Journal of Applied Mathematics and Physics, 2024, 12, 944-981
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Extension of Paraconsistent Many-Valued Similarity Method to Group Decision

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How to cite this paper: Inusah, A. (2024) Extension of Paraconsistent Many-Valued Similarity Method to Group Decision. *Journal of Applied Mathematics and Physics*, **12**, 944-981.
https://doi.org/10.4236/jamp.2024.123059

Received: January 26, 2024
Accepted: March 26, 2024
Published: March 29, 2024

Abstract

The Paraconsistent Many-Valued Similarity (PMVS) method for multi-attribute decision making will be incomplete as a decision model if it is not extended to the realm of group decision-making. Therefore, in this article, our primary objective is to show how the paraconsistent many-valued similarity method can be used to solve group decision-making problems involving choice making or ranking of a finite set of decision alternatives. Moreover, since weights are very important parameters in multi-attribute decision-making, we have introduced the Borda rule to calculate the weights of experts and that of every criterion under consideration. To demonstrate how the proposed method works, a numerical example on energy sources of an economy from the points of view of a group of experts is investigated. Further, we compare the results of this new approach with that of fuzzy TOPSIS group decision-making method to illustrate the robustness and effectiveness of the former.

Keywords

Multiple Criteria Evaluation, Group Decision-Making, Paraconsistent, Borda Rule, Energy Sources, Global Strength, Global Weakness, Aggregated Evidence Couples

1. Introduction

Multi-attribute decision making may simply be regarded as the making of choices in the midst of several and sometimes conflicting attributes or criteria or goals [1]. Existing literature abounds with methods and applications of multi-attribute decision making for individual decision-makers [2] [3] [4] [5] [6]. Group decision analysis is a continuation of these preceded efforts to help decision makers efficiently and systematically unravel their daily multiple criteria decision problems, be they individual or group problems [5]. Multi-Attribute Group Decision-Making (MAGDM) refers to the process whereby a group of individuals particularly decision-makers based on their interests, experience, judgement and preferences evaluates a given number of options vis-à-vis several usually conflicting criteria to be able to rank the options or choose the optimal option from the list of options [7] [8]. A synthesised choice is a binding choice for the whole group and not just for a single member of the group.

Problem statement: as a result of technological advances in today's world (in the areas of electricity, laser, semiconductor chips, quantum computing, elevator, human genome project, automobile, global positioning systems, smartphones and many more); multipersonal decision-making has become an indispensable tool for resolving complex decision problems posed by this advancement. Due to this reality, quite a lot of outranking methods have been extended into the field of group decision-making by adjusting those models in ways that will enable them deal with the various decision problems encountered at the group level. These extensions reflect the world's appreciation of the superiority of collective efforts over single efforts and the superiority of collective decisions over that of an individual in some situations [9] [10]. Put differently, synthesised group decisions in some circumstances turn out to be more factual, equitable and reflective of the reality on the ground than the decisions made by a perceived experienced individual for a group. Similarly, the motivation for extending the PMVS method to group decision-making is to help users to address decision-making problems fraught with partially conflicting and vague information at the group level. The existence of conflicting and vague information is often a result of a lack of information, or the presence of inadequate (incomplete) information or the availability of too much information or the existence of incomplete, yet contradictory information [11]. These shortcomings manifest themselves in the process of group decision-making due to group members unequal knowledge, understanding, perception, experience, judgment, and discernment about the phenomenon around which the decision problem revolves. In other words, each member of the decision-making group has a unique degree of knowledge, understanding, perception, experience, and other characteristics about the decision problem. This uniqueness of group members leads to the generation of unique and sometimes conflicting, incomplete, and imprecise or vague information by each member. Faced with data sets characterised by such inconsistencies, conventional outranking methods, namely TOPSIS, PROMETHEE, AHP, ANP and so on that obey the principle of non-contradiction cannot be applied to these data sets until those data sets have been made free of the inconsistencies. Applying these methods to these revised data sets engenders results that do not actually mirror reality. The PMVS method, however, (thanks to its paraconsistent logic component) does not obey the non-contradiction principle, and therefore, could analyse these data in their raw and inconsistent form to generate results that objectively and truly reflect reality. Thus, PMVS is a model that is tolerant to contradiction, yet coherent and non-trivial.

Further, weight is one of the most important features of every mult-attribute decision-making model. Weights of criteria and that of experts or decision-makers can influence considerably the findings of any group decision-making operation. For this reason, these weights must be fixed with great caution. However, no hard and fast rules have been provided for the establishment of the weights of criteria for all decision models—In most cases, it is assumed that the decision maker is able to conjure or generate intuitively the weights of the criteria [12]. This way of inducing weights of criteria could lead to erroneous decisions. Hence, the essence of the methods of establishing the relative weights of criteria and sometimes the relative weights of decision-makers under group decision context cannot be over emphasised. Therefore, in this paper which is an extension of our previous article [13], we present a systematic procedure on how to use the Paraconsistent Many-Valued Similarity (PMVS) method to address group decision-making challenges (*i.e.*, challenges characterised by numerous goals or attributes in relation to a fixed number of alternative courses of action), and by means of the Borda count, we have determined the relative weights of criteria and decision experts which is a sine qua non parameter in every outranking model. The rational for choosing the Borda rule for the calculation of weights in group decision-making includes the facts that it is simple, easy to understand and calculate as well as effective in determining weights that accurately reflect the importance of each criterion and each expert. It is also less cumbersome and less prone to errors.

