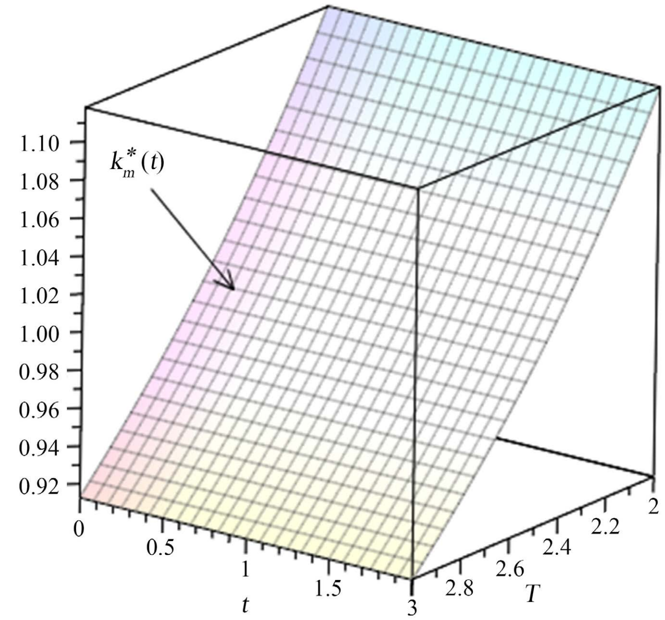

Figure 5. Effects of t and T on $k_m^*(t)$. Note: $\chi = 0.5$.

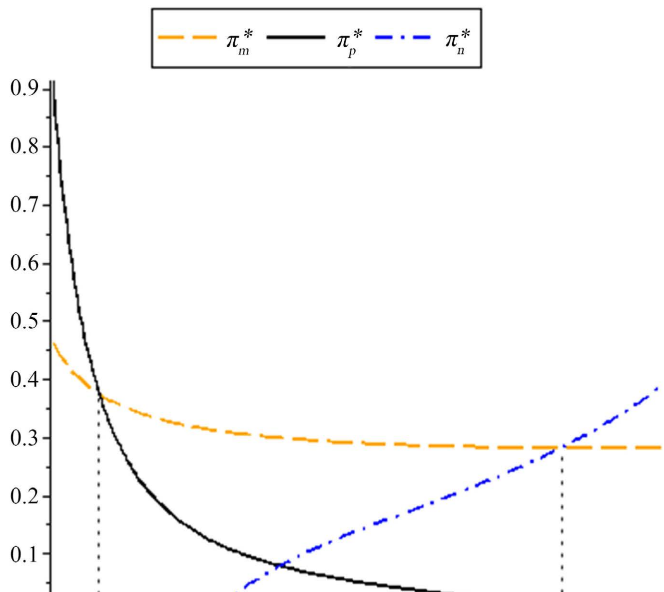

Figure 6. Effect of r on π_m^*, π_p^* and π_n^*.

Figure 4 shows that in mixed model, the impact of the cumulative effect of promotion on the demand in the platform market is related to the ratio r of the consumer's sensitivity to price and sensitivity to the volume of ads. For the market demand of free video, only in case of higher r (*i.e.*, $r > \frac{q_n}{\kappa(2q_p - q_n)}$), the cumulative effect of promotion has a positive impact on the market demand;

otherwise, the cumulative effect of promotion has a negative impact on the market demand. For the market demand of paid video, the cumulative effect of promotion has a positive impact on the market demand only when r is low (*i.e.*, $r < \frac{1}{\kappa}$); otherwise, the cumulative effect of promotion has a negative impact on the market demand. This is due to the fact that when r is low, consumers are more sensitive to the amount of ads in free model video, and consumers prefer to watch the video by paid, so the platform is not able to increase the market demand for free video through promotional strategies; similarly, when r is high, consumers are more sensitive to the price of the video, and consumers prefer to watch the video through free model, so the platform is not able to increase the market demand for paid video through promotional strategies. The above results validate the results of Corollary 3(ii).

Figure 5 shows that the optimal promotion investment intensity after the video is launched in the market decreases with the extension of the promotion period (broadcasting period). In addition, the promotion investment intensity remains constant over time during the promotion period. The optimal promotion investment intensity before the video is launched in the market has similar characteristics as described above, so we omit the numerical examination. In summary, the platform should determine the promotion investment intensity based on the length of the promotion period and distribute the promotion costs evenly over the promotion period. The above results are consistent with Corollary 3(iv).

