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The Impulse Response of Domestic and Foreign Interest Rate in Output, Price, Exchange Rate Model, a Deconstructed Derivation and Economic Calibration of Vector Error Correction Model

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Abstract

How to assess external shocks, whether they are inflationary shocks or interest rate shocks without warning, is crucial to construct smooth and predictable financial market. The purpose of this paper is to analyze the short-run and long-run effects of shocks on an economy under chaotic conditions of uncertainty using the Vector Error Correction Model (VECM). A comprehensive and detailed derivation process is presented for VECM and reveals the dynamics of interest rates and inflation in the face of external shocks, through Cholesky ordering and impulse response decomposition. With a constraint, the VECM model is also used to derive its two applications in well-known theoretical fields, the Fisher model and the uncovered interest rate parity (UIP) theory.

Keywords

Vector Error Correction Model, Interest Rate, Impulse Response, Cholesky Ordering, Fisher Model, Uncovered Interest Rate Parity

1. Introduction

A dip in consumer confidence of less than 80 may signal the start of a recession anywhere from six months to one year in advance, given the outsized impact that private spending has on the economy. As a result of the ongoing conflict between Russia and Ukraine and the subsequent disruption of global supply lines, the Michigan Consumer Sentiment Index has been weighed down by inflation, and as a result, it has dropped below the recessionary readings of 1991 and 2001, putting it in the second place, behind only the Great Stagflation of the 1970s and the financial crisis of 2008. The Federal Reserve of the United States will eventually be forced to hike interest rates rapidly in order to maintain price stability, and it will come at a time when consumers are already feeling fatigued. Nevertheless, maintaining low prices will have an effect on the production of the economy, the currency rate, and the nominal interest rate via a variety of routes, including increases in borrowing costs, wealth, and income. How to analyze external shocks, whether they are inflationary shocks or interest rate shocks without notice, would be vital to develop smooth and predictable market attitude. Using the Vector Error Correction Model (VECM), the goal of this paper is to conduct an analysis of the short-run and long-run impacts of shocks on the economy while simultaneously taking into account the chaotic circumstances of uncertainty.

Suppose we are in a system of four domestic variables, denoted by $Y_t = \left(\Delta p_t, y_t, i_t, e_t\right)'$, and one foreign variable denoted by $X_t = i_t^*$, in all subsequent analyses, items superscripted by "*" denote foreign variables contrasting domestic ones. We design a micro environment to mimic the external shock to the system, in which we first build a vector error correction model (VECM) for all variables and list the lag length and cointegrating rank of the model. After that, we assume 2 cointegrating vectors based on nominal and real exchange rate equation, and limit β in a certain space corresponding to the Fisher theory and uncovered interest rate parity (UIP) theory. We explain why this model corresponds to the nominal effective exchange rate (NEER) theory and do an item-by-item analysis, use Cholesky ordering and analyze external shocks to domestic and foreign interest rates, in terms of inflation and exchange rate changes. We will comprehensively analyze the short-term and long-term effects of sudden changes in interest rates on the various items of the model.

In this study, we employ quantitative analysis to determine the amount to which transitory interest rate shocks influence the economic model's components and the model itself within a standard theoretical framework. The study gives not only an intuitive grasp of the economics of the process, but also a tangible derivation procedure for a more thorough comprehension of the VECM model. By coupling interest rates, currency rates, inflation, and potential production, the likelihood of an economic crisis in the actual world is simulated. We find that confronted with a positive external shock, when there is a positive trend in the 3 Month London Interbank Offered Rate in US Dollars (libor3m), domestic interest rates rise if London is selected as the domestic position, hence luring more individuals to invest in the local currency. Concurrently, the domestic currency "tightens," which results in a drop in inflation. Then investors invest domestically, causing the already-lower inflation to increase more.

This volatility spiral illustrates the dynamic nature of inflationary interest rates. Over time, foreign money will leave and flee to nations with higher interest rates, and long-term inflation will drop. When there is a positive shock to foreign interest rates as measured by the Three Month Euro Interbank Offered Rate (euribor3m), the local currency becomes less appealing to investors in comparison to the foreign currency. The foreign exchange market sells the domestic currency and purchases foreign assets. The effect of the inflation shock on international short-term interest rates is a decrease in domestic investment, capital outflows, and a decline in inflation. However, in the long run, both capital and inflation resume.

2. Literature Review

Reference [1] and [2] verified the use of unit root and vector error correction models for panel data, where the cells in the panel are independently homogeneous and heterogeneous factors may induce deviations. The maximum likelihood method developed by [3] is able to demonstrate the cointegration relationship between the variables of interest to determine the presence of ambiguous long-run dynamic equilibrium, while the equation of [4] permits the presence of heterogeneity for each variable. Reference [5] argues further that cointegration tests might be viewed as pre-tests in order to avoid false regression scenarios. Unit root tests are used to determine the order of integration [6] [7], and the Augmented Dickey Fuller (ADF) test is employed to determine the existence and quantity of unit roots [8].

VECM and related models have been utilized in several economic and management applications. Reference [9] examined the impact of transportation to economic growth using vector auto-regression. Reference [10] discovered stochastic trends by examining time series for the presence of unit roots. Reference [11] utilized VECM to determine the influence of exchange rate fluctuations on the price volatility of Nigerian agricultural goods. In contrast, [12] investigated the causal relationship between cement output and demand in India. The relationship between financial depth and economic growth via VECM connection has also been confirmed by academics in many national settings [13] [14].

