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# Proton-Proton Collisions in View of Thermo-Statistical Approach

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## Abstract

The data of charged particles produced in proton-proton collisions extracted from Durham particle data group at energy ranges $\sqrt{s}$ = 6.3 - 17 GeV and at 0.9 - 7 TeV are investigated in the framework of Tsallis thermo-statistics and the Vlasov time dynamics. The analysis can describe the experimental data well all-over the considered energies and rapidity intervals. The variation of the collision parameters (chemical potential, entropy index and the time of evolution) is studied and discussed as a function of the final state temperature. According to the obtained result, a scenario, and a script of the time evolution for the particle production is simulated by the pp collision.

## Keywords

High Energy Particle Collisions, Statistical Thermodynamic Analysis, Phase Transition

## 1. Introduction

The measurable physical quantities of produced particles at high energy are helpful information to recognize the states of phase transition in nuclear matter. In a wide energy range, there are some common aspects of interaction dynamics. However, when Quark-Gluon Plasma is formed, the number of quanta increases to a large extent [1] [2] and can reveal much about the physics of particle production.

Over the past two decades, numerous experimental collaborations at the RHIC and LHC have collected a wealth of data on the transverse momentum spectra of particles, spanning from p-p collisions through Pb-Pb collisions at various energies and centralities. The study of the experimental data of hadron collisions or heavy ion collisions which are carried out in complex experiments like RHIC and LHC, is not a direct study for extracting the system information. Measurable quantities such as the transverse momentum spectra and the rapidity of the produced particles could provide us with the transverse flow velocity of produced particles, kinetic freeze-out temperature, and volume of the interacting system. These could also indicate the excitation degree of the interacting system [3]. To describe the transverse momentum spectra of the particles, numerous theoretical models have been considered such as the Tsallis statistics which has recognized much attention due to its successful applications to the particle transverse momentum spectra and the pseudo-rapidity distribution of charged particles produced in high energy proton-proton and heavy-ion collisions [4] [5] [6] [7].

On the other hand, the physics of particle reaction is a rapid dynamical process. The question of phase transitions in a dynamical system is still an open field of research. Particle collisions may contribute to this field at two points: the dynamics of the phase transitions in "small" systems, and the dynamics of the phase transitions in ultra-relativistic systems where the energy of the system is much higher than the rest mass of the produced particles. This will give a clue to understand the symmetry problem which is important to describe the equation of state inside a thermodynamic nuclear system. In addition, it may give some information on the natural structure of QCD.

In a recent published paper by our research group [5], we studied the signals of phase transition at high-energy proton-proton (pp) collisions. We focused on investigating the possible states of matter and the location of phase boundaries between hadronic gas and the quark-gluon plasma QGP. The boundary at the hadron freeze-out was also considered. Proton-proton collisions at a wide range of center of mass energies were used to examine the phase transition using (entropy-temperature) phase diagram. In this technique, we assumed local thermodynamic equilibrium at different intervals of rapidity space. The values of the critical temperatures were found at the boundaries of the phases with a quite clear description of the states. To be able to separate different phases, we need more thermodynamic observables, which serve as indicators for the corresponding phase. For this purpose, the observables must be related to properties of the system, which change when passing from one phase to another.

In the present work, we will use a recent thermo-statistical tool called "Tsallis statistics" which is a generalization of the Boltzmann-Gibbs (BG) distribution [6]. It is characterized by an additional parameter "$q$" that measures the degree of non-equilibrium of the system. It approaches the usual Boltzmann or Gibbs as the system comes to equilibrium. It has been successfully used in high-energy physics over the past few years to describe the physics of p-p collisions [7] [8].

