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Quantum Physics Has a New, and Remarkable, Expansion

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Abstract

Canonical quantization has taught us great things. A common example is that of the harmonic oscillator, which is like swinging a ball on a string back and forth. However, the half-harmonic oscillator blocks the ball at the bottom and then it quickly bounces backwards. This second model cannot be correctly solved using canonical quantization. Now, there is an expansion of quantization, called affine quantization, that can correctly solve the half-harmonic oscillator, and offers correct solutions to a grand collection of other problems, which even reaches to field theory and gravity. This paper has been designed to introduce affine quantization: what it is, and what it can do.

Keywords

Canonical and Affine Quantization, Different Simple Examples, Comments Regarding Field Theory and Gravity

1. Selected Features of Quantum Mechanics

There are rules that the classical variables, e.g., p & q, need to follow when dealing with quantization. The most important requirement is that $-\infty < p \,\&\, q < \infty$. Certain functions of p & q, are often accepted variables provided that they follow Poisson's bracket, which means that $\overline{p} = f(p,q)$ while $\overline{q} = g(p,q)$, obeys

$$\{\overline{q},\overline{p}\} \equiv \left(\partial g(p,q)/\partial q\right)\left(\partial f(p,q)/\partial p\right) - \left(\partial g(p,q)/\partial p\right)\left(\partial f(p,q)/\partial q\right) = 1. \quad (1)$$

While we might find that $\{q,p\} = 1 = \{\overline{q},\overline{p}\}$, along with $-\infty < p \,\&\, q < \infty$, may now find $-\infty < \overline{p} \,\&\, \overline{q} < \infty$. They both have passed the requirements to create the quantum operators, $p \Rightarrow P \,\&\, q \Rightarrow Q$ along with $\overline{p} \Rightarrow \overline{P} \,\&\, \overline{q} \Rightarrow \overline{Q}$, which they obey, $[Q,P] = i\hbar\mathbb{1}$ and $[\overline{Q},\overline{P}] = i\hbar\mathbb{1}$. While the bar-pair and the no bar-pair can be very different, they were designed for the equality of $\overline{H}(\overline{p},\overline{q}) = H(p,q)$, but find a *non*-equality in $\overline{\mathcal{H}}(\overline{P},\overline{Q}) \neq \mathcal{H}(P,Q)$. The correct basic operators are the only ones to lead to valid results. How can we find the proper quantum operators? We now show how that is possible.

We now introduce coherent states for canonical quantization which are defined by

$$|p,q\rangle = e^{-ipQ/\hbar} e^{iqP/\hbar} |\omega\rangle \equiv U(p,q)|\omega\rangle,$$

where we choose $[Q+iP/\omega]|\omega\rangle = 0$, which leads to $\langle\omega|P|\omega\rangle = \langle\omega|Q|\omega\rangle = 0$, while $\langle\omega|\omega\rangle = 1$. It follows that $U(p,q)^\dagger (aP+bQ)U(p,q) = a(P+p)+b(Q+q)$. We will find that coherent states provide a useful link between the classical and quantum realms, specifically, a link between a classical Hamiltonian $H(p,q)$ and the correct quantum Hamiltonian $\mathcal{H}(P,Q)$.

We next introduce the relation

$$\begin{aligned} H(p,q) &= \langle p,q|\mathcal{H}(P,Q)|p,q\rangle = \langle\omega|\mathcal{H}(P+p,Q+q)|\omega\rangle \\ &= \mathcal{H}(p,q)+\mathcal{O}(\hbar;p,q). \end{aligned} \tag{2}$$

At this moment, the term $H(p,q)$ may have a hidden dependence on $\hbar$, but that will disappear very soon. Choosing the quantum Hamiltonian as a general polynomial, then the term $\mathcal{H}(p,q)$ is independent of $\hbar$. Therefore, letting $\hbar \to 0$, it follows that $\mathcal{O}(\hbar;p,q) \to 0$, and $H(p,q)$ looses any $\hbar$ contribution. Therefore, we find that now $H(p,q) = \mathcal{H}(p,q) \Rightarrow \mathcal{H}(P,Q)$, showing how to pass from a classical function to the proper quantum function. It is still possible that a special ordering of $\mathcal{H}(P,Q)$ is needed, but this is a standard issue. The final result is well known, but it has been derived, because there will be a similar story below in which an operating order is a real issue, which needs to be understood.