3. Methodology

We used a public economic dataset from Gretl's database of *chdat 2001.gdt*, which covers the period from the first quarter of 1974 to the first quarter of 2020. In this database and our subsequent analysis, we will use UK economic data as a starting point, with the UK as "domestic" and other countries as foreign. For example, "*i*" which represents the domestic interest rate, in this study it represents the interest rate of the domestic currency. We use "euribor3m" to represent foreign nominal interest rates. We use "libor3m" to represent the home country nominal interest rate. We use l_{neer} to represent the log of exchange rate, use l_{gdp} to represent the log of gdp (output), and those 5 variable $\Delta p_t, y_t, i_t, e_t, i_t^*$ to represent domestic inflation rate, the log of gdp, domestic nominal interest rate, the log of exchange rate and foreign nominal interest rate.

First, we need to extend the likelihood arrangement of i and i^* through the vector error correction model, hereinafter referred to as: VECM. Through the model time-series multivariate var-lag selection process, we get a perspective display of information criteria, where AIC stands for Akaike criterion: $\text{AIC} = 2k - 2\ln(L)$, and BIC/SIC stands for Bayesian information criterion/ Schwarz information criterion: $\text{BIC} = k\ln(n) - 2\ln(\hat{L})$ and $\text{HQC} = -2L_{\max} + 2k\ln(\ln(n))$ for Hannan-Quinn criterion where $L_{\max}$ is the log-likelihood, k is the number of parameters, and n is the number of observations.

Reference [15] analyzed and contrasted the Bayesian Information Criterion (BIC) and the Akaike Information Criterion (AIC), analyzing their rationale and estimate level as approximations for various target values. Reference [16] concluded that the Bayesian Information Criterion (BIC) is more effective for selecting the best model, while the Akaike Information Criterion (AIC) is superior for predicting future data. While [17] determined that the Hannan-Quinn criteria (HQC) is the most accurate for quarterly VAR models, it is more appropriate to apply the Schwarz Information Criterion (SIC)/BIC when the sample size is small, as demonstrated by [18]. To simplify the computation and save time, we set the highest lag threshold to 4 and discover that when the lag is 2 stages, AIC, BIC, and HQC yield the least results. We do not pay attention to the distinctions between these various information criteria since the three distinct information criteria all provide the same result: a 2-period lag.

To obtain the cointegration rank, we employ the Johansen trace test and select the lag duration for the VAR model that minimizes AIC, SIC, or HQC. The ideal lag stage has been noted with a star in **Table 1**. Therefore, we've settled on "two" as the length of the lag. We conduct the Johansen test and obtain the Trace statistics: H_0: rank = (Π) = r; H_A: rank = (Π) > r.

Note that the null hypothesis for Johansen trace test is: there are r cointegration ranks while the alternative hypothesis is that there are more than r ranks. The number of ranks equals the number of cointegration relationships between the variables. Here we have:

rank = (Π) = 0 and reject H_0;

rank = (Π) = 1 and reject H_0;

rank = (Π) = 2 and reject H_0;

rank = (Π) = 3 and fails to reject H_0.

Table 1. The minimized estimated values of the information criteria. The asterisks indicate the best fitted value.

Lags	Loglik	P (LR)	AIC	BIC	HQC
1	815.71911		−8.778984	−8.244786	−8.562371
2	901.03706	0.00000	−9.452928*	−8.473564*	−9.055803*
3	913.60129	0.45519	−9.313981	−7.889451	−8.736345
4	937.68246	0.00357	−9.303715	−7.434019	−8.545568

Therefore, depending on the outcome of the trace test's p-value, we have three cointegration rankings. But we only barely reject the hypothesis: rank = (Π) = 2 marginally. The p-value for the Lmax test reveals that:

rank = (Π) = 0 and reject H_0;

rank = (Π) = 1 and reject H_0;

rank = (Π) = 2 and fails to reject H_0 (Because 0.06 is great than 0.05).

Therefore, according to the outcome of the trace test's p-value in **Table 2**, we set two cointegration rankings. Since the trace test provides a cointegration rank of 3 only by a narrow margin (rejection p-value of 0.0403, barely below the rejection criterion of 0.05), the Lmax test yields a cointegration rank of 2, indicating that there are two cointegration relationships and two cointegration vectors. Now the Vector error correction model (VECM) model is defined, when $\Pi = \alpha\beta'$, $Z_t = \left[Y_t' X_t'\right]'$:

$$\Delta Z_t = \mu + \prod Z_{t-1} + \sum_{j=1}^{p} \Gamma_j \Delta Z_{t-j} + \varepsilon_t,\ \varepsilon_t \sim N(0, \Sigma) \tag{1}$$

where the $\Pi = \alpha\beta'$, α is adjustment vector, β is the cointegration vector. In the vector of β, different value of β measures the proportion of different variables in a cointegration relationship (long-term equilibrium). α is adjustment vector to close the deviation from the long-run equilibrium of different variables. Remember β and α now are all vectors. Previously we define 5 variables: $Y_t = (\Delta p_t, y_t, i_t, e_t)$, $X_t = i_t^*$, and we write the VECM equation in matrix in details as:

$$\begin{bmatrix} \Delta\Delta p_t \\ \Delta y_t \\ \Delta i_t \\ \Delta e_t \\ \Delta i_t^* \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} + \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \\ \alpha_{41} & \alpha_{42} \\ \alpha_{51} & \alpha_{52} \end{bmatrix} ec_{t-1} + \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \gamma_{14} & \gamma_{15} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & \gamma_{24} & \gamma_{25} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} & \gamma_{34} & \gamma_{35} \\ \gamma_{41} & \gamma_{42} & \gamma_{43} & \gamma_{44} & \gamma_{45} \\ \gamma_{51} & \gamma_{52} & \gamma_{53} & \gamma_{54} & \gamma_{55} \end{bmatrix} \begin{bmatrix} \Delta\Delta p_{t-1} \\ \Delta y_{t-1} \\ \Delta i_{t-1} \\ \Delta e_{t-1} \\ \Delta i_{t-1}^* \end{bmatrix} + \varepsilon_t \tag{2}$$

or in a simpler way:

$$\begin{bmatrix} \Delta Y_t \\ \Delta X_t \end{bmatrix} = \begin{bmatrix} \mu_y \\ \mu_x \end{bmatrix} + \begin{bmatrix} \Pi_y \\ \Pi_x \end{bmatrix} Z_{t-1} + \begin{bmatrix} \Gamma_y \\ \Gamma_x \end{bmatrix} \Delta Z_{t-1} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix} \tag{3}$$

where we have the cointegration relationship as:

$$ec_{t-1} = \beta' Z_{t-1} = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} & \beta_{15} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} & \beta_{25} \end{bmatrix} + \begin{bmatrix} \Delta p_{t-1} \\ y_{t-1} \\ i_{t-1} \\ e_{t-1} \\ i_{t-1}^* \end{bmatrix} \tag{4}$$

and for every specific variables, we have their expanded forms as:

$$\begin{aligned}
\Delta\Delta p_t &= \mu_1 + \alpha_{11} ec_{1,t-1} + \alpha_{12} ec_{2,t-1} + \gamma_{11}\Delta\Delta p_{t-1} + \gamma_{12}\Delta y_{t-1} + \gamma_{13}\Delta i_{t-1} + \gamma_{14}\Delta e_{t-1} + \gamma_{15}\Delta i_{t-1}^* + \varepsilon_t \\
\Delta y_t &= \mu_2 + \alpha_{21} ec_{1,t-1} + \alpha_{22} ec_{2,t-1} + \gamma_{21}\Delta\Delta p_{t-1} + \gamma_{22}\Delta y_{t-1} + \gamma_{23}\Delta i_{t-1} + \gamma_{24}\Delta e_{t-1} + \gamma_{25}\Delta i_{t-1}^* + \varepsilon_t \\
\Delta i_t &= \mu_3 + \alpha_{31} ec_{1,t-1} + \alpha_{32} ec_{2,t-1} + \gamma_{31}\Delta\Delta p_{t-1} + \gamma_{32}\Delta y_{t-1} + \gamma_{33}\Delta i_{t-1} + \gamma_{34}\Delta e_{t-1} + \gamma_{35}\Delta i_{t-1}^* + \varepsilon_t \\
\Delta e_t &= \mu_4 + \alpha_{41} ec_{1,t-1} + \alpha_{42} ec_{2,t-1} + \gamma_{41}\Delta\Delta p_{t-1} + \gamma_{42}\Delta y_{t-1} + \gamma_{43}\Delta i_{t-1} + \gamma_{44}\Delta e_{t-1} + \gamma_{45}\Delta i_{t-1}^* + \varepsilon_t \\
\Delta i_t^* &= \mu_5 + \alpha_{51} ec_{1,t-1} + \alpha_{52} ec_{2,t-1} + \gamma_{51}\Delta\Delta p_{t-1} + \gamma_{52}\Delta y_{t-1} + \gamma_{53}\Delta i_{t-1} + \gamma_{54}\Delta e_{t-1} + \gamma_{55}\Delta i_{t-1}^* + \varepsilon_t
\end{aligned} \tag{5}$$

Table 2. Cointegration rank test with unrestricted constant.

Rank	Eigenvalue	Trace test	P-value	Lmax	P-value	Trace test (corrected)	P-value
0	0.28970	123.02	[0.0000]	61.913	[0.0000]	123.02	[0.0000]
1	0.15520	61.106	[0.0014]	30.527	[0.0171]	61.106	[0.0017]
2	0.10692	30.579	[0.0403]	20.467	[0.0607]	30.579	[0.0424]
3	0.053084	10.112	[0.2772]	9.8727	[0.2249]	10.112	[0.2820]
4	0.0013214	0.23933	[0.6247]	0.23933	[0.6247]	0.23933	[0.6277]

With the aforementioned formula, a detailed analysis of every component of the VECM model, we may precisely describe how each object responds to shocks. We leave the theoretical model at this point for the time being, and through the subsequent regression analysis, we will get precise numbers and incorporate them into the theoretical model for interpretation.

4. Result

4.1. Regression Display

Now, we examine the unrestricted VECM regression results to investigate cointegration connections and adjustment vector vectors. The unrestricted VECM conclusions do not account for a particular economic theory but pave the way for further analysis.

Remember that β values in the two columns represent the two cointegration relationships we find. The value of α below represents the amount of deviation closed in a period once a certain data deviates from the long-term equilibrium cointegration relationship in VECM. That is why we call α the adjustment vector. In the first column, the beta coefficient is: 1, 0, −0.0908, 0.0805, −0.0478. β shows the weight of the 5 variables in the cointegrating relationship. We can rewrite it here:

$$\begin{cases} \beta_{11}\Delta p_t + \beta_{12} y_t + \beta_{13} i_t + \beta_{14} e_t + \beta_{15} i_t^* + \varepsilon_t = 0 \\ 1\Delta p_t + 0 y_t - 0.0908 i_t + 0.0805 e_t - 0.0478 i_t^* + \varepsilon_t = 0 \end{cases} \tag{6}$$

This is the estimated cointegration relationship in **Table 3**, Column (1). We now identify the five equations for the five variables each. Due to space constraints, we only display and explain Δe_t and Δi_t^* in Equation (7) and Equation (8) and their VECM Regression in **Table 4** and **Table 5**.

