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Strategies for Indexed Stock Option Hedgers with Loss-Risk-Minimizing Criterion Based on Monte-Carlo Method

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Abstract

Unlike traditional options, indexed stock options use market performance as a benchmark reference index, and the option exercise price is a variable that changes with market performance. This paper, by taking the expected loss at the end of the hedging period as a risk measure, conducts a study on the hedging strategies for indexed stock option hedgers. Empirical analysis shows that, firstly, it is more conducive for indexed stock options to play an incentive role by adjusting the exercise price according to changes in market conditions, secondly, when the frequency of hedging position adjustment is relatively high, it can better cope with the price fluctuations in the market, thereby reducing the risk of possible loss and achieving a better hedge effect, but the hedging costs will increase for because of the existence of transaction costs.

Keywords

Indexed Stock Option Hedgers, Loss-Risk-Minimizing, Monte-Carlo Method

1. Introduction

All kinds of contingent claims may be perfectly replicated by self-finance strategy if the market is complete, and the cost to replicate is the fair price of the contingent claim; in an incomplete market, investors may also perfectly replicate a contingent claim by super hedging strategy (Bayraktar & Zhou, 2017), however, with the exception of being costly, super hedging causes the loss of chance to get more profits, thus, many investors are unwilling to do this. In fact, most people only want to pay a small quantity of initial cost to hedge the terminal contingent claim,

but they have to endure a level of risk, how to find the optimal hedging strategy for such kind of hedging has been a hot topic in finance. Before seeking for the optimal strategy, we should decide a criterion to measure the risk, a simple and exclusively accepted method is the minimal variance hedging (Last & Penrose, 2011; Makogin, Melnikov, & Mishura, 2017), even though it's shortcoming to simultaneously punish the profit and the loss; another method to measure risk is VaR (Cong, Tan, & Weng, 2014; Soloviev, 2016; Capiński, 2015), which anticipates the heaviest lost under given criterion level, however, VaR may be given different value for different investors. As for hedging, only potential shortfall will be considered, with an example European claim, investors' goal is to seek the optimal strategy to minimize the expected loss $E\left(\left[H_T - V_T(\varphi)\right]^+\right)$, which had been originally researched by Follmer & Leukert (Follmer & Leukert, 2000), and many subsequently research results have achieved (Kim, 2012; Kabaila & Mainzer, 2018).

Unlike traditional options, indexed stock options use market performance as a benchmark reference index, and the option exercise price is a variable that changes with market performance, i.e. $K_T = S_0 I_T / I_0$, where I_T represents the market performance, or the overall market trend of the stock market, or the performance of competitors in the same industry at the end of the hedging period, while S_0 and I_0 represent the initial price of the underlying stock and the initial market price, respectively. In this way, even in a bull market, a rise in the market will cause the benchmark reference index to rise, and drive the option exercise price to rise, thereby filter the stock price changes in the market due to non-manager efforts, if the performance level of the company is lower than the benchmark index, the value of stock options may still be zero in a bull market, and managers will not receive huge profits when they execute stock options; Conversely, even in a bear market, if the corporate performance is higher than the benchmark index, the option value can also be positive, and the manager can still get incentives for option returns.

This paper, by taking the expected loss i.e. $\min_{\varphi} E\left[H_T - V_T(\varphi)\right]^+$, at the end of the hedging period as a risk measure, and using the Monte Carlo method, conducts a study on the hedging strategies for indexed stock option hedgers. Enlightened by Longstaff and Schwartz (Longstaff & Schwartz, 2001), Potters M. et al. (Potters, Bouchaud, & Sestovic, 2001), who priced the option with numerical method, we firstly generate many asset price paths by Monte-Carlos simulation and look on the averaged terminal shortfall as the expected loss, then, basis functions are introduced to estimate hedging positions and finally the optimal strategy is achieved through an algorithm (Seydel, 2017, Monte-Carlo-Simulation [M]).

