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Astrophysical Applications of a Variant to Kepler's Third Law

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Abstract

It is verified that the Nebula Hypothesis is applicable to the Solar System by way of a straightforward generalization of Kepler's third law which also confirms that angular momentum transport is achieved by way of the self-gravity of the protoplanetary disk itself as it coalesces into planetesimals. The masses of the planets may then be approximately determined (within 10% error, for three planets) by way of this methodology, using the radius as well as the rate of rotation of the particular planet being considered. This would only be possible, not only in light of the Nebula Hypothesis, but also due to angular momentum transport (as these three planets most ideally express the expectations of angular momentum conservation from the protoplanetary disk). Also in this regard, the rotation of the Sun at its equator is discussed as it is found to be closely related to the planetary issue as it pertains to the evolution and structure of the body. A modified technique from that used in planetary study is then applied to the Galaxy for the purpose of the calculation of dark matter mass, presupposes treating the Galaxy as a homogeneous sphere (of dark matter) that is rotating. The model provides clear evidence of not only flat rotation-curves, but also the lack of centrifugal ejection of stars from galaxies as well as the configuration of the arms of spiral galaxies, along with a sound basis for black hole creation at the center of spiral galaxies.

Keywords

Planet Formation, Nebula Hypothesis, Conservation of Angular Momentum, Rotation of the Sun at the Equator, Dark Matter, Galactic Dynamics and Structure

1. Introduction

In the following, a simple relationship between the rate of rotation of a planetary body on its own axis and its surface gravity demonstrates a direct proportionality between these quantities within the Solar System. This result is fundamentally significant insofar as it establishes not only the validity of the Nebula Hypothesis, but also establishes that angular momentum transport as being determined by way of the self-gravity of the debris field of the protoplanetary disc. The equations prove to be a generalization of Kepler's third law that may now be interpreted within a much broader context that includes phenomena not normally associated with the application of the original expression, but which may now be viewed as being a consequence of the modified format.

Application of the new expression to the Galaxy was compelling as the flat rotation-curves may now be incorporated into the calculation of dark matter mass leading to values much greater than those anticipated by purely dynamical considerations alone. These methods proved to be wholly consistent with the Newtonian expression for calculating the surface gravity of a celestial body based on its mass and radius, but in the case of the Galaxy, the radius and the acceleration due to gravity were used. This methodology is identical to a double application of the generalized laws as they may be used to precisely calculate the mass of a planet given its radius and surface gravity. And to the extent that dark matter mass is accurately reflected by the scenario here presented, the associated requirements of the methods of calculation presume treating the Galaxy as a homogeneous sphere endowed with torsional properties. This is important, as dark matter would now have to be viewed as being indistinguishable from the vacuum, a physical field that is not only rotationally dynamic but that also conforms to the observed circumstances of not only the rotational dynamics of the Galaxy, but also the structural.

The goals of the present treatment, however, are focused on addressing various phenomena by way of a single expression that encapsulates angular momentum conservation at its core. The relationship of this expression to Kepler's third law is largely based on the aspect of being able to calculate the mass of a planet using its radius and rate of rotation on its own axis, though by way of elementary analysis, a deeper connection is established. And while this circumstance is both interesting and unique, its real value becomes clear when analyzing the evolution of the planets in terms of their likely conservation (preservation) of the angular momentum of the protoplanetary disk. As is explored herein, it now becomes reasonable to conjecture as to the inner processes of certain planets, e.g., Mars and Saturn. The former has likely experienced a cooling of its molten core, and the latter most certainly is manifesting a collapsed core.

Application of the modified law to the Galaxy for the purpose of the calculation of dark matter mass, still entailed using the expression twice in two different forms which proved to be equivalent to employing the Newtonian format for determining the surface gravity of a homogenous sphere. This proved to be a fortuitous circumstance, as it not only greatly simplified the calculation of mass, but also provided the further basis for the construction of a highly applicable dark matter model. The use of the planetary study in this capacity was, of course, unusual as it was not constructed for this purpose, however, given that the existence of dark matter is partly demonstrated by way of a very slight acceleration, this then became the best analytic basis for the calculation of mass.