We will throw some light at the states of phase transition by keeping track to the chemical potential-temperature ($\mu$-$T$) phase diagram. We will follow the Tsallis statistics [9] [10] [11] [12] to estimate the possible macroscopic variables found in the Tsallis distribution function that is appropriate for the non-equilibrium thermodynamic states. Moreover, the non-equilibrium thermodynamics will be considered by solving Vlasov time-dependent differential equation [13] [14] to predict the time at which the final state could be recognized. Based on that, the paper is organized as follows:

Section 1: Refers to the introduction

Section 2: Concerns the data acquisition and statistical analysis

Section 3: Time evolution of particle production

Section 4: Focuses on results and discussion

Finally, in section 5 we recount the conclusive remarks.

## 2. Data Acquisition and Statistical Analysis

Recently, many published articles have revealed that transverse momentum and rapidity distributions carry much information about the state and the nature of the sources producing the particles at the final states of high energy pp collision. Data in hand, represents the produced charged particles in pp collisions at high energy of 6.3, 7.7, 8.8, 12.3, 17.3 GeV [15] and at ultrahigh energy 0.90, 2.3 and 7.0 TeV [16] [17]. These were collected through private communication with Durham Particle Data Group [18]. From now and then, briefly we shall call them GeV group and TeV group. We will start first with examining the features of the produced particles along the whole range of the examined energy. Then will try to construct the time-dependent phase diagram ($\mu$-$T$) for the non-equilibrium system. In addition to the QGP phase, we will pay attention to other phases such as the pion condensation and the phase at which heavy bosons may be produced as well.

The LHC at the CERN has produced data of proton-proton collisions at center-of-mass energy (per nucleon pair) up to 14 TeV. Only data up to 7 TeV is already allowed to be handled by any of researchers outside the CERN members. As the collision energy increases, a much broader and deeper study of QGP could appear. It leads to a significant extension of the kinematic range in longitudinal rapidity and transverse momentum. A systematic study of charged hadron multiplicities is very important in understanding the production mechanism of hadrons in pp collision experiments. Moreover, in a forthcoming article, we will deal with the data of heavy-ion depending crucially on the comparison with the present results of proton-proton (pp) as smaller collision systems.

The bulk of created matter in high-energy collisions can be quantitatively described in terms of hydrodynamic and thermo-statistical models [19] [20], which are governed mainly by the chemical freeze-out temperature and the baryochemical potential. These models provide an accurate description of the data over a wide range of center-of-mass energies [21].

Tsallis distribution could be formulated from several standard distributions as mentioned before. It is a general form of the Boltzmann-Gibbs (BG) distribution. The effectiveness of Tsallis distribution appears clearly in describing the thermal fluctuation of the system especially when this system has several sources may exist at high energy hadrons collision. The used Tsallis form is one of the simplest forms to describe the probability density function of transverse momentum and rapidity [9] [22]. Let us apply the Tsallis double differential distribution $\frac{\mathrm{d}^2 N}{\mathrm{d}y\mathrm{d}P_T}$ defined as:

$$\frac{\mathrm{d}^2 N}{\mathrm{d}y\mathrm{d}P_T} = \frac{gV}{(2\pi)^2} P_T \sqrt{P_T^2 + m_0^2} \cosh y \left[1 + \frac{q-1}{T}\left(\sqrt{P_T^2 + m_0^2} \cosh y - \mu\right)\right]^{-\frac{q}{q-1}} \quad (1)$$

where, "$g$" is the degeneracy factor, "$V$" is the volume of the source of particle emission, "$T$" is the temperature representing the average temperature in a few sources which can describe local equilibrium states, and "$q$" is entropy index which represents the degree of non-equilibrium among different states. The value of "$q$" larger than 1 indicates a system far from the equilibrium state [23] and approaches equilibrium as "$q$" tends to 1. "$\mu$" is the chemical potential and $m_0$ is the rest mass of the produced hadron due to quark recombination during the interaction. For simplicity and to be appropriate to the fitting technique, we use the following distribution form:

$$f(P_T, y) = CP_T \sqrt{P_T^2 + m_0^2} \cosh y \left[1 + \frac{q-1}{T}\left(\sqrt{P_T^2 + m_0^2} \cosh y - \mu\right)\right]^{-\frac{1}{q-1}} \quad (2)$$

While $C = \frac{gV}{(2\pi)^2}$ is the normalization constant and the factor $\frac{q}{q-1}$ can be replaced by $\frac{1}{q-1}$ as "$q$" value is usually close to 1. Then to fit the transverse momentum distribution of produced hadrons, we used Equation (2) to deduce the values of $\mu$, $T$ and $q$ as fitting parameters. We apply this procedure to proton-proton collisions at GeV and TeV groups at all possible rapidity intervals. Accordingly, we get for each rapidity interval three fitting parameters to describe its transverse momentum distribution. The results of the fitting will be discussed in the following section.

