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Photon and Graviton: Similarity and Distinctions

Andrey Nikolaevich Volobuev

Department of Physics, Samara State University, Samara, Russia
Email: volobuev47@yandex.ru

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Abstract

Two elementary particles—a photon and a graviton—responsible for interaction with physical objects of two long-range action fields electromagnetic and gravitational are considered. The similarity and distinction of these quantum particles are investigated. It is shown that these particles essentially differ from each other. First of all, they are in different spaces. The photon is in photon space, and the graviton is in Riemann's space-time. Interaction of a photon and a mass body cannot be calculated in Euclidian space. Interaction of a graviton and a mass body can be calculated in the Euclidian space. Polarizing properties of a gravitational wave both in plane Minkowski's space, and in curved Riemann's space-time are in detail considered. The differential equation of a gravitational quantum oscillator is found and its solution is analyzed. Also, the quantum metrics of Riemann's space-time in presence of graviton and its quantum numbers are found.

Keywords

Photon, Graviton, Gravitational Wave, Riemann's Space–Time, Scalar Curvature, Polarization, Quantization, Metric Tensor

1. Introduction

During research of quantum properties of particles frequently, there is a desire to compare, at first sight, very similar particles: a photon and graviton.

The photon and the graviton are two quantum particles responsible for interaction with physical objects of two long-range action fields: electromagnetic and gravitational.

Perhaps, the main distinction between these particles consist that the first is real, and the second is hypothetical. But in the given work, we shall not pay attention to this distinction.

First of all, we shall notice that a photon—a vector particle in the sense that the electromagnetic field is described by the vector Maxwell's equations, and the graviton—a tensor particle because the gravitational field is described by the tensor Einstein's equation. Therefore, polarization of a photon has a vector character, and polarization of a graviton has a tensor character.

2. Properties of a Photon

We list the basic properties of a photon.

Photon (Greek "light") or quantum of electromagnetic radiation—an elementary particle with which help electromagnetic interaction between charges is carried out.

The photon—absolutely stable particle, *i.e.* in vacuum spontaneously does not disintegrate.

The photon—really neutral particle, *i.e.* a charge of a photon is equal to zero. For a photon there is no antiparticle.

The mass of a photon is always equal to zero.

The photon in vacuum moves with light velocity $c = 299792458\,\frac{\mathrm{m}}{\mathrm{c}}$.

The energy of a photon is equal $E = \hbar\omega$, where $\hbar = 1.054589 \times 10^{-34}\,\frac{\mathrm{J}}{\mathrm{Hz}}$ there is reduced Planck's constant, ω—cyclic frequency of a photon.

The impulse of a photon is equal $\boldsymbol{p} = \hbar\boldsymbol{k}$, where $k = \frac{2\pi}{\lambda}$ there is the module of a wave vector of a photon (or wave number), λ—a photon length of a wave.

Spin of the photon is directed along a direction of a photon movement and is equal $S = \pm\hbar$. The sign ± reflects two possible helicity of a photon. From the point of view of classical electrodynamics it is two possible circular polarizations: right-polarized and left-polarized.

The magnetic moment of a photon is equal to zero.

The photon concerns to bosons. As against fermions (for example, electrons) for photons the Pauli's exclusion principle does not operate. In the same energy state there can be an infinite set of photons. For example, in a laser beam theoretically all photons are identical and coherent.

For photons it is characteristic corpuscular—wave dualism. On the one hand the photon behaves as a particle. It is absorbed and radiated by particles of substance (for example, atoms) entirely. Using corpuscular properties of a photon Einstein has explained the phenomenon of a photo effect. On the other hand for photons the wave phenomena are inherent: diffraction and interference.

The theory of a photon and an electron interaction it is a basis of the part of physics—quantum electrodynamics. It is well developed due to R. Feynman's works which has offered so-called "a diagram method" of this interaction calculation [1]. However the Feynman's theory is approximated. We shall not stop on essence of these approximations, and we shall consider a simple model task.

Let's consider as an example interaction of a photon and a mass particle in an elementary case when the mass particle initially is rested. The law of preservation of an impulse at absorption of a photon by a motionless particle looks like:

$$\frac{\hbar\omega}{c} = mV\ , \tag{1}$$

where c there is a light velocity in vacuum, V—a velocity of a particle after absorption of a photon, m—its relativistic mass, ω —frequency of a photon.

The law of energy conservation in the relativistic form can be written down as:

$$\hbar\omega = mc^2 - m_0c^2 = mc^2\left(1-\frac{m_0}{m}\right) = mc^2\left(1-\sqrt{1-\frac{V^2}{c^2}}\right), \tag{2}$$

where m_0—rest mass of a particle.

Having divided the law of energy conservation on the law of an impulse preservation we shall receive $\frac{V}{c} = \left(1-\sqrt{1-\frac{V^2}{c^2}}\right)$, *i.e.* $V = c$. Thus after absorption of a photon the particle starts to move with speed of light. This at first sight the absurd result is not casual. It reflects fact that the photon is not in Euclidian space, as against a mass particle. Photon goes in so-called photon space [2]. Since the reference system in photon space moves with a light velocity the calculation demands also the mass particle moved with light velocity that is impossible. Therefore accurately to solve above the offered task it is impossible. R. Feynman has offered the good approximated method of the solution of similar problems.

3. Properties of a Graviton

In connection with that quantum electrodynamics is in detail developed science all further attention we shall give a graviton.

