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The Geometric Model of Particles (The Origin of Mass and the Electron Spin)

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Abstract

The geometrization process of physics could involve, in addition to space and time in General Relativity (GR), even elementary particles. Our starting point is the formulation of an original hypothesis about particles, compatible with the basic assumptions of the Standard Model (SM): a massive particle is a geometric structure of a set of elastically coupled quantum oscillators that propagates along a line of a non-massive base field (in impulse eigenstate). We show that the propagation equation of an oscillation associated with the geometric shape representing an electron propagates following Dirac's wave equation. Thus, one gives a foundation to a geometric model of massive particles (GMP) which would explain the physical origin of the mass, spin, and the magnetic moment of the electron.

Keywords

Mass, Coupling, IQuO, Sub-Oscillator, Semi-Quantum, Spin, Moment, Electron

1. Introduction

This article shows a new way of describing elementary particles and some of their fundamental properties, which is by no means an alternative to that of the Standard Model (SM) but rather complementary to it and, in some respects, completes it. In the current physical research, some pressing topics are addressed, such as the nature of dark matter and dark energy, the origin of the matter-antimatter asymmetry in cosmology, while in particle phenomenology, we have the internal structure of nucleons, the intrinsic meaning of quarks and gluons, etc. All this shows us that the standard model is still incomplete. To solve this problem of incompleteness, the current research merely proposes the existence of new particles and forces. Instead of looking at the "outside" of a particle, studying its dual behaviours, and looking for new types of particle fields, we look inside it, to try to glimpse properties that then explain its "external" behaviour and the relative problems that follow. This internal look leads us to understand fundamental issues of physics better, and more deeply, such as the concepts of mass and energy, electric charge, color charge, and spin, associated with the Fermion and Boson aspects of a particle.

The starting point of the new investigation is the concept of mass, a fundamental characteristic of some particles, see sect. 2.1. This concept is reformulated, considering the formal analogy between a scalar wave equation describing a massive particle, eq. of Klein-Gordon, and the wave equation describing a lattice of pendulums connected by springs. If we introduce the idea of an additional and transversal coupling (as in pendulums) to that present between the oscillators of a non-massive scalar field, Ξ, we realize a lattice-field with mass Ξ, satisfying eq. by K-G. In this way, the mass turns out to be the expression of the presence of an additional or "massive" coupling. Defining the origin of mass allows us to validate the principle of equivalence between inertial mass and gravitational mass, sect. 2.2, and to deny the existence of particles with negative mass. The massive coupling also introduces the concept of lattice-field and consequently the idea of a massive particle as a geometric structure of field oscillator couplings, see sect. 2.3, in analogy with the system of pendulums connected by springs. Oscillators that build a structure are called **IQuO** (an acronym for **I**ntrinsic **Qu**antum **O**scillator) due to their aggregation properties and because they are a redefinition of the quantum oscillator. In this, we realize the intent of looking inside a particle, seen as a "field" of coupled quantum oscillators. In sect. 2.4 we state the hypothesis of structure which is at the basis of the geometric model of the particles. Thus, we formulate the two possibilities of aggregation of quantum oscillators that give rise to massive particles: the leptons and quarks. The first emergent important consequence of the hypothesis of structure, see sect. 2.5, is given by the fact that Dirac's equation of a particle coincides with the equation describing a structure of couplings that is in rotation around its axis of propagation: this reveals the arduous concept of spin, sect. 3.1, thus giving it a physical dimension and not just math. In sect. 3.2, we show that the magnetic moment, associated with a geometric structure, results in a value very close to that experimentally detected and predicted by the QM for the electron. The second emergent aspect, see sect. 4.1, is given by the fact that for an IQuO to aggregate, it must be made up of sub-oscillators that contain semi-quanta, even if we reiterate that the energy exchange between different particles always takes place by "whole" quanta. The new notation at semi-quanta and sub-oscillators introduces a new paradigm in Quantum Mechanics because it allows us to explain the origin of electric charge and color charge, as well as the same geometric structures of massive particles. This introduces a new idea of physics geometrization, following that of space-time in general relativity: the geometrization of physical particles. In fact, in the sect. 4.2, we so show the triangular geometric structure of the electron and the internal property, sect. 4.3 of the flow of semi-quanta inside the structure.

2. Hypothesis of Structure

2.1. The Massive Coupling

In relativity, a massive particle, with mass m, is an "object" to which one can associate a reference frame S° where the object is at rest; in S°, we assign a "proper time" τ, or a "clock". In the 4-dim Space-Time (x, ict) of the S°, we could speak of an "imaginary" velocity ic in time, so as, in the "dual" space Impulse-Energy (p_x, imc) where ($p_4 = imc = (ic)m$), we could speak of energy at rest E_0, with ($E_0 = -ip_4c$), as "a movement energy in time". This "uniform motion" in time recalls once again the "clock": then, we can state that exists a "τ—*proper* time" inside every massive particle originated by a periodic "motion" with proper frequency ω_0: [$\omega_0 \Leftrightarrow \tau$]. To this periodic motion, we could associate the "energy of movement in time" or energy at rest; it follows $E_0 = \eta\omega_0$. So, we will have, from the Quantum Mechanics (QM) [1]:

$$\begin{Bmatrix} E_0 = mc^2 \\ E_0 = \hbar\omega_0 \end{Bmatrix} \Rightarrow \left\{ m = \frac{\hbar\omega_0}{c^2} \right\} \tag{1}$$

By the Equation (1) we can assert that the mass m is the physical expression of a proper frequency ω_0 inherent in a particle. If the frequency ω_0 generates the proper time τ of a massive particle ($\tau = \eta/mc^2$), then for symmetry, in the undulatory mechanics exists a wavelength $\lambda\!\!\!^{-}_0$ that originates the "proper space" of the particle. Following De Broglie, we have [1]:

$$\begin{cases} p_0 = mc \\ p_0 = \hbar\dfrac{2\pi}{\lambda_0} \Rightarrow \lambda\!\!\!^{-}_0 = \dfrac{\hbar}{mc} \equiv \lambda\!\!\!^{-}_c \end{cases} \tag{2}$$

where $\lambda\!\!\!^{-}_c$ is the Compton wavelength. Now, combining the equation of the relativistic energy with the equations of De Broglie and Einstein, we have:

$$\left\{ E^2 = m^2c^4 + p^2c^2 \quad \Leftrightarrow \quad \omega^2 = \omega_0^2 + k^2c^2 \right\} \tag{3}$$

The second equation is the dispersion relationship of waves, as described by the Klein-Gordon (K-G) equation [2]:

$$\left\{ \frac{\partial^2\Psi(x,t)}{\partial t^2} = c^2\frac{\partial^2\Psi(x,t)}{\partial x^2} - \omega_0^2\Psi(x,t) \right\} \tag{4}$$

As is well known, this equation describes scalar fields associated with massive particles with zero spin:

$$\left\{ \frac{\partial^2\Psi(x,t)}{\partial x^2} - \frac{\partial^2\Psi(x,t)}{c^2\partial t^2} = \left(\frac{mc}{\hbar}\right)^2 \Psi(x,t) \right\} \Leftrightarrow \left\{ \nabla^2\Psi(x,t) = \left(\frac{mc}{\hbar}\right)^2 \Psi(x,t) \right\} \tag{5}$$

However, this equation also describes oscillations in a set of "pendulums" coupled through springs [2]. In analogy with this system, we can connect the ω_0 of a particle to a particular elastic coupling, which is in addition to the one already existing between the oscillators of a massless scalar field (Ξ), as the springs. This "additional coupling", which gives the origin to mass in a scalar field (Ξ), is referred to as a "**massive coupling**" [3]. So, we conjectured that the field of the massive particle (Ψ) has its origin in a "transversal coupling" (T_0), see the pendulums, between the chains of oscillators of the scalar base field (Ξ). All that can be represented in way figurative in **Figure 1**.

Therefore, a massive particle can be represented by a "lattice" field with a transversal coupling on a basic Ξ-scalar field. This coupling can so originate a "structure" of "coupled oscillators", see **Figure 1**, on the base field Ξ. In this way, we introduce a "**new paradigm**" in physical field theory: the m mass as oscillation frequency ω_0 of a structure of coupled quantum oscillators. This structure can also have a geometric aspect, whether it be an extended lattice of field oscillators or a well-defined group of oscillators. The model that will be developed considering this "geometric" aspect of a particle will be referred to as the "**Geometric Model of Particles**", with the acronym (**GMP**). When we observe only the oscillation with frequency ω_0 in all points x, then we are at rest with the massive particle ($m \Leftrightarrow \omega_0 \Leftrightarrow T_0$) and this aspect is coincident with that in which the "springs" do not are involved. Instead, when the springs are involved, the wave becomes progressive with frequency ω and wavelength λ and represents a massive particle with velocity v, see Equation (3). Recalling the (£) Lagrange density [4] of the Klein-Gordon equation

$$£ = \left[\partial_\mu \Psi(x_i)\right]^2 + m^2 \Psi^2(x_i) \tag{6}$$

We can assert that the term of mass ($m\Psi^2$) is an additional component to the basic field Ξ, which makes pass it from a descriptive Ξ-function to the Ψ-function. Note that in the Lagrange theory of fields, the Ξ-field is "hidden": therefore, the Lagrange theory does not consider a possible internal structure of an elementary massive particle, seeing instead this as a point particle.