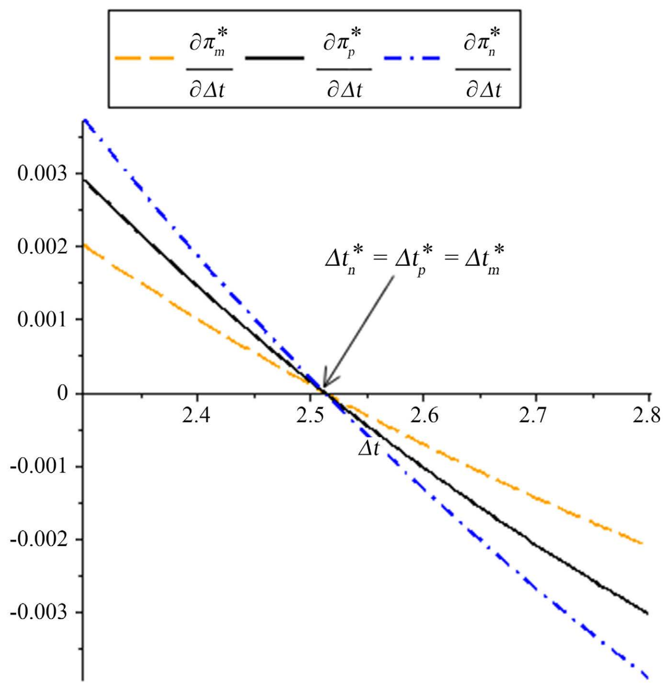

Figure 7. Effect of Δt on π_m^*, π_p^* and π_n^*. Notes: (m) $r = 1$; (p) $r = 0.55$; (n) $r = 1.4$.

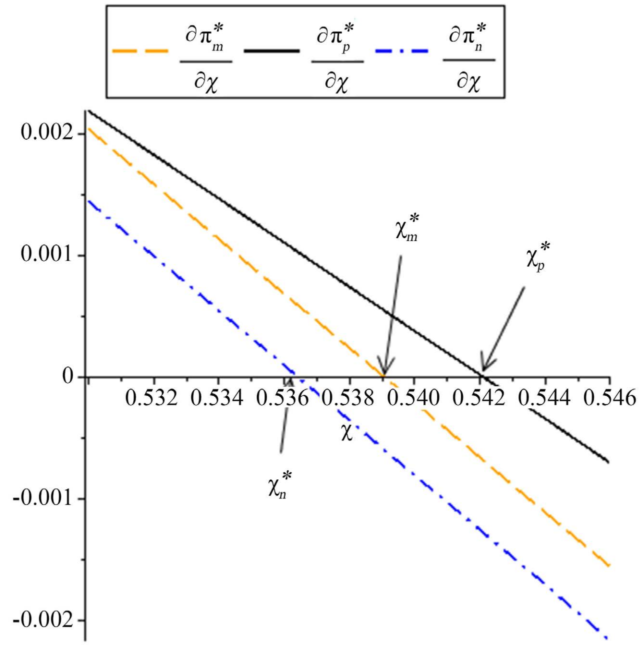

Figure 8. Effect of χ on π_m^*, π_p^* and π_n^*. Notes: (m) $r=1$; (p) $r=0.55$; (n) $r=1.4$.

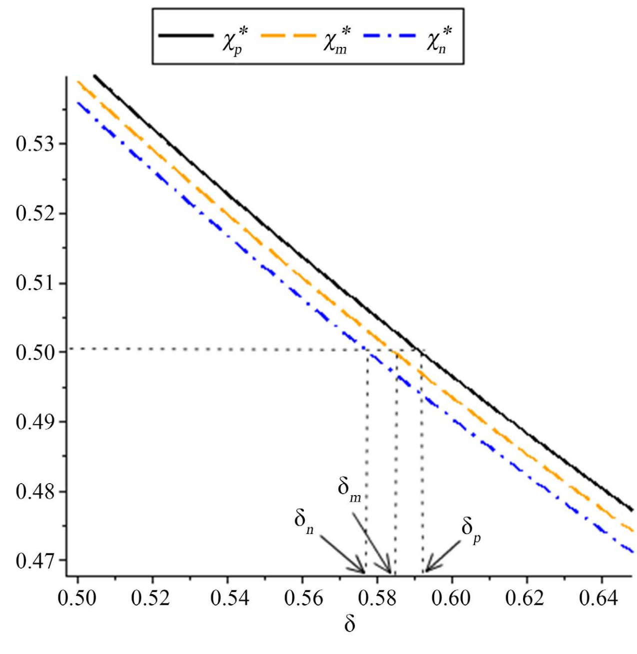

Figure 9. Effect of δ on χ_m^*, χ_p^* and χ_n^*. Notes: (m) $r=1$; (p) $r=0.55$; (n) $r=1.4$.