There are basic properties of a graviton.

A graviton is a quantum particle a carrier of gravitational interaction.

A graviton is boson, it is stable. Spin of a graviton is equal to two. A graviton has two directions of polarization. Polarization of a graviton will be in detail considered below.

The electric charge of a graviton is equal to zero.

At a graviton there is no antiparticle.

Energy of a graviton is equal $E = 2\hbar\omega$ [3], where ω there is cyclic frequency of a graviton—frequency of the Riemann's space-time fluctuations.

The impulse of a graviton is equal $p = \frac{2\hbar\omega}{c}$.

Physically the graviton represents local wave perturbation of the Riemann's space-time. Therefore, as against a photon, the graviton cannot be in photon space. It is in Riemann's space-time. In it the main difference of a graviton from a photon.

Let's solve the previous task only instead of interaction of a photon and initially motionless mass body we shall consider interaction of a graviton and a mass body (**Figure 1**).

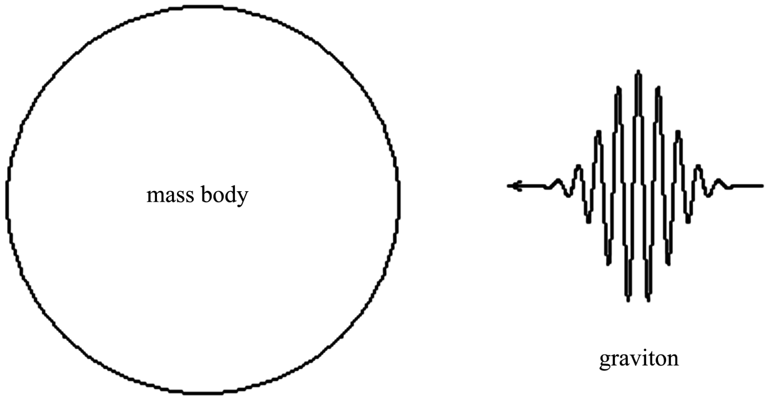

Figure 1. Interaction of initially motionless mass body and a graviton.

The law of preservation of an impulse at interaction of a graviton and a mass particle we shall write down as:

$$\frac{2\hbar\omega}{c}=\left(m+m_g\right)V\,, \tag{3}$$

where ω there is frequency of a graviton at the moment of interaction with a mass body, m—mass of a body, m_g—a part of graviton mass joined to a body, V—velocity of a body after absorption of a graviton. Other part of graviton mass is use as energy for movement of system.

The law of conservation of energy looks like:

$$2\hbar\omega=\left(m+m_g\right)c^2-m_0c^2=\left(m+m_g\right)c^2\left(1-\frac{m_0}{m+m_g}\right), \tag{4}$$

where m_0 there is initial mass of a motionless body. Besides $E=2\hbar\omega>m_gc^2$.

If to accept $E=2\hbar\omega=m_gc^2$ then $m=m_0$, hence interaction of a graviton and a mass body is absent.

Let's divide the Formula (4) into the Formula (3):

$$c=\frac{c^2}{V}\left(1-\frac{m_0}{m+m_g}\right). \tag{5}$$

Let's transform (5) with use $m=\dfrac{m_0}{\sqrt{1-\dfrac{V^2}{c^2}}}$:

$$\frac{m\sqrt{1-\dfrac{V^2}{c^2}}}{m+m_g}=\left(1-\frac{V}{c}\right). \tag{6}$$

Carrying out the further transformations we shall find:

$$\frac{1+\dfrac{V}{c}}{1-\dfrac{V}{c}}=\left(1+\frac{m_g}{m}\right)^2. \tag{7}$$

At approach of a graviton to a mass body its frequency increases due to the longitudinal Doppler's effect under the relativistic formula $\omega = \omega_0 \sqrt{\frac{1+\frac{V}{c}}{1-\frac{V}{c}}}$ [4], hence:

$$\omega = \omega_0 \left(1 + \frac{m_g}{m}\right). \tag{8}$$

where ω_0 there is frequency of a graviton in own reference system.

At removal from a body the frequency of a graviton decreases (so-called "a red displacement") under formula $\omega = \omega_0 \sqrt{\frac{1-\frac{V}{c}}{1+\frac{V}{c}}}$, hence:

$$\omega = \frac{\omega_0}{1 + \frac{m_g}{m}} \approx \omega_0 \left(1 - \frac{m_g}{m}\right). \tag{9}$$

where m_g it is necessary to interpret, as reduction of a graviton mass in process of removal from a mass body, also decrease and a graviton frequency $\omega < \omega_0$.

The relative Doppler's shift of frequencies is equal $\frac{\Delta\omega}{\omega_0} = \pm \frac{m_g}{m}$, where is $\Delta\omega = \omega - \omega_0$, plus means approach of a graviton to a mass body, and minus removal from a body.

Thus, a task of interaction of a graviton and a mass body completely solved as against a task of interaction of a photon and a mass body. It is connected by that a graviton, as against a photon, as well as a mass body, are in uniform Riemann's space.

4. Polarization of a Gravitational Wave and a Graviton

Let's consider a problem of a gravitational wave and a graviton polarization.