2.2. Theinertial Mass m_i and Gravitational m_g

We see, in **Figure 1**, that the Ξ-field has a lattice structure: it is thus possible that two field lines Ξ couple transversely with elastic tension T_0. We so speak about "junction" oscillators which operate between field oscillators. Then, we wonder

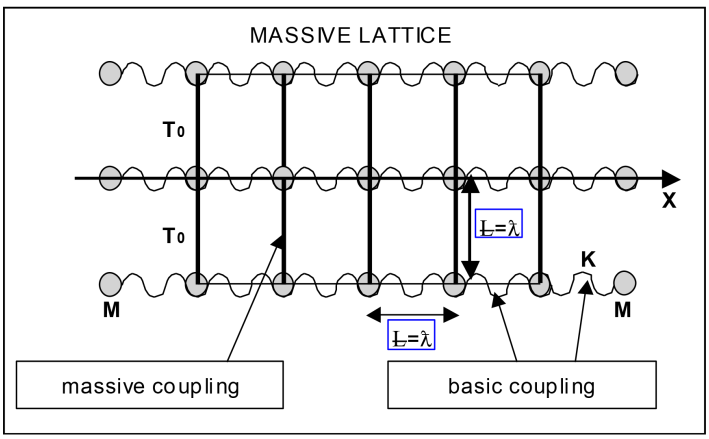

Figure 1. The massive scalar field as a lattice of "pendulums" with springs.

what causes the transversal coupling T_0. We could think about the intersection between two lattice fields Ξ_i, $(\Xi_1 \underline{\otimes} \Xi_2)$, where the symbol $\underline{\otimes}$ represents, in purely formal terms, the "transversal" intersection of the two fields which realizes the additional coupling with tension T_0. We can also suppose a simultaneous intersection of several lattice-fields Ξ_i with angles different from zero, thus simultaneously determining several transversal couplings between two lines of a fixed field $(\Xi_i)^\circ$. This intersection could originate a "**closed structure**" of field oscillator couplings $(\Xi_i)^\circ$. But as soon as a closed coupling structure is formed (triangular in shape and due to "phase agreement" between the oscillators, now internal of the new structure) it happens that the latter detaches (due to phase shift with the "external" oscillators of the field $(\Xi_i)^\circ$) from one of the two lines (sides). It follows that, like any field oscillation, the particle with its structure propagates along a field line $(\Xi_i)^\circ$ which we could define as a "guideline". In our opinion, the signal of this detachment and formation of a closed structure, in the "universe" set of fields Ξ_i is a "gravitational" field Γ. In the latter case, the mass m assigned to the closed structure would become a "gravitational" mass m_g, the source of a gravitational field [5]: ($T_0 \Leftrightarrow \omega_0 \Leftrightarrow m_g \Leftrightarrow \Gamma$). We can thus assert that a massive field is like a field at "double" coupling: two contiguous chains of Ξ-field oscillators are elastically connected by additional coupling with tension T_0. One more observation we must add. Having defined that ω_0, originating from T_0, is part of relation (3), where it appears, see the literature, the "inertial" mass m_i, it follows that ($\omega_0 \Leftrightarrow m_i$); in this case, we would have that ($m_i \Leftrightarrow m_g$): this recalls the Equivalence Principle [5] in General Relativity (GR). Therefore, in GMP the inertial mass and gravitational are "coincident": $\boldsymbol{m_i = m_g}$. This aspect is in full compatibility with the GR; nay, it fulfils the wish expressed by Einstein, see ref. [5]: "*It is evident, however, that science will find a plausible justification for the numerical equality* ($\boldsymbol{m_i = m_g}$) *only when this can be brought back to an equality of the real nature of the two concepts*". There is another consequence of the two relations [($\omega_0 \Leftrightarrow m_i$), ($\omega_0 \Leftrightarrow m_g$)]: since there are no "negative" frequencies (see the definition of angular frequency ω_0) there are no negative values of gravitational mass and inertial mass. The mass in GMP is always positive. This would determine that the correct way to explain the acceleration of the universe expansion is not that of setting the existence of an "antigravity" which would have an origin in inertial (and gravitational) masses at negative values ($m < 0$).

2.3. The Intrinsic Quantum Oscillator of Structure (IQuO)

We remember, see the Equation (6), that in fields theory [6] is:

$$L = -\frac{c^2}{2}\int\left[\left(\frac{\partial \Psi^2}{\partial x_\mu}\right) + m^2\Psi^2\right]\mathrm{d}^3x = \int £\mathrm{d}^3x \Rightarrow L = \frac{1}{2}\sum_k\left(\dot{q}_k\dot{q}_{-k} - \omega^2 q_k q_{-k}\right) \quad (7)$$

where q_k is the generalized coordinate of an oscillation way associated with the wave, with ($\lambda \Leftrightarrow k$), propagating along a line of field oscillators (Ξ_i) coupled elastically, but with an additional coupling T_0, each placed in a point (x_i) of space (the reference frame of laboratory S_{lab}), see **Figure 1**. In this way, we associate to a point (x_i) in S_{lab} an oscillator $[\Psi(x_i)]\Xi_i$. In the "ordinary" field-particle theory, the background Ξ-field would be so "hidden", while the mass m, given by the transversal additional coupling on Ξ-field, would "transform" the latter into the field Ψ. Recall that the quantum field Ψ-field is a set of coupled quantum oscillators expressed by operators (a, a^+) [7] [8]:

$$\begin{gathered}\Psi(x_i)=\left(\frac{1}{\sqrt{V}}\right)\sum_k q_k(t)\mathrm{e}^{i(\vec{k}\cdot\vec{x})}=\left(\frac{1}{\sqrt{V}}\right)\sum_k\left[\left(\sqrt{\frac{\hbar}{2\omega_k}}\right)\left(a_k+a_{-k}^+\right)\right]\mathrm{e}^{i(\vec{k}\cdot\vec{x})}\\ \varepsilon_{(n_1,n_2,\cdots,n_k)}=\sum_k\left[\hbar\omega_k\left(n_k+\frac{1}{2}\right)\right],\quad P_{(n_1,n_2,\cdots,n_k)}=\sum_k\left[\hbar k_k\left(n_k\right)\right]\end{gathered}\tag{8}$$

where the Ψ-Field becomes a field operator to which we associate, in each oscillation way (k_i), a particle (quanta) of impulse ($k_i \Leftrightarrow p_i$), through the creation operator a^+ and annihilation a. Besides, it is:

$$\left[q_k(t)\right]_{op}=\left(\sqrt{\frac{\hbar}{2\omega_k}}\right)\left[a_k(t)+a_{-k}^+(t)\right]=\left(\sqrt{\frac{\hbar}{2\omega_k}}\right)\left[a_{0k}+a_{0(-k)}^+\right]\mathrm{e}^{i(\omega_k t)}\tag{9}$$

If now we introduce a particle with an "internal structure" of coupled oscillators, the operators (a, a^+) need to express this aspect. The unique possibility, in this case, is to insert the information about the internal structure in the components not dependent on the time (a_0, a_0^+) of operators (a, a^+); it follows the operators (a_0, a_0^+) will be expressed by matrices (A_0, A_0^+) with elements describing the structure of coupled quantum oscillators:

$$\left(\hat{A}_{0k}+\hat{A}_{0(-k)}^+\right)=\left[\begin{pmatrix}\hat{I}_{11} & \cdots & \hat{I}_{1n}\\ \vdots & \ddots & \vdots\\ \hat{I}_{1m} & \cdots & \hat{I}_{nm}\end{pmatrix}\otimes\left(a_{0k}+a_{0(-k)}^+\right)\right]\tag{10a}$$

The term I_{ij} is a "projector" operator [9] on the spatial axes (X, Y, Z) as well as a projector on the time axis (t). We will thus have that, for example, $I_{ij}a_0 = I_X a_0 = (a_0)_x$, where $(a_0)_x$ is the action of a_0 on the X-axis. In n-dim spaces, we will have I_{nm}. The symbol ⊗ formally indicates, for the moment, the operation of combining the elements of the set $\{I\}$ with the set $\{a, a^+\}$, so as to highlight the particular geometric structure assigned to a massive particle. Explicitly we will have:

$$\begin{aligned}\left(\hat{A}_{0k}+\hat{A}_{0(-k)}^+\right)\equiv\begin{pmatrix}\hat{A}_{0k}\\ \hat{A}_{0(-k)}^+\end{pmatrix}&=\left[\begin{pmatrix}\hat{I}_{11}a_0\\ \vdots\\ \hat{I}_{1n}a_0\end{pmatrix}\otimes\cdots\otimes\begin{pmatrix}\hat{I}_{1m}a_0\\ \vdots\\ \hat{I}_{nn}a_0\end{pmatrix}\right]_k\otimes\left[\begin{pmatrix}\hat{I}_{11}a_0^+\\ \vdots\\ \hat{I}_{1n}a_0^+\end{pmatrix}\otimes\cdots\otimes\begin{pmatrix}\hat{I}_{1m}a_0^+\\ \vdots\\ \hat{I}_{nn}a_0^+\end{pmatrix}\right]_{-k}\\ &=\begin{bmatrix}\begin{pmatrix}(a_0)_{11}\\ \vdots\\ (a_0)_{1n}\end{pmatrix}\otimes\cdots\otimes\begin{pmatrix}(a_0)_{1m}\\ \vdots\\ (a_0)_{nm}\end{pmatrix}\\ \begin{pmatrix}(a_0^+)_{11}\\ \vdots\\ (a_0^+)_{1n}\end{pmatrix}\otimes\cdots\otimes\begin{pmatrix}(a_0^+)_{1m}\\ \vdots\\ (a_0^+)_{nm}\end{pmatrix}\end{bmatrix}\end{aligned}\tag{10b}$$