The result in the last paragraph chose the classical variables, but how can that be the proper choice that leads to the correct Hamiltonian? Dirac has stated that the proper phase-space variables should be Cartesian, *i.e.*, $d\sigma^2 = \omega^{-1}dp^2 + \omega dq^2$. The phase space does not include that relation, and a Cartesian requirement is not to be found there. Indeed, this rule is semi-classical needing that $\hbar > 0$. Using coherent states, a special metric, known as the Fubini-Study metric, leads to

$$d\sigma^2 = 2\hbar\left[\|d|p,q\rangle\|^2 - |\langle p,q|d|p,q\rangle|^2\right] = \omega^{-1}dp^2 + \omega dq^2, \tag{3}$$

which has become Cartesian. The minus sign term is designed to eliminate any phase factor, *i.e.* $|p,q;f\rangle = e^{if(p,q)}|p,q\rangle$, which is good physics.

Briefly stated, the coherent states have *created* the correct variables!

2. A Standard Approach to Canonical Quantization

The classical variables must be $-\infty < p\,\&\,q < \infty$ for a good reason. This permits the basic operators to be self-adjoint, *i.e.* $P^\dagger = P$ & $Q^\dagger = Q$, which is very important.

To understand what is self-adjoint, consider this simple example. For simplicity our example requires that

$$\int_a^b (\mathrm{d}/\mathrm{d}x)\left[f(x)g(x)\right]\mathrm{d}x = f(b)g(b) - f(a)g(a)(=0), \tag{4}$$

which we ask to be zero. This can be by choosing $g(a)=g(b)=0$, but that leaves $f(a)$ and $f(b)$ free to be anything. That is the case where $P^\dagger \neq P$. However, if we need $P^\dagger = P$, that requires $f(a)=f(b)=0$ as well. For canonical quantization, the wave functions span the whole real line. In that case, where $P=-i\hbar(\partial/\partial x)$, then

$$\int_{-\infty}^{\infty} P\left[\Phi(x)^*\Psi(x)\right]\mathrm{d}x = -i\hbar\left[\Phi(x)^*\Psi(x)\right]\Big|_{-\infty}^{\infty} = 0 \tag{5}$$

since Hilbert space requires finite elements, e.g., $\int_{-\infty}^{\infty}|\Psi(x)|^2\,\mathrm{d}x < \infty$, which forces each wave function, $\Psi(x)$, to vanish at $x=\pm\infty$, *i.e.*, $\Psi(-\infty)=\Psi(\infty)=0$.

What happens if the available space *does not span* the entire real line? Canonical quantization has adopted including a "virtual infinite wall", or just "v-wall" for short, which is designed to squash wave functions to zero imitating, but not removing, and simply ignoring that portion of space.

As an example of that procedure, let us examine the harmonic oscillator with the classical Hamiltonian $H=\left(p^2+q^2\right)/2$. But now we require that $0<q<\infty$, for which it is now called the half-harmonic oscillator. Its classical behavior is that of a ball, hanging on a string, bouncing backwards at $q=0$. Now, using canonical quantization, the quantum Hamiltonian is still $\mathcal{H}=\left(P^2+Q^2\right)/2$, but the "v-wall" is located throughout $q<0$. The region where $q>0$ is free to accept only the odd, not even, solutions of the harmonic oscillator that become zero at $q=0$, and then join those eigenfunctions, with the portion that has been squashed, to ensure a full, and continuous function. After two derivatives it will again be zero at $q=0$, and thus there could be a second continuous function. That would be the accepted story in this case.

However, the first derivative, on the way toward the second derivative, is *not* a continuous function which immediately implies that the second derivative reaches infinity at the point $q=0$. Such a wave function cannot be part of any Hilbert space, because its normalization would be infinity. This problem will be correctly solved in the following section.

Another commonly studied example, using canonical quantization, is called "The particle in a box". and its classical Hamiltonian is simply $H=p^2/2$. In this example, the box consists of $0<q<L<\infty$. In this case, the customary procedure is to adopt *two* "v-walls", one throughout $q<0$ and the other throughout $q>L$. By shifting the box to the space $-b<q<b<\infty$, for convenience, this example will be considered in the next section.