2. Planet Formation and Angular Momentum Transport

The Nebula Hypothesis presumes that the planets of the Solar System evolved from a protoplanetary disk orbiting the Sun. The conservation of the angular momentum of the debris field as the planets form would lead to an expectation of a proportionality between the tangential velocity of the planet on its own axis and its mass, more specifically to its surface gravity. Writing out the tangential velocities of rotation of the planets in increasing order of mass: Mercury 3.0 (meters per second), Mars 241.0, Venus 2.0, Earth 465.0, Uranus 2492.0, Neptune 2251.0, Saturn 10283.0, and Jupiter at 12680.0, illustrates the relationship. In order to set up such an expression, we measure the tangential velocity of the planet (on its own axis) by way of

$$2\pi r/T = v_{tan},$$

where T is the period of rotation of the planet on its own axis. The tangential velocity is measured along the equator of the sphere in units of meters per second. And for the purpose of this work, escape velocity is the parameter chosen to represent the gravitational field intensity as it proved to be the most direct means by which to achieve the desired results. The escape velocity is given by

$$(2GM/r)^{1/2} = v_{es},$$

where G is the gravitational constant whose value is $G = 6.67 \times 10^{-11}\ \text{kg}^{-1}\cdot\text{m}^3\cdot\text{s}^{-2}$, M is the mass of the planet and r is the radius. At this time, it is convenient to review **Figure 1**, and study the plot of the tangential velocity of the planets vs. the escape velocity for each respective planet and note the generally parabolic shape of the curve that is formed. With the notable exception of Saturn (planet number 7), the plot implies a relation of the second order in the escape velocity and indeed, for three of the planets within the Solar System, the relationship

$$v_{tan}/v_{es}^2 = k, \tag{1}$$

where k is a constant, displays a remarkable degree of accuracy. Equation (1) has been plotted on **Figure 1**, though the scale of the graph is insufficient to display anything beyond a general impression of the parabolic nature (a relationship of the second order) of the relationship being outlined. However, in **Table 1**, Column C, the data from Equation (1) has been tabulated, where the planets of the Solar System have been cataloged in increasing order of mass, shows a notable similarity between three of the planets, in particular, from the Earth, Neptune and Jupiter. We have for an average value of k:

$$k = 3.7889 \times 10^{-6}\ \text{s}\cdot\text{m}^{-1}.$$

Introducing the surface gravity into Equation (1) is most easily accomplished by writing it out in full:

$$2\pi r/T = 2GMk/r$$

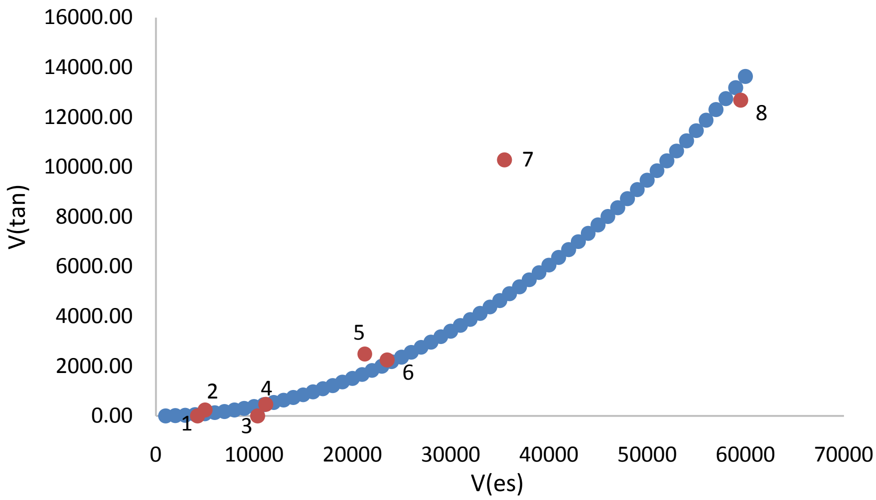

Figure 1. Consists of the planets of the Solar System where the rates of rotation of each planet are presented vs. the escape velocity of each respective planet (with all units being in meters per second). The parabolic graph is of Equation (1) and represents an average, in terms of the proportionality factor, for the Earth, Neptune and Jupiter, being planets 4, 6 and 8. The planets are arranged in the order of increasing mass where Mercury, Mars and Venus are planets 1, 2 and 3. Uranus is planet 5 and Saturn is 7. The generally parabolic arc to the data points suggests a relation of the second order in the escape velocity and justifies the construction of Equation (1).