The overall shape of the matrix $\hat{A}$, with $\{\hat{A} = [I \otimes (a, a^{+})]\}$, describes to the geometric structure of couplings between quantum oscillators. This quantum oscillator of the closed structure is called "**IQuO**", an acronym for "**I**ntrinsic **Qu**antum **O**scillator". We establish the matrix $\hat{A}$ is composed of IQuO associated with an additional or massive coupling which builds the internal structure of a massive particle; thus, we will have, see the Equation (9):

$$\left[q_k(t)\right]_{op} = \left(\sqrt{\frac{\hbar}{2\omega_k}}\right)\left[a_k(t) + a_{-k}^{+}(t)\right] \Rightarrow \left[\hat{q}_k(t)\right]_{op} = \left(\sqrt{\frac{\hbar}{2\omega_k}}\right)\left[\hat{A}_{0k} + \hat{A}_{0(-k)}^{+}\right]\mathrm{e}^{i(\omega_k t)} \quad (11)$$

with $\hat{A}$ ($\hat{I}$; (a, a^{+})). It is evident that the matrix ($\hat{A}$), the new "internal degree of freedom" of the particle, is not involved in the "external" interactions and, thus, in the calculation of the scattering matrix T_{if} (which describes the external behavior of particles). Later, in one of our publication proposals, currently being submitted, we will give mathematical details on the elements I_{ij} and matrices Â. It should be noted that now we are not yet provided with the mathematics that can describe these new physical elements expressed in the hypothesis of structure; therefore, in this article, we are forced to proceed in an intuitive way and formal, but ever with "rigor of logic", waiting for new mathematical developments.

2.4. The Hypothesis of Geometric Structure

Looking at Figure 1, we can think that a field-particle is represented by a lattice of field oscillators with an additional or massive coupling that gives to the lattice a "geometric" form. Note that it is easy to describe a scalar particle by means of a field-lattice of coupled oscillators; if, on the other hand, we begin to consider vector fields and spinor, that is with spin, then it becomes difficult to represent the relative particles by a lattice. However, having argued that a massive particle is a geometric structure of coupled quantum oscillators, we must still look for its "geometric" representation, whatever the field associated with it. We facilitate the attempt by considering a particle in an eigenstate of impulse and energy. The local aspect of the lattice-particle would be given by its quantum energy and momentum associated with the propagation of the oscillation (wave) with additional coupling along a line of the base field. At this point, we have two descriptive possibilities of a "structure" particle:

1) to conjecture that a massive particle (m) can be represented by a geometric structure of a finite number (group) of elastically coupled quantum oscillators, which can propagate along a line of a field as if this were some sort of a "guide-rail".

2) see a base field with an additional coupling geometrically structured at lattice form, where along a field line a vibration quantum would propagate, see Figure 1.

In the first case, wave mechanics associates with an internal oscillation frequency ω_0, that is the mass (m), as well as an "external" frequency relating to the wave ω, associated with it, see Equation (3), (remember that the propagation of an oscillation is a wave). In the second case, the quantum can be described by the creation operators and annihilation (a, a^+). We immediately observe that there is a descriptive equivalence between the two cases since they respectively represent the "first quantization" and the "second" of the QM [9]. In this way, the description that we will make of geometric structures representing massive particles is perfectly equivalent to the dualistic representation associated with particles. Now, we should see how an (additional) coupling with geometrically shaped structure of quantum oscillators (IQuO) can be made. Let us consider a triangle structure of three Vertex-IQuO (I_A, I_B, I_C) with three junctions IQuO (I_{AB}, I_{BC}, I_{CA}) that couple the vertices, see **Figure 2**.

Recall a quantum oscillator has a length L_n, in its various energy eigenstates: [$L_n = (l_0/2)(2n + 1)^{1/2}$]. For $n = 0$ one has [$L_0 = (l_0/2)$] while for $n = 1$ one has [$L_1 = (l_0/2)(3)^{1/2}$]. The triangle in **Figure 2(a)** can be made according to the scheme:

$$\left\{\left[L_1\left(I_{A,B,C}\right)=\left(l_0/2\right)\sqrt{3}\right],\left[L_1\left(I_{AB,BC,CA}\right)=\left(l_0/2\right)\sqrt{3}\right]\right\} \tag{12}$$

We ask ourselves how a vertex oscillator physically binds to a junction oscillator to build a reciprocal coupling. QM tells us nothing about how a quantum oscillator is constituted "inside" when it is in one of its energy eigenstates E_n, nor how two oscillators "couple" [10]. In fact, the theory of interactions describes an interaction or a bond between the interacting particles (seen as "point" objects) in an "external" way, through an intermediary particle that plays the role of exchanging an energy quantum [8] [9]. It is clear, however, that the intermediary particle must "couple" elastically with the representative oscillators of the interacting particle fields. Thus, in the interactions, we can consider that these are always realized through couplings of oscillators representative of the interacting fields. However, we can attempt to understand how two oscillators bind together even if we do not know how a quantum oscillator is constituted "inside" it. It is enough to think that two quantum oscillators can be considered coupled when they exchange their respective energy quanta. During this exchange, we could even think of a sharing of the respective quanta (or a part of its quanta). All this implies the existence of an overlapping area of the two oscillators. This leads us to think that a quantum oscillator can be made of "parts" and that between these parts we can identify a "peripheral zone". We find this aspect in beats of two coupled pendulums by an intermediary spring: between them, the energy exchange happens when one of the two pendulums is next to the equilibrium point of another pendulum. We need therefore think that two oscillators, the vertex,

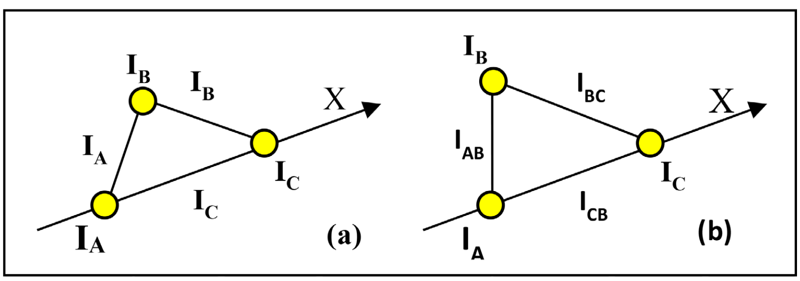

Figure 2. The triangles-particles.

and the junction one, overlap each other in a peripheral part of them: only in this way can the reciprocal sharing of quanta be achieved through an exchange. It should be noted that the partial overlapping of the oscillators with quantum-energy exchange creates a "zone" with "double" coupling. This double coupling reminds us of the additional coupling (the massive one) and therefore we could assume that a structure of coupled oscillators would describe a massive particle only if all the coupled oscillators are "doubly" coupled. The last difficulty to be overcome in building the coupling between two oscillators is to connect an IQuO-vertex to the two lateral IQuO of junction, placed in two different directions, see **Figure 2**. In these cases, the IQuO-vertexes are "bent" along their oscillation amplitude and simultaneously are bound to two different oscillators, which oscillate along two different sides. This aspect, in a classic oscillator, does not raise problems: the classic oscillator can very well be decomposed into two oscillating parts or sub-oscillators and therefore bendable in their joint [9]. We point out that in Quantum Mechanics (QM) an oscillator should be a non-deformable and inseparable whole. However, to physically admit a coupled IQuO structure we must think that the IQuO-vertexes are "folded" in two break-up parts, like a classical oscillator "hinged" in a point. This means that even a vertex IQuO must be made up of two oscillating parts, that is, two "**sub-oscillators**" as in the classical case. For continuity, this last characteristic must be possessed by any IQuO: also, the junction IQuOs. The structures of coupled IQuOs thus lead us to admit that any quantum oscillator could be decomposed into several oscillating or sub-oscillators. This would constitute a novelty in the panorama of the foundations of quantum mechanics: the quantum oscillator could be constituted by a set of coupled sub-oscillators [3] [11] [12]. The same needs to happen in the triangle-particle of **Figure 2(b)**. Observe that the triangle in **Figure 2(a)** is an isosceles rectangle triangle while in **Figure 2(b)** it is only an isosceles triangle. This last could be given by the relations:

$$\left\{\left[L\left(I_{A,B,C}\right)=\left(l_0/2\right)\sqrt{3}\right],\left[L\left(I_{AB,BC,CA}\right)=\left(l_0/2\right)\sqrt{5}\right]\right\} \tag{13}$$

We refer to a later work the insights on this new possibility, Equation (13), of representing a structure of coupled oscillators. However, in precedent publications [13] [14] [15] [16] we assigned to d-quark the structure in **Figure 2(b)**.