$$\begin{cases} \Delta e_t = \mu_4 + \alpha_{41} ec_{1,t-1} + \alpha_{42} ec_{2,t-1} + \gamma_{41}\Delta\Delta p_{t-1} + \gamma_{42}\Delta y_{t-1} \\ \qquad + \gamma_{43}\Delta i_{t-1} + \gamma_{44}\Delta e_{t-1} + \gamma_{45}\Delta i_{t-1}^* + \varepsilon_t \\ \Delta e_t = -0.274 + 0.006 ec_{1,t-1} + 0.039 ec_{2,t-1} + 0.0039\Delta\Delta p_{t-1} \\ \qquad - 0.065\Delta y_{t-1} + 0.007\Delta i_{t-1} + 0.288\Delta e_{t-1} - 0.0049\Delta i_{t-1}^* + \varepsilon_t \end{cases} \tag{7}$$

$$\begin{cases} \Delta i_t^* = \mu_5 + \alpha_{51} ec_{1,t-1} + \alpha_{52} ec_{2,t-1} + \gamma_{51}\Delta\Delta p_{t-1} + \gamma_{52}\Delta y_{t-1} + \gamma_{53}\Delta i_{t-1} \\ \qquad + \gamma_{54}\Delta e_{t-1} + \gamma_{55}\Delta i_{t-1}^* + \varepsilon_t \\ \Delta i_t^* = 1.912 + 0.056 ec_{1,t-1} - 0.29 ec_{2,t-1} + 0.12\Delta\Delta p_{t-1} + 24.18\Delta y_{t-1} \\ \qquad + 0.108\Delta i_{t-1} + 0.105\Delta e_{t-1} + 0.249\Delta i_{t-1}^* + \varepsilon_t \end{cases} \tag{8}$$

Table 3. VECM regression results without restriction.

Beta (cointegrating vectors, standard errors in parentheses)			Alpha (adjustment vectors)		
	(1)	(2)		(3)	(4)
dp	1.00000 (0.00000)	0.00000 (0.00000)	dp	−0.61834	−0.06008
l_gdp	0.00000 (0.00000)	1.00000 (0.00000)	l_gdp	−7.2970e-005	−0.00946
libor3m	−0.09088 (0.03764)	0.11261 (0.02749)	libor3m	0.12218	−1.3047
l_neer	0.08053 (0.19821)	−0.98749 (0.14478)	l_neer	0.00602	0.03908
euribor3m	−0.04786 (0.03835)	−0.08863 (0.02801)	euribor3m	0.05688	−0.29018

Table 4. VECM regression results for exchange rate change.

	Coefficient	Std. error	T-ratio	P-value
const	−0.274189	0.081428	−3.367	0.0009***
d_dp_1	0.003949	0.005158	0.766	0.4449
d_l_gdp_1	−0.065600	0.304526	−0.215	0.8297
d_libor3m_1	0.007005	0.003425	2.045	0.0424**
d_l_neer_1	0.288388	0.079582	3.623	0.0004***
d_euribor3m_1	−0.004926	0.004210	−1.170	0.2436
EC1	0.006015	0.005654	1.064	0.2889
EC2	0.039079	0.011443	3.415	0.0008***
Mean dependent var	0.006780	S.D. dependent var		0.024851
Sum squared resid	0.091803	S.E. of regression		0.023103
R-squared	0.174148	Adjusted R−squared		0.135737
rho	0.003098	Durbin−Watson		1.991934

Table 5. VECM regression results for interest rate change.

	Coefficient	Std. error	T-ratio	P-value
const	1.912580	1.655850	1.155	0.2497
d_dp_1	0.121826	0.104888	1.161	0.2471
d_l_gdp_1	24.185200	6.192510	3.906	0.0001***
d_libor3m_1	0.108691	0.069658	1.560	0.1205
d_l_neer_1	0.105856	1.618440	0.065	0.9479
d_euribor3m_1	0.249863	0.085619	2.918	0.0040***
EC1	0.056876	0.114993	0.495	0.6215
EC2	−0.290181	0.232706	−1.247	0.2141
Mean dependent var	−0.055448	S.D. dependent var		0.584531
Sum squared resid	37.961270	S.E. of regression		0.469793
R-squared	0.382761	Adjusted R−squared		0.354052
rho	0.017903	Durbin−Watson		1.963244

4.2. Theoretic and Economic Intuition

Following the preparation of the mathematical foundation, we replace particular numbers for the economic interpretation and demonstrate the extent to which the VECM model simplifies the understanding of intuitive meaning. If we enter the following restrictions and show full restricted estimation, we will get the following result where [*x*, *y*] means the *y*th variable in the *x*th cointergrating vector. For example, the "*b* [1, 3] = 1" indicates the coefficient of the 3rd variable in the 1st cointergrating vector. We have five variables in the issue setting, hence the maximum number of *y* is five.

$$b[1,1]=0$$
$$b[1,2]=0$$
$$b[1,3]=1$$
$$b[1,5]=-1$$
$$b[2,1]=-1$$
$$b[2,2]=0$$
$$b[2,3]=1$$

which is

$$\beta' = \begin{bmatrix} 0 & 0 & 1 & \beta_{14} & -1 \\ -1 & 0 & 1 & \beta_{24} & \beta_{25} \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} & \beta_{15} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} & \beta_{25} \end{bmatrix} \tag{9}$$

Why do we assume this and specify the first two columns of the model's first row as 0? Through the analysis that follows, it will become clear that the parameters given above are consistent with Fisher's theory and the notion of uncovered interest rate parity (UIP). The coefficients of the VECM model will display the following interpretation space and magnitude if we adhere to this criterion.

$$\begin{cases} \beta_{11}\Delta p_t + \beta_{12} y_t + \beta_{13} i_t + \beta_{14} e_t + \beta_{15} i_t^* + \varepsilon = 0 \\ 0\Delta p_t + 0 y_t + 1 i_t - 2.938 e_t - 1 i_t^* + \varepsilon = 0 \end{cases} \tag{10}$$

What is the economics intuition behind Equation (10)? That is, $i_t - i_t^* - 2.9381 e_t = \varepsilon_t$. The long-run historical connection that we estimate (the cointegrating coefficients) indicates that appreciation shocks had a positive influence (higher libor or lower euribor) on the interest rate differential. A stochastic trend influencing, for instance, the exchange rate or one of the interest rates should afterwards have an effect on all variables. The higher the domestic interest rate, or the lower the foreign interest rate, the higher e_t. Pay attention: What is the e_t here, it is the log (neer), where the "neer" means norminal effective exchange rate. Using an uncorrected weighted-average computation, the nominal effective exchange rate (NEER) measures a currency's nominal exchange rate relative to a basket of other currencies. Reference [19] has compiled an international database of effective exchange rates. A NEER coefficient greater than 1.0 indicates that the native currency is more valuable than foreign currencies. If the NEER coefficient is less than 1, it indicates that the domestic currency is valued less than foreign currencies. Therefore, if we consider the United Kingdom to be home nation, a greater NEER suggests the local currency increases (worth more). Attention is required, however, as the term "exchange rate" often refers to the amount of domestic currency required to purchase a foreign currency. Therefore, an increase in the local currency exchange rate "*E*" depreciates the domestic currency.