Some of the group decision-making methods and those multi-attribute single decision-making methods that have been expanded to the field of group decision-making include Analytic Hierarchy Process (AHP) [14], the Analytic Network Process (ANP) [15], the Delphi method [5] [16] [17]; the Preference Ranking Organisation MeTHod for Enrichment Evaluation (PROMETHEE) [18], the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [19] [20] [21], PL-TOPSIS [22], Probabilistic Linguistic—organisation rangement et synthèse de donnèes relationnelles (PL-ORESTE) [23], the VIse-Kriterijumska Optimizacija I Kompromisno Resenje, which translates as: Multicriteria Optimization and Compromise Solution (VIKOR) method [24], the Multi-Objective Optimisation by Ratio analysis plus the full Multiplicative form (MULTIMOORA) method [25]. An agent model for group ranking problems has also been developed by Fernandez and Olmedo in [26]. Their model is an extension of the concepts of concordance and discordance in the Elimination and Choice Translating Reality (ELECTRE) method. They modelled group preferences with fuzzy binary relations that are devoid of paradoxes with respect to the imprecise idea of group preferences. A lot more methods can be found in [5].

In the literature, several, yet different weight calculation procedures have been proposed to facilitate a fair and an objective distribution of weight among a set of criteria and usually a group of experts or decision-makers in diverse suitable decision-making methods. Among these are, namely [27], where Yue used an extended version of TOPSIS to calculate the relative weights of decision-makers;

the same author developed an approach by means of extended projection technique to establish the individual weight of decision makers [28]. Based on the influence of group relations between group members, French advanced a technique for calculating the relative weights of group members [29]. Xu by means of bodily's method designed an approach for determining the relative weights of group members [30]. Khélifa and Martel developed an approach for the establishment of the weights of members of a group via individual outranking indices [31]. Macharis *et al.*, in [12] adopted the AHP to among other things determine the weights of criteria. A novel approach based on Ordered Weighted Averaging (OWA) for weights determination in industrial decision making was proposed by Renaud *et al.*, in [32]. More information on weight calcuation can be in [33].

So, as mentioned in the problem statement, we introduced in this paper the Borda rule to calculate the relative weights of a group of experts and the relative weights of criteria and then incorporate these two sets of values of weight into our novel group decision-making model.

The remaining part of the article is partitioned as follows: Section 2 recalls the concept of the Borda count and describes the steps of the Borda rule; Section 3 recalls some other relevant concepts, definitions and properties; Section 4 gives a brief description of the fuzzy TOPSIS group decision-making method; Section 5 presents the algorithm of the extension of the paraconsistent many-valued similarity method to group decision-making; Sections 6 presents a numerical example; Section 7 discusses the outcome; and Section 8 concludes the article.

2. The Borda Count or Rule

The Borda rule was devised in 1770 by a French Mathematician and naval engineer called Jean-Charles de Borda for the purpose of electing members in a fair manner to the French Academy of Sciences [4] [34] [35] [36]. It is adopted today by countries and organisations the world over for similar purposes. At the polls, every voter is given a ballot paper that contains the list of candidates and each of these candidates is given a number of marks that is equivalent to the number of fellow contenders that have been ranked below each candidate. In this case, the most preferred candidate will be the highest ranked candidate and he or she is given $k-1$ marks, where k denotes the number of candidates. The lowest ranked candidate has no any other candidate below him or her and so he or she gets 0 marks and the one next to the lowest candidate gets 1 mark and so on. The aim of the Borda count is to elect candidates generally endorsed by the electors instead of those preferred by a majority. Hence, this voting system is more of consensus than majority. Currently, this voting system is used by Slovenia to elect representatives of ethnic minorities in the National Assembly of Slovenia. A modified version of the system is also employed to select members of the parliament of Nauru. Finland too in the sixties to early seventies used another variant of the Borda rule to elect candidates in party lists. Today, private organisations and competitions around the world adopt this system for various selection purposes.

Use of the Borda Approach to Calculate Weights of Criteria

In fact, details of the Borda approach can be found in [5] [36]. For any finite set *A* containing *k* members or criteria,

1) Rank the members or criteria ordinally as 1st, 2nd, 3rd, $\cdots$, *k*th.

2) Assign to the ranking-oder: 1st, 2nd, 3rd, $\cdots$, *k*th the values $k-1, k-2, k-3, \cdots, k-k=0$ respectively.

3) a) Calculate the Borda score (F_B) for each member or criterion of the set *A* by finding the sum of the separate values of each member or criterion. This can be done via the formula

$$F_B(a) = \sum_{b \in A} number(i : aP_i b),$$

where i denotes an individual member, $aP_i b$ denotes the individual's preference of *a* over *b*; *P* denotes preference and *a*, *b* are any pair of members or criteria from *A*. In short, the notation $number(i : aP_i b)$ represents the value of the individual *i* preference of *a* over *b*, and that of $\sum_{b \in A} number(i : aP_i b)$ denotes the sum of the values of all individuals' preference of *a* to *b*.

b) In case there is a tie in rank between two or more members or criteria, assign to each member or criterion in the tie the average of the values they would have been assigned if they were to occupy different ranks, and after this you then determine the value of F_B as given in (3) (a) [36] [37].

4) Refer to the F_B values for the various members or criteria as the relative weights of the members or criteria of the set *A*. This means the member or criterion with the highest F_B value is regarded the most important member or criterion and the least value of F_B corresponds to the least important member or criterion in the finite set *A*.

Example 2.1. Assume 4 decision-makers have been asked to rank 6 experts, namely *a*, *b*, *c*, *d*, *e* and *f* according to their relative importance or weights, and in response the 4 decision-makers provided 4 rankings for the experts as shown in **Table 1**.

Then, the values of $k-i$ for $i=1,\cdots,k$ and $k=6$ are displayed in **Table 2**.

Table 1. The ranking order of experts.

	DM_1	DM_2	DM_3	DM_4
a	1st	1st	1st	1st
b	2nd	2nd	2nd	2nd
c	3rd	3rd	3rd	3rd
d	4th	5th	6th	6th
e	5th	4th	5th	5th
f	6th	6th	4th	4th

Table 2. The values of $k-i$ and the F_B for the ordinal ranking in **Table 1**.