2.5. Wave Equation of a Massive Structure of Coupled Field Oscillators

Remember, in this work, the mass has the origin from the presence of a "transverse" coupling between two or more chains of field oscillators [$(m = (h/c^2)\omega_0$] which realizes a structure to geometric form. Note that the well-known Higgs boson, which gives mass to the leptons in the Higgs-Weinberg mechanism (H-W), also has mass [4]. This means that the Higgs boson cannot explain the origin of the mass in particles. Therefore, we can intuit that the Higgs field present in the H-W Mechanism provides mass to the leptons of the weak interactions, thanks ever to its additional or massive transverse coupling present between its field oscillators. Let us go back to the K-G equation in the relativistic form:

$$\left(\frac{\hbar^2}{c^2}\right)\frac{\partial^2\Psi(x_i)}{\partial t^2}=\left[\hbar^2\nabla^2-m^2c^2\right]\Psi(x_i) \tag{14}$$

And in covariant form:

$$\left[\tilde{p}_i^2+m^2c^2\right]\Psi(x_i)=0 \tag{15}$$

where $\left\{\left[x_i\equiv(\vec{x},ict),\ p_i\equiv(\vec{p},iE/c)\right],\left[\tilde{p}_i\equiv\left(-i\hbar\frac{\partial}{\partial\vec{x}}\right),\left(i\hbar\frac{\partial}{\partial ct}\right)\right]\right\}$

We also remember that if $E\equiv H$, with H the Hamiltonian, we have by Equation (14):

$$\left\{\left[\tilde{H}_t\Psi=\hat{H}^2\Psi\right],\left[\left(\tilde{H}_t=i\hbar\frac{\partial}{\partial t}\right),\left(\hat{H}^2=\tilde{p}^2c^2+m^2c^4\right)\right]\right\} \tag{16}$$

As we all know, quadratic equations (Equation (16)) admit plane wave solutions with negative frequencies or negative energies. This led Dirac to look for a relativistic equation with "negative" solutions (particles) without negative energies [8] [17]. If this aspect is not properly addressed, one risks setting field theories by considering negative energy values and mass, as it happens in the antigravity hypothesis, see the sect. 2.2. In the QM theory, see the eq. 16, the presence of negative energies (frequencies), splits the solution Ψ of eq. of K-G in two components [18]: Ψ ≡ (φ, χ). This implies that the matrices operating on the wave function Ψ are all 2 × 2 matrices that can be expressed as combinations of the Pauli matrices:

$$\hat{\Psi}=\begin{pmatrix}\varphi\\ \chi\end{pmatrix},\ \sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\ \sigma_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\ \sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix},\ \sigma_0=\begin{pmatrix}1&0\\0&1\end{pmatrix} \tag{17}$$

It is shown, see Davidov in the ref. [18] [19], then that Equation (14) can be written as:

$$\left\{\begin{aligned}i\hbar\frac{\partial\phi}{\partial t}&=-\left(\frac{\hbar^2}{2m}\right)\nabla^2(\varphi+\chi)+mc^2\varphi\\ i\hbar\frac{\partial\chi}{\partial t}&=\left(\frac{\hbar^2}{2m}\right)\nabla^2(\varphi+\chi)-mc^2\varphi\end{aligned}\right\}\Leftrightarrow\left\{\left(i\hbar\frac{\partial}{\partial t}-H_f\right)\hat{\Psi}=0\right\} \tag{18}$$

These two equations are not separated in the components (φ, χ): we can think of introducing the Pauli matrices, Equation (17), to resume the treatment in an only equation. By making H_f explicit, Equation (16), we obtain:

$$\begin{gathered}\left\{\begin{aligned}&\left(i\hbar\frac{\partial}{\partial t}-H_f\right)\hat{\Psi}=0\\ &H_f=(\sigma_3+i\sigma_2)\left(\frac{\tilde{p}^2}{2m}\right)+mc^2\sigma_3\end{aligned}\right\}\\ \Leftrightarrow\left\{\left(i\hbar\frac{\partial}{\partial t}-(\sigma_3+i\sigma_2)\left(\frac{\tilde{p}^2}{2m}\right)+mc^2\sigma_3\right)\hat{\Psi}=0\right\}\end{gathered} \tag{19}$$

Note the matrix σ_3 with eigenvalues (±1); it follows $\sigma_3(mc^2) \to \pm mc^2$. Let us rewrite this Equation (19) associating to the differential operator H_t the identity matrix σ_0, see the Equation (16):

$$\left\{\left(\sigma_0\left(i\hbar\frac{\partial}{\partial t}\right)-\left(\sigma_3+i\sigma_2\right)\left(\frac{\tilde{p}^2}{2m}\right)+mc^2\sigma_3\right)\hat{\Psi}=0\right\} \tag{20}$$

We call this equation the "dual equation of K-G". If matrix σ_3 was assigned to time it would give us a time ± t with the negative energy solutions going backward in time. We can also write the Equation (20) in the following way:

$$\begin{gathered}\left\{\left(\sigma_0\tilde{H}\right)\hat{\Psi}=\left[\left(\sigma_t+i\sigma_x\right)\left(\frac{\tilde{p}^2}{2m}\right)+\sigma_t mc^2\right]\hat{\Psi}\right\} \\ \Rightarrow\left\{\left(\sigma_0\tilde{H}\right)\hat{\Psi}=\left[\left(\sigma_{tx}\right)\left(\frac{\tilde{p}^2}{2m}\right)+\sigma_t mc^2\right]\hat{\Psi}\right\}\end{gathered} \tag{21}$$

where $\sigma_3 \equiv \sigma_t$ is the matrix associated with the time-energy component and $\sigma_2 \equiv \sigma_x$ is the matrix associated, see the Equation (20), with the space-impulse component, along the propagation axis X. We can think that σ_{tx}, with [$\sigma_{tx} = (\sigma_t + i\sigma_x)$], is the matrix associated with the Space-Time component (S-T) of the wave propagation in a reference frame S(x, ict)). The Equation (21) is thus the projection of eq. K-G into the S-T. We recall the role of Pauli's matrices as rotation generators [6] [9] in S^2 ($Q_\theta = I + i\sigma_\theta(\theta/2)$), which are in relation to rotations in R^3: ($R_\theta = I + iL_\theta\theta$). In this space S^2, the spinor is given by $\Psi \equiv (\varphi, \chi)$. This tells us that the dual equation of K-G is a "spinor" equation. In this case, the matrix σ_{tx} can be understood as the generator of a Lorentz rotation that describes the propagation of the wave Ψ along the X axis in the Space-Time of the two reference frames given by S_{lab} (x, ict), where the wave propagates (see the matrix σ_x), and S_{wave} (x', ict'), where the wave oscillation is at rest (see the matrix σ_t). This recalls the procedure which draws the form of a wave function describing the propagation of an oscillation along the X-axis [2]. Thus, the matrix σ_{tx} describes the evolution in S-T of spinor with components (φ, χ). However, note that the Equation (20), the same is the Equation (21), is still not symmetric in the temporal and spatial components and is quadratic in the p operator: this prevents it from being linear and suitable for a relativistic description in the S-T, as Dirac notes, see ref. [17]. At this point Dirac imposed the linearity on the dual equation, thus finding the correct relativistic equation that bears his name. However, in this paper, we do not proceed according to the Dirac, but we will follow another path. If we suppose that a massive particle is expressed by a geometric structure of coupled IQuO, then we can think that the plane (χ, ζ) of the structure (see **Figure 3**) can rotate around the X axis in which the oscillation associated with the particle propagates. Affirming this is not so senseless because the "massive" structure is associated with a Compton length that gives a "space" dimension to the particle, thus setting aside the hypothesis of a point-like particle. Here, we must relieve that a structure of coupled oscillators, despite having a