## 3. Time Evolution of Particle Production in View of Vlasov Equation

From another thermo-statical point of view, giving more attention to the physics of nonequilibrium thermodynamic systems, we solve the Vlasov time-dependent equation [13]:

$$\frac{\mathrm{d}f(t,r,P)}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \frac{\boldsymbol{P}}{m} \cdot \nabla_t f + \nabla_t U \cdot \nabla_t f \quad (3)$$

where $f(t,r,P)$ is the time-dependent wave function, while $U(r)$ is a scalar potential acting among the particles. We assume that the created quarks and gluons are produced in a spherical cloud with expandable volume according to the thermodynamic environment. We will add some approximations so that we can solve Equation (3). Let us consider a pre-equilibrium state where the time derivative $\mathrm{d}f/\mathrm{d}t$ may be approximated as $(f - f_0)/t_c$, where $f_0$ is the equilibrium distribution, and $t_c$ is the time interval required by the system to approach the equilibrium state $f_0$. So, we replace $f$ with $f_0$ in the RHS of Equation (3). Moreover, let us consider the particles as almost free so that we neglect the potential $U$ in this stage of approximation. The solution will be a power series, and for simplicity, we will be satisfied by the first order only. So that Equation (3) becomes:

$$f_1 = f_0 + t_c \frac{\boldsymbol{P}}{m} \cdot \nabla_t f_0 \tag{4}$$

$$f_1(r) = f_0(r) + t_c \frac{p_t}{m} \cos\theta \frac{\partial f_0(r)}{\partial r} \tag{5}$$

For the problem at hand, we consider the equilibrium distribution as transverse momentum distribution:

$$f_0(P_t, r) = \gamma \exp\left[-\frac{P_t}{T(r)}\right] \tag{6}$$

As the radial dependence of $f_0(r)$ is included in the temperature gradient $\partial T(r)/\partial r$.

Assuming Gaussian density for the quark-Gluon cloud with the form of:

$$\rho(r) = A(\pi r)^{-\frac{3}{2}} \exp\left(-\frac{r^2}{R^2}\right) \tag{7}$$

with $R$ is the virtual radius of the cloud and $r$ is the distance measured from the center of the cloud. Now the temperature is very sensitive to the quark-gluon density $\rho(r)$. So that the temperature gradient is written as:

$$\frac{\partial T(r)}{\partial r} = C \frac{\partial \rho(r)}{\partial r} \tag{8}$$

$C$ is constant including the constants of Equation (7) and the average kinetic energy of the particles in the cloud. Then, Equation (5) could be written as:

$$f(r) \approx f_0(r) + t_c \frac{P_t}{m} m \cos\theta \frac{\partial f_0}{\partial T} \frac{\partial T(r)}{\partial r} \tag{9}$$

Equation (9) is the power series solution of Equation (3) with only a first order approximation. The first term of Equation (9) is the zero-order approximation. It represents the equilibrium distribution. The next term is the first correction term. The value of $t_c$ is found by comparing Equation (9) with the experimental $p_t$ distribution at each rapidity interval.

## 4. Results and Analysis

In this work, we used the Tsallis statistics to fit with the experimental data of the double differential distribution $\mathrm{d}^2N/\mathrm{d}y\mathrm{d}p_t$ to obtain the values of the three parameters $T$, $\mu$ and $q$ at each rapidity interval for the pp collisions, each at corresponding center of mass energy $\sqrt{s_{NN}}$ for the GeV and TeV groups.