	DM_1	DM_2	DM_3	DM_4	F_B
a	5	5	5	5	20
b	4	4	4	4	16
c	3	3	3	3	12
d	2	1	0	0	3
e	1	2	1	1	5
f	0	0	2	2	4

Note that, the values of F_B are obtained by summing the row values of each expert; and the weight of each expert say *a* denoted by $w(a)$ is determined by the equation

$$w(a)=\frac{F_B(a)}{\sum F_B}. \tag{2.1}$$

Hence, the weights or importance of the experts: *a*, *b*, *c*, *d*, *e* and *f* are 0.33, 0.27, 0.2, 0.05, 0.08, and 0.07 respectively.

Example 2.2. This example illustrates an instance of a tie. Suppose 5 decision-makers have been asked to rank 5 experts, namely *a*, *b*, *c*, *d* and *e* based on their weights. To this end, the 5 decision-makers gave 5 rankings for the experts one by each decision-maker as shown in **Table 3**.

So, the values of $k-i$ for $i=1,\cdots,k$ and $k=5$ are displayed in **Table 4**.

So, by Equation (2.1), we have the weights of the experts: *a*, *b*, *c*, *d* and *e* as 0.24, 0.26, 0.20, 0.13, and 0.17 respectively.

However, it is worthwhile to mention at this point that a major drawback of the Borda rule is the fact that it has not been able to satisfy the independence of irrelevant alternatives (IIA) condition as expressed in Arrow's impossibility theorem; and this gives rise to rank reversal situations. IIA: given for instance three alternatives say *a*, *b* and *c*, the combined or overall ranking of *a*, *b* is said to be *independent of irrelevant alternatives* if the ranking of *a*, *b* does not change in the event that the third alternative, *c*, is either removed or added to this ranking [38]. A rank reversal, on the other hand, refers to the situation where by the relative order of two alternatives in a chain (ranking) reverses when an alternative is removed or added to the chain [39]. Nonetheless, except for theoretical considerations, in practical applications, this consequence does not significantly undermine the validity of the rankings that emanate from Borda rule [39]. This is because if even rank reversals do occur they usually occur at the tail end of the ranking where the order of alternatives are greatly affected by the loss of information due to the deletion or addition of an alternative.

Example 2.3. Assume 5 decision-makers by means of the Borda method generated the following 5 rankings: one for each decision-maker for 4 alternatives say *a*, *b*, *c*, and *d*.

Table 3. The ranking order of experts.

	DM_1	DM_2	DM_3	DM_4	DM_5
a	1st	2nd	1st	3rd	2nd
b	2nd	1st	1st	2nd	3rd
c	3rd	1st	3rd	1st	4th
d	4th	3rd	2nd	1st	5th
e	5th	4th	1st	2nd	1st

Table 4. The values of $k-i$ (averaged in case of tie) and the F_B for the ordinal ranking in **Table 3**.

	DM_1	DM_2	DM_3	DM_4	DM_5	F_B
a	4	2	3	0	3	12
b	3	3.5	3	1.5	2	13
c	2	3.5	0	3.5	1	10
d	1	1	1	3.5	0	6.5
e	0	0	3	1.5	4	8.5

$$a \succ b \succ c \succ d$$
$$c \succ a \succ b \succ d$$
$$b \succ d \succ a \succ c$$
$$d \succ c \succ a \succ b$$
$$c \succ d \succ a \succ b,$$

where $\succ$ denotes preference. So, by Borda rule, we calculate the value of option a as a: 3 + 2 + 1 + 1 + 1 = 8. The values of the rest are 6 for b, 9 for c and 7 for d. Hence, the ranking of these four alternatives is

$$c \succ a \succ d \succ b.$$

Now, if alternative b is removed, then the resulting rankings are as follows:

$$a \succ c \succ d$$
$$c \succ a \succ d$$
$$d \succ a \succ c$$
$$d \succ c \succ a$$
$$c \succ d \succ a.$$

Again by Borda count, we have the value of a to be 4, the value of c to be 6 and that of d to be 5. So, the ranking here stands as, $c \succ d \succ a$. Thus, there is a rank reversal between options a and d since in the first ranking option a is preferred to d and in the second ranking option d is preferred to a. Moreover, this example amply demonstrates how Borda count violates the IIA requirement.

Therefore, using the Borda rule, the removal or addition of an alternative to a set of alternatives can lead to rank reversals.

3. Preliminaries

We recall some concepts, definitions and properties relevant to this study. However, for more details about these concepts, definitions and properties see [13].

3.1. Paraconsistent Logic

Paraconsistent logic is one of the logical ways by which classical logic can be generalised. Paraconsistent logic is a logical system that contravenes the principle of non contradiction. It is an inconsistency tolerant field of logic that does not out rightly reject contradictions, but instead accept and deal with them in a discriminating way. Given three logical statements say α, $\neg\alpha$, and β; the the logical consequence relation $\vdash$ is said to be *explosive* if it holds that $\{\alpha,\neg\alpha\}\vdash\beta$. Meaningful conclusions can be drawn from contradictions if only the consequence relation does not explode into triviality. Unlike other logical systems such as classical logic and intuitionistic logic, paraconsistent logic does not explode, hence, it is the most appropriate logical framework to deal with inconsistencies. The type of paraconsistent logic we deal with in this study is the one proposed by Belnap and was subsequently extended by Perny and Tsoukias and further advanced by Turunen in [11] into what is known as *paraconsistent Pavelka style fuzzy logic* [13]. According to Belnap, based on the *evidence* available to us, a statement say α can take one of the following four states: *falsehood*, *contradiction*, *unknown* and *truth* but not always one of the usual two truth values—*completely true* or *completely false*. This means every statement α can be assigned a pair of values say $\langle a,b\rangle$ called the *evidence couple*, where the first component a indicates the degree of truth associated with α and the second component b indicates the degree of falsehood associated with α. Hence, the four states are defined as follows:

1) α is true if we have evidence in support of α and no evidence against α. This state may be denoted by $T(\alpha)$, and so $T(\alpha)=\langle 1,0\rangle$.