Figure 6 shows that when the ratio of price sensitivity and advertising volume sensitivity is small (*i.e.*, $r<r^*$), the platform can gain the maximum profits by adopt paid model; when the ratio of price sensitivity and advertising volume sensitivity is in the middle (*i.e.*, $r^*<r<r_2^*$), the platform can gain the maximum profits by adopt mixed model; when the ratio of price sensitivity and ad-

vertising volume sensitivity is large (*i.e.*, $r > r_2^*$), the platform can gain the maximum profits by adopt free model. This is because when advertising volume sensitivity is high and price sensitivity is low (*i.e.*, r is small), consumers are more willing to pay to watch the video, so the platform adopt paid model is the optimal strategy; when advertising volume sensitivity and price sensitivity are not high (*i.e.*, r is in the middle), consumers are insensitive to the choice of viewing model, so the platform adopt mixed model is the optimal strategy; when advertising volume sensitivity is low and price sensitivity is high (*i.e.*, r is large), consumers are more willing to watch the video for free, so the platform adopt free model is the optimal strategy. The above results validate the results of Lemma 4.

Figure 7 shows that there exists optimal promotion period for the platform before the video is launched in the market under different business models. In addition, the optimal promotion period is the same under different business models. Therefore, the platform is able to increase its profits by strategizing the promotion period before the video is launched in the market, while the costs of promotion remain the same. Moreover, the optimal promotion period is consistent regardless of the platform's business model. These results agree with those of Propositions 1(i), 3(i), 5(i), and 7(i).

Figure 8 indicates that, on the one hand, there exists an optimal proportion of promotion investment by the platform before the video is launched in the market under different business models and constant promotion costs; on the other hand, the optimal proportion of promotion investment is largest for paid model, followed by the optimal proportion of promotion investment in mixed model, and the smallest proportion of optimal promotion investment in free model. Therefore, unlike the optimal promotion period before the video is launched in the market, the platform should allocate the promotion investment before and after the video is launched in the market according to the business model it adopts in order to enhance the video profits. The above results are consistent with those of Propositions 1(ii), 3(ii), 5(ii), and 7(ii).

Figure 9 shows that the greater the degree of consumer forgetting of the impact of promotion (the decay factor), the lower the optimal proportion of promotion investment before the video launched in the market, under different business models. When the degree of forgetting of the impact of the promotion by consumers increases up to a certain level, the platform should invest more promotion costs after the video is launched in the market; conversely, the platform should invest more promotion costs before the video is launched in the market. The above results are consistent with those of Propositions 2, 4, and 6.

8. Conclusions

In this paper, we discuss the advertising volume, pricing and promotion strategies of the monopoly online video platform. We construct a decision model for the profits maximization problem of the monopoly platform under free model, paid model and mixed model respectively. By solving the model through optimal control theory, we obtain the optimal advertising volume strategy and the optimal promotion investment intensity strategy under free model, the optimal pricing strategy and the optimal promotion investment intensity strategy under paid model and the optimal pricing and optimal advertising volume strategy and the optimal promotion investment intensity strategy under mixed model. We also analyse the impact of promotion effects on optimal advertising volume and optimal pricing. On this basis, firstly, we compare the optimal profits of the platform under different business models, get the optimal business model choice of the platform, and provide management suggestions for the business model choice of the platform. Then, we discuss the optimal promotion period strategy before the video is launched in the market and the optimal promotion investment allocation strategy before and after the video is launched in the market under the optimal business model of the platform to satisfy different situations. Finally, we examine the relevant results obtained through visual analysis of numerical simulations. Our study differs from previous studies on online video platforms' marketing strategies in that we discuss the optimal promotional investment intensity strategy before and after the video is launched in the market, the optimal promotional period strategy before the video is launched in the market, and the optimal allocation strategy of promotional investment before and after the video is launched in the market under different business models of the platform, which fills in the blanks in the literature on online video promotional strategies.

Through this paper's research on the advertising volume, pricing and promotion strategies of monopolistic online video platform, the following three managerial implications can be obtained. First, advertising volume, price and business model aspects: the platform should assess the range of parameters for consumer price sensitivity and advertising volume sensitivity based on the type of the video being broadcast, using big data from previous videos of similar types, and then make decisions such as advertising volume, pricing or business model selection based on the parameters. Second, the promotion period: the platform only needs to decide on the promotion period before the video is launched in the market based on the degree of forgetting of the impact of the promotion by consumers. No matter what business model the platform adopts and whether it adjusts the total budget for promotion, it will not affect the decision of promotion period. For example, platforms can make full use of big data and artificial intelligence and other high-tech means to assess the "consumer forgetting parameters", and then decide on the optimal promotion period before the video is launched in the market. Third, allocation of promotion investment: after the total budget of the video promotion is determined, the platform should reasonably allocate the promotion investment before and after the video is launched in the market according to the business model it adopts. Among free model, paid model and mixed model, paid model should have a bigger promotion investment before the video is launched in the market; mixed model should have a middle promotion investment before the video is launched in the market; and free model should have the smallest promotion investment before the video is launched in the market. In addition, the platform should also decide on the focus of the promotional period based on the degree of consumer forgetting of the impact of the promotion; in case of a greater "consumer forgetting parameters", the platform should focus on the promotion of the video after the video has been launched in the market; conversely, the platform should focus on the promotion of the video before the video has been launched.