Table 1. Displays the eight planets of the Solar System in increasing order of mass where Column A contains the radii and Column B the escape velocity's of the planets presented to the nearest significant figure. Column C shows the proportionality factor "*K*" for each planet (Equation (1)), where the results for the Earth, Neptune and Jupiter are seen as being representative of the applicability of the Nebula Hypothesis and will permit the approximate calculation of these planets respective masses.

	A	B	C
	Radius (m)	v_{es} ($\text{m}\cdot\text{s}^{-1}$)	k ($\text{s}\cdot\text{m}^{-1}$)
1. Mercury	2,439,500	4239.0	1.6513×10^{-7}
2. Mars	3,397,000	5010.0	9.6006×10^{-6}
3. Venus	6,052,000	10360.0	1.9306×10^{-8}
4. Earth	6,378,150	11178.0	3.7216×10^{-6}
5. Uranus	25,560,000	21262.0	5.5122×10^{-6}
6. Neptune	24,765,000	23532.0	4.0668×10^{-6}
7. Saturn	60,270,000	35479.0	8.1683×10^{-6}
8. Jupiter	71,490,000	59528.0	3.5782×10^{-6}

which readily leads to

$$T = \pi / gk ,$$

where g is the surface gravity of the planet:

$$g = GM / r^2 . \tag{2}$$

Equation (1) may also be written as

$$v_{tan} = 2grk , \tag{3}$$

and upon substituting

$$v_{tan} = r\omega$$

for v_{tan} in Equation (3), where ω is the angular velocity, we then have

$$\omega = 2gk\,.$$

In the following, however, Equation (3) and

$$v_{tan} = 2GMk/r \tag{4}$$

will prove to be the most useful forms of Equation (1) for this study.

Starting from Equation (4), we may approximately calculate the masses of the three main planets of this study by employing the rate of rotation of an individual planet, its radius as well as the proportionality factor, *k*. These data have been compiled in **Table 2**, where in Column C we find the percentage error between the observed mass of the individual eight planets of the Solar System and the corresponding calculated value per each planet. The values range from negative 5% to positive 10% error for the three most viable planets. While these errors are acceptable as far as establishing the general applicability of the Nebula Hypothesis, the other five planets are noteworthy for the considerable degree of divergence from the relative continuity displayed by the Earth, Neptune and Jupiter. Mercury's rate of rotation is clearly affected by the Sun's gravitational field whereas Venus is generally regarded as having experienced one or more collisions, most likely by former moons. Mars, on the other hand, with its enhanced rate of observed rotation and a calculated mass 150% greater than the observed, could very well be an example of a formerly volcanically active body whose molten core has cooled and thus solidified into a sphere of reduced radius thus

Table 2. Where the planets of the solar system are arranged in increasing order of mass, displays the observed mass of each planet as well as the approximate mass calculated by way of Equation (4) in Columns A and B respectively. The percentage error between the two mass figures is shown in Column C as a plus or minus percentage from the observed to the calculated mass. The particularly diminished error for the Earth, Neptune and Jupiter are here considered to be the best reflection of angular momentum transport, showing that the angular momentum of the protoplanetary disk being at least approximately conserved in the formation of these three planets.