2) α is considered false if we have no evidence in support of α but we have evidence against α. This state may be denoted by $F(\alpha)$, so $F(\alpha)=\langle 0,1\rangle$.

3) α is said to be contradictory if simultaneously we have evidence in support of α and we have evidence against α. This state is denoted by $C(\alpha)$, and $C(\alpha)=\langle 1,1\rangle$.

4) α is said to be unknown if we neither have evidence in support of α nor evidence against α. This is denoted by $U(\alpha)$, and $U(\alpha)=\langle 0,0\rangle$.

This four valued logic by Belnap [40] was further developed and extended to cover the whole real unit interval [0, 1] by Perny and Tsoukias [41]. As a result, the definitions of the four states above were modified as follows

$$T(\alpha)=\min(1-b,a), \tag{3.1}$$

$$F(\alpha)=\min(b,1-a), \tag{3.2}$$

$$C(\alpha)=\max(0,a+b-1), \tag{3.3}$$

$$U(\alpha) = \max(0, 1 - a - b). \tag{3.4}$$

Following this extension, Turunen *et al.* [11] further developed it into paraconsistent Pavelka style fuzzy logic in which they associated the real unit interval [0, 1] with the Łukasiewicz algebraic structure which is an *injective MV-algebra*. Per this development, these four states were re-expressed by the authors as

$$T(\alpha) = a \wedge b^*, \tag{3.5}$$

$$F(\alpha) = a^* \wedge b, \tag{3.6}$$

$$C(\alpha) = a \odot b, \tag{3.7}$$

$$U(\alpha) = a^* \odot b^*. \tag{3.8}$$

3.2. MV-Algebras

MV-algebras are to fuzzy logic what Boolean algebras are to classical logic [42]. Chang [43] introduced the MV-algebras to prove algebraically the completeness theorem of the Łukasiewicz logic.

Definition 3.1. [44] [45] An algebraic structure $L = \langle L, \oplus, \odot, *, 0, 1 \rangle$ with two constants 0, 1, two binary operations $\oplus$, $\odot$ and a unary operation $*$ is called an *MV-algebra* if for all $x, y, z \in L$ the following equations hold

$$x \oplus y = y \oplus x, \ \ x \odot y = y \odot x, \tag{3.9}$$

$$x \oplus (y \oplus z) = (x \oplus y) \oplus z, \ \ x \odot (y \odot z) = (x \odot y) \odot z, \tag{3.10}$$

$$x \oplus x^* = 1, \ \ x \odot x^* = 0, \tag{3.11}$$

$$x \oplus 1 = 1, \ \ x \odot 1 = x, \tag{3.12}$$

$$x \oplus 0 = x, \ \ x \odot 0 = 0, \tag{3.13}$$

$$(x \oplus y)^* = x^* \odot y^*, \ (x \odot y)^* = x^* \oplus y^* \tag{3.14}$$

$$x^{**} = x, \ 1^* = 0, \ 1 = 0^*. \tag{3.15}$$

Given any MV-algebra *L*, we define the binary operation $\rightarrow$ as $x \rightarrow y = x^* \oplus y$ for all $x, y \in L$. Moreover, if we equip *L* with the binary relation $\leq$ defined as $x \leq y$ if and only if $x \rightarrow y = x^* \oplus y = 1$, then the relation $\leq$ is an order relation on *L* where 1 and 0 are the top and the bottom elements, respectively in *L*. If *L* is further equipped with the binary operations $\vee$ and $\wedge$, then *L* becomes a lattice where for any elements $x, y \in L$,

$$x \vee y = (x \rightarrow y) \rightarrow y, \tag{3.16}$$

$$x \wedge y = (x^* \vee y^*)^*, \tag{3.17}$$

and the bi-residuum operation $\leftrightarrow$ when associated with *L* is defined as

$$x \leftrightarrow y = (x \rightarrow y) \wedge (y \rightarrow x). \tag{3.18}$$

Hence, in any MV-algebra *L*, the following additional equations hold

$$x \wedge y = y \wedge x, \ \ x \vee y = y \vee x, \tag{3.19}$$

$$x \wedge (y \wedge z) = (x \wedge y) \wedge z,\ x \vee (y \vee z) = (x \vee y) \vee z, \tag{3.20}$$

$$x \oplus (y \wedge z) = (x \oplus y) \wedge (x \oplus z),\ x \odot (y \vee z) = (x \odot y) \vee (x \odot z). \tag{3.21}$$

A typical example of an MV-algebra is the Łukasiewicz structure also called the *standard MV-algebra* [13]. This structure is defined on the real unit interval [0, 1] and has a natural order and given any two elements $x, y \in L$ in the Łukasiewicz structure the corresponding operations, namely $\oplus$, $\odot$, $*$, $\vee$, $\wedge$ and $\rightarrow$ are defined as follows

$$5x \oplus y = \min(x + y, 1), \tag{3.22}$$

$$x \odot y = \max(0, x + y - 1), \tag{3.23}$$

$$x^* = 1 - x, \tag{3.24}$$

$$x \vee y = \max(x, y), \tag{3.25}$$

$$x \wedge y = \min(x, y), \tag{3.26}$$

$$x \rightarrow y = \min(1, 1 - x + y). \tag{3.27}$$

3.3. Injective MV-Algebras

[44] Any MV-algebra that is described as injective must be complete and divisible.