This study only analyses the advertising volume, pricing and promotion strategies of the platform in monopoly situations. However, there is also a competitive situation between video platforms, and we will follow up on the issue of advertising volume, pricing and promotional strategies for videos in competitive markets.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix 1 for Corollary 1

Proof. According to Equations (11), (13) and (14), we get

$$\frac{\partial n^*(t)}{\partial A^*(t)} = \frac{\eta}{2\alpha_n}$$

$$\frac{\partial D_n^*(t)}{\partial A^*(t)} = \frac{\eta}{2q_n}$$

$$\frac{\partial \pi_n^*(t)}{\partial A^*(t)} = \frac{\eta\kappa}{2\alpha_n q_n}\left(q_n + \eta A^*(t)\right)$$

Therefore, $\frac{\partial n^*(t)}{\partial A^*(t)} > 0$, $\frac{\partial D_n^*(t)}{\partial A^*(t)} > 0$ and $\frac{\partial \pi_n^*(t)}{\partial A^*(t)} > 0$.

According to Equations (10) and (12), we get

$$\frac{\partial k_n^*(\tau)}{\partial \Delta t} = -\frac{\chi C}{2k_n^*(\tau)\Delta t^2}$$

$$\frac{\partial k_n^*(t)}{\partial T} = -\frac{(1-\chi)C}{2k_n^*(t)T^2}$$

Therefore, $\frac{\partial k_n^*(\tau)}{\partial \Delta t} < 0$ and $\frac{\partial k_n^*(t)}{\partial T} < 0$. Obviously $\frac{\partial k_n^*(\tau)}{\partial \tau} = 0$, $\frac{\partial k_n^*(t)}{\partial t} = 0$. In summary, Corollary 1 holds.

Appendix 2 for Corollary 2

Proof. According to Equations (25), (27) and (28), we get

$$\frac{\partial p^*(t)}{\partial A^*(t)} = \frac{\eta}{2\alpha_p}$$

$$\frac{\partial D_p^*(t)}{\partial A^*(t)} = \frac{\eta}{2q_p}$$

$$\frac{\partial \pi_p^*(t)}{\partial A^*(t)} = \frac{\eta}{2\alpha_p q_p}\left(q_p + \eta A^*(t)\right)$$

Therefore, $\frac{\partial p^*(t)}{\partial A^*(t)} > 0$, $\frac{\partial D_p^*(t)}{\partial A^*(t)} > 0$ and $\frac{\partial \pi_p^*(t)}{\partial A^*(t)} > 0$.

Similar to the proof of Lemma 1, according to Equations (24) and (26), we get $\frac{\partial k_p^*(\tau)}{\partial \Delta t} < 0$, $\frac{\partial k_p^*(t)}{\partial T} < 0$, $\frac{\partial k_p^*(\tau)}{\partial \tau} = 0$, $\frac{\partial k_p^*(t)}{\partial t} = 0$. In summary, Corollary 2 holds.

Appendix 3 for Corollary 3

Proof. According to Equations (39), (40), (42) and (43), we have

$$\frac{\partial p_m^*(t)}{\partial A^*(t)} = \frac{\eta\kappa\left(\kappa\alpha_p + \alpha_n\right)\left(q_p - q_n\right)}{M}$$

$$\frac{\partial n_m^*(t)}{\partial A^*(t)} = \frac{2\eta q\kappa\alpha_p\left(q_p - q_n\right)}{M}$$

$$\frac{\partial D_{mp}^*(t)}{\partial A^*(t)} = \frac{\eta\kappa\alpha_p(\alpha_n - \kappa\alpha_p)}{M}$$

$$\frac{\partial D_{mn}^*(t)}{\partial A^*(t)} = \frac{\eta\alpha_n\left[2\kappa q_p\alpha_p - q_n(\kappa\alpha_p + \alpha_n)\right]}{q_n M}$$

$$\frac{\partial \pi_m^*(t)}{\partial A^*(t)} = \frac{\kappa\eta(q_p - q_n)(\kappa\alpha_p q_n + \alpha_n q_n + 2\kappa\eta\alpha_p A^*(t))}{q_n M}$$