	A	B	C
	Mass observed (kg)	Mass calculated (kg)	Percentage error between observed and calculated mass (%)
1. Mercury	3.3×10^{23}	1.5×10^{22}	−95
2. Mars	6.4×10^{23}	1.6×10^{24}	+150
3. Venus	4.9×10^{24}	2.2×10^{22}	−100
4. Earth	6.0×10^{24}	5.9×10^{24}	−2
5. Uranus	8.7×10^{25}	1.3×10^{26}	+49
6. Neptune	1.0×10^{26}	1.1×10^{26}	+10
7. Saturn	5.7×10^{26}	1.2×10^{27}	+111
8. Jupiter	1.9×10^{27}	1.8×10^{27}	−5

increasing the angular velocity of the body due to angular momentum conservation. Saturn, as it is commonly thought, would almost certainly have a collapsed core leading to an enhanced degree of rotation and thus a calculated mass 111% greater than the observed value. This impression is also supported by the likelihood that a collision would most likely not have created such an extreme effect. The density of Saturn is nearly 50% that of Jupiter, which would also suggest a scenario in which a collapsed core would have to be regarded as a serious possibility. As for Uranus, the error in the calculated mass is not that extreme, and thus leaves open the possibility of either a collision and or a collapsed core. Thus, at least in principle, for the Earth, Neptune and Jupiter, the angular momentum of the protoplanetary disk has been at least approximately conserved as the commonality of the proportionality factor would illustrate. This is significant, as it makes clear the possible applicability of using Equation (4) to eventually correlate data from exoplanets as the methods of observation continue to improve. It is, however, unclear as to whether or not the proportionality factor is accurately applicable to all solar systems. One additional point that also deserves to be made in regard to the issue of angular momentum conservation of debris fields that is wholly consistent with this discussion: We note the spherically truncated configuration of the Sun as well as the accumulation of matter along the plane of rotation, both of which are consistent with the impression of a rapidly spinning collapsed core. Due to the angular momentum of the swirling gas cloud that led to the formation of the Sun being conserved, leads directly to the conclusion that the plane of rotation of the Sun would have a greater angular velocity than those regions north and south of the equator due to the accumulation of matter along the plane of rotation, as is, in fact, found to be the actual case. This point is worth making as the data pertaining to the planets as well as the issue with respect to the Sun may now support one another. It is also reasonable to assert, given the extreme generality of the conditions, that this circumstance is most likely common to all stars of this class.

Employing both Equations (3) and (4) in order to make precise calculations of mass, it is necessary to determine a value for k from Equation (3) by taking the rate of rotation of a planet, the Earth for example, the radius as well as the surface gravity. Compiling these data:

$$v_{tan} = 465.1\,\mathrm{m}\cdot\mathrm{s}^{-1}$$

$$r = 6378150\,\mathrm{m}$$

as well as

$$g = 9.81\,\mathrm{m}\cdot\mathrm{s}^{-2},$$

we then obtain for k

$$k = 3.7167\times10^{-6}\,\mathrm{s}\cdot\mathrm{m}^{-1}.$$

Using this value for k in Equation (4), then leads to a calculated mass of the Earth of

$$M = 5.98 \times 10^{24}\ \text{kg}.$$

while the accuracy of this result is certainly desirable, it does lead to further inquiry as to the nature of Equations (3) and (4). If we equate the two expressions, as follows:

$$2grk = 2GMk/r$$

immediately clarifies the issue as

$$g = GM/r^2$$

results. This circumstance serves to help in not only establishing the viability of this methodology, but will prove to be most useful in the following section.

In order to establish that angular momentum transport [1] is achieved by way of the self-gravity of the debris field itself, we construct an imaginary celestial body of mass 10^{26} kg and a radius of

$$r = 23000000\ \text{m}.$$

Using Equation (4), a tangential velocity of

$$v_{tan} = 2197.5620\ \text{m}\cdot\text{s}^{-1}$$

is obtained. Writing out the expression for the angular momentum of a homogeneous sphere:

$$L = 2/5\,Mr^2\omega,$$

we eliminate the angular velocity by using the earlier stated definition of it, and then obtain

$$L = 2/5\,Mrv_{tan}.$$

Calculating the angular momentum:

$$L = 2.0217 \times 10^{36}\ \text{kg}\cdot\text{m}^2\cdot\text{s}^{-1}$$

results. Also at this point of time, the surface gravity of the imaginary body is determined to be