Reference [20] suggests that a country's effective exchange rate should take into account the nominal exchange rate and inflation differentials. If we go back to our model, we see $i_t - i_t^* = 2.9381e_t$, it means that according to our historical data, we observe the following phenomenon: the higher the domestic interest rate (or the lower the foreign interest rate), the greater "*e*" for the domestic currency, which indicates that the domestic currency appreciates. This makes sense if we consider the investor's short-term decision: he invests in the more appealing currency. If domestic currency interest rates are higher, investors will invest and the local currency will rise.

This reminds us of the uncovered interest rate parity-UIP theory, which describes the link between domestic interest rates, foreign interest rates, and exchange rate fluctuations. However, take caution that it does not precisely depict the UIP theory. We shall observe the distinction. Interest rate parity clarifies the link between local and international interest rates as well as the exchange rates of the two nations [21]. If E^e represents the future exchange rate, the current exchange rate is denoted by *E*. Then, the following formula holds:

$$\begin{cases} 1+i = \left(1+i^*\right)\dfrac{E^e}{E} \\ \dfrac{E^e}{E} = 1+\dfrac{E^e - E}{E} = 1+\Delta E^e \end{cases} \tag{11}$$

we combine the above two equations and we get:
$1+i = \left(1+i^*\right)\left(1+\Delta E^e\right) = 1+i^* + \Delta E^e + i^*\Delta E^e$, where the $i^*\Delta E^e$ is very small and we can ignore it. So, the above Equation (10) becomes:

$$i - i^* = \Delta E^e \tag{12}$$

The letter "*E*" represents the exchange rate that is often discussed. The greater "*E*" for local currency indicates that you must spend more to purchase a foreign currency (domestic currency depreciates). Remember that a greater "*e*" (nominal exchange rate compared to a basket of other currencies) indicates that the domestic currency is appreciating. The greater "*E*" (exchange rate) depreciates the indigenous currency. Here, it can be seen that, according to the UIP theory, the difference between domestic and international interest rates is equivalent to the anticipated increase in the exchange rate (depreciation of the local currency) [22].

When domestic interest rates increase, foreign currencies purchase domestic currency, leading the domestic currency to appreciate (*E* to decrease) in the near term. In the long run, the domestic currency depreciates (*E* increases) as a result of the withdrawal of foreign capital following the expiry of foreign funds. So, according to the UIP hypothesis, the native currency will devalue over time (*E* rises). Our model has no issues with the UIP theory. Our model $i_t - i_t^* = 2.9381e_t$

shows that the positive difference of *i* and *i** means the domestic "*e*" increases (domestic currency appreciates). The UIP theory shows, in the short run people buy the domestic currency and domestic currency appreciates, but in the long run, the "*E*" increases. If we observe **Table 6**, Column (2), we may deduce the following equation:

$$\begin{cases} \beta_{11}\Delta p_t + \beta_{12} y_t + \beta_{13} i_t + \beta_{14} e_t + \beta_{15} i_t^* + \varepsilon = 0 \\ -1\Delta p_t + 0 y_t + 1 i_t - 2.7353 e_t - 0.8590 i_t^* + \varepsilon = 0 \end{cases} \tag{13}$$

That is $i_t - \Delta p_t - 0.8590 i_t^* = 2.7353 e_t$, reminding us of Fisher parity of real interest rate and real exchange rate. With inflation, the Fisher equation explains the connection between nominal and real interest rates [23]. According to the Fisher formula, [24] present a dynamic model of long-term interest rates and price changes. Reference [25] additionally verified the concept with a cointegration and error correction model. "*r*" represents the real interest rate, "*i*" represents the domestic nominal interest rate, and "$dp(\Delta p_t)$" represents the inflation rate in the preceding formulae and charts.denoted by Δp_t: $r = i_t - \Delta p_t$.

The second cointegration relationship shows: $i_t - \Delta p_t - 0.8590 i_t^* = 2.7353 e_t$ meaning that there is a positive relationship between (the difference of domestic interest rate, domestic inflation rate, foreign interest rate) and (the nominal exchange rate "*e*"). In the long run, shocks in the "*e*" and in the euribor were reflected in the real interest rate. This equation shows a long-term relationship. That reminds us of the theory of real interest rates: $r - r^* = (i_t - \Delta p_t) - (i^* - \Delta p_t^*) = \Delta E^e$. The difference of the real interest rate shows the expectation of the "*E*". The higher domestic real interest rate, the higher "*E*"(exchange rate) we will have. While the higher "*E*" means the domestic currency depreciates. The difference between our theory and the UIP is that: we subtract the inflation rate for both domestic *i* and foreign *i**. The *α* in **Table 6** means the magnitude of adjustment. The greater the absolute value of *α*, the faster the adjustment. The smaller the absolute value of α, the slower the adjustment. In the long-term equilibrium, if the long-term parameter *β* is positive, its adjustment parameter *α* is negative. We see that in the first column, *α* and *β* have opposite signs. In **Table 6** Column (2), some *α* and *β* have opposite signs while others have the same signs, showing the model accuracy still leave room to improve.

Table 6. VECM regression results with restriction.