2. Some Preliminaries

Assume there are two kinds of assets: risky asset (Security) and riskless asset (Bond). Let (Ω, F, P) be a complete probability space with filtration $F = (F_t)_{t \in [0,T]}$,

and the price of risky asset $S = (S_t)_{t \in [0,T]}$ and the market index (in this paper, which is CSI) $I = (I_t)_{t \in [0,T]}$ be nonnegative and adapted to F, satisfying:

$$\begin{cases} dS_t = S_t \left(\mu dt + \sigma dw_t + q_t dN_t \right) \\ dI_t = I_t \left(\mu^{(I)} dt + \sigma^{(I)} dw_t^{(I)} + q^{(I)} dN_t^{(I)} \right) \end{cases}, \quad (1)$$

where w_t, w_t^I are standard Brownian motions, $N_t, N_t^{(I)}$ are Poisson processes with Poisson strength $\lambda, \lambda^{(I)}$ and respectively independent with w_t and $w_t^{(I)}$, $q^{(I)}$, $q_t > -1$ are amplitudes of price jumps and $J_t = \ln(1 + q_t) \sim N(\mu_J, \sigma_J^2)$, $J_t^{(I)} = \ln(1 + q_t^{(I)}) \sim N\left(\mu_J^{(I)}, (\sigma_J^I)^2 \right)$.

Let $B = (B_t)_{t \in [0,T]}$ be riskless asset's price process, satisfying:

$$dB_t = rB_t dt, \quad r \text{ denotes riskless interest rate.} \quad (2)$$

As for random sequence $l = (l_t)_{t=0,1,\cdots,T}$, let $\Theta(l)$ be a space consisting of all predictable sequences $(\vartheta_t)_{t=0,1,\cdots,T}$ which satisfies $\vartheta_t \Delta l_t \in L^2(P)$.

Call a 2-demension stochastic process $\varphi = (\vartheta_t, \delta_t)_{t=0,\cdots,T-1}$ be a investment strategy and $\vartheta_t \in \Theta(S)$, δ_t is an adapted process, satisfying:

$$V_t(\varphi) := \vartheta_t S_t + \delta_t B_t \in L^2(P), \quad t \in \{0,1,\cdots,T-1\}, \quad (3)$$

where $V(\varphi)$ is the value of strategy φ, when (ϑ_t, δ_t) denotes the hedging position held at time t.

Furthermore, we call it a self-finance strategy when $\varphi = (\vartheta_t, \delta_t), t = 0, \cdots, T-1$ satisfies:

$$\vartheta_t S_{t+1} + \delta_t = \vartheta_{t+1} S_{t+1} + \delta_{t+1}, \quad t = 0,1,\cdots,T-1. \quad (4)$$

As to strategy $\varphi = (\vartheta_t, \delta_t)_{t=0,1,\cdots,T-1}$, we define its shortfall risk as the following:

$$R_t(V_t, S_t, \varphi) = E\left\{ \left[H_T - V_T(\varphi) \right]^+ \mid F_t \right\}, \quad t = 0,1,\cdots,T-1, \quad (5)$$

where V_t, S_t denote the portfolio value and the stock price at time t respectively, $H_T = (S_T - K_T)^+$ is a F_T-measurable and nonnegative random variable which denotes the hedger's payment reliability at the expiration.

Suppose an investor has initially written a share of European Call Option with the exercising price K and T horizon, in order to minimize the terminal shortfall, he hedges the option by self-financing at discrete time $\{0,1,\cdots,T-1\}$ with $V_0 = \vartheta_0 S_0 + \delta_0$ as his initial cost, thus, we can express the hedging model as following:

$$\begin{cases} \min_{\phi} E\left\{ \left[H_T - V_T(\varphi) \right]^+ \right\} \\ \text{s.t. } \vartheta_t S_{t+1} + \delta_t B_{t+1} = \vartheta_{t+1} S_{t+1} + \delta_{t+1} B_{t+1} \\ \quad t = 0,1,\cdots,T-1 \end{cases}. \quad (6)$$

3. Solution

3.1. The Parameter-Estimation of the Price Process

According to the expression (1):

$$y_t = \left(\mu - \frac{\sigma^2}{2} \right) + \sigma \varepsilon_t + \sum_{i=N(t-1)+1}^{N(t)} J_i , \qquad (7)$$

where $\varepsilon_t = w_t - w_{t-1} \sim N(0,1)$, $Z_t = N_t - N_{t-1} \sim p(\lambda)$, $J_i \sim IIDN\left(\mu_J, \sigma_J^2 \right)$.