**Figure 1** shows the dependence of the chemical potential $\mu$, on the temperature $T$. One would notice that the values of the chemical potential change with the temperature follow nicely a fitting straight line given as follows:

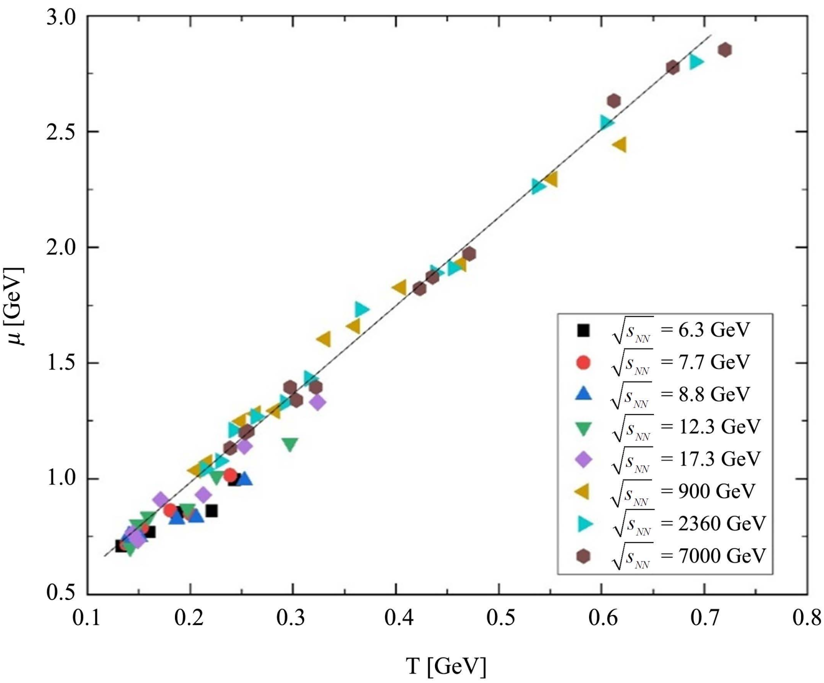



**Figure 1.** The $\mu$-$T$ phase diagram of particles produced in the final state of pp collisions at center of mass energies GeV and TeV groups. The solid line is a linear fit to the data.

$$\mu = bT + \mu_0 \tag{10}$$

with fitting parameters, $b = 3.8$ and $\mu_0 = 0.62$.

$\mu_0$ is a reference chemical potential and $b$, the slope of the line, reveals a constant chemical potential change per unit change of the temperature of the nuclear medium. This fact may point to a continuous transition, in analogous to a second order of QCD phase. This feature might also tell that as the centrality of interaction increases, more energy would be deposited into the system, leading to a rise of its temperature and creation of more particles. This could also be physically accepted on the bases that in the phase through which the nuclear matter is passing in the considered range of energy, more kinetic energy is gained by the created particles, thus increasing the free energy of the system for higher interaction energy. This occurs when chemical potential exceeds much more than the pion mass and corresponds to the pion condensation [24].

However, one may observe from **Figure 1** that some data points do not stick well to the fitting line in the temperature range ~0.2 to 0.3 GeV. This may be attributed to one or more than one factor. These factors could include mixed phases that exist in this temperature range, or more than one species of particles are produced in this region or unbalanced state between the absorbed and emitted energy of the particles content (e.g. a latent heat energy of releasing more sea quarks or even condensate of sea quarks recombining to form particles).