Obviously, $\frac{\partial p_m^*(t)}{\partial A^*(t)} > 0$, $\frac{\partial n_m^*(t)}{\partial A^*(t)} > 0$, $\frac{\partial \pi_m^*(t)}{\partial A^*(t)} > 0$. Let $\frac{\partial D_{mp}^*(t)}{\partial A^*(t)} = 0$, we get $\frac{\alpha_p}{\alpha_n} = \frac{1}{\kappa}$, when $\frac{\alpha_p}{\alpha_n} < \frac{1}{\kappa}$, $\frac{\partial D_{mp}^*(t)}{\partial A^*(t)} > 0$; when $\frac{\alpha_p}{\alpha_n} > \frac{1}{\kappa}$, $\frac{\partial D_{mp}^*(t)}{\partial A^*(t)} < 0$. Let $\frac{\partial D_{mn}^*(t)}{\partial A^*(t)} = 0$, we get $\frac{\alpha_p}{\alpha_n} = \frac{q_n}{\kappa(2q_p - q_n)}$, when $\frac{\alpha_p}{\alpha_n} > \frac{q_n}{\kappa(2q_p - q_n)}$, $\frac{\partial D_{mn}^*(t)}{\partial A^*(t)} > 0$; when $\frac{\alpha_p}{\alpha_n} < \frac{q_n}{\kappa(2q_p - q_n)}$, $\frac{\partial D_{mn}^*(t)}{\partial A^*(t)} < 0$. The proof process for Corollary 3(iv) is the same as that for Corollary 1(iv), so we omit it. In summary, Corollary 3 holds.

Appendix 4 for Lemma 4

Proof. By Equations (14) and (28), we get

$$\pi_p^*(t) - \pi_n^*(t) = \frac{1}{4\alpha_p}\left[\frac{1}{q_p}(q_p + \eta A^*(t))^2 - \frac{\kappa r}{q_n}(q_n + \eta A^*(t))^2\right]$$

Let $\pi_p^*(t) - \pi_n^*(t) = 0$, we have that there exists a critical value $r^* = \frac{q_n(q_p + \eta A^*(t))^2}{\kappa q_p(q_n + \eta A^*(t))^2}$. Also, since $r^* \in (r_1^*, r_2^*)$, it follows that the $\pi_p^*(t) > \pi_n^*(t)$ when $r \in (r_1^*, r^*)$; $\pi_p^*(t) < \pi_n^*(t)$ when $r \in (r^*, r_2^*)$. In case $r \in (r_1^*, r^*)$, by Equation (43), we have

$$\frac{\partial \pi_m^*(t)}{\partial \alpha_p} = \frac{\alpha_n\kappa^2(q_p - q_n)\left[\alpha_n(2q_p + \eta A(t)) + \kappa\eta\alpha_p A(t)\right]}{M^2} \times\left[\kappa r(q_n + \eta A(t)) - (2q_p - q_n + \eta A(t))\right]$$

Since $r \in (r_1^*, r^*)$ and therefore $\frac{\partial \pi_m^*(t)}{\partial \alpha_p} < 0$. Substituting $r = r_1^*$ into Equation (43), we have $\pi_m^*(t)\big|_{r=r_1^*} = \pi_p^*(t)$. Therefore, in case $r^* \in (r_1^*, r^*)$, we have $\pi_p^* > \max\{\pi_n^*, \pi_m^*\}$. In case $r \in (r^*, r_2^*)$, and similarly in case $r^* \in (r_1^*, r^*)$, we have $\pi_m^* > \max\{\pi_n^*, \pi_p^*\}$ when $r \in (r^*, r_2^*)$; $\pi_n^* > \max\{\pi_p^*, \pi_m^*\}$ when $r > r_2^*$. In summary, Lemma 4 holds.

Appendix 5 for Proposition 1

Proof. By Equation (14), we have

$$\frac{\partial \pi_n^*}{\partial \Delta t} = \frac{\eta \kappa k_n^*(\tau)}{4\delta \Delta t \alpha_n q_n}\left[(2\delta \Delta t + 1)\mathrm{e}^{-\delta \Delta t} - 1\right]\int_0^T \mathrm{e}^{-\delta t}\left(q_n + \eta A^*(t)\right)\mathrm{d}t \quad \text{(A.1)}$$

According to the optimality first order condition, let $\frac{\partial \pi_n^*}{\partial \Delta t} = 0$, we can get the optimal Δt_n^* which satisfies the following function form, *i.e.*

$$(2\delta \Delta t_n^* + 1)\mathrm{e}^{-\delta \Delta t_n^*} - 1 = 0 \quad \text{(A.2)}$$

Furthermore, taking the second order derivative with respect to Δt_n^* for Equation (A.1) yields

$$\begin{aligned}\frac{\partial^2 \pi_n^*}{\partial \Delta t^2} &= \frac{\eta \kappa k_n^*(\tau)}{4\delta \Delta t \alpha_n q_n}\int_0^T \mathrm{e}^{-\delta t}\left(q_n + \eta A^*(t)\right)\mathrm{d}t \frac{\partial\left[(2\delta \Delta t + 1)\mathrm{e}^{-\delta \Delta t} - 1\right]}{\partial \Delta t} \\ &+ \left[(2\delta \Delta t + 1)\mathrm{e}^{-\delta \Delta t} - 1\right]\frac{\partial\left[\frac{\eta \kappa k_n^*(\tau)}{4\delta \Delta t \alpha_n q_n}\int_0^T \mathrm{e}^{-\delta t}\left(q_n + \eta A^*(t)\right)\mathrm{d}t\right]}{\partial \Delta t}\end{aligned} \quad \text{(A.3)}$$