3. An Introduction to Affine Quantization

The focus is to seek examples without a full line coordinate space, such as the first example.

Choosing $0<q<\infty$

In this example, the point $q=0$ is removed, and then discarding $-\infty<q<0$, while keeping $0 < q < \infty$. Immediately, it follows that $P^\dagger \neq P$. That is accepted, and now introduce the dilation variable and also its coordinate, *i.e.*, $d = pq \,\&\, q > 0 \Rightarrow D = \left[P^\dagger Q + QP\right]/2 = D^\dagger \,\&\, Q = Q^\dagger > 0$. Hereafter, D & Q become the principal operators, not P & Q, although P will still have an important role to play.

The first usage of these variables is by examining the kinetic factor for classical Hamiltonians, *i.e.*, $p^2 = d^2/q^2$, in which

$$d^2/q^2 \Rightarrow D\left(Q^{-2}\right)D = P^2 + (3/4)\hbar^2/Q^2 = \hbar^2\left[-\left(\mathrm{d}^2/\mathrm{d}x^2\right) + (3/4)/x^2\right]. \tag{6}$$

We now introduce coherent states for affine quantization which are defined by

$$|p;q\rangle = \mathrm{e}^{ipQ/\hbar}\mathrm{e}^{-i\ln(q)D/\hbar}|\beta\rangle \equiv U(p;q)|\beta\rangle,$$

where we choose $\left[(Q-\mathbb{1}) + iD/\beta\hbar\right]|\beta\rangle = 0$, which leads to $\langle\beta|Q|\beta\rangle = 1$ and $\langle\beta|D|\beta\rangle = 0$, while $\langle\beta|\beta\rangle = 1$. It follows that $U(p;q)^\dagger(aD + bQ)U(p;q) = a(D + pqQ) + b(qQ)$. We will find that coherent states provide a useful link between the classical and quantum realms, specifically, a link between a classical Hamiltonian-like $H'(pq,q)$ and the correct quantum Hamiltonian-like $\mathcal{H}'(D,Q)$.

We now introduce the relation

$$\begin{aligned} H'(pq,q) &= \langle p;q|\mathcal{H}'(D,Q)|p;q\rangle = \langle\beta|\mathcal{H}'(D+pqQ,qQ)|\beta\rangle \\ &= \mathcal{H}(pq,q) + \mathcal{O}'(\hbar;pq,q). \end{aligned} \tag{7}$$

The result in the last paragraph chose the classical variables, but how can that be the proper choice that leads to the correct Hamiltonian? No longer should they be Cartesian, but, using the Fubini-Study metric, we find that they are constant negative curvature with a value of $-2/\beta\hbar$, specifically it is $\mathrm{d}\sigma'^2 = (\beta\hbar)^{-1}q^2\mathrm{d}p^2 + (\beta\hbar)q^{-2}\mathrm{d}q^2$. Once again, phase space does not include that relation, and now, our constant negative curvature requirement is not to be found there. As was the prior case, this rule is semi-classical, again needing that $\hbar > 0$.

Once again, and briefly stated, the affine coherent states have *created* the correct variables! This treatment of canonical and affine quantization has been deliberately designed to be very similar to prove they belong together.

These equations and their variations are a fascinating feature of affine quantization. As a first example here, we adopt the classical Hamiltonian $H = p^2/2 + V(q)$, with $0 < q < \infty$, it follows that the quantum Hamiltonian becomes

$$\mathcal{H}(P,Q) = \frac{1}{2}\left[P^2 + (3/4)\hbar^2/Q^2\right] + V(Q). \tag{8}$$

As two important examples, we choose the harmonic oscillator and the half-harmonic oscillator. Adopting $m = \omega = 1$ for simplicity, the classical harmonic oscillator Hamiltonian is $H_{ho} = \left(p^2 + q^2\right)/2$, and using canonical quantization, the quantum Hamiltonian becomes $\mathcal{H}_{ho} = \left(P^2 + Q^2\right)/2$. The eigenvalues of this operator are well known, and are $E_{n\,ho} = \hbar(n + 1/2)$, for $n = 0,1,2,\cdots$. It is quite unique in having *equal spacing*, here at $1\hbar$.