Let $\Theta = \left(\mu, \sigma^2, \lambda, \mu_J, \sigma_J^2 \right)$, $y = \{ y_1, y_2, \cdots, y_T \}$, $X = \{ (Z_t, J_t) \}$ be jump times and jump amplitude respectively, MCMC technology has been used to estimate all parameters, i.e. a Markovian chain of each parameter has been drawn from $p(\Theta, X \mid y) \propto p(y \mid \Theta, X) p(X \mid \Theta) p(\Theta)$ with observations $\{ y_1, y_2, \cdots, y_T \}$ and prior distribution $p(\Theta)$, and averaging the chain as estimator of parameter (Johannes & Polson, 2006).

With the estimated parameters we can produce:

$$y_t = \ln \left(S_t / S_{t-1} \right) \sim N\left(\left(\mu - \frac{\sigma^2}{2} \right) + \lambda \mu_J, \sigma^2 + \lambda \left(\sigma_J^2 + \mu_J^2 \right) \right). \qquad (8)$$

3.2. The Monte-Carlo Simulation of Price Process

Let $f(x_1, \cdots, x_n)$ be the union density function of random vector $(X_1, \cdots, X_n)$, $f(x_{i+1} \mid x_1, \cdots, x_i)$ be conditional density function of X_{i+1} with $X_1, \cdots, X_i$ being known, according to the Bayesian theory:

$$f(x_1, \cdots, x_n) = f(x_1) f(x_2 \mid x_1) \cdots f(x_n \mid x_1, \cdots, x_{n-1}).$$

Theoretically, we can draw $(X_1, \cdots, X_n)$ from the union density function $f(x_1, \cdots, x_n)$, but in fact, we do the following:

Firstly, drawing $\tilde{X}_1 \sim f(x_1)$; then, $\tilde{X}_2 \sim f(x_2 \mid \tilde{X}_1)$ and so on, drawing $\tilde{X}_n \sim f(x_n \mid \tilde{X}_1, \cdots, \tilde{X}_{n-1})$, and we can deduce that the drawn sequence $(\tilde{X}_1, \cdots, \tilde{X}_n)$ has a union density function $f(x_1, \cdots, x_n)$.

3.3. Strategy Decision

On discrete time $\{ 0, 1, \cdots, T-1 \}$, assuming Bond's price $B \equiv 1$, the stock price has been discounted. Under the constraint of self-financing (4), we have:

$$\begin{aligned} V_T &= V_{T-1} + \vartheta_{T-1} \Delta S_{T-1} \\ &= V_{T-2} + \vartheta_{T-2} \Delta S_{T-2} + \vartheta_{T-1} \Delta S_{T-1} , \\ &= \cdots = V_0 + \sum_{t=0}^{T-1} \left(\vartheta_t \Delta S_t \right) \end{aligned} \qquad (9)$$

where $\Delta S_t = S_{t+1} - S_t, t = 0, 1, \cdots, T-1$, V_0 is the initial cost. Now, substituting (6) with (9), the optimizing problem (6) becoming into:

$$\min_{\varphi} E \left\{ \left[H_T - V_0 - \sum_{t=0}^{T-1} \left(\vartheta_t \Delta S_t \right) \right]^+ \right\}, \qquad (10)$$

Up to now, our goal is to find a self-financing strategy for optimization problem (10), however, it is a stochastic programming problem, and $H_T - V_0 - \sum_{t=0}^{T-1} \left(\vartheta_t \Delta S_t \right)$ depends on the whole price path, so, it is difficult to directly solve (10). Having generated M independent price paths by Monte-Carlo simulation method, solving problem (10) is equivalent to solving the following optimizing problem:

$$\min_{\varphi} \sum_{m=1}^{M} \left[H_T^{(m)} - V_0 - \sum_{t=0}^{T-1} \left(\vartheta_t^{(m)} \Delta S_t^{(m)} \right) \right]^+ . \tag{11}$$