Substituting Equation (A.2) into Equation (A.3), we get

$$\left.\frac{\partial^2 \pi_n^*}{\partial \Delta t^2}\right|_{\Delta t = \Delta t_n^*} = \frac{\mathrm{e}^{-\delta \Delta t_n^*}\eta \kappa k_n^*(\tau)\left(1 - 2\delta \Delta t_n^*\right)}{4\Delta t_n^* \alpha_n q_n}\int_0^T \mathrm{e}^{-\delta t}\left(q_n + \eta A^*(t)\right)\mathrm{d}t$$

Since $(2\delta \Delta t_n^* + 1)\mathrm{e}^{-\delta \Delta t_n^*} - 1 > 0$ when $\Delta t_n^* < \frac{1}{2\delta}$. Therefore, $\Delta t_n^* > \frac{1}{2\delta}$ under $\Delta t = \Delta t_n^*$. As a result, we can get $\left.\frac{\partial^2 \pi_n^*}{\partial \Delta t^2}\right|_{\Delta t = \Delta t_n^*} < 0$. In summary, Δt_n^* is the optimal solution.

Taking the first order derivative with respect to χ for Equation (14) under the condition $\Delta t = \Delta t_n^*$ yields

$$\left.\frac{\partial \pi_n^*}{\partial \chi}\right|_{\Delta t = \Delta t_n^*} = \frac{\eta \kappa C}{4\alpha_n q_n}\int_0^T \left(q_n + \eta A^*(t)\right)\frac{T k_n^*(t)\left(1 - \mathrm{e}^{-\delta \Delta t_n^*}\right)\mathrm{e}^{-\delta \tau} - k_n^*(\tau)\Delta t_n^*\left(1 - \mathrm{e}^{-\delta \tau}\right)}{k_n^*(\tau)k_n^*(t)\delta \Delta t_n^* T}\mathrm{d}t \quad \text{(A.4)}$$

According to the optimality first order condition, let $\left.\frac{\partial \pi_n^*}{\partial \chi}\right|_{\Delta t = \Delta t_n^*} = 0$, we can obtain the optimal χ_n^* satisfying the following Equation as

$$\chi_n^* = \frac{1}{\frac{\Delta t_n^*}{T\left(1 - \mathrm{e}^{-\delta \Delta t_n^*}\right)^2}\left[\frac{\int_0^T \left(q_n + \eta A^*(t)\right)\mathrm{d}t}{\int_0^T \mathrm{e}^{-\delta \tau}\left(q_n + \eta A^*(t)\right)\mathrm{d}t} - 1\right]^2 + 1} \quad \text{(A.5)}$$

Furthermore, taking the second order derivative with respect to χ for Equation (14) under the condition $\Delta t = \Delta t_n^*$ yields

$$\left.\frac{\partial^2 \pi_n^*}{\partial \chi^2}\right|_{\Delta t=\Delta t_n^*} = -\frac{e^{-\delta\Delta t_n^*}\eta\kappa C^2}{8\alpha_n q_n \Delta t_n^* T\delta^2 k_n^*(\tau)k_n^*(t)\chi(1-\chi)}\int_0^T \Big\{\chi\delta q_n \Delta t_n^* e^{\delta\Delta t_n^*}\left(1-e^{-\delta\Delta t_n^*}\right) + \left(e^{\delta\Delta t_n^*}-1\right)\left[T\delta q_n k_n^*(t)(1-\chi)e^{-\delta t}+\eta C e^{-\delta t}\left(1-e^{-\delta t}\right)\right]\Big\}dt$$

Substituting Equation (A.5) into $\left.\frac{\partial^2 \pi_n^*}{\partial \chi^2}\right|_{\Delta t=\Delta t_n^*}$, we get $\left.\frac{\partial^2 \pi_n^*}{\partial \chi^2}\right|_{\substack{\Delta t=\Delta t_n^* \\ \chi=\chi_n^*}} < 0$. As can be seen, χ_n^* is the optimal solution. In summary, Proposition 1 holds.

Appendix 6 for Proposition 2

Proof. According to Equation (A.5), we have when

$f_n < \frac{T}{\left[\frac{\int_0^T (q_n+\eta A^*(t))dt}{\int_0^T e^{-\delta\tau}(q_n+\eta A^*(t))dt}-1\right]^2}$, $\chi_n^* > \frac{1}{2}$; otherwise, $\chi_n^* < \frac{1}{2}$. Where

$f_n = \frac{\Delta t_n^*}{\left(1-e^{-\delta\Delta t_n^*}\right)^2}$. By Equation (A.2), we have $\frac{\partial \Delta t_n^*}{\partial \delta} = -\frac{\Delta t_n^*}{\delta} < 0$. Therefore, there exists $f_n(\delta_n) = \frac{T}{\left[\frac{\int_0^T (q_n+\eta A^*(t))dt}{\int_0^T e^{-\delta\tau}(q_n+\eta A^*(t))dt}-1\right]^2}$, when $\delta < \delta_n$, $\chi_n^* > \frac{1}{2}$; when $\delta > \delta_n$, $\chi_n^* < \frac{1}{2}$. In summary, Proposition 2 holds.