$$g = 12.6086\ \text{m}\cdot\text{s}^{-2}.$$

with this result, the tangential velocity of the celestial body may then be determined by Equation (3) and then compared to the results as determined by way of angular momentum conservation as the celestial body undergoes radial contraction. Holding the angular momentum constant, and assuming that the radius of the body contracts to

$$r = 20000000\ \text{m},$$

leads to an enhanced tangential velocity of

$$v_{tan} = 2527.1963\ \text{m}\cdot\text{s}^{-1}.$$

Calculating a new surface gravity of the planet given the reduced radius, we obtain

$$g = 16.6750\ \text{m}\cdot\text{s}^{-2},$$

and then from Equation (3) a new tangential velocity is found to be identical to the one given above. Given the uniqueness of this result, it is most compelling to accept it as an accurate reflection of a fundamental physical process whereby the changes in tangential velocity of a celestial body undergoing radial contraction due to angular momentum conservation precisely parallel the same results as dictated by changes in the value of the surface gravity of the same body. It is here accepted that this fact represents, by way of this numerical simulation, the phenomenon of angular momentum transport. Also of fundamental importance, is the fact that the Nebula Hypothesis and angular momentum transport support one another to such an extent that there cannot be one without the other, as so outlined by this study.

To the extent that Equations (3) or (4) represent a generalization of Kepler's third law, the transition from a debris field obeying Kepler's laws to a vortex coalescing into a planetesimal in the protoplanetary disk describable by the system presented here, may most simply be demonstrated by substituting the centripetal acceleration v^2r^{-1} for the surface gravity in Equation (3), which leads to the identity

$$k = (2v)^{-1}, \tag{5}$$

which, when inserted into either Equations (3) or (4), for instance, leads directly to Kepler's third law:

$$v = (GM/r)^{1/2}.$$

Equation (5) may also be obtained by equating Equation (4), for example, with Kepler's third law however, the method used here would seem to be somewhat more revealing as to the true nature of the problem. In particular, Equation (4) may be used to determine the mass of a rotating planetary body whose rate of rotation and radius are known, analogous to the way in which Kepler's third law is used to ascertain the mass of a central body. Thus, at least in principle, the methodology as well as the intent of the classical system carries on over to the present format. Equation (5) still signifies, as an identity, the relationship between Kepler's third law and the variant for individual bodies (as it follows from equating the two). And while it appears to have no physical meaning of its own, it still ties the variant to the original expression due to its following from Equation (3) when the gravitational acceleration is substituted by the centripetal acceleration (keeping in mind, of course, that equating the two results in Kepler's third law).

3. Dark Matter and Its Role in Galactic Structure and Dynamics

In this section, a model of Galactic rotation and how it is determined by dark matter is developed based solely on the application of Equations (3) and (4) to the Galaxy which leads to an effective construct which incorporates the Galactic flat rotation-curves into a calculation of dark matter mass. We already know from the previous section that the equations incorporate much of Newtonian physics and thus this new application will not be in violation of those principles. The basis of the model presumes that the assumptions upon which Equations (3) and (4) were constructed carry on over to the Galaxy, in spite of the fact that the equations were not constructed for this purpose, and that the calculation of mass leading to results that would be considered excessive for the case of dark matter. Given this circumstance, it is here accepted, nevertheless, as a valid methodology to allow the presumptions inherent within the calculations to determine the nature of as well as the behavior of dark matter. Moreover, this methodology would obviously conform to the most peculiar feature of the rotational dynamics of the Galaxy by employing the flat rotation-curves into the calculation of mass. What is particularly important in this regard is that as k is allowed to diminish in value as r increases in Equation (3), a situation that is equivalent to k approaching zero as r approaches infinity, and employing the basic value of the gravitational acceleration of interstellar space within the Galaxy:

$$1.2 \times 10^{-8}\ \mathrm{cm{\cdot}s^{-2}}$$

which we will use in the form

$$1.2 \times 10^{-10}\ \mathrm{m{\cdot}s^{-2}}$$

will always guarantee a constant tangential velocity with the increase in radius (flat rotation-curves). And in order to arrive at a determination of mass, while treating the Galaxy as a homogenous sphere, we introduce the rate of rotation of the Galaxy of 200 $\mathrm{km{\cdot}s^{-1}}$, and writing it out in meters per second, we have