The volumetric density of energy of a gravitational wave [5] [6] is equal:

$$W = T + U = \frac{c^2}{16\pi G}\left(\dot{h}_+^2 + \dot{h}_\times^2\right), \tag{10}$$

where it is designated $h_+ = h_{22} = -h_{33}$ and $h_\times = h_{23}$—wave gravitational components of metric tensor $\begin{Vmatrix} h_+ & h_\times \\ h_\times & -h_+ \end{Vmatrix}$, $T = \frac{c^2}{16\pi G}\dot{h}_+^2$, $U = \frac{c^2}{16\pi G}\dot{h}_\times^2$, $G = 6.67 \times 10^{-8} \frac{\text{m}^3}{\text{kg} \cdot \text{c}^2}$—a gravitational constant. Components h_+ and $h_\times$ describing two independent polarization of a gravitational wave there are dimensionless sizes. Points above are derivatives on time.

Taking into account (10), Lagrangian of a free gravitational field we shall write down as:

$$l = \frac{c^2}{16\pi G}\left(\dot{h}_+^2 - \dot{h}_\times^2\right). \tag{11}$$

For Lagrangian (11) the Lagrange equation is correct:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial l}{\partial \dot{q}}\right)=\frac{\partial l}{\partial q}, \tag{12}$$

where q and $\dot{q}$ there are the generalized coordinate and speed of system. We accept the generalized coordinate $q = h_+$, hence the generalized speed $\dot{q}=\dot{h}_+$.

At a choice of the generalized coordinates and speeds the known formula [6] should be carried out:

$$\begin{aligned} W &= \dot{q}\frac{\partial l}{\partial \dot{q}} - l = \dot{h}_+\frac{\partial l}{\partial \dot{h}_+} - l = \dot{h}_+\frac{c^2}{16\pi G}2\dot{h}_+ - \frac{c^2}{16\pi G}\left(\dot{h}_+^2-\dot{h}_\times^2\right) \\ &= \frac{c^2}{16\pi G}\left(\dot{h}_+^2+\dot{h}_\times^2\right). \end{aligned} \tag{13}$$

The Lagrangian of a gravitational field also it is equal [6]:

$$l=\frac{c^4}{16\pi G}R, \tag{14}$$

where R there is the scalar curvature of space arising due to presence of mass bodies.

Let's consider in more detail the Formula (14). In [7], it is marked that for particles and gravitational field the action looks like:

$$S=\int l\sqrt{g}\,\mathrm{d}\Omega=-\sum mc\int \mathrm{d}s-\frac{c^3}{16\pi G}\int R\mathrm{d}\Omega=-\sum mc\int \mathrm{d}s-\frac{c^3}{16\pi G}\int R\mathrm{d}V\mathrm{d}(ct), \tag{15}$$

where $\mathrm{d}s$ there is four-dimensional space-time interval, $\mathrm{d}\Omega=\mathrm{d}V\mathrm{d}(ct)$—an element of space-time volume, $\sqrt{-g}$—the size determining dependence of a normalizing element of volume on curvature of space-time, $g<0$—a determinant of a metric tensor. Size $\sqrt{-g}$ on account of its small size for the account of a gravitational wave we believe to equal unit. The second addend in (15) concerns to a gravitational field.

At absence of mass bodies $m=0$ the action is equal:

$$S=-\frac{c^4}{16\pi G}\int_V R\mathrm{d}V\mathrm{d}t, \tag{16}$$

that also results in the Formula (14). The scalar curvature R is negative sizes.

Four-dimensional Riemann's space-time can be in three states. If in Riemann's space-time there are no mass bodies, fields, first of all gravitational, such space is plane or Minkowski's space -time. If in Riemann's space-time there are no mass bodies but there is a gravitational field the Riemann's space-time is curve. It has scalar curvature *R*. To this case the Formula (14) concerns. If in the Riemann's space-time there are mass bodies the action of such space is expressed by the Formula (15).

Thus, at the presence of a gravitational field it is possible to write down:

$$l=\frac{c^2}{16\pi G}\left(\dot{h}_+^2-\dot{h}_\times^2\right)=\frac{c^4}{16\pi G}R. \tag{17}$$

Hence:

$$\left(\dot{h}_+^2 - \dot{h}_\times^2\right) = Rc^2 . \tag{18}$$

The given formula it is similar to the formula $E = mc^2$, physically reflects equivalence of curvature the Riemann's spaces and energy or mass. From this point of view Einstein's equation for gravitation [6] can be considered also as a principle of equivalence of space-time curvature and mass.

At absence of a gravitational field and mass bodies, and also neglecting of a cosmological constant Λ *i.e.* believing the space plane (space of Minkowski) it is possible to accept $R = 0$. Also neglect a curvature Riemann's space-time due to a gravitational wave or graviton, *i.e.* approximately we believe that a gravitational wave do not create curvature of space-time. In this case there is $\dot{h}_+ = \dot{h}_\times$ and $h_+ = h_\times$. The given equality is average on the period of fluctuations of the components h_+ and $h_\times$. The instantaneous values of the components can be not equal.