Appendix 7 for Proposition 3

Proof. By Equation (28), we have

$$\frac{\partial \pi_p^*}{\partial \Delta t} = \frac{\eta k_p^*(\tau)}{4\delta\Delta t\alpha_p q_p}\left[(2\delta\Delta t+1)e^{-\delta\Delta t}-1\right]\int_0^T e^{-\delta t}\left(q_p+\eta A^*(t)\right)dt \quad \text{(A.6)}$$

According to the optimality first order condition, let $\frac{\partial \pi_p^*}{\partial \Delta t} = 0$, we can get the optimal Δt_p^* which satisfies the following function form, *i.e.*

$$\left(2\delta\Delta t_p^*+1\right)e^{-\delta\Delta t_p^*}-1=0 \quad \text{(A.7)}$$

The proof of the existence and uniqueness of Δt_p^* is similar to Proposition 1, so we omit the discussion.

Taking the first order derivative with respect to χ for Equation (28) under the condition $\Delta t = \Delta t_p^*$ yields

$$\left.\frac{\partial \pi_p^*}{\partial \chi}\right|_{\Delta t=\Delta t_p^*} = \frac{\eta C}{4\alpha_p q_p}\int_0^T 2\left(q_n+\eta A^*(t)\right) \times \frac{Tk_p^*(t)\left(1-e^{-\delta\Delta t_p^*}\right)e^{-\delta\tau} - k_p^*(\tau)\Delta t_p^*\left(1-e^{-\delta\tau}\right)}{2k_p^*(\tau)k_p^*(t)\delta\Delta t_p^* T}dt$$

According to the optimality first order condition, let $\left.\frac{\partial \pi_p^*}{\partial \chi}\right|_{\Delta t=\Delta t_p^*}=0$, we can obtain the optimal χ_p^* satisfying the following Equation as

$$\chi_p^*=\frac{1}{\frac{\Delta t_p^*}{T\left(1-\mathrm{e}^{-\delta\Delta t_p^*}\right)^2}\left[\frac{\int_0^T\left(q_p+\eta A^*(t)\right)\mathrm{d}t}{\int_0^T \mathrm{e}^{-\delta\tau}\left(q_p+\eta A^*(t)\right)\mathrm{d}t}-1\right]^2+1} \tag{A.8}$$

The proof of the existence and uniqueness of χ_p^* is similar to Proposition 1, and we omit the discussion. In summary, Proposition 3 holds.

Appendix 8 for Proposition 4

Proof. The proof of Proposition 4 is similar to that of Proposition 2, *i.e.*, there exists $f_p\left(\delta_p\right)=\frac{T}{\left[\frac{\int_0^T\left(q_p+\eta A^*(t)\right)\mathrm{d}t}{\int_0^T \mathrm{e}^{-\delta\tau}\left(q_p+\eta A^*(t)\right)\mathrm{d}t}-1\right]^2}$, $\chi_p^*>\frac{1}{2}$ when $\delta<\delta_p$; $\chi_p^*<\frac{1}{2}$ when $\delta>\delta_p$. The detailed proof is omitted. In summary, Proposition 4 holds.

Appendix 9 for Proposition 5

Proof. By Equation (43), we have

$$\frac{\partial \pi_m^*}{\partial \Delta t}=\frac{\eta\kappa k_m^*(\tau)\left(q_p-q_n\right)\left[\left(2\delta\Delta t+1\right)\mathrm{e}^{-\delta\Delta t}-1\right]}{2\delta\Delta t q_n M}\times\int_0^T \mathrm{e}^{-\delta\tau}\left[2\kappa\alpha_p\eta A^*(t)+q_n\left(\alpha_n+\kappa\alpha_p\right)\right]\mathrm{d}t$$

According to the optimality first order condition, let $\frac{\partial \pi_m^*}{\partial \Delta t}=0$, we can get the optimal Δt_m^* which satisfies the following function form, *i.e.*

$$\left(2\delta\Delta t_m^*+1\right)\mathrm{e}^{-\delta\Delta t_m^*}-1=0 \tag{A.9}$$

The proof procedure for the existence and uniqueness of Δt_m^* is similar to Propositions 1 and 3, so we omit it.