$$2.0 \times 10^{5}\ \mathrm{m{\cdot}s^{-1}}.$$

Employing a radius of 10 kpc or,

$$3.0842 \times 10^{20}\ \mathrm{m},$$

a value for k of

$$k = 2.7019 \times 10^{-6}\ \mathrm{s \cdot m^{-1}}$$

would then follow. We now have the required information to make a calculation of mass from Equation (4) that meets the stipulated requirement of the flat rotation-curves being incorporated into it. Employing the above value for k, we obtain from Equation (4) a dark matter mass of

$$M = 1.7114 \times 10^{41}\ \mathrm{kg}$$

or,

$$M = 8.6 \times 10^{10} M_{\odot}.$$

The above value follows from the dual application of Equations (3) and (4) which was found to be equivalent to applying

$$g = GM/r^2$$

to the Galaxy where the dark matter is now assumed to infuse the vacuum, that is to say, is to now be regarded as being indistinguishable from it. Also in this regard, it is to be noted that as k diminishes in value in direct proportion to the increase in radius, leads to an interpretation of an intrinsically torsional medium due to the equation

$$T = \pi/gk$$

where, T increases in value as k diminishes in turn implying rotation as the radius increases. The above expression is equivalent to Equations (3) and (4) and would then be expected to provide accurate data as to the period of rotation of the Galaxy at varying radii. Of course, Equations (3) and (4) were constructed on the basis of a homogeneous sphere that is rotating, but the above equation makes the matter explicitly clear that we are dealing with a specifically torsional medium. The above expression measuring the surface gravity is then applicable with respect to not only a situation incorporating flat rotation-curves, but also to a homogeneous sphere endowed with torsional properties. Imposing the conditions of flat rotation-curves on Equations (3) and (4) would also require the interpretation of a torsional medium.

Employing the methodologies developed thus far, we may now compare dark matter mass to various estimates given in the contemporary literature of the field. For instance, a projected mass of combined baryonic plus dark matter mass [2] at a radius of 50 kpc, based on dynamical considerations, is given as

$$M = 4.9 \times 10^{11} M_{\odot}.$$

Using a baryonic mass of the Galaxy of [3]

$$M = 6.0 \times 10^{10} M_{\odot}$$

and then calculating a dark matter mass using Equation (2), we obtain for the total mass of the galaxy at a radius of 50 kpc

$$M = 1.2 \times 10^{12} M_{\odot}.$$

This result is 145% greater than the figure arrived at by the author's, which would be expected from a model such as this, in particular, does not employ any dynamical nuance in the determination of mass. The previously mentioned author's also provide a dark matter density within the Solar System of

$$\text{Density} = 5 \times 10^{-25}\ \text{g} \cdot \text{cm}^{-3},$$

whereas we obtain by way of this model a density of (evaluated at a radius of 10 kpc)

$$\text{Density} = 1.39 \times 10^{-24}\ \text{g} \cdot \text{cm}^{-3}.$$

On the one hand, it is not surprising that the density would be an order of magnitude greater but given the considerably higher value of the masses calculated here, one would have likely anticipated even greater densities than the above. The estimated dark matter mass of $10^{12} M_{\odot}$ at a radius of 200 kpc [4] is exceeded by this model by 34 times with a value of

$$M = 3.4 \times 10^{13} M_{\odot}.$$

while this result could be regarded as a possibly serious problem with respect to the viability of this model, it must be remembered that we are in such an early stage of our appreciation and understanding of dark matter related issues that reasonable standards by which to gauge the accuracy of any particular outlook have not yet even been developed. It is for this reason that any new developments must be patiently evaluated over time as this entire field is still very much in its infancy. And regardless of how we view the result embodied in the previous statement of mass, it was derived on the basis of a system used to evaluate planetary data but which soon became apparent that the flat rotation-curves of the Galaxy could be incorporated into a calculation of mass while employing the basic gravitational acceleration of the Galaxy (which is also surely due to dark matter), in such a manner as to preclude the introduction of imagination or prejudice into the construction of a workable model. Thus we are left with a model whose construction depended solely upon interpreting the equations in a seemingly objective fashion in spite of whatever notions we may find intuitively objectionable.

