

The Quantum Chromodynamics Gas Density Drop and the General Theory of Relativity Ether

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Abstract

β decay is one of the most fundamental and thoroughly studied nuclear decay. Surprisingly, the β decay rates were found to have a periodic time variability [1]. However, others argued that there is no evidence for such cyclic deviation from the exponential first order kinetics decay law [2]. Here we propose that the β decay is a pseudo-first order exchange reaction triggered by $u\bar{d}\bar{d}\bar{u}$ exotic mesons and propose a QCD gas theory. In analogy to the atmospheric gas density, the proposed QCD gas density drops with elevation from the sun. Accordingly, we propose that the β decay rate periodic variability is due to the pseudo-first order exchange reaction kinetics and the QCD gas atmospheric density drop. The proposed QCD gas may be a possible candidate for Einstein's general theory of relativity ether [3]. Our main results are the derived formulas for calculating the effective mass of the QCD gas and the cosmology perfect fluid equation of state dimensionless parameter, based on the measured ratio of the β decay rates at the earth trajectory aphelion and perihelion dates.

Keywords

Nuclear Decay, β Decay Rate Variability, Atmospheric Density, Quantum Chromodynamics (QCD), Exotic Mesons, General Theory of Relativity (GR), Ether, Dark Energy

1. The β Decay and the $u\bar{d}\bar{d}\bar{u}$ Exotic Meson

In 1930, Wolfgang Pauli postulated the neutrino as a desperate remedy to the energy crisis of the time—the continued energy spectrum of electron emitted in the negative β decay [4]. Pauli assumed that nuclear β decay emits a neutrino to-

gether with the electron such that the sum of energies is constant since sensitive measurements of the energy and momentum of the β decay electrons and the recoiling nuclei in cloud chambers indicated that substantial quantities of energy and momentum were missing.

$$n \rightarrow p^+ + e^- + \tilde{\nu}_e \tag{1}$$

In 1932, Enrico Fermi provided a theoretical framework for the β decay [5], but it took another 25 years before the neutrino was detected. In 1957, Reines and Cowan made the first observation of the free antineutrino $\tilde{\nu}_e$ through the positive β decay utilizing the flux of $\tilde{\nu}_e$ from a nuclear reactor.

$$\tilde{\nu}_e + p^+ \rightarrow e^+ + n \tag{2}$$

Reines and Cowan measured the Υ rays of the positron-electron annihilation (Equation (3)) for the positive β decay.

$$e^+ + e^- \rightarrow 2\Upsilon \tag{3}$$

Pauli and Fermi were unaware in the 1930s of the sub-nucleon quark particles discovered in 1964 by Murray Gell-Mann [6] and George Zweig. The β decay equations in terms of the quarks and charged W^- boson decay channels are

$$udd(n) \rightarrow udu(p^+) + w^- \tag{4}$$

$$w^- \rightarrow d\tilde{u}(\pi^-) \tag{5a}$$

$$w^- \rightarrow e^- + \tilde{\nu}_e \tag{5b}$$

The first issue with the β decay equations is that it is not balanced in terms of quarks. A down quark is annihilated and an up quark and a w^- boson are created (Equation (4)). Then, the w^- charged boson decays into a $d\tilde{u}$ charged pion (Equation (5a)), or with a higher rate, to two leptons, $e^- + \tilde{\nu}_e$, (Equation (5b)) that are created without their antiparticle pairs. The second issue with the β decay equations is that neutrons of stable atoms do not decay according to Equation (1) hinting that the β decay is not a simple first order kinetic reaction. The third issue is the observed periodic time variability of the β decay rates [7].

In this paper we propose an alternative pseudo-first order exchange reaction kinetics [8] for the β decay triggered by a $d\tilde{d}u\tilde{u}$ exotic meson. The proposed β decay pseudo-first order quark and antiquark pair exchange reaction equation is:

$$udd(n) + \tilde{u}\tilde{d}d\tilde{u}(\text{exotic meson}) \rightarrow udu(p^+) + d\tilde{u}\tilde{d}\tilde{d}(\pi^{-*}) \tag{6}$$

$$d\tilde{u}\tilde{d}\tilde{d}(\pi^{-*}) \rightarrow e^- + \tilde{\nu}_e \tag{7}$$

Equation (6) describes a second order reaction, but since we further assume that the $\tilde{u}\tilde{d}d\tilde{u}$ exotic mesons fill space and are in a huge excess over neutrons, the reaction behaves like a pseudo-first order kinetic reaction. Equation (6) is balanced in terms of quarks and antiquarks that appear on the left-hand-side and the right-hand-side. The $\tilde{u}\tilde{d}d\tilde{u}$ exotic meson triggers the β decay by capturing a down quark from the neutron by its antidown quark (\tilde{d}) and then providing

an up-quark replacement (u) via a pair exchange reaction as illustrated below. The proposed pseudo first order quark and antiquark pair exchange reaction may explain why neutrons in atoms are stable and do not decay to protons if the approach of the QCD gas $u\tilde{d}\tilde{d}\tilde{u}$ exotic meson to the neutron's quarks is blocked (**Figure 1**).