Now, the half-harmonic oscillator has the identical classical Hamiltonian, $H_{h-ho} = \left(p^2 + q^2\right)/2$, *but now*, $0 < q < \infty$. This forces any particle to bounce off a "virtual-wall" at $q = 0$, and immediately turn around. The quantum Hamiltonian for the half-harmonic oscillator is given by $\mathcal{H}_{h-ho} = \left[P^2 + (3/4)\hbar^2/Q^2 + Q^2\right]/2$. The eigenvalues of this operator have been determined, and remarkably, they are $E_{n\,h-ho} = 2\hbar(n+1)$, for $n = 0,1,2,\cdots$, and again they are *equally spaced*, now at $2\hbar$ [1]. This usage of the number "2 = twice" is everywhere, even in the fact that $-\infty < q < \infty$ is "twice" $0 < q < \infty$! These remarkable results seem as if they both act like "pals".

It is well established that the harmonic oscillator has had a valid quantization, and now there is no reason to doubt that the half-harmonic oscillator also has a valid quantization.

4. A Rich Realm of Examples Using Affine Quantization

The harmonic operator story opened the door to a great many more Hamiltonians and their promotions to valid operators. For example,

1) $0 < q < \infty \;\&\; H = \frac{1}{2}p^2 + V(q) \Rightarrow \mathcal{H} = \frac{1}{2}\left[P^2 + (3/4)\hbar^2/Q^2\right] + V(Q)$

2) $-b < q < \infty \;\&\; H = \frac{1}{2}p^2 + V(q) \Rightarrow \mathcal{H} = \frac{1}{2}\left[P^2 + (3/4)\hbar^2/(Q+b)^2\right] + V(Q)$

3) $-b < q < b \;\&\; H = \frac{1}{2}p^2 \Rightarrow \mathcal{H} = \frac{1}{2}\left[P^2 + \hbar^2\left(2Q^2 + b^2\right)/\left(b^2 - Q^2\right)^2\right]$

4) $0 < b < |q| < \infty \;\&\; H = \frac{1}{2}p^2 + V(q)$
$\Rightarrow \mathcal{H} = \frac{1}{2}\left[P^2 + \hbar^2\left(2Q^2 + b^2\right)/\left(b^2 - Q^2\right)^2\right] + V(Q)$

5) $0 < |q| < \infty \;\&\; H = \frac{1}{2}p^2 + V(q) \Rightarrow \mathcal{H} = \frac{1}{2}\left[P^2 + 2\hbar^2/Q^2\right] + V(Q)$

The term P^2 in each of these equations is safe to say $P^2 = -\hbar^2\left(\mathrm{d}^2/\mathrm{d}x^2\right)$ and $Q = x$, both in the standard Schrödinger representation. In each one of these examples, the q-limitation becomes that of an appropriate x-limitation. The new $\hbar$-term in number 3 was derived in [2]. Note that number 5. Comes from number 4. by letting $b \to 0$. It means that only $q = 0$ has been removed, but still $P^\dagger \neq P$.

Example 3. can be viewed as that of "The particle in a box", which is often used as a simple teaching example, because it uses canonical quantization, and cos and sin functions for its solutions. As an example, the ground state for our box is $\cos(\pi x/2b)$ because at $x = \pm b$, it reaches zero to match its zero wave function with that of a "v-wall". After 2 derivatives, leading again to a similar term, the needed zero is there again. However, the first derivative, taken before the second derivative, encounters a non-continuous function that leads to an infinity, and which prevents that function being accepted in any Hubert space. Briefly stated, canonical quantization fails "The particle in a box", while affine quantization succeeds. It may be helpful to define the eigenfunctions, which have the form $\phi(x)=\left(b^2-x^2\right)^{3/2} f(x)$. Afterwards, focus on finding some solutions for $f(x)$, which, at the present time (Nov. 2022), are still all unknown.

Introducing Vector Expressions

Just as a pointer, if there are vectors involved, it follows that in many places it is correct to simply change single letters from $p\,\&\,q \rightarrow \vec{p}\,\&\,\vec{q}$, and then $P\,\&\,Q \rightarrow \vec{P}\,\&\,\vec{Q}$. Now for $0<b^2<\vec{q}^2$ and $0<b^2<\vec{Q}^2$, we find that we have a classical Hamiltonian, $H=\frac{1}{2}\vec{p}^2+V(\vec{q})$, which leads to the quantum Hamiltonian, $\mathcal{H}=\frac{1}{2}\left[\vec{P}^2+\hbar^2\left(2\vec{Q}^2+b^2\right)\Big/\left(\vec{Q}^2-b^2\right)^2\right]+V(\vec{Q})$.