Definition 3.2. An *injective MV-algebra* is an MV-algebraic structure $L = \langle L, \oplus, \odot, *, 0, 1 \rangle$ in which L is both *complete* as a lattice and *divisible* [13].

We say that L is complete if it is closed with respect to the infimum and supremum of each subset of L. Every complete MV-algebra is infinitely distributive. This means that in every complete MV-algebra the following two conditions hold [11]:

$$x \wedge \bigvee_{i \in \Gamma} y_i = \bigvee_{i \in \Gamma} (x \wedge y_i), \tag{3.28}$$

$$x \vee \bigwedge_{i \in \Gamma} y_i = \bigwedge_{i \in \Gamma} (x \vee y_i) \tag{3.29}$$

for any $x \in L$, and $\{y_i \mid i \in \Gamma\} \subseteq L$. This implies in a complete MV-algebra another adjoint couple $\langle \wedge, \Rightarrow \rangle$ can be defined with another residual operation $\Rightarrow$ defined by

$$x \Rightarrow y = \bigvee \{z \mid x \wedge z \leq y\}, \tag{3.30}$$

and $x^\star = x \Rightarrow 0$ defines what is called a *weak complementation* in a complete MV-algebra [11]. The Łukasiewicz structure L is a complete MV-algebra.

On the other hand, L is *divisible* if for every non-zero element x in L and $n \in \mathbb{N}$ there exists an element y in L called *non-zero n-divisor* of x so that $ny = x$ and $(x^* \oplus (n-1)y)^* = y$, where $ky = (k-1)y \oplus y$ (*i.e.*, $ky = y \oplus \cdots \oplus y$ (k times)) for all $k \in \mathbb{N}$. Every n-divisor is unique. Any injective MV-algebra L is structurally isomorphic to a collection say $\mathbb{F}$ of fuzzy sets [44]. The Łukasiewicz structure above is an injective MV-algebra. Another example of an injective MV-algebra which forms the bedrock of this study is the structure $\mathcal{M} = \langle \mathcal{M}, \oplus, ^\perp, 0 \rangle$, where $\mathcal{M}$ is a set of 2-by-2 matrices generated by pairs of evidence couples $\langle a,b\rangle$ of an injective MV-algebra L ($\langle a,b\rangle \in L\times L$). The associated binary operation $\oplus$ is defined for any *evidence matrices M*, *N* in $\mathcal{M}$ by

$$M\oplus N=\begin{bmatrix} a^*\wedge b & a\odot b \\ a^*\odot b^* & a\wedge b^* \end{bmatrix}\oplus\begin{bmatrix} p^*\wedge q & p\odot q \\ p^*\odot q^* & p\wedge q^* \end{bmatrix}$$
$$=\begin{bmatrix} (a\oplus p)^*\wedge(b\odot q) & (a\oplus p)\odot(b\odot q) \\ (a\oplus p)^*\odot(b\odot q)^* & (a\oplus p)\wedge(b\odot q)^* \end{bmatrix},$$

where

$$M=\begin{bmatrix} a^*\wedge b & a\odot b \\ a^*\odot b^* & a\wedge b^* \end{bmatrix},\ N=\begin{bmatrix} p^*\wedge q & p\odot q \\ p^*\odot q^* & p\wedge q^* \end{bmatrix}.$$

Note that the evidence matrices *M*, *N* are obtained from the evidence couples $\langle a,b\rangle$, $\langle p,q\rangle$, respectively and so the evidence couple for the evidence matrix $M\oplus N$ is $\langle a\oplus p, b\odot q\rangle$. For any $\langle a,b\rangle\in L\times L$, the evidence matrix $M^\perp$ is derived from the evidence couple $\langle a^*,b^*\rangle$ and is defined by

$$M^\perp=\begin{bmatrix} a\wedge b^* & a^*\odot b^* \\ a\odot b & a^*\wedge b \end{bmatrix}.$$

Finally, the element $\mathbf{0}\in\mathcal{M}$ is the bottom element of $\mathcal{M}$ and is generated by the couple $\langle 0,1\rangle$ and the corresponding matrix is $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. The element $\mathbf{1}\in\mathcal{M}$ is the top element of $\mathcal{M}$ and is generated by the evidence couple $\langle 1,0\rangle$ and the corresponding evidence matrix is $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

Proposition 1. [11] If L is an injective MV-algebra and the evidence couple $\langle a,b\rangle$ is in $L\times L$ then, the set of all evidence couples in $L\times L$ induces a corresponding set of evidence matrices $\mathcal{M}$ written as

$$\mathcal{M}=\left\{\begin{bmatrix} a^*\wedge b & a\odot b \\ a^*\odot b^* & a\wedge b^* \end{bmatrix} \mid \langle a,b\rangle\in L\times L\right\},$$

It is important to add that there is a bijective relation between the evidence couples and the corresponding evidence matrices. This means given any two evidence matrices denoted by *M*, *N*, $M=N$ provided $a=p$ and $b=q$.

3.4. Fuzzy Similarity Relations

Details of the ideas presented in this subsection are all in [13].

Let us suppose that X is a non void set and A is an injective MV-algebra. Then, the binary operation S defined on X is a fuzzy similarity relation if for any $x,y,z\in X$, S satisfies the following three equations

$$S(x,x)=1, \tag{3.31}$$

$$S(x,y)=S(y,x), \tag{3.32}$$

$$S(x,y)\odot S(y,z)\le S(x,z). \tag{3.33}$$

Thus, S is reflective, symmetrical and weakly transitive as found in Equations (3.31), (3.32) and (3.33), respectively. Besides, S is a fuzzy equivalence relation and so it is a generalisation of the equivalence relation in standard logic. Moreover, in any residuated lattice L, the binary operation $\leftrightarrow$ is defined for any $x, y \in L$ by Equation (3.18).