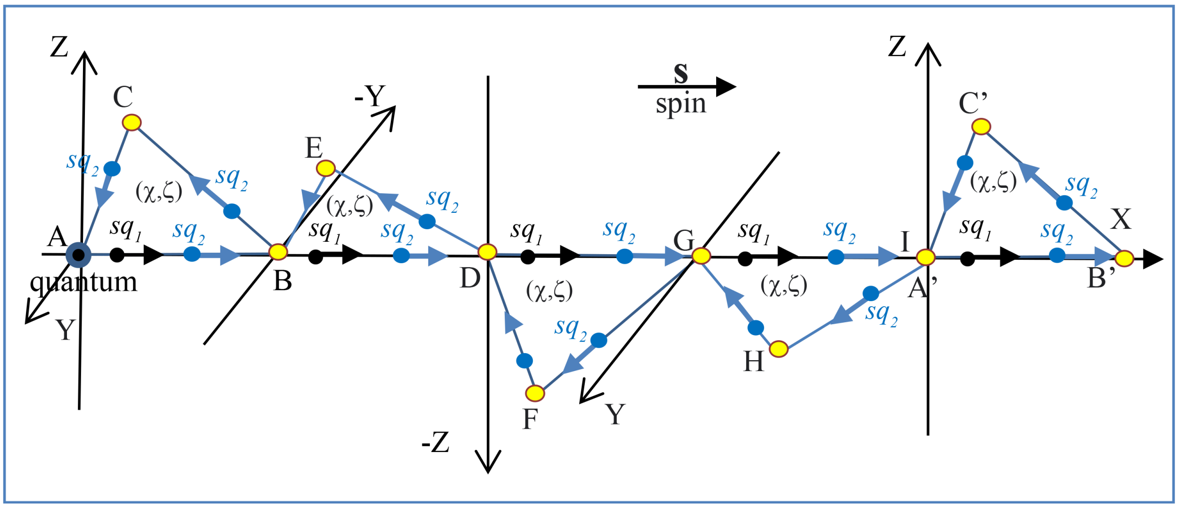

Figure 3. The configurations of triangle-quark in the propagation along X-axis.

time dimension ($m \Leftrightarrow \tau$) and a spatial dimension (λ_c) cannot be considered a particle composed of "bonded" parts or composed of separable oscillating parts: it is an inseparable "unity" that is cannot be separable into more basic subunits. Even if the IQuOs are compositions of sub-oscillators, the coupling between these sub-oscillators realize makes a structure inseparable and unique. Note the structure plain (χ, ζ) can be positioned in any orientation with respect to the propagation axis: this rotational symmetry leads us to associate the particle with a rotation around the propagation axis, that is a "**spin**". Let us then try to describe a wave function of a massive particle with a triangular structure that "rotates" around the propagation axis. Note, the (χ, ζ) plane rotation origins a spin vector **s**. The configuration, along the propagation axis X will be, see Figure 3.

We observe that during the propagation of the geometric structure along the propagation axis X, the internal oscillation associated with the "quantum" $(\bullet)_A$ of particle massive occurs in different planes for the rotation of the structure, with phase period $\tau_0(\tau_0 = 2\pi/\omega_0)$; the succession in time and space of the oscillation planes will be: (ZX, −YX, −ZX, YX, ZX). It is thus evident that we must combine the oscillations along the sides of the triangular structure with the oscillations along the X propagation axis: the oscillation Ψ_Δ which propagates along the triangular structure must be coherent (in phase) with the oscillation Ψ_x propagating along the X-axis. The presence of "two" oscillations in a single physical object implies that the oscillating energy associated with the quantum ($\bullet$) must "split" into the two paths: $\Psi \equiv \Psi_\Delta + \Psi_x$. Obviously, the two states are not quantum separated, see the "quantum entanglement". We thus associate a structure propagation with a quantum ($\bullet$) of oscillation energy that flows along the sides of the structure ($\bullet_2$) and at the same time propagates along the X axis ($\bullet_1$) together with the structure, see Figure 3. Note in Figure 3 that the ($\bullet$) "quantum" is so X is made up of two semi quanta [$sq(\bullet_1)$, $sq(\bullet_2)$]: [$\bullet \equiv \bullet_1 + \bullet_2$]. This is coherent with the hypothesis of the sub-oscillators in an IQuO of the triangular structure. We observe that the "semi-quantum" sq_2 proceeds at the phase velocity $v > c$, while sq_1 proceeds at the velocity $v_x < c$. This is because sq_2 moves along the triangle structure with an elastic Tension T_0 and frequency ω_0 while sq_1 propagates along the guideline X with an elastic tension T_X, ($T_X \neq T_0$), and frequency ω. The relationship between the two frequencies will be given by Equation (3). The path of the representative quantum ($\bullet_2$), along X-axis, is given by:

$$\begin{aligned}&(\mathrm{AB},\mathrm{BC},\mathrm{CA},\mathrm{AB})_{(\mathrm{Z},\mathrm{X})} \to (\mathrm{BD},\mathrm{DE},\mathrm{EB},\mathrm{BD})_{(-\mathrm{Y},\mathrm{X})}\\ &\to (\mathrm{DG},\mathrm{GF},\mathrm{FD},\mathrm{DG})_{(-\mathrm{Z},\mathrm{X})} \to (\mathrm{GI},\mathrm{IH},\mathrm{HG},\mathrm{GI})_{(\mathrm{Y},\mathrm{X})}\end{aligned}$$

After this path, the primary configuration (ABC) resumes in (A'B'C') after an phase period $\underline{T} = 2\underline{T}_0 = 4\pi$. Remember that this phase period of 4π is an oscillating characteristic of "Fermions" [9]. If we (observers in the ZX plane of the Frame Reference S_{lab}) follow (by instrumentation of polarization) the oscillation of the quantum belonging to our ZX-plane, then we would notice that this oscillation disappears when the triangle particle moves on the other -YX plane; the same when it goes from the -ZX plane to the YX plane. The rotation (spin) of the particle-triangle thus appears discontinuous (appears and disappears) for the various observers of the reference planes and the wave function associated with the quantum oscillation (•) along the propagation axis X will have a phase period $\underline{T}_{phase} = 4\pi$. It could be this aspect that determines a "halved" spin because, for an observer, the complete rotation in a phase plane takes place in $T = 4\pi$. The representation of the wave function Ψ will be given in **Figure 4**.

Therefore, the oscillation with phase period $T_{phase} = 4\pi$ will have the angles of the phase rotation "halved" and the operator of this rotation, "spin" $\boldsymbol{s}$, will be expressed by half angles and will have eigenvalues $s' = \pm(1/2)$: the rotation of the structure-particle (plane (χ, ζ)) can thus be expressed in a space $\mathcal{S}^2$. There must then be a matrix R_S that describes the rotation of the oscillation planes (see the (χ, ζ) plain) around the X axis. In this way, we must determine the application of a rotation matrix R_S, at "half-angles", on the (χ, ζ) plain, where there are the internal oscillations associated with the structure particle, described in turn by Pauli matrices σ_s (recall "oscillation ⇔ phase rotation"). As we know, the theory of rotations sets in correspondence rotations in an ordinary R^3 space with rotations

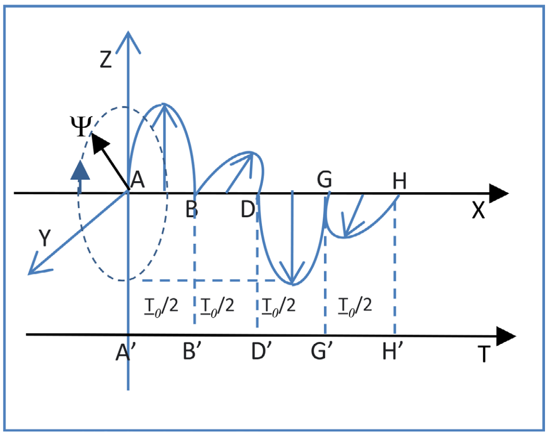

Figure 4. Wave function at variable oscillation planes with phase period $\underline{T} = 4\pi$.

(Q) in S^2, the complex 2-dim space: $R_S = I + iL_S(\theta) \Leftrightarrow Q_S = I + i\sigma_S(\theta/2)$. Recall the column vectors operating in S^2 are spinors, therefore, the wave function "vector" $\mathbf{\Psi}$ in **Figure 4** is a spinor. This means that the generators of the rotations of the structure-particle are Pauli matrices σ_S operating in Q^2. These matrices must be inserted into the dual equation of K-G, which describes the propagation of the oscillation along the X-Axis in S_{lab}. We will have so the combination of two "movements": the propagation of a "rotating" oscillation. Therefore, each solution (φ, χ) of dual equation is doubled into the two components associated with the "left" and "right" rotation of the particle-triangle (φ_l, φ_d; χ_l, χ_d) during propagation along the X axis. In this way the "spinor-vector" will be a column matrix (1 × 4): $\Psi_{(1\times4)} \equiv (\varphi_l, \varphi_d; \chi_l, \chi_d)$. The dual equation, Equation (21), must therefore transform into a new form with a wave vector at 4-component and must have all the matrices (4 × 4) indicating both the "oscillating" translation (rotation in S-T in S_{lab}) of quantum (•) and both the rotation of triangle particle around the axis of propagation X. So, the excitation quanta (•) pass (oscillating) from one quantum oscillator to the next along the bases (AB, BD, DG, GI, …), and after passing along the bases (A'B', B'D', etc.). All this pushes us to admit an invariance in the form of the dual equation of K-G. Furthermore, according to Dirac's observations, the new form of the dual equation must be relativistically covariant and symmetrical in the spatial and temporal components. The application of a rotation matrix (Q_s) on a "structure with an internal rotation (Q_r)" will be described by an "external product" between matrices (Q_s, Q_r), which represents an application of Pauli matrix $\sigma_s(\varphi)$ on matrices of Pauli $\sigma_f(\theta)$; in formal terms, we will write $\sigma_s(\varphi) \times \sigma_f(\theta) = \alpha_r$, where Xrepresent a particular application about "matrix on a matrix" which gives as result a matrix 4 × 4. Then, the matrices α_r will describe a new Hamiltonian operator H_D which expresses the rotational reality to which the particle-structure is subjected. The operator H_f of the "dual" equations of K-G (Equation (20) and Equation (21)) by the matrices α_r will become so the operator H_D; by the invariance in the form of the dual equation, we will have $[H_{KG}(\sigma)\Psi_{KG} = 0] \to [H_D(\alpha)\Psi_D] = 0$. In this case, the matrices α_r are the 4-dim form of the Pauli matrices and "**isomorphic**" to the σ_s, in space ($Q_s \times Q_r$). The Pauli matrices σ_i expressed in the dual equation must thus be replaced by matrices (4 × 4) α_r functions of the matrices σ_i, that is:$\alpha_r(\sigma_i) = \sigma_i(s) \times \sigma_j(\theta)$