Equation (7) completes the description of the proposed pseudo-first order β decay and is also interesting since the exotic charged pion $d\tilde{u}\tilde{d}\tilde{d}(\pi^{*-})$ that further decays into an electron and antineutrino hints that the leptons may not be elementary particle as suggested recently by Davidson [9].

The negative β decay pseudo first order quark and antiquark exchange reaction equation is triggered similarly by the $u\tilde{d}\tilde{d}\tilde{u}$ exotic meson where an excited positively charged pion π^{*+} is created, $u\tilde{d}\tilde{u}\tilde{u}(\pi^{*+})$ that further decays into a positron and electron neutrino in addition to the neutron.

$$udu(p^+) + u\tilde{d}\tilde{d}\tilde{u}(\text{exotic meson}) \rightarrow udd(n) + u\tilde{d}\tilde{u}\tilde{u}(\pi^{*+}) \quad (8)$$

$$\tilde{d}\tilde{u}\tilde{u}(\pi^{*+}) \rightarrow e^+ + \nu_e \quad (9)$$

The excited positively charged pion that decays into a positron and neutrino may capture an internal electron like in Sodium nuclei that captures an electron and transmutes to Neon nuclei.

The impact of clustering inside exotic tetraquarks was studied by Sazdjian [10] concluding that a weakly bound compact two mesons Van-der-waals molecule-like structure is energetically favorable for the exotic meson comparing to two separate mesons [11]. We propose that the $u\tilde{d}\tilde{d}\tilde{u}$ exotic meson may condense into a new strongly bound peculiar positronium/quarkonium state introduced by Crater and Wong's Two Body Dirac Equation (TBDE) approximate solution [12] [13]. The peculiar positronium binding energy is about 300,000 times bigger than the more familiar positronium hydrogen-like solution having a binding energy of about only 6 eV. The peculiar positronium bond length is much shorter than the hydrogen-like solution, on the scale of the Compton length.

We propose that the condensation to the $u\tilde{d}\tilde{d}\tilde{u}$ exotic meson peculiar state may be generated according to the following kinetic equation

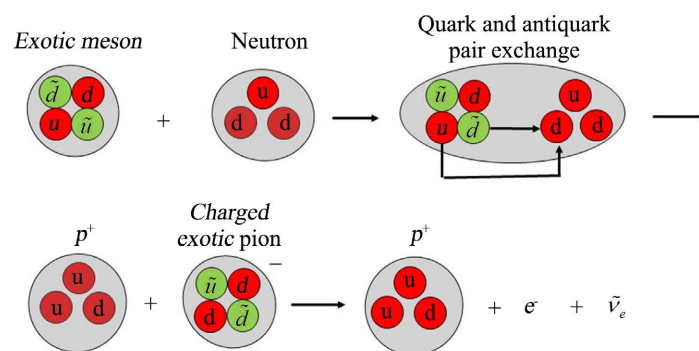


Figure 1. The pseudo first order β decay quark and antiquark pair exchange reaction of a neutron with the QCD gas exotic mesons.

$$d\tilde{d}u\tilde{u}(\text{molecule like}) \xrightleftharpoons[k_2]{k_1} u\tilde{d}(\pi^+) + d\tilde{u}(\pi^-) \xrightleftharpoons[k_4]{k_3} u\tilde{d}d\tilde{u}(\text{condensed}) + 2\Upsilon \quad (10)$$

The first configuration on the left is the compact exotic meson comprised of weakly bound two mesons Van-der-waals molecule-like state suggested by Sazdjian [10] and on the right is the condensed configuration, which is much more stable in the peculiar quarkonium state suggested by Crater [12] [13], and two emitted photons Υ rays. The short-lived charged pions $u\tilde{d}(\pi^+) + d\tilde{u}(\pi^-)$ act as an extremely short lived transition state.