The binary operation $\leftrightarrow$ satisfies the following

$$x \leftrightarrow x = 1, \tag{3.34}$$

$$x \leftrightarrow y = y \leftrightarrow x, \tag{3.35}$$

$$(x \leftrightarrow y) \odot (y \leftrightarrow z) \le (x \leftrightarrow z), \tag{3.36}$$

$$x \leftrightarrow 1 = x. \tag{3.37}$$

Hence, the operation $\leftrightarrow$ is reflexive as seen from equation (3.34), symmetric as seen from (3.35) and weakly transitive as found in (3.36). A fuzzy subset $\tilde{B}$ is an ordered pair $(X, \mu_{\tilde{B}})$, where X is a set of elements and $\mu_{\tilde{B}} : X \to L$ is a function referred to as a *membership function*. A fuzzy subset $\tilde{B}$ with the membership function $\mu_{\tilde{B}}$ of any set X measures the degree to which an element $x \in X$ is an element of $\tilde{B}$. Hence, each fuzzy subset $\tilde{B}$ with the membership function $\mu_{\tilde{B}}$ defined on a non void set X generates a fuzzy similarity relation S on the set X through

$$S_{\tilde{B}}(x, y) = \mu_{\tilde{B}}(x) \leftrightarrow \mu_{\tilde{B}}(y) \text{ for any } x, y \in X. \tag{3.38}$$

Especially, if the fuzzy subset $\tilde{B}$ is given and A is the Łukasiewicz structure on the interval [0, 1], then for any $x, y \in X$,

$$S_{\tilde{B}}(x, y) = 1 - \left|\mu_{\tilde{B}}(x) - \mu_{\tilde{B}}(y)\right|. \tag{3.39}$$

In Multi-Criteria Decision Making, each fuzzy subset stands for a criterion. This means a five-criteria decision-making problem has five fuzzy subsets. The elements of X constitute the set of decision alternatives in the decision problem. The value of $S_{\tilde{B}}(x, y)$ shows the degree to which any two elements x, y in X are similar in relation to the fuzzy subset $\tilde{B}$. Therefore, if the element y in X has a full membership grade of 1 in $\tilde{B}$, then our interest will be to calculate the degree to which each element other than y in X is identical to y in $\tilde{B}$. Hence, for each element $x \in X$, it holds that

$$S_{\tilde{B}}(x, y) = \mu_{\tilde{B}}(x) \leftrightarrow \mu_{\tilde{B}}(y) = \mu_{\tilde{B}}(x) \leftrightarrow 1 = \mu_{\tilde{B}}(x). \tag{3.40}$$

So, having k criteria and m decision alternatives in X in a given decision-making problem implies we have k fuzzy subsets and k fuzzy similarity relations $S_j(x, y)$, $j = 1, \cdots, k$ covering all the m alternatives in X. Hence, in an injective MV-algebra, the *total fuzzy similarity relation* denoted by $S(x, y)$ between every two elements or alternatives in X over all the k fuzzy similarity relations $S_j(x, y)$ is calculated as

$$S(x, y) = \frac{S_1(x, y)}{k} \oplus \cdots \oplus \frac{S_k(x, y)}{k}, \tag{3.41}$$

where the binary operation $\oplus$ is the MV-addition operation and the expression $\frac{S_j(x,y)}{k}$ is the k-divisor of every similarity relation value $S_j(x,y)$, $j=1,\cdots,k$.

Particularly, if A is the Łukasiewicz structure, then,

$$S(x,y)=\frac{1}{k}\sum_{j=1}^{k}S_j(x,y). \tag{3.42}$$

furthermore, if different weight values are given to the fuzzy sets or criteria, then the total fuzzy similarity relation is derived through the weighted mean

$$S(x,y)=\frac{w_1S_1(x,y)}{W}\oplus\cdots\oplus\frac{w_kS_k(x,y)}{W}, \tag{3.43}$$

where $W=\sum_{j=1}^{k}w_j$, and $w_j\in\mathbb{N}$. Again, if A is the Łukasiewicz structure then,

$$S(x,y)=\frac{1}{W}\sum_{j=1}^{K}w_jS_j(x,y). \tag{3.44}$$

As an illustration, suppose X is the set of decision alternatives over $k$criteria which are expressed in fuzzy subsets as $\tilde{B}_1,\cdots,\tilde{B}_k$. Let us further assume that each $\tilde{B}_j$, $j=1,\cdots,k$ contains y as an ideal solution. That is, $\mu_{\tilde{B}_j}(y)=1$. Then, to measure the similarity of each decision alternative $x\in X$ to y, we calculate the magnitude of the total similarity between x and y via the weighted mean

$$S(x,y)=\frac{1}{W}\sum_{j=1}^{k}w_j\mu_{\tilde{B}_j}(x), \tag{3.45}$$

where $\mu_{\tilde{B}_j}(x)$ is the membership grade of x in the fuzzy subset $\tilde{B}_j$, $j=1,\cdots,k$.

4. Fuzzy TOPSIS Method for Group Decision-Making

Suppose a group of r experts with varied weights, w_k, for $k=1,\cdots,r$ wants to rank from the best to the worst a set of m options denoted by $A=\{A_i \mid i=1,\cdots,m\}$ in relation to a set of n criteria denoted by $C=\{C_j \mid j=1,\cdots,n\}$ and a set of weight of criteria denoted by $W=\{w_j \mid j=1,\cdots,n\}$.