Let is the general relation for the generalized coordinate $q = f\left(h_+, h_\times\right)$ then from the Equation (12) follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial l}{\partial \dot{q}}\right) = \frac{c^2}{16\pi G}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \dot{h}_+^2}{\partial \dot{h}_+}\right) = \frac{\partial l}{\partial q} = \frac{c^2}{16\pi G}\left(\frac{\partial \dot{h}_+^2}{\partial q} - \frac{\partial \dot{h}_\times^2}{\partial q}\right). \tag{19}$$

Hence:

$$\frac{\mathrm{d}\dot{h}_+}{\mathrm{d}t} = \left(\dot{h}_+\frac{\partial \dot{h}_+}{\partial q} - \dot{h}_\times\frac{\partial \dot{h}_\times}{\partial q}\right). \tag{20}$$

Writing down a full derivative $\frac{\partial \dot{h}_+}{\partial t} + \frac{\partial \dot{h}_+}{\partial q}\dot{q} = \left(\dot{h}_+\frac{\partial \dot{h}_+}{\partial q} - \dot{h}_\times\frac{\partial \dot{h}_\times}{\partial q}\right)$ and reducing similar members, we shall find:

$$\frac{\partial \dot{h}_+}{\partial t} = -\dot{h}_\times\frac{\partial \dot{h}_\times}{\partial q}. \tag{21}$$

Distance between two points in the polarized wave [7] in polar coordinates equally, **Figure 2**:

$$l = r\left(1 + \frac{1}{2}h_+\cos 2\theta + \frac{1}{2}h_\times \sin 2\theta\right), \tag{22}$$

where θ there is a polar angle across a wave, r – polar radius.

If to consider cross-section of a gravitational wave the Formula (22) receives a kind:

$$l = \frac{1}{2}h_+\cos 2\theta + \frac{1}{2}h_\times \sin 2\theta = l_+ + l_\times , \tag{23}$$

where it is accepted $r = 1$.

In the Formula (23), the first term l_+ characterizes displacement of points in the polarized gravitational wave for component h_+, and the second addend $l_\times$ for component $h_\times$, **Figure 2**.

For the components h_+. At angles $\theta = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}$ the size $\cos 2\theta = 0$ therefore

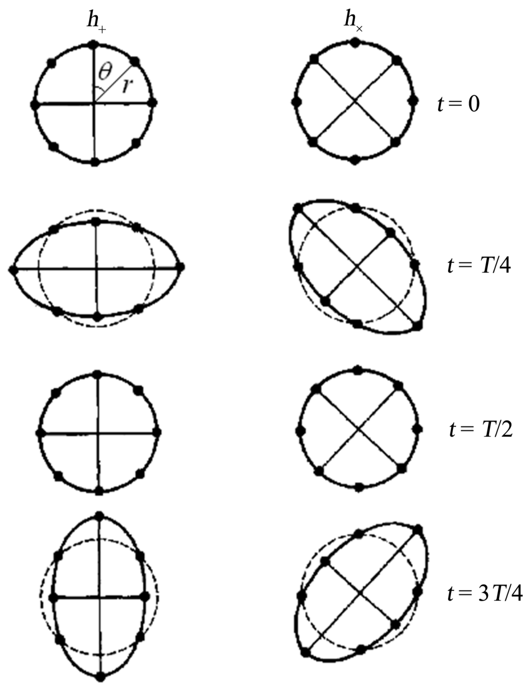

Figure 2. Polarization of the two wave gravitational components h_+ and $h_\times$ of metric tensor.

displacement is $l_+ = 0$. At angles $\theta = 0, \pm\frac{\pi}{2}, \pm\pi$ the size $\cos 2\theta = \pm 1$ therefore displacement l_+ makes harmonious fluctuations.

For the components $h_\times$. At angles $\theta = 0, \pm\frac{\pi}{2}, \pm\pi$ the size $\sin 2\theta = 0$ therefore displacement is $l_\times = 0$. At angles $\theta = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}$ the size $\sin 2\theta = \pm 1$, therefore displacement $l_\times$ makes harmonious fluctuations.

As follows from **Figure 2** the wave additives to metric тензору h_+ and $h_\times$ at any moment are shifted on a phase on a quarter of the period $\frac{T}{4}$. To this there corresponds a ratio:

$$\dot{h}_\times = \frac{2\pi h_+}{T} = \omega h_+ \quad \text{or} \quad h_+ = \frac{1}{\omega}\dot{h}_\times . \tag{24}$$

where ω there is a cyclic frequency of the field oscillation in the gravitational wave.

Substituting in (21) Formula (24) with use $q = h_+$ we shall find:

$$\frac{\partial \dot{h}_+}{\partial t} = \frac{\partial^2 h_+}{\partial t^2} = -\dot{h}_\times \omega \frac{\partial h_+}{\partial h_+} = -\dot{h}_\times \omega \tag{25}$$

or

$$\frac{1}{\omega}\frac{\partial^3 h_\times}{\partial t^3} = -\omega \frac{\partial h_\times}{\partial t} . \tag{26}$$

From (26), we find:

$$\frac{\partial^3 h_\times}{\partial t^3} + \omega^2 \frac{\partial h_\times}{\partial t} = 0\,. \tag{27}$$

Integrating (27) we have:

$$\frac{\partial^2 h_\times}{\partial t^2} + \omega^2 h_\times = 0\,. \tag{28}$$

Using in (27) Formula (24), we receive:

$$\frac{\partial^2 h_+}{\partial t^2} + \omega^2 h_+ = 0\,. \tag{29}$$

Thus in empty plane space wave additives to components of the metric tensor make harmonious fluctuations according to the differential Equations (28) and (29). Solutions of these equations are functions of harmonious fluctuations:

$$h_+ = A\sin(\omega t) \quad \text{and} \quad h_\times = A\sin\left(\omega\left(t \pm \frac{T}{4}\right)\right). \tag{30}$$

where A there is an amplitude of fluctuation.