Equations (6), (8) and (10), maintain strict quark balance. The numbers of down (d), up (u), antidown (\tilde{d}) and antiup (\tilde{u}) quarks are strictly conserved in each equation, which may be a fundamental principle of quark and antiquark pair exchange reactions. Accordingly, quarks and antiquarks exchange partners with other quarks of composite particles, e.g. with mesons, baryons [6] and maybe also with leptons that may be composite particles [9].

2. QCD Gas Density Drop and the GR Ether

Based on the observed periodic variability of the β decay rates [1] [14] and the new pseudo-first order β decay quark and antiquark pair exchange reaction kinetics proposed in the first part, we suggest here in the second part that the exotic mesons, comprised of even number of quarks and antiquarks in a condensed peculiar state [12] [13], that we already assumed fill space with high density, has a small effective mass, m_{QCD_gas} , and hence will have a density drop with elevation from massive stars like the sun similar to earth's ideal gas atmospheric density drop. Boltzmann's gas kinetic theory [15] is based on non-reactive collisions between inert atoms and molecules that collide and reach thermodynamic equilibrium in the gas phase. The proposed QCD gas comprised of exotic mesons is assumed to be a reactive gas that performs rapid quark and antiquark pair exchange reactions with itself and with matter particles that can accelerate, decelerate or conserve the velocity of the interacting quark and antiquark-based particles. Both gas theories are driven by the underlying particle collision dynamics, quarks and antiquarks pair exchange exchanges with the QCD gas and atoms and molecules collisions with Boltzmann's kinetic theory [16].

The atmospheric density formula describes the drop in the atmospheric density as a function of elevation above sea level due to gravity. In analogy, we propose that the QCD gas in the solar system has a similar density drop as a function of elevation from the sun surface as shown below (Figure 2).

The force balance on an infinitesimal box of volume $A * dh$ taking into account the sun gravitational force acting on the QCD gas in the box is described by Equation (11a)

$$P_{up}A - P_{bottom}A = -\frac{\rho A dh GM_s}{r^2} \quad (11a)$$

The QCD gas density ρ is related to the gas pressure using the ideal gas equation

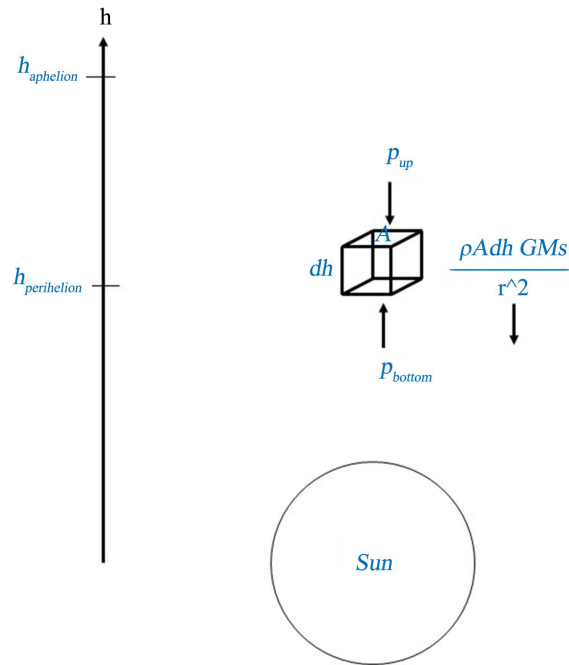


Figure 2. The force balance of a QCD gas in a virtual box of volume $A \cdot dh$ at elevation $h_{perihelion}$ from the sun.

$$\rho = \frac{m_{QCD_gas} n}{V} = \frac{m_{QCD_gas} P}{k_B T} \tag{11b}$$

A differential equation for the pressure as a function of the elevation interval dh is obtained by dividing Equation (11a) by the surface area A and substituting $dp = P_{up} - P_{bottom}$

$$\frac{dp}{p} = -\frac{m_{QCD_gas} GM_s}{r^2 k_B T} dh \tag{11c}$$

The solution of Equation (11c) is

$$p(h) = p(h_0) e^{\frac{-m_{QCD_gas} GM_{sun}(h-h_0)}{h_0^2 k_B T}} \tag{11d}$$

And the QCD gas density is similarly

$$\rho(h) = \rho(h_0) e^{\frac{-m_{QCD_gas} GM_{sun}(h-h_0)}{h_0^2 k_B T}} \tag{11e}$$

where m_{QCD_gas} is the effective mass of the QCD gas exotic meson; M_{sun} is the sun mass; G is the gravitation constant; k_B is Boltzmann constant; and T is the QCD gas temperature.