The steps involved in the implementation of the fuzzy TOPSIS group decision-making method to address a decision problem with r-decision makers are [20] [46]

Step 1. Construct the decision matrices denoted by D^k, $k=1,\cdots,r$ for the m alternatives over the n criteria:

$$D^k=\begin{bmatrix} x_{11}^k & x_{12}^k & \cdots & x_{1n}^k \\ x_{21}^k & x_{22}^k & \cdots & x_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1}^k & x_{m2}^k & \cdots & x_{mn}^k \end{bmatrix} \tag{4.1}$$

Step 2. Normalise each decision matrix D^k to get the matrix denoted by $\boldsymbol{R}^k$ using the formula:

$$\boldsymbol{r}_{ij}^k=\frac{x_{ij}^k}{\sqrt{\sum_{i=1}^{n}\left(x_{ij}^k\right)^2}}.$$

Thus,

$$\boldsymbol{R}^k = \begin{bmatrix} \boldsymbol{r}_{11}^k & \boldsymbol{r}_{12}^k & \cdots & \boldsymbol{r}_{1n}^k \\ \boldsymbol{r}_{21}^k & \boldsymbol{r}_{22}^k & \cdots & \boldsymbol{r}_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{r}_{m1}^k & \boldsymbol{r}_{m2}^k & \cdots & \boldsymbol{r}_{mn}^k \end{bmatrix} \tag{4.2}$$

Step 3. Construct the weighted normalised matrix denoted by V^k from each decision matrix $\boldsymbol{R}^k$ by multiplying each column in $\boldsymbol{R}^k$ by the weight (w_j) of the corresponding criterion. That is,

$$V^k = \begin{bmatrix} w_1\boldsymbol{r}_{11}^k & w_2\boldsymbol{r}_{12}^k & \cdots & w_n\boldsymbol{r}_{1n}^k \\ w_1\boldsymbol{r}_{21}^k & w_2\boldsymbol{r}_{22}^k & \cdots & w_n\boldsymbol{r}_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ w_1\boldsymbol{r}_{m1}^k & w_2\boldsymbol{r}_{m2}^k & \cdots & w_n\boldsymbol{r}_{mn}^k \end{bmatrix} \tag{4.3}$$

Note that in our method, we are dealing with two types of weight, namely the weights of criteria and experts' weights and so this step must be treated with caution.

Step 4. Construct an aggregated collective matrix D of the individual weighted normalised matrices through the weighted mean

$$x_{ij} = \frac{1}{r}\sum_{k=1}^{r} w_j \boldsymbol{r}_{ij}^k.$$

$$D = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \tag{4.4}$$

Step 5. Determine the ideal solution A^+ and the negative-ideal solution A^- which are defined as

$$A^+ = \left\{\left(\max_i D_{ij} : j \in J\right), \left(\min_i D_{ij} : j \in J'; i = 1, \cdots, m\right)\right\} = \left\{D_1^+, \cdots, D_n^+\right\}. \tag{4.5}$$

and

$$A^- = \left\{\left(\min_i D_{ij} : j \in J\right), \left(\max_i D_{ij} : j \in J'; i = 1, \cdots, m\right)\right\} = \left\{D_1^-, \cdots, D_n^-\right\}, \tag{4.6}$$

where $J = \left\{ j = 1, \cdots, n \mid j \text{ is a maximising criterion} \right\}$ and $J' = \left\{ j = 1, \cdots, n \mid j \text{ is a minimising criterion} \right\}$.

Step 6. Calculate the separation measures as follows:

The ideal separation and the negative ideal separation denoted by S_i^+ and S_i^-, respectively are defined by

$$S_i^+ = \sqrt{\sum_{j=1}^{n}\left(D_{ij} - D_j^+\right)^2}, \text{ for } i = 1, \cdots, m.$$

$$S_i^- = \sqrt{\sum_{j=1}^{n}\left(D_{ij} - D_j^-\right)^2}, \text{ for } i = 1, \cdots, m.$$

Step 7. Compute the relative closeness to the ideal solution as:

$$C_i^* = \frac{S_i^-}{S_i^+ + S_i^-}, \tag{4.7}$$

where $0 \le C_i^* \le 1$ and $i = 1, \cdots, m$.

Step 8. Establish a linear order for the alternatives in A according to the value of C_i^*, $i = 1, \cdots, m$.

5. An Algorithm of the Group Decision Model

To resolve the decision problem as given in Section 4, we first of all partition the set C into two subsets denoted by C^+ and C^- such that $C^+ \cap C^- = \varnothing$ and $C^+ \cup C^- = C$. The set C^+ is the set of the pros criteria and the set C^- is the set of the cons criteria. Similarly, the weights are divided into two parts so that $W^+ = \sum w_j$ is the sum of the weights w_j of all criteria in C^+, and $W^- = \sum w_j$ is the sum of the weights w_j of all criteria in C^-.

Furthermore, we used the weighted average or the weighted mean to combine the values of group decision tables to derive the final outcomes that represent the conclusive decision of the whole group. Hence, in conformity with the weighted average approach, the steps involved in applying PMVS to group decision scenarios are as follows,

Step 1. Determine the weight of every expert and every criteria by means of the Borda's approach as explained in the preceded section.

Step 2. From the decision table of each expert, determine the global strength, a_i, from the pros criteria and the global weakness, b_i, from the cons criteria for each alternative A_i in line with Equation (3.45). This means

$$a_i = \frac{1}{W^+} \sum_{j=1}^{n} w_j \mu_{C_j}(A_i), \tag{5.1}$$

where $C_j \in C^+$; $j = 1, \cdots, n$ and $i = 1, \cdots, m$. Similarly,

$$b_i = \frac{1}{W^-} \sum_{j=1}^{n} w_j \mu_{C_j}(A_i), \tag{5.2}$$

where $C_j \in C^-$.