5. What Affine Quantization Can Do, and Has Already Done, for Physics

Applications of affine quantization are still developing, and this section will only suggest reasons that affine quantization may be useful. A wider study of those proposals may be found in several of the author's articles, e.g., [3] [4] [5].

5.1. Quantum Field Theories, and Affine Quantization

Conventional scalar field theories include classical Hamiltonians, such as

$$H=\int\left\{\frac{1}{2}\left[\pi(x)^2+\left(\vec{\nabla}\varphi(x)\right)^2+m^2\varphi(x)^2\right]+g\varphi(x)^p\right\}\mathrm{d}^s x. \tag{9}$$

While the Hamiltonian $H<\infty$, this may still permit the fields to reach infinity. An example of that is $\pi(x)^2=A(x)\big/\left|x-x_0\right|^{s/2}$, with $A(x)>0$. Such behavior for fields of nature offers unwelcome behavior, and such field infinites should be excluded. In that case, the affine variables can be helpful.

Step 1: Following the expression $d=pq$ leads to the dilation field $\kappa(x)=\pi(x)\varphi(x)$, which then leads to

$$H=\int\left\{\frac{1}{2}\left[\kappa(x)^2\big/\varphi(x)^2+\left(\vec{\nabla}\varphi(x)\right)^2+m^2\varphi(x)^2\right]+g\varphi(x)^p\right\}\mathrm{d}^s x. \tag{10}$$

Using these variables, $0<|\varphi(x)|<\infty$ and $0\le|\kappa(x)|<\infty$ so that $\pi(x)$ is well represented. As a bonus, it follows that $\varphi(x)^p<\infty$ for every, even, $p<\infty$, as well as $\vec{\nabla}\kappa(x)=\left(\vec{\nabla}\pi(x)\right)\left(\varphi(x)\right)+\left(\pi(x)\right)\left(\vec{\nabla}\varphi(x)\right)$, and the gradient term should exhibit no divergence lest it disturbs the other terms. *Nature is respected!*

Step 2: Following the expression $D=\left[P^\dagger Q+QP\right]\big/2$ for $Q\neq 0$ leads to $\hat{\kappa}(x)=\left[\hat{\pi}(x)^\dagger\hat{\varphi}(x)+\hat{\varphi}(x)\hat{\pi}(x)\right]\big/2$ and following $D\left(Q^{-2}\right)D=P^2+2\hbar^2\big/Q^2$ leads to $\hat{\kappa}(x)\left(\hat{\varphi}(x)^{-2}\right)\hat{\kappa}(x)=\hat{\bar{\pi}}(x)^2+2\hbar^2\delta(0)^{2s}\big/\hat{\varphi}(x)^2$. That last term has many infinities, because several of Dirac's delta function, *i.e.*, $\delta(0)=\infty$, designed so that $\int_{-a}^{a}\delta(x)\mathrm{d}x=1$ for any $a>0$, and they need to be removed!

Step 3: Scaling is a tool of quantum field theory already, and treating $\delta(0)\equiv\mathcal{D}$, temporarily as huge, but not infinity, we let scaling work its magic. First, choose $\hat{\pi}(x)\rightarrow\mathcal{D}^{s/2}\hat{\pi}(x)$, and then $\hat{\varphi}(x)\rightarrow\mathcal{D}^{s/2}\varphi(x)$. This leads us to

$\mathcal{D}^s\hat{\pi}(x)^2 + 2\hbar^2\mathcal{D}^{2s}/\mathcal{D}^s\hat{\varphi}(x)^2$. Finally, multiply the whole last term by $\mathcal{D}^{-s}$, which will lead to the first two factors in the following quantum Hamiltonian, and which has added the remaining terms, using scaling, including that for *g*, as needed, and using Schrödinger's representation, to become

$$\mathcal{H} = \int\left\{\frac{1}{2}\left[\hat{\pi}(x)^2 + 2\hbar^2/\varphi(x)^2 + \left(\vec{\nabla}\varphi(x)\right)^2 + m^2\varphi(x)^2\right] + g\varphi(x)^p\right\}\mathrm{d}^s x. \quad (11)$$