According to the assumption that the nuclear β decay is a pseudo-first order kinetic reaction (Equation (6)), the β decay rate depends on the QCD gas density inversely [8]

$$t_{1/2} = \frac{1}{k_\beta \rho [u\bar{d}\bar{d}\bar{u}]}(h) \tag{12}$$

We propose to measure the β decay half-life time, $t_{1/2}$, in summertime and wintertime at the aphelion and perihelion of earth's elliptical trajectory around the sun [17].

The QCD gas density, $\rho[u\bar{d}\bar{d}\bar{u}](h)$, at the aphelion is

$$\rho(h_{\text{aphelion}}) = \rho(h_{\text{perihelion}}) e^{\frac{-m_{\text{QCD gas}} GM_{\text{sun}} (h_{\text{aphelion}} - h_{\text{perihelion}})}{h_{\text{perihelion}}^2 k_b T}} \tag{13}$$

where $h_{\text{perihelion}}$ and h_{aphelion} are earth's perihelion and aphelion trajectory distances from the sun. Thus, the ratio of the β decay half-life times at the two elevations is

$$\frac{t_{\frac{1}{2}}(\text{perihelion})}{t_{\frac{1}{2}}(\text{aphelion})} = e^{\frac{-m_{\text{QCD gas}} GM_{\text{sun}} (h_{\text{aphelion}} - h_{\text{perihelion}})}{h_{\text{perihelion}}^2 k_b T}} \tag{14}$$

If we assume that the effective mass of the QCD gas exotic meson, $m_{\text{QCD gas}}$, is on the scale of the electron mass, 1.0^{-31} kg, and that the QCD gas temperature is about 2.7 Kelvin, we get that the β decay half-life time ratio according to Equation (14) is 0.9211. Due to the inverse relation on the QCD gas density, the β decay half-life time will be longer at the aphelion

$$\frac{t_{\frac{1}{2}}(\text{perihelion})}{t_{\frac{1}{2}}(\text{aphelion})} = e^{\frac{-1.0^{-31} * 1.989^{30} * (152093251 - 147098925) * 1000}{2.1637^{18} * 1.38^{-23} * 2.7}} = 0.9211 \tag{15}$$

The effective mass of the QCD gas exotic mesons, positive or negative (possible indication for dark energy), can be calculated from the β decay rates measured at two extreme points of earth's elliptical trajectory, the aphelion and perihelion

$$m_{\text{QCD gas}} = \frac{-k_b T h_{\text{perihelion}}^2 \ln\left(\frac{t_{\frac{1}{2}, \text{perihelion}}}{t_{\frac{1}{2}, \text{aphelion}}}\right)}{GM_{\text{sun}} (h_{\text{aphelion}} - h_{\text{perihelion}})} \tag{16}$$

It should be noted that the observed periodic time variability of the β decay rates was suggested to be due to the sun's neutrino flux by Sturrock [1] and Steintz [14]. Pomee argued however that the evidence does not suggest that radioactive decay is triggered by neutrinos and that there are no cyclic deviations from the exponential (first order kinetics) decay law [2]. We want to emphasize that the first order β decay reaction of Equation (1) does not depend on incoming neutrino flux that contradict Sturrock [1] and Steintz [14] explanation for the β decay rate periodic variability. We propose that the observed β decay periodic variability is due to the β decay pseudo-first order exchange reaction kinetics and to the QCD gas exotic meson density drop from the sun surface. We further propose to determine the QCD gas effective mass using the observed β decay rates according to Equation (16).

Does ether exist, or in other words, is space physically empty? Albert Einstein said in a lecture at the University of Leiden “The Ether and the Theory of Relativity” on May 5, 1920, 5 years after he published the general theory of relativity [18] and six years before Schrödinger [19], and later Heisenberg [20] and Dirac [21] published their quantum mechanics theories, that the GR theory disposed the view that space is physically empty and that space without ether is unthinkable. Einstein explained his view in favor of the ether in the following few paragraphs cited from his lecture [3]:

“The recognition of the fact that ‘empty space’ in its physical relation is neither homogeneous nor isotropic, compelling us to describe its state by ten functions (the gravitation potentials g_{mn}) has, I think, finally disposed of the view that space is physically empty.”