	Beta (cointegrating vectors, standard errors in parentheses)			Alpha (adjustment vectors)	
	(1)	(2)		(3)	(4)
dp	0.0000 (0.00000)	−1.0000 (0.00000)	dp	−0.559660	0.618360
l_gdp	0.0000 (0.00000)	0.0000 (0.00000)	l_gdp	−0.00097603	0.00033185
libor3m	1.0000 (0.00000)	1.0000 (0.00000)	libor3m	−0.098398	−0.075756
l_neer	−2.9381 (0.75134)	−2.7353 (0.70093)	l_neer	0.011192	−0.007507
euribor3m	−1.0000 (0.00000)	−0.85906 (0.01709)	euribor3m	0.052627	−0.053628

4.3. Cholesky Ordering and Impulse Response

After connecting the VECM model with Fisher's interest rate inflation model and the uncovered interest rate parity (UIP) theory, we are able to test to what extent external interest rate shocks specifically affect each economic factor and verify the previous theoretical assumptions using computer software. This allows us to test whether or not uncovered interest rate parity (UIP) holds true. The Cholesky decomposition needs to be performed first, which imposes a recursive structure [26], and the ordering needs to go from the variable that is the most exogenous to the variable that is contemporaneously affected by the greatest number of shocks. Altering the order in which things happen will result in distinct impulse response functions [27]. Only then can the impulse response be tested. We rank the variables, place the variables with the highest level of exogeneity at the top of the list and utilize the Cholesky decomposition to apply the Impulse Response (IR). The foreign interest rate is the most exogenous variable among the five that are being considered. Therefore, we placed it at the very beginning of the Cholesky order. Now, we find the impulse response of l_{neer} and dp to euribor3m and libor3m respectively. We first show the response image and then we illustrate the intuition behind it.

In **Figure 1**, we see the response of l_{neer} to shock in libor3m. When there is a positive shock in libor3m, the domestic interest rate increases, and attracting more people invest in domestic currency, thus domestic currency appreciates. **Figure 2** shows the response of l_{neer} to shock in euribor3m. When there is a positive shock to euribor3m, foreign currencies become more desirable, and individuals sell home currency to invest in foreign nations. Domestic currency depreciates.

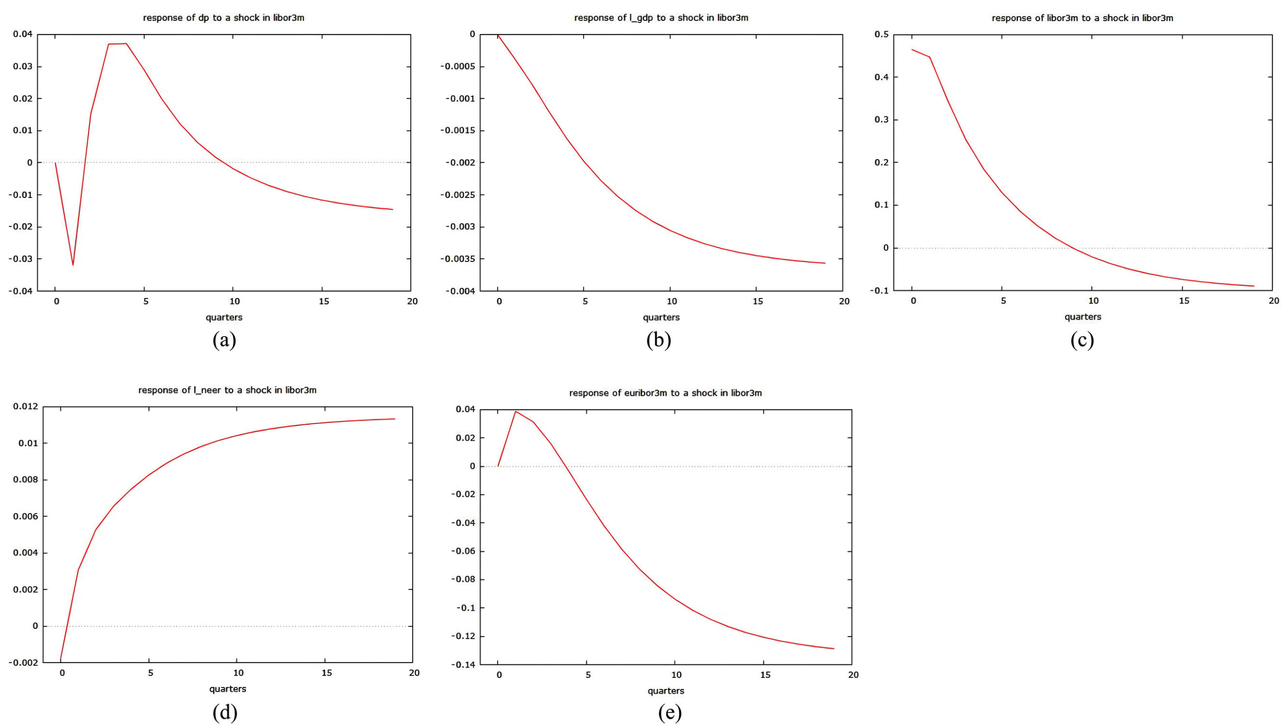

Figure 1. Impulse response of a shock in Libor3m.