In **Figure 2**, we see that the temperature decreases gradually with the rapidity from the central collision region with low rapidity. This is common for the collisions at all center of mass energies considered in this study. The temperature

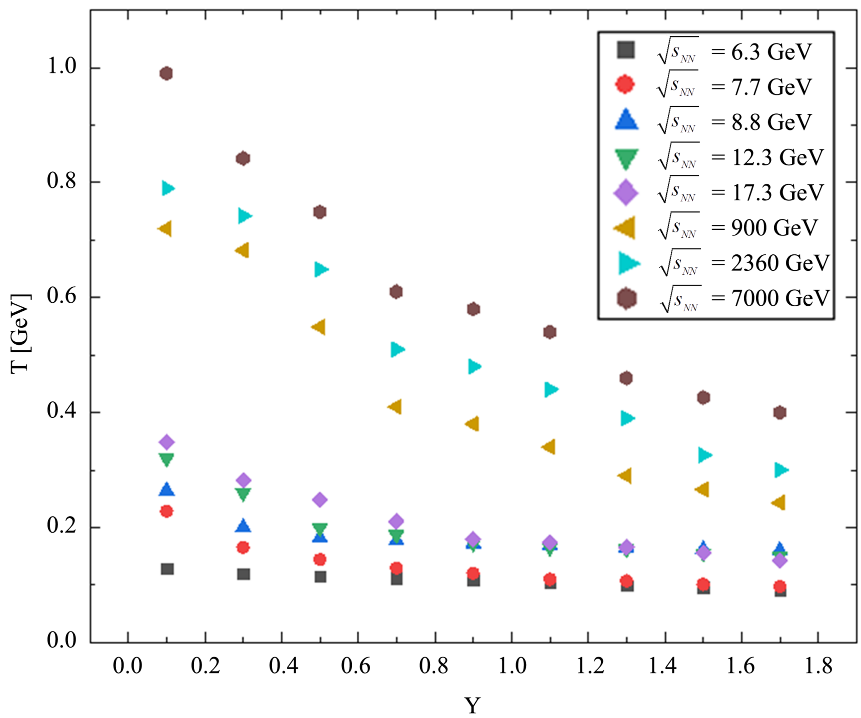
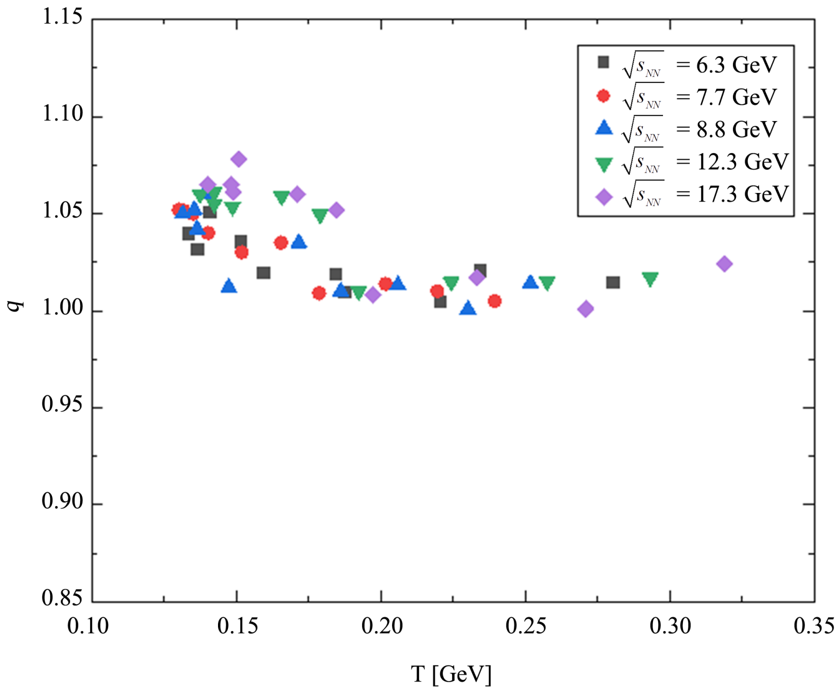



**Figure 2.** The variation of the temperature against the rapidity interval of particles produced in pp collision for the GeV and for TeV groups.