About the special theory of relativity and the ether of the general theory of relativity Einstein said:

“The special theory of relativity forbids us to assume the ether to consist of particles observable through time, but the hypothesis of ether in itself is not in conflict with the special theory of relativity. Only we must be on our guard against ascribing a state of motion to the ether. The ether of the general theory of relativity is a medium which is itself devoid of all mechanical and kinematical qualities, but helps to determine mechanical (and electromagnetic) events.”

Einstein defined what should be the difference between the general theory of relativity ether opposed to Lorentz ether:

“What is fundamentally new in the ether of the general theory of relativity as opposed to the ether of Lorentz consists in this, that the state of the former is at every place determined by connections with the matter and the state of the ether in neighboring places, which are amenable to law in the form of differential equations.”

Einstein summarized the “Ether and Relativity Theory” lecture: *“Recapitulating, we may say that according to the general theory of relativity space is endowed with physical qualities; in this sense, therefore, there exists an ether. According to the general theory of relativity space without ether is unthinkable; for in such space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring-rods and clocks), nor therefore any space-time intervals in the physical sense. But this ether may not be thought of as endowed with the quality characteristic of ponderable media, as consisting of parts which may be tracked through time. The idea of motion may not be applied to it.”*

The proposed QCD gas comprised of even number of quarks and antiquarks exotic mesons may be a possible candidate for the general theory of relativity ether Einstein thought must exist. Note that the QCD gas is not comprised of regular matter particles. Since it is comprised of even number of matter and antimatter particles exotic mesons, $u\bar{d}\bar{d}u$, in a condensed peculiar state [12] [13], we do not assign coordinates and velocities to the QCD gas and we only observe its density drop via its pseudo-first order reactions with neutrons of unstable

atom nuclei via the β decay rate periodic variability [1] [14].

An expression for the Friedmann-Robertson-Walker (FRW) metric [22] scale factor dimensionless equation-of-state parameter, w , [23] dependent on the ratio of the measured β decay rates at the earth trajectory aphelion and perihelion using Equations (11b) and (16) above is

$$\rho = \frac{P}{wc^2} = \frac{m_{QCD_gas} P}{k_B T} \tag{17a}$$

$$w = \frac{k_B T}{m_{QCD_gas} c^2} = \frac{GM_{sun} (h_{aphelion} - h_{perihelion})}{-h_{perihelion}^2 \ln \left(\frac{t_{\frac{1}{2},perihelion}}{t_{\frac{1}{2},aphelion}} \right) c^2} \tag{17b}$$

$$w \sim \frac{-3.4091^{-10} t_{\frac{1}{2},aphelion}}{\Delta_t}, \Delta_t = t_{\frac{1}{2},aphelion} - t_{\frac{1}{2},perihelion} \tag{17c}$$

Equations 17(a)-(c) allows determining by the proposed β decay measurements if we are in a matter dominated era (if w is close to 0), in a radiation dominated era (if w is close to 1/3) or in a dark energy dominated era (if w is close to -1). The evolution of the metric scale factor $a(t)$ is controlled by the dominating energy era [23]

$$a(t) \sim t^{\frac{2}{3(1+w)}} \tag{18}$$

Carbon 14 β decay may be measured for example at the aphelion date on July and perihelion date on January and the calculation of the metric scale factor dimensionless parameter w according to Equation (17c) with the expected ratio values for the decays for the three dominating era's are:

$t_{\frac{1}{2},aphelion} - t_{\frac{1}{2},perihelion}$	The metric scale factor dimensionless parameter w	Dominating Era
50.196 hours	3.4091 ⁻⁴	Matter
3.08 min	1/3	Radiation
-61.6 sec	-1.00	Dark Energy

Assuming we are in the dark energy dominated era, $w = -1$, and taking the aphelion carbon 14 decay rate to be 5730 years, the expected decay rate time difference between the aphelion and perihelion will be -61.6 seconds as shown above in the third row.

Note that a sign change is expected for the dark energy era and that variations due to the distance to the moon and the temperature between the aphelion and perihelion measurements should be considered.

3. Summary

We propose that a QCD gas comprised of exotic mesons may be a possible can-

didate for the GR ether where its density drops with elevation from massive stars like the sun. We provide an alternative explanation for the β decay rate periodic variability based on a pseudo-first order β decay quark and antiquark pair exchange reaction of neutrons with the QCD gas exotic mesons. Our main results are the derived formulas for calculating the effective mass of the QCD gas and the cosmology perfect fluid equation-of-state dimensionless parameter, w , based on the measured ratio of the β decay rates of Equations (16) and (17c).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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