Therefore, we will have:

$$\left(H_f = \sigma_0 p_0 - \sigma_k p_k - \sigma_3 mc^2\right) \to \left(H_D = \alpha_0 p_0 - \alpha_k p_k - \alpha_3 mc^2\right) \tag{22}$$

where (p_0, p_k) are derivative operators. We look for the shape of the matrices α. Let's consider the operation $\alpha_r(\sigma_i) = \sigma_i(s) \times \sigma_j(\theta)$. If $\Psi(x_i, ict)$ is a column matrix (1 × 4) then the α will be matrices (4 × 4) and represent the connection between the spinors $\Psi_{K\text{-}G}$ of the dual equation of K-G and those of the rotation of the structure Ψ_s: then, we will have ($\Psi_D = \Psi_{K\text{-}G} \cap \Psi_s$), where the sign $\cap$ indicates the union of the states. However, we have said that any rotation can be expressed by Pauli matrices σ_j, so that even a rotation of a rotation α_r will be expressed by

Pauli matrices, that is its elements α_{ij} will be Pauli matrices: $\alpha_r \to \alpha_{ij} = f_i(\sigma_j)$. Since the α_r is isomorphic to σ_j it follows that the matrices α_r will have the same shape as the Pauli matrices (2 × 2) but with elements given by Pauli matrices; it follows that:

$$\alpha_0 = \begin{pmatrix} \sigma_0 & 0 \\ 0 & \sigma_0 \end{pmatrix}, \ \alpha_1 = \begin{pmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{pmatrix}, \ \alpha_2 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}, \ \alpha_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}$$

$$\alpha_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \ \alpha_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \tag{23}$$

$$\alpha_2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \ \alpha_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The equation ($H_D \Psi_D = 0$) becomes:

$$\left(\alpha_0 p_0 - \alpha_k p_k - \alpha_3 mc^2\right)\Psi_D = 0 \tag{24}$$

The new Equation (24) coincides with Dirac's equation. The $\Psi_D \equiv (\varphi_b, \varphi_d, \chi_b, \chi_d)_D$ represents, therefore, a fermion wave of a massive particle with semi-inter spin s_F. We have thus shown how the propagation of a "triangle-particle" structure can be described by a 4-component (spinor) or by wave function "associated" with Pauli 4 × 4 matrices (α_D Dirac's matrices) and satisfactory to the first order and relativistic wave equation $H_D\Psi = 0$. A last note: if we want to admit as possible that a particle is a triangular structure of coupled quantum oscillators, then, we are forced to change our vision about the quantum oscillator. We need to think that each quantum oscillator (called IQuO) that belongs to a structure is composed of sub-oscillators with energy (quantum •) composed of semi-quanta (*sq*): (•) = ($\bullet_1$, $\bullet_2$). We already indicated, see the ref. [11] [12], just by the acronym IQuO, a quantum oscillator at semi-quanta and sub-oscillators, therefore we can assert that structure-particle can be built just by "IQuO" oscillators.

3. The Aspects of the Geometric Model

3.1. The Spin

Thanks to the "IQuO" idea and the structure hypothesis, we can build a representative model (GMP) of the particles that geometrically expresses the Standard Model and its phenomenology, in-depth and explanatory as well as clarifying terms. One of the insights of GMP is given by the concept of spin. The spin, in GMP, is no longer a property of a particle that can only be defined "mathematically". In fact, if we include an intrinsic rotation in a structure rotating around a propagation axis, then we can obtain particles described by the Dirac equation, see the sect. 2.5. Not only that, but we have obtained that the particles are described by wave functions with phase period $\underline{T} = 4\pi$, a characteristic aspect of particles whose rotations are at semi-angles and therefore at spin (±1/2). Dirac, just speaking of fermions (see ref. [17]), points out that if we consider spin operators σ (operators indicating rotations) and combine them with an operator ζ that anti-commutes, then we can obtain quantum oscillators of field described by operators (η, η^+) that anti-commute. Dirac posed the following hypothesis:

$$\eta = \frac{1}{2}\varsigma\left(\sigma_x - i\sigma_y\right), \quad \eta^+ = \frac{1}{2}\varsigma\left(\sigma_x + i\sigma_y\right) \tag{25}$$

With ζ operator which anti commute ($[(\zeta^+, \zeta)]_{(+)} = i\eta$,), commutes with σoperators and $(\zeta)^2 = 1$. Then one proves:

$$\left\{\eta^2 = 0, \eta^{+2} = 0, \left(\eta\eta^+\right)^2 = \eta\eta^+, \left(\eta\eta^+\right) = n\right\}, \left[\eta^+\eta + \eta\eta^+\right]_{(+)} = 1 \tag{26}$$

The Dirac showed $(\eta\eta^+)' = n' = (0, 1)$, that is ($\eta$, η^+) are operators describing quantum oscillators associated with fermion particles. Thus, the anti-commutation property of ζ combined with the properties of rotations result in particles with spin (1/2). Recall that in the quantum (harmonic) oscillator the operators (a, a^+) can be placed in the form [9]:

$$a_t = a_0 e^{i\omega t}, \quad a_t^+ = a_0^+ e^{-i\omega t}.$$

These relations represent the time evolution of the operators (a, a^+) where ($i\omega t$) is the phase φ of the oscillation. Since (η, η^+) represent quantum oscillators, then we can write:

$$\eta_t = \eta_0 e^{i\omega t}, \quad \eta_t^+ = \eta_0^+ e^{-i\omega t} \tag{27}$$

where η_t is the operator η at time t and η_0 the operator at the initial time $t = 0$. From (26d) it follows that:

$$\left.\begin{cases} \eta\eta^+ = n \\ \eta\eta^+ = \eta_0\eta_0^+ \end{cases}\right\} \Rightarrow \eta_0\eta_0^+ = n \tag{28}$$

where n is self-adjoint, $n = n^+$. For an isolated oscillator, we have ($n_t = n_0$). We could thus posit, as Dirac did [20], also see the Equation (28), that, see also ref. [18]:

$$\eta = \eta_0 e^{i\omega t} = \sqrt{n} e^{i\omega t}, \quad \eta^+ = \eta_0^+ e^{-i\omega t} = \sqrt{n} e^{-i\omega t} \tag{29}$$

In this way, Dirac obtains that $\eta_0 = (n)^{-1/2}$. We note that if the phase $\exp(i\omega t)$ is a "number" then $(n)^{-1/2}$ must be an operator. We recall from the properties of the spin that: $\sigma_\pm = \sigma_x \pm i\sigma_y$. We can then express ($\eta$, η^+), see Equation (25), such as:

$$\eta = \frac{1}{\sqrt{2}}\varsigma\sigma_-, \quad \eta^+ = \frac{1}{\sqrt{2}}\varsigma\sigma_+ \tag{30}$$

By means of the properties of matrices σ $[(\sigma_x)^2 = (\sigma_y)^2 = 1]$, we find that $\eta, \eta^+ = \zeta^2 = n$, that is $\zeta = (n)^{1/2}$. We thus find Equation (29) if we place the operator σ in exponential form: $\sigma \to \exp(i\omega t)$. Comparing Equation (27) with Equation (30) we obtain:

$$\begin{gathered} \eta = \eta_0 e^{i\omega t} = \frac{1}{\sqrt{2}}\varsigma\sigma_-, \quad \eta^+ = \eta_0^+ e^{-i\omega t} = \frac{1}{\sqrt{2}}\varsigma\sigma_+ \\ \sigma \Leftrightarrow e^{i\omega t}, \quad \varsigma \Leftrightarrow \left(\eta_0 \equiv \sqrt{n}\right) \end{gathered} \tag{31}$$