At presence of curvature of space-time due to mass bodies according to (18) we have $\dot{h}_+ = \sqrt{\dot{h}_\times^2 + c^2 R}$. Substituting this formula in the Equation (29) differentiated on time, we shall find:

$$\frac{\partial^2 \sqrt{\dot{h}_\times^2 + c^2 R}}{\partial t^2} + \omega^2 \sqrt{\dot{h}_\times^2 + c^2 R} = 0\,. \tag{31}$$

Carrying out simple but awkward transformations we shall receive the equation:

$$\frac{c^2 R}{\dot{h}_\times}\left(\frac{1}{\dot{h}_\times^2 + c^2 R}\left(\frac{\partial \dot{h}_\times}{\partial t}\right)^2 + \omega^2\right) + \frac{\partial^2 \dot{h}_\times}{\partial t^2} + \omega^2 \dot{h}_\times = 0\,. \tag{32}$$

Using (18) we shall write down:

$$\frac{c^2 R}{\dot{h}_\times}\left(\left(\frac{\ddot{h}_\times}{\dot{h}_+}\right)^2 + \omega^2\right) + \frac{\partial^2 \dot{h}_\times}{\partial t^2} + \omega^2 \dot{h}_\times = 0\,. \tag{33}$$

Taking into account formula $h_+ = \frac{1}{\omega}\dot{h}_\times$ (24) which correct at $R \to 0$ we have the equation:

$$\frac{\partial^2 \dot{h}_\times}{\partial t^2} + \omega^2 \dot{h}_\times + \frac{2c^2\omega^2 R}{\dot{h}_\times} = 0\,, \tag{34}$$

which at $R = 0$ is transformed to the Equation (28).

All results of the given paragraph can be attributed to separate graviton as to local wave perturbation of Riemann's space-time.

5. Equation of Gravitational Quantum Oscillator

If to investigate the Equations (33) or (34) in a paradigm of quantum gravidynamics theirs needs to consider as the equations for gravitational quantum oscillator by analogy to the equation for standard quantum-mechanical oscillator in quantum mechanics [8].

Let's analyze the Equation (34). We execute once integration of this equation. For this purpose we shall designate $\frac{\partial^2 \dot{h}_\times}{\partial t^2}$. Hence $\frac{\partial^2 \dot{h}_\times}{\partial t^2} = \frac{\partial Y}{\partial \dot{h}_\times} Y = \frac{1}{2}\frac{\partial Y^2}{\partial \dot{h}_\times}$. Substituting this formula in (34) we shall find:

$$\frac{1}{2}\frac{\partial Y^2}{\partial \dot{h}_\times} + \omega^2 \dot{h}_\times + \frac{2c^2\omega^2 R}{\dot{h}_\times} = 0\,. \tag{35}$$

Integrating (35) we have:

$$\frac{Y^2 + \omega^2 \dot{h}_\times^2}{2} + 2c^2\omega^2 R \ln C\dot{h}_\times = 0\,, \tag{36}$$

where C there is a constant of integration.

Taking into account according to Formula (24) $Y = \ddot{h}_\times = \omega \dot{h}_+$ we shall find:

$$\omega^2 \frac{\dot{h}_+^2 + \dot{h}_\times^2}{2} + 2c^2\omega^2 R \ln C\dot{h}_\times = 0\,. \tag{37}$$

Using the Formula (10) for volumetric density of energy of gravitational wave W we have:

$$\frac{8\pi G W}{c^2} + 2c^2 R \ln C\dot{h}_\times = 0\,. \tag{38}$$

From (38) follows:

$$C\dot{h}_\times = \mathrm{e}^{-\frac{4\pi G W}{c^4 R}}\,. \tag{39}$$

The size C is a scaling multiplier for wave additives to components metric тензора h_+ and $h_\times$. Proceeding from dimensions in (39) we shall accept $C = \frac{1}{\alpha\omega}$ and, taking into account (24), we shall write down:

$$h_+ = \alpha \mathrm{e}^{-\frac{4\pi G W}{c^4 R}}\,. \tag{40}$$

The scaling condition for C is accepted proceeding from position: at $W = 0$ (*i.e.* at absence of gravitational waves) size $h_+ = \alpha$—a dimensionless constant equal to zero. Size of scalar curvature is $R \neq 0$ due to ineradicable curvature of space-time of gravitational vacuum [3].

Till now we did not enter into the analysis longitudinal coordinate X of quantum oscillator spreading, and actually a graviton. We research longitudinal spreading of a graviton in a direction of an axis X.

The Equation (34) is possible to write down for the component of metric tensor h_+ using a Formula (24) $\dot{h}_\times = \omega h_+$:

$$\frac{\partial^2 h_+}{\partial t^2} + \omega^2 h_+ + \frac{2c^2 R}{h_+} = 0\,, \tag{41}$$

where the size h_+ depends on two arguments, time t and longitudinal coordinate X.