Results from Several Monte Carlo Studies

At the start of affine Monte Carlo (MC) investigations, around 2018, the factor $2\hbar^2$ in the last equation, was replaced by $(3/4)\hbar^2$, adopting a natural, but incorrect, classical reasoning. However, the $(3/4)\hbar^2$ term has performed very well. The MC results have been for φ_n^p models, where $n = s+1$, and *s* is the number of spatial dimensions, while 1 represents time. The model φ_4^4, already examined in the 1980s, and started again around 2019, that used canonical quantization, found that their results were as if the interaction term was missing, while the similar studies, using affine quantization found that their results were that the interaction term appeared as it should [6]. Additionally, the model φ_3^{12} had stronger positive results, as it should, see [7]. Studies using $2\hbar^2$ should lead to even better results.

5.2. Quantum Gravity, and Affine Quantization

This story follows the previous one in certain respects, which allow it to be shorter. The traditional classical variables are the momentum, $\pi^{ab}(x)$, and the metric, $g_{ab}(x)$. The dilation field becomes $\pi_b^a(x) = \pi^{ac}(x)g_{bc}(x)$, with summation by *c*. All $a,b,c,\cdots$ run over 1, 2, 3, and "$ab = ba$" in the fields. The metric field helps measure distance, such as $\mathrm{d}\sigma^2(x) = g_{ab}(x)\mathrm{d}x^a\mathrm{d}x^b > 0$ provided $\Sigma_{a=1}^3\left(\mathrm{d}x^a\right)^2 > 0$. The quantum operators then force $\hat{\pi}^{ab}(x)^\dagger \neq \hat{\pi}^{ab}(x)$, because "$\hat{g}_{ab}(x) > 0$". The dilation operator field is $\hat{\pi}_b^a(x) = \left[\hat{\pi}^{ac}(x)^\dagger\hat{g}_{bc}(x) + \hat{g}_{bc}(x)\hat{\pi}^{ac}(x)\right]/2$. At this point, the classical ADM Hamiltonian [8], now in affine variables, is

$$H = \int\left\{g(x)^{-1/2}\left[\pi_b^a(x)\pi_a^b(x) - \frac{1}{2}\pi_a^a(x)\pi_b^b(x)\right] + g(x)^{1/2}\,{}^{(3)}R(x)\right\}\mathrm{d}^s x, \quad (12)$$

where $g(x) = \det\left[g_{ab}(x)\right]$ and ${}^{(3)}R(x)$ is the Ricci scalar for 3 spatial coordinates. The quantum ADM Hamiltonian, using Schrödinger's representation, then becomes

$$\mathcal{H} = \int\left\{\left[\hat{\pi}_b^a(x)g(x)^{-1/2}\hat{\pi}_a^b(x) - \frac{1}{2}\hat{\pi}_a^a(x)g(x)^{-1/2}\hat{\pi}_b^b(x)\right] + g(x)^{1/2}\,{}^{(3)}R(x)\right\}\mathrm{d}^s x, \quad (13)$$

but there is a miracle in that $\hat{\pi}_b^a(x)g(x)^{-1/2} = 0$, for all *a* & *b*, and there are *no* Dirac delta functions as they appeared before. There are constraints involved in a full quantization of gravity, for which there is special information in [9].

A Comment Regarding a Path Integration of Gravity

Since valid path integrations employ Wiener measures, an important element for a canonical field quantization would be $\omega^{-1}\dot{\pi}(x,t)^2+\omega\dot{\varphi}(x,t)^2$, which ensures Cartesian behavior, while for an affine field quantization it would be $\beta^{-1}\varphi(x,t)^2\,\dot{\pi}(x,t)^2+\beta\varphi(x,t)^{-2}\,\dot{\varphi}(x,t)^2$, which ensures that it follows a constant negative curvature.

Now, it happens that the gravity Wiener measure element needs to be $\gamma^{-1}\left[g_{ab}(x,t)\dot{\pi}^{ab}(x,t)\right]^2+\gamma\left[g^{ab}(x,t)\dot{g}_{ab}(x,t)\right]^2$, which also ensures a constant negative curvature, and an affine quantization, as seen in [10].

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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