Since σ are operators, we expect from Equation (30) that $\exp(i\omega t)$ should also be an operator (!), that is the oscillation phase should become an operator. This last aspect, as Davidov [18] pointed out referring to Susskind's works and Glogower [21], could lead to some "unclear" aspects in the theory of the quantum oscillator, if not even to contradictions. Nevertheless, we specify that in the cited article [20], Dirac does not make well it clear whether the phase should be treated as a "true" operator. This "unclearness" could derive from an incompleteness: something is missing in the description of the quantum oscillator. Something that Dirac did not want to explore but which should, however, be connected to "hidden" degrees of freedom, as hypothesized by Dirac himself in his text [17], when he introduced the concept of spin in QM: "*The angular momentum of spin can be imagined as an internal motion of the particle itself associated with degrees of freedom different from those describing the motion* ("external") *of the particle*". This "internal motion", we have identified in the proper rotational motion of the structure of the coupled IQuOs, see section 2.5, while one of the "internal" degrees of freedom we have identified in the direction $\boldsymbol{r}$ of phase rotation, see ref. [11] [12]. In support of our choice, we report what Fain demonstrated [18]: the eigenstate of energy (eigenstate of n) is degenerate with respect to the sign relative to the direction of phase rotation φ; the sign is associated with the operator $\boldsymbol{I}$ (we have called it $\boldsymbol{r}$) with eigenvalues ±1, which commutes with the Hamiltonian. Therefore, to complete the QM of an oscillator, in indicated previous studies we have associated an operator $\boldsymbol{r}$ with the phase φ in $\exp(i\omega t)$, whose eigenvalues are $r' = \pm 1$ relative to the two rotation directions of the phase. Besides, we have diversified η_0 into $[((\eta_0)_-, (\eta_0)_+]$, see again ref. [11] [12]. Then, we assume that:

$$\varsigma_\eta \to \sqrt{n_-}, \quad \varsigma_{\eta^+} \to \sqrt{n_+}$$
$$\sigma_- \equiv e^{ir\omega t}, \quad \sigma_+ \equiv e^{-ir\omega t}$$

And so

$$\left[\eta = \sqrt{n_-}\, e^{ir\omega t}\right], \quad \eta^+ = e^{-ir\omega t}\sqrt{n_+} \tag{32}$$

These relationships define the IQuO. Through the concept of IQuO we manage to avoid the contradiction of considering the phase as an operator, adding, instead, in the $\exp(i\omega t)$ term the phase rotation direction operator $\boldsymbol{r}$**:** $\exp(i\boldsymbol{r}\omega t)$. The difference $[(\eta_0)_- \neq (\eta_0)_+]$ allows us after to describe the oscillator as composed of semi-quanta and sub-oscillators, while $\boldsymbol{r}$ allows an external "agent" to read the direction of rotation of the phase of the oscillator (unthinkable aspect for classical mechanics) but which is then interpreted phenomenologically as the electric charge (exactly its sign ±) of the particle represented by a oscillators field. Furthermore, as we have already expressed in this article, the "IQuO" would be that "particular" quantum oscillator which is suitable for building geometric structures representing massive particles. So, we believe that only through the structure hypothesis and the idea of the "IQuO" oscillators, one can able to complete the QM.

3.2. The Magnetic Moment of the Geometric Structure of Electron

The “proper” rotation associated with the geometric structure of the electron determines the intrinsic spin, to which a spinor is associated. As we know, the spinor is related to intrinsic rotations made up of “semi-angles” which originate an oscillation-rotation period $\underline{T} = 4\pi$. To the rotating structure around the axis of propagation, we could also assign a magnetic moment [22] if this structure has an electric charge with a sign corresponding to the direction of rotation of the phase. The internal flow $\mathbf{\Phi}$ of the sq inside the structure can be considered as an “electric current” along a circuit (the triangular structure), see **Figure 3**; however, since the structure, in turn, rotates around the axis of propagation X, its projection onto the plane perpendicular to the X-axis can be compared to a circular loop in which a current circulates. Well, the magnetic moment of this structure can be traced back to this circular coil and since the rotation period of the structure is double that of 2π, thenit follows that the path of the current results as if it was composed of a double turn, see **Figure 5**.

Thus, we have the presence of a factor of 2 in the magnetic moment that we associate with the loop relating to the structure:

$$\begin{aligned} &\mu_{(structure)} = 2\mu_{(loop)} \\ &\mu_{(loop)} = \mu_{(spin)} = s\mu_B = \left(\frac{1}{2}\right)\left[\frac{e\hbar}{2m_e c}\right] \end{aligned} \tag{33}$$

It follows $\mu_e(str) = 2\mu_e(loop) = 2(s\mu_B)$.

But in literature it is assumed that $\mu_e(spin) = gs\mu_B$, where g is the gyromagnetic ratio. Therefore, we find in Geometric Model that $g = 2$. The relativistic treatment shows a correction of (0.1)%, with $(\mu_e)_{rel} > (\mu_e)_{cl}$. Finally, in the geometric model, the intrinsic magnetic moment of the electron is consistent with the experimental one [23].

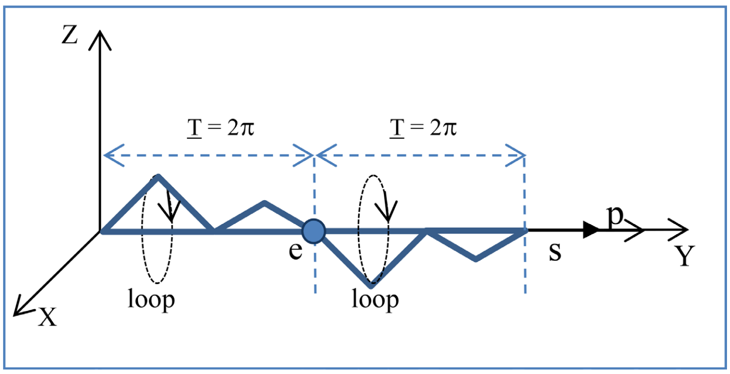

Figure 5. The double loop associated with the electron.

4. Geometric Structure of Electron

4.1. The Quantum Oscillator at Semi-Quanta

In the Schrödinger’s representation the quantum oscillator, in the energy eigenstates with occupation number n is represented by a wave function $F_n(x)$ and

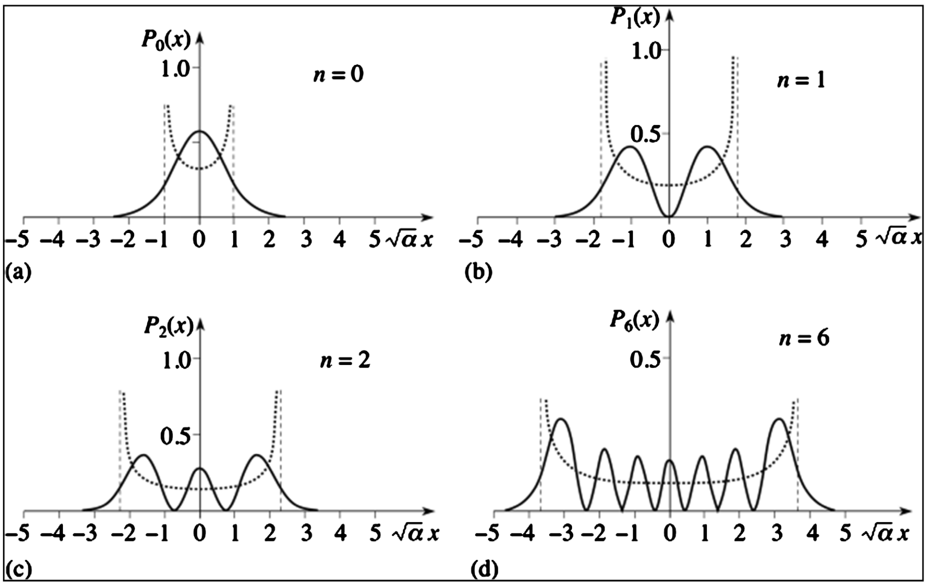

Figure 6. Probability function of quantum oscillator.

its probability function $P_n(x)$ [24], see **Figure 6**.

The function $P_n(x)$ shows some peaks which indicate where is more probable to find the energy quanta of the oscillation at time t_0. QM tells us that when the oscillator, with ($n = 1$), is not observed the quantum is in a "non-local" state Ψ of the two probable "zones" of the position (Ψ_L ($x < 0$), Ψ_R ($x > 0$)), see **Figure 6(b)**. We must not forget that the oscillator is always described by the two components, the elastic one (U_{el}) and the inertial one (K_{in}), with total energy $E_{osc} = (U_{el}) + (K_{in})$. We can thus say that the quantum oscillator, in whatever state and eigenstate it is, is always given in all its "wholeness" with its "two" energetic components. However, looking at the two peaks and considering this "wholeness" we might so think that the oscillator is composed of two not-separable oscillating parts. The same in other values of n: the oscillator is composed of ($n + 1$) not-separable oscillating parts. In this way, the two components (elastic and inertial) and the two "parts" of position x encourage us to believe that the energy of the quanta flows between the two oscillating components not separable, see two peaks. From these arguments, we can conjecture that the two peaks of $P_1(x)$ could represent two distinct parts of the same quantum oscillator, each always with elastic and inertial characteristics: this would allow us to physically treat the quantum oscillator as if it made up of two elastically coupled "**sub-oscillators**". Then, we conjecture that the quantum oscillator, in an eigenstate of value n, is given by ($n + 1$) sub-units of oscillation or "sub-oscillators" S (sub-osc.). We think the presence of two or more oscillating components in an oscillator causes the "splitting" of its energy quanta into two energetic components in each sub-oscillators: this introduces the idea of "**semi-quanta**" (or individually "**semi-quantum**" (sq)). When we consider the value ($n = 0$), and therefore only one sub-osc. with the energy of ($\varepsilon = 1/2h\nu$), then the two sq must each have an energy of [($\varepsilon = 1/4h\nu$)]. It follows for ($n = 1$) that:

$$\begin{aligned}\left[H_{(n)}\right]&=\left[U_{(n)}+K_{(n)}\right]=\left[\left(U_{(n)}\right)_{el}+\left(K_{(n)}\right)\right]\\&=\left[(2n+1)\left(\frac{1}{4}\hbar\omega\right)_{el}+(2n+1)\left(\frac{1}{4}\hbar\omega\right)_{in}\right]\\&=\left[(2n)\left(\frac{1}{4}\hbar\omega\right)_{el}+(2n)\left(\frac{1}{4}\hbar\omega\right)_{in}\right]+\left[\left(\frac{1}{4}\hbar\omega\right)_{el}+\left(\frac{1}{4}\hbar\omega\right)_{in}\right]\end{aligned} \tag{34}$$