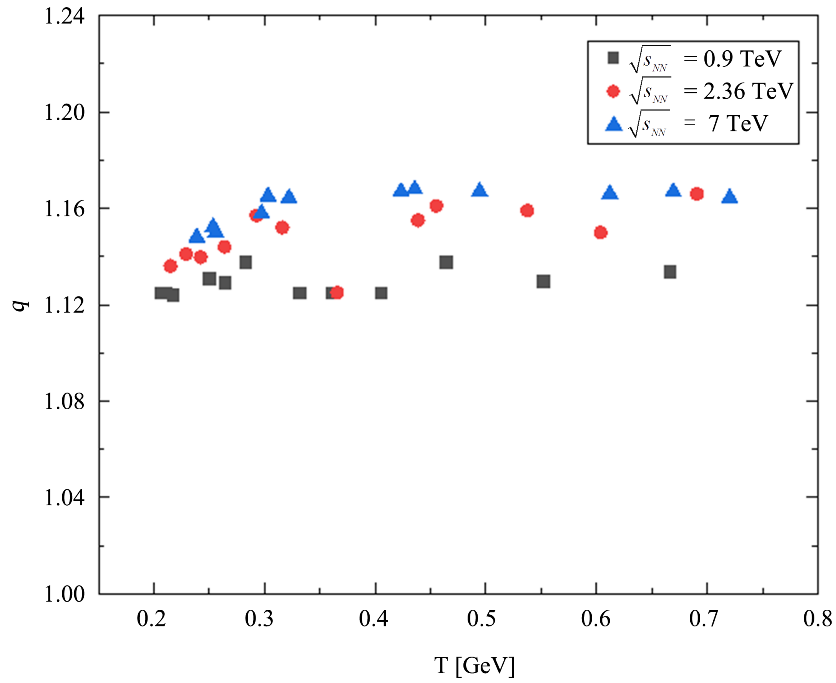
falls more rapidly with the rapidity in TeV region than in the GeV region. The $T$-$Y$ curves shifts up for the TeV group where more energy is available. One may come up with the fact that particles produced in the most forward direction (high rapidity intervals) are produced at low temperature and far from equilibrium. As particles spread out toward the central region with small rapidity, the final state is attained with larger temperature and approach the pre-equilibrium. This means that as time goes on, the particles are produced in larger cone angle (small rapidity) and due to the successive quarks-gluons interactions both the temperature and entropy increase, hence the system approaches equilibrium.

Let us now study the dependence of the entropy index $q$ on the temperature as found by fitting of Tsallis distribution with the experimental data. The results are presented in Figure 3(a) and Figure 3(b).

What is observed in this figure might be fairly consistent with what came out from Figure 2. In Figure 3(a), at the beginning of particle emission, the system has relatively large $q$ values for temperature below 0.2 GeV, which refers to a state far from equilibrium. As time goes on, the temperature rises, and the system approaches equilibrium ($q \approx 1$). This trend looks the same for the data of center of mass energies at the GeV group. In Figure 3(b), the $q$-$T$ relations show steady behavior for the TeV group far from equilibrium with larger values of $q$ at higher interaction energies. In the temperature ranging between 0.2 and 0.3 GeV, $q$ shows lower values tending to be somehow relatively closer to equilibrium.




(a)




(b)

**Figure 3.** The change of the non-equilibrium parameter $q$ with the temperature $T$ estimated as fitting parameters in Tsallis statistics for particles produced in pp collision for the GeV group (a) and TeV group (b).

Reading the data of Figure 1 and Figure 2 and combining with that of Figure 3, one found that particles produced in the most forward direction (high rapidity intervals) are produced at low temperature and far from equilibrium. As particles spread out toward the central region with small rapidity, the final state is detected with larger temperature and approach the pre-equilibrium. This means that as time goes on, the particles are produced in larger cone angle (small rapidity) and due to the successive quarks-gluons interactions both the temperature and entropy increase, hence the system approaches the equilibrium. This is clear in Figure 3 where: at the beginning of particle emission, the system appears with large $q$ value far from equilibrium with low temperature. As time goes on, the temperature increases, and the system approaches the equilibrium. The same trend fits the data al all center of mass energies at few GeVs. The time taken by the system to approach the equilibrium state, $t_c$ was found by comparing Equation (9) with the experimental values of the transverse momentum, $p_t$ distribution at each rapidity interval.