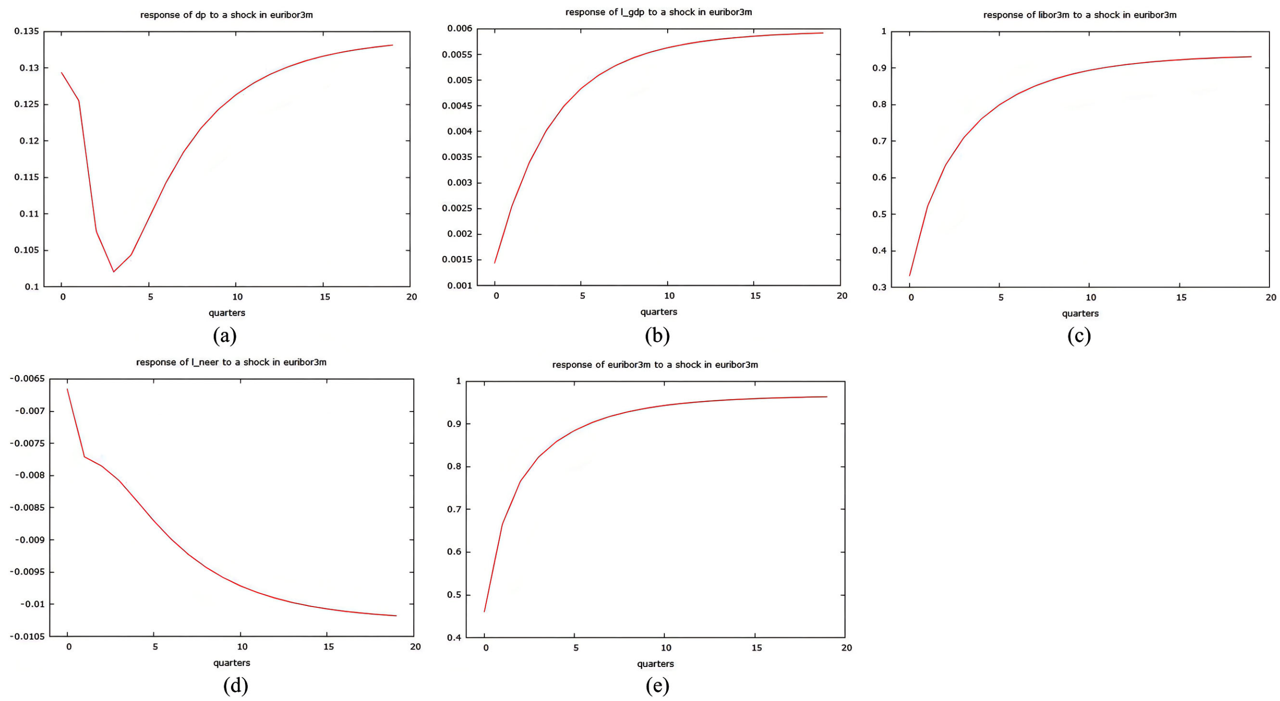

Figure 2. Impulse response of a shock in Euribor3m.

When there is a positive shock in libor3m (such a raise in interest rates by the Federal Reserve), the money in the domestic economy may be "tight," which results in a fall in inflation as seen in **Figure 1**. While this is going on, higher domestic nominal interest rates make the local currency a more desirable investment. As a result of people purchasing domestic currency and investing in their own nation, the inflation rate has once again increased. On the other hand, over the course of several years, foreign investment will progressively decrease and go to other nations where the interest rates are higher. The rate of inflation will also fall on a more gradual basis. **Figure 2** illustrates the response of the inflation rate to a shock in the Euribor3m. During this time, interest rates on investments made in other countries rise, making such investments look more appealing. As a result, domestic investment and capital outflows fall, then the overall rate of inflation falls as well. Over the course of time, both capital and inflation will make their way back.

5. Conclusions

In this study, we make use of quantitative analysis in conjunction with a standard theoretical framework to investigate the ways in which temporary shifts in interest rates have an effect not only on the component sections of the economic model but also on the model as a whole. The VECM method searches varied models of heterogeneity for possible correlations over the long run. Because of its one-of-a-kind benefits, academics have made extensive use of this model to investigate a variety of topics, including agricultural economics, financial development, and international commerce. This paper provides a brief overview of the research conducted by earlier scholars. Furthermore, it combines the examples presented in this paper to demonstrate the computational mathematical process of deriving VECM in detail, to enhance our comprehension of the origin and history of each component of VECM.

In addition to an intuitive understanding of the economics of the process, this study gives a practical derivation approach for a deeper understanding of the VECM model. By incorporating interest rates, currency rates, inflation, and potential output, the probability of a global economic catastrophe is simulated and we discover that by adjusting the parameters and constraints of the VECM model, the VECM can be used to demonstrate the Fisher rate model and the uncovered interest rate parity (UIP) theory, thereby aiding in the comprehension of the relationships between interest rates, exchange rates, and inflation. Using Cholesky ordering, we finally describe the intrinsic and extrinsic interest rate impulse responses in terms of output, inflation rate, the domestic interest rate, the foreign interest rate, and the domestic real interest rate. Combining derivation and economic intuition, this study is instructive and interpretive.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Breitung, J. (2001) The Local Power of Some Unit Root Tests for Panel Data. In: Baltagi, B.H., Ed., *Nonstationary Panels*, *Panel Cointegration*, *and Dynamic Panels*, Emerald Group Publishing Limited, Bingley.

[2] Levin, A., Lin, C.F. and Chu, C.S.J. (2002) Unit Root Tests in Panel Data: Asymptotic and Finite-Sample Properties. *Journal of Econometrics*, **108**, 1-24. https://doi.org/10.1016/S0304-4076(01)00098-7

[3] Johansen, S. and Juselius, K. (1990) Some Structural Hypotheses in a Multivariate Cointegration Analysis of the Purchasing Power Parity and the Uncovered Interest Parity for UK (No. 90-05). University of Copenhagen, København. https://econpapers.repec.org/paper/kudkuiedp/9005.htm

[4] Pedroni, P. (1999) Critical Values for Cointegration Tests in Heterogeneous Panels with Multiple Regressors. *Oxford Bulletin of Economics and Statistics*, **61**, 653-670. https://doi.org/10.1111/1468-0084.61.s1.14