Then we conjecture that the energy value of [($\varepsilon = 1/4h\nu$)], indicated as **"empty"** semi-quantum with symbol (**o**), while another value [$\varepsilon = (1/2h\nu)$] indicates as **"full"** semi-quantum and symbol (•). So, an oscillator [$\varepsilon_n = (n + 1/2)h\nu$] will be represented by empty *sq*(**o**) and full *sq*(•):

[$\varepsilon(\bullet) = (1/2h\nu)$]; ($\varepsilon(\mathbf{o}) = 1/4h\nu$), then [$\varepsilon_n = (n + 1/2)h\nu) = (n(1/2 + 1/2) + (1/4 + 1/4))h\nu$]

The treatment quantum oscillator at semi-quanta pushes us to consider any quantum oscillator like an IQuO [11] [12].

4.2. The Charge Leptons of Base: The Electron

Considering Let us try to build the lepton structure. We consider the most elementary figure that a coupling of IQuO can build: this structure will be composed entirely of IQuO$_{(n=1)}$. There could be two possibilities physically equivalent, see **Figure 2**:

1) [(I_A, I_B, I_C), (I_{AB}, I_{BC}, I_{CA})]$_{(n=1)}$, 2) [(I_{AB}, I_{BC}, I_{CA})]$_{(n=1)}$

where (I_A, I_B, I_C) are IQuO-vertices while (I_{AB}, I_{BC}, I_{CA}) are IQuO-junctions. In option 2) each IQuO will be an IQuO-vertex and, at the same time, IQuO-junction. This option can be considered as a simplification of the first option. Having all IQuO$_{(n=1)}$ we will have IQuOs with 2 sub-osc. Two contiguous sub-oscillators (S_1, S_2) could be considered as "coupled" oscillators when the respective pairs of semi-quanta exchange a full semi-quantum (•) at least in a point:

$$\left[(\mathbf{o},\bullet)_{S_1}\leftrightarrow(\mathbf{o},\mathbf{o})_{S_2}\right]\rightarrow\left[(\mathbf{o},\mathbf{o})_{S_1}\leftrightarrow(\bullet,\mathbf{o})_{S_2}\right] \tag{35}$$

where the symbol ↔ indicates the elastic coupling, achieved through the exchange of a *sq* (•), which maintains the locality and individuality of S. The overlapping of the respective sub-oscillators of the IQuOs [(I_A, I_B, I_C) ⊕ (I_{AB}, I_{BC}, I_{CA})] realizes of so the coupling, see sect. 2.2, between the IQuOs and therefore builds the particle-structure. We indicate by [$s = (S_i)_{Iv} \oplus S_j)_{Ij}$] the overlapping of two sub-oscillators. In this case, the distribution of full *sq* assigns two full *sq* (•, •) in each side with vertex. This means that in our particle-structure there are all "double" sub-oscillators and the component IQuO are all the F type. Not only that, but the closed chain of IQuO with "double" sub-oscillators appears as a chain with additional coupling, that is, massive: we could assert that all particle-structures with F-IQuO represent massive particles of the "fermion" type. Thus, we will have, see **Figure 7**.

This overlapping structure of IQuO determines the structure of the most basic lepton, that is the "**electron**". With S_{em} we indicate the sub-osc. of the line of electromagnetic field oscillators that guides the "triangle-structure". The junction IQuO I_{CA}, by its *sq* (•), can slide along the guideline. This is because the sub-oscillators (S_e, S_{em}) exchange a *sq* (•) and the "internal" S_{em} sub-osc. is coupled to an external S_{em} sub-osc.: it follows that S_e couples even to external S_{em} and, thus, all the structure transfer forward along the X-axis.

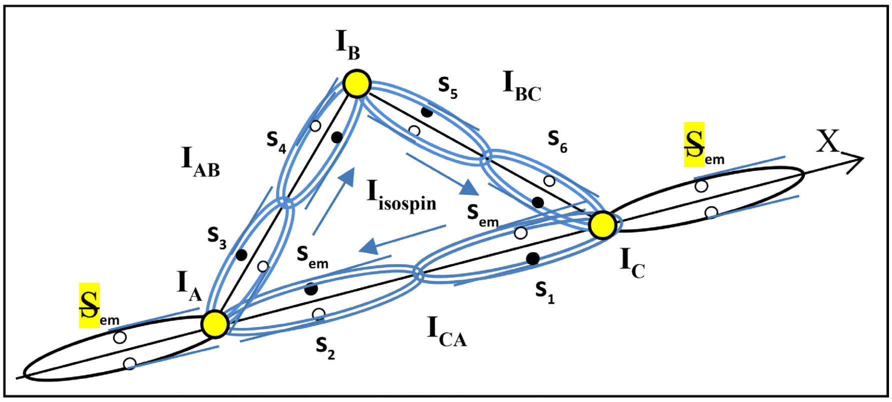

Figure 7. The IQuO-structure with sub-oscillators of an elementary lepton and the electromagnetic guideline.

4.3. An Internal Degree of Freedom: The *sq* Vector Flow of the Electron

Note the *sq* flow along the sides; we observe that the full *sq*(•), can have two rotation directions. To flow we assign a new physics quantity called as "*sq* Vector Flow" (**Φ**(•)). The flow operator commute with Hamiltonian and it has two eigenvalues (±1). The structure is a triangle isosceles, and rectangle and the coupling determine a "real" particle because it can exchange a full quantum (•) with external IQuO. The triangle-electron transfer (propagation) along the X-axis, with a spin (1/2), see the sect. 2.2. We can represent the flow **Φ** in a triangle structure, see Figure 8.

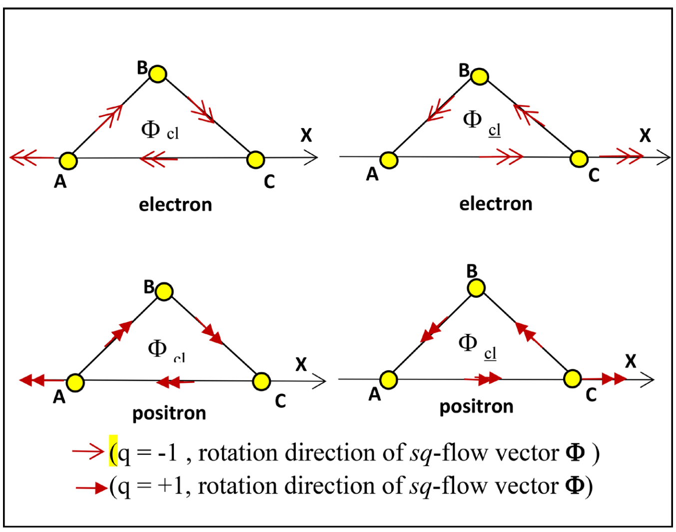

Figure 8. The configuration of the electron and positron and *sq*-vector flow **Φ**.

The crossed-out arrow indicates the direction of the flow of *sq* inside the structure. This internal degree of freedom will be defined by us as "***sq*-vector flow**" and its two eigenvalues will correspond to clockwise or right-handed (R), and counter clockwise or left-handed (L). The electron only involves the vertices (A, B, C), see **Figure 7**, that is $e = \{[(\bullet, \mathbf{o}), (\bullet, \mathbf{o})]_{AC} \equiv (\bullet)_X\}$ along the propagation X-axis.

5. Conclusion

In light what has been demonstrated, that is a structured particle with triangular geometry propagates according to an equation coinciding with that of Dirac, we can deduce that any massive particle with a triangular structure is a fermion and the oscillators that define a geometric structure are quantum oscillators composed of sub-oscillators with semi-quantum that is they are IQuO. In previous studies and already published articles it has been demonstrated that in an IQuO it is possible to detect the direction of the phase rotation as a sign of the electric charge [11] [12]; besides, the structure with sub-oscillator of an IQuO is physically highlighted through the color charge [3]. So, through the IQuO we can represent electrons (positrons) and quarks. Ultimately, thanks to the "IQuO" idea, we can build a representative model of the particles that geometrically expresses the Standard Model and its phenomenology, in in-depth and explanatory as well as clarifying terms.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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