Obviously, the unknown variables in (11) equal $M \cdot T$, where M denotes the number of scenarios and T denotes the adjusting frequency, thus, it is computationally challenging to directly solve problem (11) when the number of scenarios is large and the adjusting is frequent. In order to simplify (11), we try to approximate holdings ϑ_t by basis functions, having done this, the number of unknowns at each hedging time is reduced to the number of parameters in the basis functions, which is typically very small.

Assume that the holding ϑ_t is decided by the underlying stock price at any time t, i.e. $\vartheta_t = \vartheta(S_t)$, by definition 1, $\vartheta_t \Delta S_t \in L^2(P)$, and the Hilbert space $L^2(P)$ is of countable orthonormal basis, so, the holding ϑ_t can be linearly represented by basis functions (Potters, Bouchaud, & Sestovic, 2001):

$$\vartheta_t = \vartheta(S_t) = \sum_{j=1}^{p} a_j^{(t)} L_j(S_t), \ t = 0,1,\cdots,T-1 , \tag{12}$$

where $L_j(S_t)$ denotes basis function and $a_j^{(t)}$ denotes the corresponding constant coefficients, p is the number of selected basis functions such as Hermite polynomial, Legendre polynomial, Chebyshev polynomial, Laguerre polynomial, hereafter we choose Laguerre polynomial as basis function which is formed as following:

$$L_n(x) = \exp\left(-\frac{x}{2}\right) \frac{\exp(x)}{n!} \frac{\mathrm{d}^n}{\mathrm{d}x^n} \left(x^n \mathrm{e}^{-x}\right).$$

After substituting (12) into (11), the optimizing problem (17) changes into:

$$\min_{\varphi} \sum_{m=1}^{M} \left[H^{(m)} - V_0 - \sum_{t=0}^{T-1} \left(\left(\sum_{j=1}^{p} a_j^{(t)} L_j\left(S_t^{(m)}\right) \right) \Delta S_t^{(m)} \right) \right]^+ . \tag{13}$$

Comparing (13) with (11), we find the number of unknowns has greatly decreased, and the holding ϑ_t may be deduced by (12) only with the selected basis functions and the simulated price scenarios, which is solved by the steepest descent method.

3.4. The Steepest Descent (Wahab & Khan, 2018)

Assuming objective function $f(x)$, $x \in R^n$, when $\nabla f(x_k) \neq 0$, at x_k, expressing $f(x)$ with:

$$f(x) = f(x_k) + \nabla f(x_k)'(x - x_k) + o(\|x - x_k\|). \tag{14}$$

Denoting $x - x_k = \alpha d_k (\alpha > 0)$, then the expression (20) can be expressed:

$$f(x_k + \alpha d_k) = f(x_k) + \alpha \nabla f(x_k)' d_k + o(\|\alpha d_k\|). \tag{15}$$

We call d_k the descent direction of $f(x)$ when $\nabla f(x_k)' d_k < 0$. As to small α, there is $f(x_k + \alpha d_k) < f(x_k)$, and with smaller $\nabla f(x_k)' d_k$, $f(x)$ has greater descent at x_k. denoting θ_k as the plane included angle of

$-\nabla f\left(x_k\right)'$ and d_k, by $-\nabla f\left(x_k\right)' d_k = \left\|\nabla f\left(x_k\right)\right\| \cdot \left\|d_k\right\| \cos\theta_k$, when $\theta_k = 0$, *i.e.*, $d_k = -\nabla f\left(x_k\right)$, $\nabla f\left(x_k\right)' d_k$ arrives at the smallest value, and $-\nabla f\left(x_k\right)$ is called as the steepest descent direction, $d_k = -\nabla f\left(x_k\right)$ is the optimal searching direction. The iterative format of the steepest descent method is as following:

$$x_{k+1} = x_k + \alpha_k \nabla f\left(x_k\right), \tag{16}$$

where α_k denotes the step length decided by linear searching method.