Solving the Vlasov time-dependent Equation (3) using the power series with only a first order approximation as Equation (9): The first term of this equation is the zero-order approximation that represents the equilibrium distribution. The next term is the first correction term. We did that for just one reaction of the investigated data, as a sample of a large amount of data. Figure 4 presents the transverse momentum distribution of the produced hadrons in p-p collisions at 6.3 GeV for different rapidity region. Different colored points belong to their corresponding rapidity value while lines are their fittings using the first order of Vlasov equation.

The above figure tells us that Vlasov equation succeeded very well in describing the experimental data at almost all the regions of rapidity. Accordingly, the deduced values of T and $t_c$ are listed in Table 1.

The values that show up in the above table say that the temperature rises up as the rapidity goes to the center. So that particles emitted in the forward direction are created in early time with low temperature and far from equilibrium (longer $t_c$).

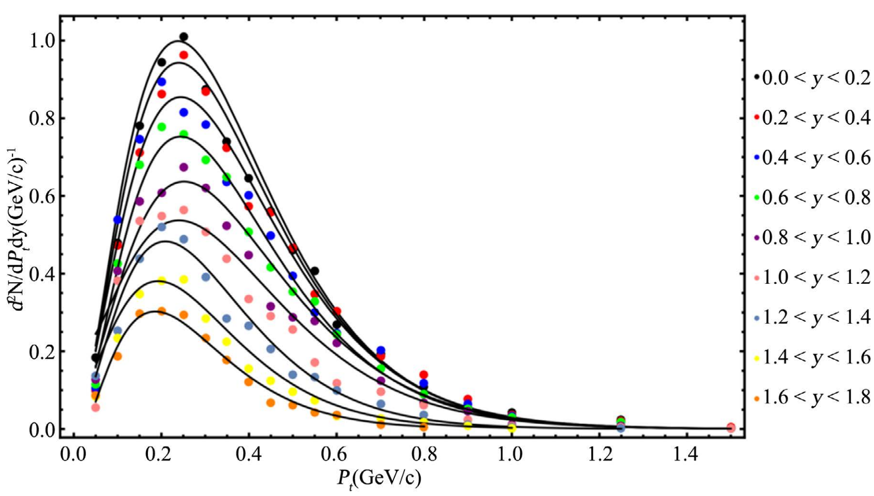



Figure 4. The transverse momentum distribution of the produced hadrons through the proton-proton collision at center of mass energy equals 6.3 GeV of produced hadrons at the rapidity region from 0 to 1.8 which has fitted using the first order approximation $f_1$ of the particle spectrum which described by Vlasov equation.

**Table 1.** Values of the parameters $T$ and $t_c$ for pions produced in pp collisions at 6.3 GeV.

| Rapidity interval | $T$ [GeV] | $t_c$ (fm/c) |
|---|---|---|
| $0.0 < y < 0.2$ | 0.129 | 3.46 |
| $0.2 < y < 0.4$ | 0.120 | 15.0 |
| $0.4 < y < 0.6$ | 0.115 | 20.3 |
| $0.6 < y < 0.8$ | 0.112 | 28.8 |
| $0.8 < y < 1.0$ | 0.110 | 32.5 |
| $1.0 < y < 1.2$ | 0.104 | 39.5 |
| $1.2 < y < 1.4$ | 0.100 | 48.0 |
| $1.4 < y < 1.6$ | 0.096 | 55.0 |
| $1.6 < y < 1.8$ | 0.091 | 66.0 |

In other words, the quarks-gluons interact together in a small volume of a cloud. The quarks passing in parallel trajectories and having nearby momentum can recombine together forming hadrons emitted in the forward direction with a relatively low temperature in a non-equilibrium state. The cloud expands gradually with increasing the frequency of collisions and the emission is produced at a wider solid angle, *i.e.* (small values in rapidity space) with increasing temperature. Hence, the system approaches an equilibrium state. More expansion of the cloud and more frequency of particle collisions may cause the stability of the temperature that reaches a maximum value.