Let in a direction of an axis X the flat harmonious gravitational wave or a graviton is spread. In this direction for simplicity of the analysis we believe space flat. In that case the equation of a wave looks like:

$$\frac{\partial^2 h_+}{\partial X^2}+\frac{\omega^2}{c^2}h_+=0\,. \tag{42}$$

Finding from the Equation (42) the size $\omega^2 h_+$, and substituting it in (41) we receive the wave equation of a graviton spreading:

$$\frac{\partial^2 h_+}{\partial X^2}-\frac{1}{c^2}\frac{\partial^2 h_+}{\partial t^2}=\frac{2R}{h_+}\,. \tag{43}$$

The Equation (43) received without use of the Einstein's equation [6]. Therefore it is interesting to compare it to the equation of a graviton [3] received on the basis of the Einstein's equation.

Interrelation of scalar curvature of space due to one graviton with its volumetric density of energy [3] is $R=-\frac{8\pi G}{c^4}W_1$ where $W_1=\frac{2\hbar\omega}{V}$ —volumetric density of energy for a one graviton. Substituting these sizes in the Equation (43), we shall find:

$$\frac{\partial^2 \chi}{\partial X^2}-\frac{1}{c^2}\frac{\partial^2 \chi}{\partial t^2}=-\frac{32\pi G\hbar\omega}{c^4 h_+}\,, \tag{44}$$

where as well as in [3] it is designated $\chi=h_+V$.

The main difference of the Equation (44) from the equation in [3] will be that the first equation nonlinear, and the second linear. Linearity of the equation in [3] based on linearity of the wave equation following from the Einstein's equation. If to assume in the right part of the Equation (44) $h_+=\exp\left(-i\left(rX-\omega t\right)\right)$ the Equation (44) and equation in [3] become identical.

Taking into account the analysis which has been carried out for a photon (see nonlinear equation of Schrodinger [2]) it is possible to assume that for the description of a graviton the nonlinear Equation (44) is more adequate than the linear equation in [3].

Concerning the Equation (43) it is necessary to make two remarks. First, in connection with Formula (24) used during a deduction the given equation is correct only for very small values of scalar curvature of space-time R. Taking into account structure of the Equations (43) and (41) it is possible to conclude that a component $h_+\sim\sqrt{-2R}$ where R size negative. We shall notice that the equation of a graviton the following from Einstein's equation, gives result $h_+\sim 2R$ [3].

Second, the Equation (43) essentially cannot describe the form of a graviton since during its deduction the Equation (42) for an infinite sine wave has been used.

In particular, for example, instead of the Equation (42) it is possible to use the linear wave equation, which on structure similarly used at the analysis of linear quantum-mechanical oscillator [8]:

$$\frac{\partial^2 h_+}{\partial X^2} + k^2 \left(1 - (kX - \omega t)^2\right) h_+ = 0, \tag{45}$$

where $k = \frac{\omega}{c}$ there is a wave number. The Equation (45) can describe envelope of a graviton as $h_+ = Ae^{-\frac{(kX-\omega t)^2}{2}}$ where A there is an amplitude of a graviton.

Uniting this equation with the Equation (41) we receive the wave nonlinear equation of hyperbolic type:

$$\frac{1}{1 - (kX - \omega t)^2} \frac{\partial^2 h_+}{\partial X^2} - \frac{1}{c^2} \frac{\partial^2 h_+}{\partial t^2} = \frac{2R}{h_+}. \tag{46}$$

The acceptable analytical solution of this nonlinear equation as a running wave even at $R = 0$ to find difficulty but it also submits to a condition $h_+ \sim \sqrt{-2R}$.

As a whole the analysis of the graviton form in particular the finding of physically more proved Equation (42) is the important problem of the future researches in quantum gravidynamics.

6. The Metrics of Space-Time at Presence of a Graviton. Number of Filling

The gravitational wave represents the set of gravitons. A graviton is boson therefore does not submit to the Pauli's principle. Let in the given place of space the gravitational wave is created due to set n gravitons. The size n is number of filling for gravitons. Similarly photons we suppose that the volumetric density of energy of a gravitational wave is equal:

$$W = W_1 \left(n + \frac{1}{2}\right), \tag{47}$$

where $W_1 = \frac{2\hbar\omega}{V}$ there is volumetric density of the one graviton energy, $W_{\text{vak}} = \frac{1}{2} W_1 = \frac{\hbar \omega_{\text{vak}}}{V}$—vacuum gravitational volumetric density of the Riemann's spaces – time energy [3].

Dependence of scalar curvature of space on its volumetric density of energy for the one graviton is [3] $R = -\frac{8\pi G}{c^4} W_1$. Scalar curvature of space—size is negative.

For vacuum gravitational volumetric density of energy we have $W_{\text{vak}} = \frac{1}{2} W_1 = -\frac{c^4 R_\Lambda}{16\pi G} = -\frac{c^4 \Lambda}{4\pi G} \approx 2 \times 10^{-7} \frac{\text{erg}}{\text{cm}^3}$ [7] where it is taken into account that vacuum scalar curvature of space-time is connected with a cosmological constant under the formula $R_\Lambda = 4\Lambda$ [3].

The Formula (47) has profounder substantiation than analogy to photons. We assume that $W = nW_1 = -n\frac{c^4 R}{8\pi G}$, *i.e.* energy of gravitational vacuum is absent.

Using (10), we have $-2nc^2R=\left(\dot{h}_+^2+\dot{h}_\times^2\right)$. Solving this equality together with (18), we find $\dot{h}_+^2=-Rc^2\left(n-\frac{1}{2}\right)$, that obviously specifies existence of a vacuum energy. Therefore, more correctly to use the Formula (47), *i.e.* instead of n to use $n+\frac{1}{2}$.