The process of the steepest descent method is as following:

Step 1: Given the initial $x_0 \in R^n$ and the terminating error $\varepsilon > 0$, set $k := 0$;

Step 2: Calculating $d_k = -\nabla f\left(x_k\right)$, stop when $\left\|d_k\right\| < \varepsilon$ and x_k is the optimal solution;

Step 3: Deciding the step length α_k by the linear searching method;

Step 4: Setting $x_{k+1} = x_k + \alpha_k \nabla f\left(x_k\right)$, $k := k+1$, transfer to the step 2.

4. Numerical Example

4.1. The Parameter Deciding of the Price Model

In this subject, we sampled 11,598 high frequency history data of Shanghai Securities Complex Index and the stock ICBC from the 2nd, January to the 28th, December, 2018, and estimated the jump-diffusion process's parameters in WinBugs1.4 by Monte Carlo technology, the estimating results are expressed in Table 1.

4.2. The Simulated Scenarios and Strategies

Firstly, according to the parameter values in Table 1, we simulated $M = 10000$ price scenarios for the underlying asset by the Monte-Carlo technology submitted in subsection 2.1; Then, deciding the number of basis function, since the computation result does not obviously improve when the number of basis function is more than 3, we choose the first 3 Lagurre polynomials as basis function to approximate the holding position; Finally, in the light of the optimization model (12), the optimal holdings are acquired through numerical algorithm presented in subsection 2.4 with the Matlab software.

4.3. Analyzing Results

We assume that a hedger has written the 1-month and 3-month expiration indexed stock option based on the stock ICBC at the 28th, December, 2018, in order to minimize the terminal expected loss, he hedges the contingent claim with stock ICBC and Bond by self-financing with daily, weekly and biweekly

Table 1. parameter valuation of the jump-diffusion process (Data source, http://quotes.money.163.com/stock).

parameters	$\mu\left(\mu^{(I)}\right)$	$\sigma\left(\sigma^{(I)}\right)$	$\mu_J\left(\mu_J^{(I)}\right)$	$\sigma_J\left(\sigma_J^{(I)}\right)$	$\lambda\left(\lambda^{(I)}\right)$
Underlying tock (S)	0.0025	0.0411	0.0183	0.0522	0.0490
Index (I)	0.0023	0.019	0.0155	0.0519	0.0453

rebalancing frequency, the initial Index price and ICBC price are $I_0 = 2493.9$, $S_0 = 5.29$, respectively, and the riskless interest rate is 0.30% equal to the current savings interest rate offered by the PBC in December, 2018 (http://www.pbc.gov.cn/), the transaction fee rate $f = 0.001, 0.002, 0.004$, respectively.

As for all hedging cases, we calculate:

Total risk:

$$\left(H_T - V_T \right)^+.$$ (17)

Total cost:

$$V_0 + \vartheta_0 f S_0 + \sum_{t=1}^{T-1} \left| \vartheta_t - \vartheta_{t-1} \right| f S_t + \vartheta_{T-1} f S_T + H_T - V_T,$$ (18)

where V_0 is the initial cost; V_T denotes the terminal portfolio valuation.

Substituting (12) into (18):

Total cost:

$$\vartheta_0 f S_0 + \sum_{t=1}^{T-1} \left| \vartheta_t - \vartheta_{t-1} \right| f S_t + \vartheta_{T-1} f S_T + H_T - \sum_{t=0}^{T-1} \left(\vartheta_t \Delta S_t \right).$$ (19)

As a matter of fact, with the exception V_0, transaction fee and the terminal H_T, no other hedging cost is required.

The averaged hedging cost and expected loss with different striking price and different strategy adjustment frequency are calculated in Table 2 and Table 3 with 10,000 price simulations.