As far as the analytic description of a torsional field is concerned, particularly one that is also indistinguishable from the vacuum, and would thus be describable as a torsional space, it is only necessary to turn to the non-symmetric gravitational theories. These works were originally intended to incorporate the gravitational and electromagnetic fields into a common mathematical/geometric scheme under the guise of a generalized affine connection which would then lead to the breakup of the Ricci tensor in such a way as to result in a Riemannian geometry with torsion. First of all, it would appear to be satisfactory to hold the metric tensor as symmetric, as follows:

$$g_{ik} = g_{ki}$$

whereas a more general type of affine connection [5] is employed, such as

$$\Delta^{i}{}_{jk} = \Gamma^{i}{}_{jk} + \delta^{i}{}_{j}\lambda_{k}\,.$$

The Ricci tensor

$$R_{ik} = \Gamma^{s}{}_{ik,s} - \Gamma^{s}{}_{is,k} - \Gamma^{s}{}_{it}\Gamma^{t}{}_{sk} + \Gamma^{s}{}_{ik}\Gamma^{t}{}_{st}$$

is then expressed in terms of the above stated connection which leads to

$$R_{ik}(\Delta) = R_{ik} - \left(\lambda_{i,k} - \lambda_{k,i}\right).$$

The above expression depicts a Riemannian geometry with torsion and figures prominently within works on non-symmetric gravitational theory [6] [7] [8]. While it is unclear as to whether or not modifying the law of gravitation for the dark matter field would prove beneficial insofar as improving the understanding of dark matter, what is important here is to demonstrate that a torsional field does not violate general relativity as the previous equation decomposes into

$$R_{ik} = 0$$

as well as

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$$^{*}R_{ik,j} + {}^{*}R_{kj,I} + {}^{*}R_{ji,k} = 0$$

where

$$^{*}R_{ik} = \lambda_{i,k} - \lambda_{k,i} .$$

within this context, the permutated non-symmetric portion of the Ricci tensor is taken to be synonymous with a circulating current (the dark matter field) and the vanishing of the symmetric portion are the field equations of general relativity consisting of ten components in a four dimensional space (which also justifies the choice of the symmetric metric tensor). By way of this analysis, it would thus be apparent that a Riemannian geometry with torsion is not inconsistent with general relativity.

Summarizing the conception of dark matter as laid out here and its role in the rotational dynamics of the Galaxy, we have according to this very elementary model a rotationally dynamic field that is indistinguishable from the vacuum. Seeking to clarify the role and possibilities of such a construct raises the question as to the actual nature of dark matter. Most notably, the role of the Weakly Interacting Massive Particle (WIMP) is brought into question as it is not clear from that model just how the WIMPs are able to self-calibrate their distribution in such a fashion as to result in flat rotation-curves. While not doubting the intelligence inherent within physical processes, it seems excessively optimistic to presume that nature would have the innate capacity to assign a density-distribution to the WIMPS in just such a fashion, possibly even including an increase in density with the increase in radius from the center of the Galaxy, to result in the peculiarities observed in galactic rotation, most notably, the lack of centrifugal ejection of stars from galaxies, which in this model would be seen as the lack of actual motion of the star with respect to the space itself, given that space has now taken on the character or texture of a hydrodynamic vortex. This circumstance would also require the appearance of the spiral arms as is common in these conditions (The spiral-arms do present as quiet-zones common in regards to intersecting wave-like phenomena, as the dark-matter vortex appears to be identical to a hydrodynamic vortex, and in such a case the possibility exists of no forces acting within this region.). On the other hand, a particle that is an inherent part of the vacuum, such as virtual particles, would seem to offer a viable alternative to the WIMP while avoiding some of the pitfalls of such a model. It may also be possible to view the flat rotation-curves as a direct reflection of dark matter as a quantized field, given that the field displays characteristic features of (quantum) entanglement (The possibility of this could well assist in the determination of the actual nature of dark matter). The behavior of test-bodies in such a field would be analogous to that of bodies placed within a centrally symmetric accelerating field of force, where motions are strictly determined by the nature of the field itself. Presumably, the dark matter would be the result of the Big Bang and that would of course imply that the singularity possessed angular momentum, and the dark matter field would then be rotationally dynamic due to angular momentum conservation, but this is merely conjecture based on an interesting model. In any event, this model would suggest that the rotational dynamics of the Galaxy is strictly determined by dark matter and that baryonic mass plays no role whatsoever in this regard. Also, Newtonian dynamics will be of no use in determining baryonic mass as per the previous statement. In closing, however, one of the most interesting possibilities of a dark matter vortex, would be that for a spiral galaxy, the concentration of mass at the galactic center would naturally occur when a baryonic particle cloud intersects a dark matter vortex, and then immediately concentrates to the center of the vortex and then distributes to the spiral arms, where a concentration of particulate matter facilitates the formation of stars. The galactic bulge, with the concentration of stellar activity and black-hole formation, would seem to naturally follow.