Having substituted (47) in (40) with the account $R=-\frac{8\pi G}{c^4}W_1$ we shall find:

$$h_{22}=\alpha\sqrt{e^{n+\frac{1}{2}}}\,, \tag{48}$$

where the previous designation a component $h_+=h_{22}=-h_{33}$ and $h_\times=h_{23}$ is used.

The interval in the Riemann's space at presence of a graviton looks like [7]:

$$\mathrm{d}l^2=\left(\mathrm{d}X^1\right)^2+\left(1+h_{22}\right)\left(\mathrm{d}X^2\right)^2+\left(1-h_{22}\right)\left(\mathrm{d}X^3\right)^2+2h_{23}\mathrm{d}X^2\mathrm{d}X^3\,. \tag{49}$$

As already it was marked earlier it is possible to assume approximately that a gravitational wave or a graviton almost do not create curvature of space-time. Therefore we accept $h_+=h_\times$ and $h_{22}=h_{23}$, hence taking into account (48) we have:

$$\begin{aligned}\mathrm{d}l^2=&\left(\mathrm{d}X^1\right)^2+\left(1+\alpha\sqrt{e^{n+\frac{1}{2}}}\right)\left(\mathrm{d}X^2\right)^2+\left(1-\alpha\sqrt{e^{n+\frac{1}{2}}}\right)\left(\mathrm{d}X^3\right)^2\\&+2\alpha\sqrt{e^{n+\frac{1}{2}}}\mathrm{d}X^2\mathrm{d}X^3.\end{aligned} \tag{50}$$

The Formula (50) also determines the quantum metrics of the Riemann's space-time [9] at presence of a graviton:

$$g_{ik}=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1-\alpha\sqrt{e^{n+\frac{1}{2}}} & -\alpha\sqrt{e^{n+\frac{1}{2}}}\\ 0 & 0 & -\alpha\sqrt{e^{n+\frac{1}{2}}} & -1+\alpha\sqrt{e^{n+\frac{1}{2}}}\end{pmatrix}. \tag{51}$$

In Formula (51) the signature of metric tensor at absence of a gravitational field, *i.e.* $\alpha=0$ equal $(+,-,-,-)$ is used.

In the tensor (51) essentially uncertain size is the parameter α. We shall estimate it on the basis of the following assumptions.

Substituting in (47) volumetric density of the gravitational wave energy (10) and a graviton $W_1=-\frac{c^4}{8\pi G}R$ we have:

$$\left(\dot{h}_+^2+\dot{h}_\times^2\right)=-2c^2R\left(n+\frac{1}{2}\right). \tag{52}$$

Solving the system of the algebraic Equations (18) and (52) we shall find expression for number of filling:

$$n=-\frac{\dot{h}_+^2}{c^2R}=-\frac{\dot{h}_{22}^2}{c^2R}\,. \tag{53}$$

Substituting (40) in (53) with account (47) we shall receive:

$$-c^2 Rn = \alpha^2 \left(\frac{4\pi G}{c^4 R}\right)^2 e^{-\frac{8\pi GW}{c^4 R}} \dot{W}^2 = \alpha^2 e^{n+\frac{1}{2}} \frac{1}{4}\left(\frac{\partial n}{\partial t}\right)^2. \tag{54}$$

Let the gravitational wave arises during time Δt due to occurrence of one graviton. In this case the Formula (54) gets a kind:

$$-c^2 R = \alpha^2 e^{\frac{3}{2}} \frac{1}{4\Delta t^2}. \tag{55}$$

Normalizing time of a graviton occurrence by a condition $\Delta t = T = \dfrac{2\pi}{\omega}$ where ω there is graviton frequency, we believe $\alpha \sim \dfrac{c}{\omega}\sqrt{-R}$. Scalar curvature of space-time is $R < 0$. Having designated $\alpha = \beta \dfrac{c}{\omega}\sqrt{-R}$ where β there is a dimensionless constant equal to zero at absence of a gravitational field. Thus, we shall receive metric tensor (51) as:

$$g_{ik} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1-\beta\dfrac{c}{\omega}\sqrt{-Re^{n+\frac{1}{2}}} & -\beta\dfrac{c}{\omega}\sqrt{-Re^{n+\frac{1}{2}}} \\ 0 & 0 & -\beta\dfrac{c}{\omega}\sqrt{-Re^{n+\frac{1}{2}}} & -1+\beta\dfrac{c}{\omega}\sqrt{-Re^{n+\frac{1}{2}}} \end{pmatrix}. \tag{56}$$

As scalar curvature of space due to a graviton is close to zero the quantum additives to a metric tensor for the account of a graviton in (56) are extremely small.