Firstly, because the price fluctuation will be heavier if option's expiration is longer, a European indexed stock option hedger may be faced with higher risk and must invest more to hedge possible loss risk, for example, in Table 3, the averaged hedging costs (with 3-month expiration) for all kinds of strategy adjustment frequencies and all kinds of transaction fee rates are more than those in Table 2 (with 1-month expiration), i.e., for daily hedging strategy adjustment frequency, 1.5011 is bigger than 1.3251, 2.7656 is bigger than 2.4930, 5.2962 is bigger than 4.6254, and for weekly and biweekly hedging strategy adjustment frequencies, there are similar results. Other more, We can also see that the expected loss may be smaller when the hedging strategy adjusting time step is shorter, for example, in Table 2, 0.702 is the least expected loss, which corresponds to daily hedging strategy adjustment frequency, while 0.8010 is the biggest expected loss, which corresponds to biweekly hedging strategy adjustment frequency, similar results in Table 3.

In addition, we can know by the expression $\left(H_T - V_T \right)^+ = \left(\left(S_T - K_T \right)^+ - V_T \right)^+$ that the higher the striking price is, the smaller the expected loss may be; and it is more impossible for the indexed stock option with higher striking price to be executed, which results in decreased hedging cost. In fact, Table 2 and Table 3 indicate the reverse relationship between expected loss with striking price, and the following Figure 1, indicating the relationship between the holding position at the middle time point with the terminal exercising price for the indexed stock

Table 2. Averaged hedging cost and expected loss with 1-month expiration over 10,000 scenarios ($K_T = S_0 I_T / I_0 = 5.29 \times 2584.57/2493.9 = 5.48$).

frequency	Daily		Weekly		Biweekly	
f	Expected loss	cost	Expected loss	cost	Expected loss	cost
0.001	0.0702	1.3251	0.0731	1.0031	0.0810	1.0005
0.002	0.0702	2.4930	0.0731	1.9821	0.0810	1.8964
0.004	0.0702	4.6254	0.0731	3.6987	0.0810	3.5102

Table 3. Averaged hedging cost expected loss with 3-month expiration over 10,000 scenarios ($K_T = S_0 I_T / I_0 = 5.29 \times 3090.76/2493.9 = 6.56$).

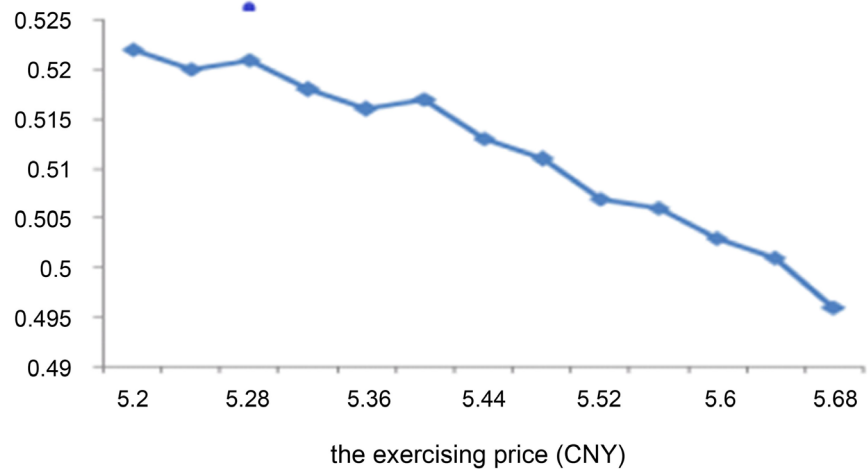| frequency	Daily		Weekly		Biweekly	
f	Expected loss	cost	Expected loss	cost	Expected loss	cost
0.001	0.0508	1.5011	0.0526	1.2513	0.0529	1.2120
0.002	0.0508	2.7656	0.0526	2.3111	0.0529	2.2589
0.004	0.0508	5.2962	0.0526	4.4596	0.0529	4.3254

Figure 1. Relationship between holdings and striking prices at middle time point.

option with single month expiration. In Figure 1, the horizontal axis data shows the end-of-period execution prices, while the data on the vertical axis represent the hedging positions that need to be held for 1 share of stock to be hedged at the middle time point with 1-month hedging period. Taking the hedging practice for the ICBC stock as an example, if the terminal exercising price is 5.2CNY, the optimal hedging position is 0.522 shares of stock index futures contract, while the terminal exercising price is 5.68CNY, the optimal hedging position is 0.496 shares of stock index futures contract.