[5] Granger, C.W.J. (1986) Developments in the Study of Cointegrated Economic Variables. *Oxford Bulletin of Economics and Statistics*, **48**, 213-228. https://doi.org/10.1111/j.1468-0084.1986.mp48003002.x

[6] Dickey, D.A. and Fuller, W.A. (1979) Distribution of the Estimators for Autoregressive Time Series with a Unit Root. *Journal of the American Statistical Association*, **74**, 427-431. https://doi.org/10.1080/01621459.1979.10482531

[7] Phillips, P. C., & Perron, P. (1988) Testing for a Unit Root in Time Series Regression. *Biometrika*, **75**, 335-346. https://doi.org/10.1093/biomet/75.2.335

[8] Dickey, D.A. and Fuller, W.A. (1981) Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root. *Econometrica*: *Journal of the Econometric Society*, **49**, 1057-1072. https://doi.org/10.2307/1912517

[9] Granger, C.W. (1988) Some Recent Development in a Concept of Causality. *Journal of Econometrics*, **39**, 199-211. https://doi.org/10.1016/0304-4076(88)90045-0

[10] Nelson, C.R. and Plosser, C.R. (1982) Trends and Random Walks in Macroeconmic Time Series: Some Evidence and Implications. *Journal of Monetary Economics*, **10**, 139-162. https://doi.org/10.1016/0304-3932(82)90012-5

[11] Obayelu, A.E. and Salau, A.S. (2010) Agricultural Response to Prices and Exchange Rate in Nigeria: Application of Co-Integration and Vector Error Correction Model (VECM). *Journal of Agricultural Sciences*, **1**, 73-81. https://doi.org/10.1080/09766898.2010.11884656

[12] Mandal, S.K. and Madheswaran, S. (2010) Causality between Energy Consumption and Output Growth in the Indian Cement Industry: An Application of the Panel Vector Error Correction Model (VECM). *Energy Policy*, **38**, 6560-6565. https://doi.org/10.1016/j.enpol.2010.07.042

[13] Kularatne, C. (2002) An Examination of the Impact of Financial Deepening on Long-Run Economic Growth: An Application of a VECM Structure to a Middle-Income Country Context. *South African Journal of Economics*, **70**, 300-319. https://doi.org/10.1111/j.1813-6982.2002.tb01185.x

[14] Safdar, L. (2014) Financial Deepening and Economic Growth in Pakistan: An Application of Cointegration and VECM Approach. *Interdisciplinary Journal of Contemporary Research in Business*, **5**, 368-382. https://journal-archieves36.webs.com/368-384apr14.pdf

[15] Kuha, J. (2004) AIC and BIC: Comparisons of Assumptions and Performance. *Sociological Methods & Research*, **33**, 188-229. https://doi.org/10.1177/0049124103262065

[16] Chakrabarti, A. and Ghosh, J.K. (2011) AIC, BIC and Recent Advances in Model Selection. In: Bandyopadhyay, P.S. and Forster, M.R., Eds., *Philosophy of Statistics* (*Handbook of the Philosophy of Science* 7), North-Holland Publishing Company, Amsterdam, 583-605. https://doi.org/10.1016/B978-0-444-51862-0.50018-6

[17] Ivanov, V. and Kilian, L. (2005) A Practitioner's Guide to Lag Order Selection for VAR Impulse Response Analysis. *Studies in Nonlinear Dynamics & Econometrics*, **9**, Article No. 1219. https://doi.org/10.2202/1558-3708.1219

[18] Warsono, W., Russel, E., Wamiliana, W., Widiarti, W. and Usman, M. (2019) Modeling and Forecasting by the Vector Autoregressive Moving Average Model for Export of Coal and Oil Data (Case Study from Indonesia over the Years 2002-2017). *International Journal of Energy Economics and Policy*, **9**, 240-247. https://doi.org/10.32479/ijeep.7605

[19] Couharde, C., Delatte, A.L., Grekou, C., Mignon, V. and Morvillier, F. (2018) EQCHANGE: A World Database on Actual and Equilibrium Effective Exchange Rates. *International Economics*, **156**, 206-230. https://doi.org/10.1016/j.inteco.2018.03.004

[20] Klau, M. and Fung, S.S. (2006) The New BIS Effective Exchange Rate Indices. *BIS Quarterly Review*, March 2006, 51-65. https://ssrn.com/abstract=1632420

[21] Camarero, M. and Tamarit, C. (1996) Cointegration and the PPP and the UIP Hypotheses: An Application to the Spanish Integration in the EC. *Open Economies Review*, **7**, 61-76. https://doi.org/10.1007/BF01886129

[22] Liu, T.Y. and Lee, C.C. (2022) Exchange Rate Fluctuations and Interest Rate Policy. *International Journal of Finance & Economics*, **27**, 3531-3549. https://doi.org/10.1002/ijfe.2336

[23] Crowder, W.J. and Hoffman, D.L. (1996) The Long-Run Relationship between No-

minal Interest Rates and Inflation: The Fisher Equation Revisited. *Journal of Money, Credit and Banking*, **28**, 102-118. https://doi.org/10.2307/2077969

[24] Muscatelli, V.A. and Spinelli, F. (2000) Fisher, Barro, and the Italian Interest Rate, 1845-93. *Journal of Policy Modeling*, **22**, 149-169. https://doi.org/10.1016/S0161-8938(99)00010-1

[25] Atkins, F.J. (1989) Co-Integration, Error Correction and the Fisher Effect. *Applied Economics*, **21**, 1611-1620. https://doi.org/10.1080/758531695

[26] Keating, J.W. (1996) Structural Information in Recursive VAR Orderings. *Journal of Economic Dynamics and Control*, **20**, 1557-1580. https://doi.org/10.1016/0165-1889(96)00914-1

[27] Dai, M., and Guo, W. (2004) Multivariate Spectral Analysis Using Cholesky Decomposition. *Biometrika*, **91**, 629-643. https://doi.org/10.1093/biomet/91.3.629