7. The Own Frequencies of a Graviton. Quantization of a Graviton

Let's consider in more detail the equation for quantum gravitational oscillator (33). It is completely equivalent to the Equation (31). The solution of the Equation (31), and hence and (33) according to (30) is function:

$$\dot{h}_+ = \sqrt{\dot{h}_\times^2 + c^2 R} = A\omega\cos(\omega t). \tag{57}$$

Thus, according to (18) it is possible to write down:

$$\dot{h}_\times = \sqrt{A^2\omega^2\cos^2(\omega t) - c^2 R}. \tag{58}$$

Taking into account $h_+ = \dfrac{1}{\omega}\dot{h}_\times$ (24), we shall copy (58) as:

$$\begin{aligned} h_+ &= \frac{1}{\omega}\sqrt{A^2\omega^2\cos^2(\omega t) - c^2 R} = A\sqrt{\cos^2(\omega t) - \frac{c^2 R}{A^2\omega^2}} \\ &= A\sqrt{\left(\cos(\omega t) + \frac{c\sqrt{-R}}{A\omega}\right)\left(\cos(\omega t) - \frac{c\sqrt{-R}}{A\omega}\right)}. \end{aligned} \tag{59}$$

In record (59) it is taken into account: scalar curvature of the Riemann's space

$R < 0$. The amplitude A can be size both positive, and negative.

Let's find the own frequencies of graviton ω. Let in (59) $t = 0$—the time of a graviton beginning occurrence, in this case at $h_+ = 0$ size $\frac{c\sqrt{-R}}{A\omega} = \pm 1$. Hence $\cos(\omega t) = \pm 1$. Size $\omega t = m\pi$ and $t = \frac{m\pi}{\omega}$—the time of a graviton end occurrence, where $m = 0, 1, 2, \cdots$—integers. Multiplying last equality for the speed of a graviton c, we shall receive feasible lengths of the gravitons $L = ct = \frac{mc\pi}{\omega}$ or $\omega = \frac{mc\pi}{L}$—the own frequencies of the gravitons. Taking into account energy of a graviton $E = 2\hbar\omega$, we shall receive:

$$E = \frac{2\pi\hbar mc}{L}. \tag{60}$$

The Formula (60) allows find feasible values of a graviton energy, m there is internal quantum number of a graviton. Energy of gravitational vacuum into the formula for the graviton energy cannot enter, since spatial vacuum for a graviton—an environment. Therefore at $m = 0$ the graviton energy $E = 0$.

The graviton amplitude is equal:

$$\pm A = \frac{c\sqrt{-R}}{\omega}. \tag{61}$$

Therefore, metric tensor of the Riemann's spaces—time at presence of a graviton (56) can be written down as:

$$g_{ik} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 - \beta A\sqrt{\mathrm{e}^{n+\frac{1}{2}}} & -\beta A\sqrt{\mathrm{e}^{n+\frac{1}{2}}} \\ 0 & 0 & -\beta A\sqrt{\mathrm{e}^{n+\frac{1}{2}}} & -1 + \beta A\sqrt{\mathrm{e}^{n+\frac{1}{2}}} \end{pmatrix}. \tag{62}$$

In the given point of the Riemann's spaces can be created a gravitational wave by the several gravitons—bosons with the own frequencies ω. We shall estimate this quantity of the gravitons. The general volumetric density of energy of a wave if to accept energy of gravitational vacuum for a reference mark, according to the Formula (47), is equal:

$$W = nW_1. \tag{63}$$

where n there are quantity of the gravitons participating in creation of a gravitational wave in the given place.

Average value components metric тензора $\overline{h}_+$ in a gravitational wave according to (40) is equal see for example [4]:

$$\overline{h}_+ = \frac{\alpha \sum_{n=0}^{\infty} n\mathrm{e}^{-\frac{4\pi GnW_1}{c^4 R}}}{\sum_{n=0}^{\infty} \mathrm{e}^{-\frac{4\pi GnW_1}{c^4 R}}} = \frac{\alpha}{\mathrm{e}^{\frac{4\pi GW_1}{c^4 R}} - 1} = \frac{\alpha}{\sqrt{\frac{1}{\mathrm{e}}} - 1}. \tag{64}$$

In (64) the formula for volumetric density of energy of a one graviton $W_1 = -\frac{c^4}{8\pi G} R$ [3] is used. Besides, formulas of the sums of infinite lines of a geometrical progression in a denominator and lines $\sum_{n=1}^{\infty} nz^n = \frac{z}{(1-z)^2}$ in numerator are used.

Equating (64) to corresponding a component of metric tensor (51) without taking into account gravitational vacuum, we shall find $\frac{\alpha}{\sqrt{\frac{1}{\mathrm{e}}-1}} = \alpha\sqrt{\mathrm{e}^n}$. Hence $n \approx 1.866$.

Thus, the gravitational wave is creating approximately to two gravitons in each point of the Riemann's spaces.

8. Conclusions

A comparison of the photon and the graviton properties shows that despite some similarity, properties of these particles essentially differ. First of all, they are in different physical spaces. The photon is in photon space, and the graviton is in Riemann's space-time. Interaction of a photon and a mass particle cannot be accurately calculated in Euclidian space. It is necessary to use the approximate "diagram method" developed by R. Feynman for this calculation.

Calculation of interaction of a graviton and a mass particle can be carried out precisely in Riemann's space-time, therefore, application of the Feynman's "diagram method" for these purposes is excessive. For the description of a graviton, the quantum wave function of probability is not necessary therefore problem of the gravitons entanglement is absent.

Principles of polarization of a photon and a graviton differ. Polarization of a photon has a vector character, and polarization of a graviton has a tensor character. Spin of a photon is equal to one and of a graviton to two, etc.

The quantum interval and metric tensor determining the metrics of the Riemann's space-time in presence of a graviton are found. As scalar curvature of space with the account of a graviton is close to zero, the quantum additives to metric tensor with the account of a graviton are extremely small.

Feasible values of the graviton energy as a quantum particle are found.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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