Figure 2 illustrates the relationship between holding positions and different executing prices for 1-month time limit indexed stock option with daily hedging
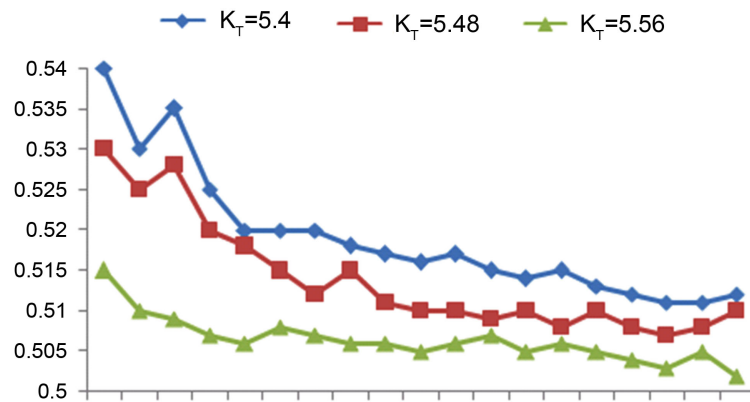
Figure 2. Monthly adjusting position plot with different striking prices.

strategy adjusting frequency. As a whole, the reverse relationship between holding position with striking price can still be found; what's more, the three dash dot lines denoting holding position changing in **Figure 2** all rightward incline, which explains that the required holding position may be decreased with time's going by. it is well known that the indexed stock option will be executed at the maturity date, because the farer away the maturity date is, the more heavily the underlying asset's price fluctuates, therefore, more hedging cost must be invested to acquire the same hedging efficiency.

Finally, it is obvious that the lowest line in **Figure 2** fluctuate mildly, especially when K_T equals to 5.56, the curve almost fluctuates around a line, however, the upper two curves denoting $K = 5.4, 5.48$, the option being in the money, fluctuate more heavily than the lowest curve, the option being out of the money, which correspond to the third column in **Table 2**, the hedging cost of option in the money augments more heavily than option out of the money corresponding to the augment of striking price K_T.

5. Conclusion

It is well-known that the goal of hedging is to decrease the risk arising from the price fluctuating, the core objective of hedging is to ascertain reasonable hedging strategies. In this paper, we construct the optimizing model to minimize the terminal shortfall risk under the constraint of self-financing, by Monte-Carlo simulation, many price scenarios are generated and are averaged to estimate the expected shortfall, then, basis functions are imported to approximate the holding positions, finally, the optimal hedging positions are acquired by numerical technology. **Table 2**, **Table 3**, **Figure 1** and **Figure 2** indicate: the technique put forward in this paper is feasible and valuable for investors to hedge risk.

1) **Table 2** and **Table 3** illustrate, the higher the hedging strategy adjusting frequency is, the more superior the hedging efficiency is.

2) **Figure 1** and **Figure 2** indicate, the holding position is in inverse proportion to the striking price, i.e., the higher executing price the European call option has, the lower holding position may be held, vice versa. In this way, we can hedge

risk and save cost at the same time.

In conclusion, because the market is changing rapidly, in order to obtain better hedging results, it is necessary to make reasonable adjustments for hedging positions based on market changes. In other words, frequent hedging strategy adjustments can reduce period-end losses, but because of the existence of transaction costs, excessively frequent hedging strategy adjustments may not be desirable, conversely, if the adjustment frequency of the hedging strategy is too low, it is difficult to achieve expected hedging effect.

Relative to existing research results, in this paper, there are two innovations and main contribution, the first is to expand the application of indexed stock options in the field of hedging, the second is to propose a solution for the nonlinear optimization problem (As shown in expression 6). However, the relevant conclusions of this study are all based on simulation data. Whether different simulated data have influence on the conclusion has not been explained theoretically. This is also our future research direction.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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