## 5. Conclusions

The analysis of the experimental data of pp collisions at wide energy range was studied in the view of Tsallis statistics to find the values of the fitting parameters $T$, $\mu$ and $q$ at each center of mass energy $\sqrt{s_{NN}}$ for the GeV and TeV groups. We found the following consequences:

For the data of the energy (GeV region), the chemical potential $\mu$ changes linearly with the temperature $T$ of particle emission. The trend is verified all over the whole center of mass energy range. Regular $\mu$-$T$ relationship for $T$ above 0.20 GeV could indicate one phase of nuclear matter in which one species of produced particle is dominating created according to one mechanism.

The linear increase in the $\mu$-$T$ phase diagram refers to a continuous transition, in analogous to a second order of QCD phase. However, below $T$ of 0.20 GeV is attributed to the existence of more than one phase of nuclear medium including more than one mechanism taking part in particle creation.

For the data of the ultra-high energy (TeV region), we found the following remarks:

In the whole range of temperature, the chemical potential of the nuclear medium increases with temperature and can be described by straight lines having almost equal slopes. This feature means that as the centrality of interaction increases, more energy deposit would be added to the system leading to rise of its temperature and creation of more particles. This could be physically accepted on the bases that as p-p interaction energy increases, more kinetic energy is gained by the created particles, thus increasing the free energy of the system. This occurs when chemical potential exceeds much more the pion mass, corresponding to the pion condensation.

The temperature-rapidity graphs show that the temperature decreases gradually with the rapidity. $T$ is maximum at the zero rapidity (central part). This is common for the collisions at all energy range. The temperature falls more rapidly with the rapidity in TeV region than in the GeV region. The $T$-$Y$ curves shift up for the TeV group, because more energy is available.

In all cases, we found that the hadronic system is produced in a non-equilibrium state which is characterized by the entropy index more than 1. The temperature decreases gradually with the rapidity from the central collision region with low rapidity. This is common for the collisions at all center of mass energies considered in this study. The temperature falls more rapidly with the rapidity in TeV region than in the GeV region. The $T$-$Y$ curves shift up for the TeV group where more energy is available.

In all cases, we found that the hadronic system is produced in a non-equilibrium state which is characterized by the entropy index $q > 1$. Meanwhile $q \to 1$ as the system approaches equilibrium.

It is found that the entropy index $q$ is relatively high below temperature 0.20 GeV indicating the presence of more than one phase of nuclear matter in other words, more than one mechanism of particle creation with more than one species of crated particles in this region.

The quark gluons are interacting in the early time interval, and then the recombined forming fast pions emitted in the forward direction.

On the other hand, particles produced in the most forward direction (high rapidity intervals) are produced at low temperature and far from equilibrium. As particles spread out toward the central region with small rapidity, the final state is detected with higher temperature and approach the pre-equilibrium. This means that as time goes on, the particles are produced in larger cone angle (small rapidity) and due to the successive quarks-gluons interactions both the temperature and entropy increase; hence, the system approaches the equilibrium.

The solution of the Vlasov equation was very successful in specifying the time of formation of the final state before approaching the equilibrium.

It is found that the temperature increases as the rapidity goes to the center. So that particles emitted in the forward direction are created in early time with low temperature and far from equilibrium (large values of $t_c$).

The values of the time parameter with both the temperature and the rapidity can clearly describe the reaction mechanism as follows:

Quarks and gluons interact together in a small volume of a cloud. The quarks passing in parallel trajectories and having nearby momentum can recombine together forming hadrons emitted in the forward direction with a relatively low temperature in a non-equilibrium state. The cloud expands gradually with increasing the frequency of collisions and the emission is produced at a wider solid angle, *i.e.* (small values in rapidity space) with increasing temperature. The system approaches toward an equilibrium state. More expansion of the cloud and more frequency of particle collisions may cause the stability of the temperature that reaches the maximum.

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## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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