Taking the first order derivative with respect to χ for Equation (43) under the condition $\Delta t=\Delta t_m^*$ yields

$$\left.\frac{\partial \pi_m^*}{\partial \chi}\right|_{\Delta t=\Delta t_m^*}=\frac{\eta\kappa C\left(q_p-q_n\right)}{q_n M}\int_0^T\left[2\kappa\alpha_p\eta A^*(t)+q_n\left(\alpha_n+\kappa\alpha_p\right)\right]\times\frac{Tk_m^*(t)\left(1-\mathrm{e}^{-\delta\Delta t_m^*}\right)\mathrm{e}^{-\delta\tau}-k_m^*(\tau)\Delta t_m^*\left(1-\mathrm{e}^{-\delta\tau}\right)}{2k_m^*(\tau)k_m^*(t)\delta\Delta t_m^* T}\mathrm{d}t$$

According to the optimality first order condition, let $\left.\frac{\partial \pi_m^*}{\partial \chi}\right|_{\Delta t=\Delta t_m^*}=0$, we can obtain the optimal χ_m^* satisfying the following Equation As

$$\chi_m^* = \frac{1}{1+\frac{\Delta t_m^*}{T\left(1-e^{-\delta\Delta t_m^*}\right)^2}\left[\frac{\int_0^T\left[2\kappa\alpha_p\eta A^*(t)+q_n\left(\alpha_n+\kappa\alpha_p\right)\right]dt}{\int_0^T e^{-\delta\tau}\left[2\kappa\alpha_p\eta A^*(t)+q_n\left(\alpha_n+\kappa\alpha_p\right)\right]dt}-1\right]^2} \quad (A.10)$$

The proof procedure for the existence and uniqueness of χ_m^* is similar to Propositions 1 and 3, so we omit it. In summary, Proposition 5 holds.

Appendix 10 for Proposition 6

Proof. The proof of Proposition 6 is similar to that of Proposition 2 and 4, *i.e.*, there exists $f_m(\delta_m)=\frac{T}{\left[\frac{\int_0^T\left[2\kappa\alpha_p\eta A^*(t)+q_n\left(\alpha_n+\kappa\alpha_p\right)\right]dt}{\int_0^T e^{-\delta\tau}\left[2\kappa\alpha_p\eta A^*(t)+q_n\left(\alpha_n+\kappa\alpha_p\right)\right]dt}-1\right]^2}$, $\chi_m^*>\frac{1}{2}$ when $\delta<\delta_m$; $\chi_m^*<\frac{1}{2}$ when $\delta>\delta_m$. Here we omit the more detailed proof. In summary, Proposition 6 holds.

Appendix 11 for Proposition 7

Proof. According to the denominator term of Equation (A.10), we let

$$g=\frac{\int_0^T\left[2\kappa\alpha_p\eta A^*(t)+q_n\left(\alpha_n+\kappa\alpha_p\right)\right]dt}{\int_0^T e^{-\delta\tau}\left[2\kappa\alpha_p\eta A^*(t)+q_n\left(\alpha_n+\kappa\alpha_p\right)\right]dt}$$

The first order derivative of g with respect to α_p is

$$\frac{\partial g}{\partial\alpha_p}=\frac{2\eta\kappa q_n\alpha_n\int_0^T A^*(t)\left(1-e^{-\delta T}-\delta Te^{-\delta\tau}\right)dt}{\delta\left\{\int_0^T e^{-\delta\tau}\left[2\kappa\alpha_p\eta A^*(t)+q_n\left(\alpha_n+\kappa\alpha_p\right)\right]dt\right\}^2}$$

Let $h=1-e^{-\delta T}-\delta Te^{-\delta\tau}$, we get $\frac{\partial h}{\partial T}=\delta\left(e^{-\delta T}-e^{-\delta\tau}\right)$, Since $\tau\le T$, therefore, $\frac{\partial h}{\partial T}\le 0$. Therefore, h takes its maximum value at $T=0$, *i.e.*, $h(T=0)=0$. Since $T>0$, therefore, $h<0$. $\frac{\partial g}{\partial\alpha_p}<0$ since $h<0$. Since $\frac{\partial\chi_m^*}{\partial g}<0$ and $\frac{\partial g}{\partial\alpha_p}<0$, hence $\frac{\partial\chi_m^*}{\partial\alpha_p}=\frac{\partial\chi_m^*}{\partial g}\frac{\partial g}{\partial\alpha_p}>0$. Similarly, we are able to proof to obtain $\frac{\partial\chi_p^*}{\partial q_p}>0$. According to the known condition in Lemma 3 that $r\in\left(r_1^*,r_2^*\right)$ (which is equivalent to $\alpha_p\in\left(\alpha_n r_1^*,\alpha_n r_2^*\right)$), therefore $\chi_p^*>\chi_m^*>\chi_n^*$. According to Δt_n^*, Δt_p^* and Δt_m^*, it is easy to know that $\Delta t_n^*=\Delta t_p^*=\Delta t_m^*$. In summary, Proposition 7 holds.