Summarizing the results of this section, we have after employing the basic equation developed within the planetary study, a scenario where it follows that the effect being measured, *i.e.*, dark matter, not only forms a homogeneous sphere with torsional properties but would also have to be a quantized field, as it not only fills the vacuum, but is also indistinguishable from it. The rotation of space itself is found to be wholly consistent with the existence of the galactic flat rotation-curves (with the likelihood of the field also manifesting entanglement), the lack of centrifugal ejection of stars from galaxies, the development of the spiral arms (in lieu of the vortex formed by the dark matter field), as well as the concentration of stellar activity (including the black hole) that constitutes the galactic bulge that occupies the centers of spiral galaxies.

4. Conclusions

Starting from basic mechanical concepts, a relationship between the rate of rotation of a planet (on its own axis) and its surface gravity is developed which shows these quantities to be proportional within the Solar System. The proportionality is intended to reflect the consequences of planetary evolution from a protoplanetary disk when angular momentum conservation (*i.e.*, angular momentum transport) is taken into account. The equations demonstrate an exact correspondence to results anticipated from angular momentum conservation and may thus be used to clarify the role of the self-gravity of the debris field in angular momentum transport. Given the role of the proportionality factor, it is now possible to calculate the masses of three planets (of the Solar System) using their radii and rate of rotation (these are the planets where the angular momentum of the debris field was approximately conserved). This result would most certainly imply the evolution of the planets from a scenario such as we would have within the nebula hypothesis. This general methodology also provides a precise technique for determining planetary mass which proves to be equivalent to employing the corresponding Newtonian expression for calculating the surface gravity of a celestial body.

Applying the previously mentioned expression to the Galaxy, whereby it is not only possible to incorporate the Galactic flat rotation-curves as well as the basic gravitational acceleration of dark matter to determine dark matter mass, proved to be most interesting as the equations were not originally designed to be applied to problems in galactic dynamics. However, it was determined that this could be done in such an un-equivocal manner as to suggest a most natural application to this unusual problem. The results are certainly in excess of current estimates of dark matter mass at various radii, but the methods presume treating the problem as that of a homogeneous sphere (with torsion), and this would certainly be expected to lead to results vastly different than those approximations achieved by way of purely dynamical considerations. First of all, the concept of the Weakly Interacting Massive Particle is seen as untenable by way of this model. According to this viewpoint, dark matter is seen as infusing the vacuum contrary to the WIMP model, and geometry, along with the field concept, establish the criteria upon which galactic dynamics are most feasibly described. Galactic structure and dynamics are addressed, whereby it is clear that the appearance of the spiral arms, flat rotation-curves, the lack of centrifugal ejection of stars along with the activity at the galactic bulge of spiral galaxies, including black-hole formation, are readily addressed by way of this model.

Starting from a simple expression pertaining to the planets, it has proven to be feasible to address issues within that field as well as clarify the peculiarities observed with respect to the rotation of the Sun at its equator. However, with respect to applications on the galactic scale, it became necessary to employ a double application of the same expression which in turn led to a far richer interpretive environment that proved to be consistent with the observed circumstances of spiral galaxies.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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