Step 3. By denoting the sum of the weights of the r experts by W_E, calculate the aggregated global strength $S_{AG}(x, y) = a_i^g$ and the aggregated global weakness b_i^g for each alternative A_i via the formula

$$a_i^g = \frac{1}{W_E} \sum_{k=1}^{r} w_k a_{i_k}, \tag{5.3}$$

where $k = 1, \cdots, r$; $i = 1, \cdots, m$ and a_{i_k} is the global strength of A_i as determined by the kth expert. The same way

$$b_i^g = \frac{1}{W_E} \sum_{k=1}^{r} w_k b_{i_k}, \tag{5.4}$$

where b_{i_k} is the corresponding global weakness for the alternative A_i as determined by the kth expert.

Step 4. Generate from every aggregated evidence couple (a_i^g, b_i^g) for the option A_i a corresponding aggregated evidence matrix

$$M_{AG_i}=\begin{bmatrix}\left(a_i^g\right)^*\wedge b_i^g & a_i^g\odot b_i^g\\ \left(a_i^g\right)^*\odot\left(b_i^g\right)^* & a_i^g\wedge\left(b_i^g\right)^*\end{bmatrix}$$

so that for $i=1,\cdots,m$; $F=\left(a_i^g\right)^*\wedge b_i^g$; $K=a_i^g\odot b_i^g$; $U=\left(a_i^g\right)^*\odot\left(b_i^g\right)^*$ and $T=a_i^g\wedge\left(b_i^g\right)^*$.

Step 5. Regard the set $\mathcal{M}_{\mathcal{AG}}=\{M_{AG}:M_{AG}\text{is an aggregated evidence matrix}\}$ as an injective MV-algebra such that any given pair of options α_k, α_l can be compared by means of their corresponding aggregated evidence matrices M_{AG_k}, and M_{AG_l}. In this sense, if $M_{AG_l}\le M_{AG_k}$, then option α_k has satisfied the given criteria better than option α_l. However, there may be cases where the evidence matrices M_{AG_k} and M_{AG_l} are incomparable which we represent by $M_{AG_k}\not\le\not\ge M_{AG_l}$. This drawback can be fixed through the following algebraic means. It holds that

$$M_{AG_l}\le M_{AG_k}\ \text{ if and only if }\ M_{AG_l}^{\perp}\oplus M_{AG_k}=M_{AG_l}\Rightarrow M_{AG_k}=\mathbf{1},$$

where

$$\mathbf{1}=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$$

is generated by the couple $\langle 1,0\rangle$.

Now, given the evidence matrix $M_{AG_l}^{\perp}\oplus M_{AG_k}$, denote the falsehood value which is the value in the first row, first column by $F\left(M_{AG_l}^{\perp}\oplus M_{AG_k}\right)$; the contradictory value which is the value in the first row, second column by $K\left(M_{AG_l}^{\perp}\oplus M_{AG_k}\right)$; the unknown value which corresponds to the value in the second row, first column by $U\left(M_{AG_l}^{\perp}\oplus M_{AG_k}\right)$; and the truth value which corresponds to the value in the second row, second column by $T\left(M_{AG_l}^{\perp}\oplus M_{AG_k}\right)$. Hence, in the evidence matrix $\mathbf{1}$ above, $F\left(M_{AG_l}^{\perp}\oplus M_{AG_k}\right)=0$; $K\left(M_{AG_l}^{\perp}\oplus M_{AG_k}\right)=0$; $U\left(M_{AG_l}^{\perp}\oplus M_{AG_k}\right)=0$ and $T\left(M_{AG_l}^{\perp}\oplus M_{AG_k}\right)=1$.

Step 6. Therefore, compare any two options, α_k, α_l by means of their corresponding aggregated matrices M_{AG_k}, and M_{AG_l} as follows,

1) if $M_{AG_l}\le M_{AG_k}$, then option α_k has satisfied the given criteria more than option α_l. Hence, α_k is better than α_l,

2) if the matrices M_{AG_k}, and M_{AG_l} are incomparable written here as $M_{AG_k}\not\le\not\ge M_{AG_l}$, then α_k is better than α_l if $T\left(M_{AG_l}^{\perp}\oplus M_{AG_k}\right)>T\left(M_{AG_k}^{\perp}\oplus M_{AG_l}\right)$,

3) if $M_{AG_k}=M_{AG_l}$, then α_k, α_l are as good as each other, and

4) if M_{AG_k}, and M_{AG_l} are incomparable but $T\left(M_{AG_l}^{\perp}\oplus M_{AG_k}\right)=T\left(M_{AG_l}^{\perp}\oplus M_{AG_k}\right)$, then α_k, α_l are weakly equal.

Details of how Step 6 is applied are in the following note.

Note: suppose that the evidence matrix M_{AG_l} is generated by the evidence couple $\langle a,b\rangle$, and the evidence matrix M_{AG_k} is induced by the evidence couple $\langle p,q\rangle$, where $\langle a,b\rangle,\langle p,q\rangle\in[0,1]\times[0,1]$. Based on the above notations for the four components of $M_{AG_l}^{\perp}\oplus M_{AG_k}$, we can logically compare any two alternatives α_k, α_l using their evidence matrices M_{AG_k}, M_{AG_l}, respectively. In this regard, it holds that $M_{AG_l} \le M_{AG_k}$ if and only if $a \le p$, $q \le b$. This means there is more evidence in favour of α_k than there is in favour of α_l and there is less evidence against α_k than there is against α_l. Similarly, from $M_{AG_l} \le M_{AG_k}$ it holds that $p^* \le a^*$, $b^* \le q^*$. Therefore,

$$T\left(M_{AG_l}^{\perp} \oplus M_{AG_k}\right) = \left(a^* \oplus p\right) \wedge \left(b^* \odot q\right)^* = 1 \ge \left(p^* \oplus a\right) \wedge \left(q^* \odot b\right)^* = T\left(M_{AG_k}^{\perp} \oplus M_{AG_l}\right)$$

. However, if $M_{AG_k} \not\le\not\ge M_{AG_l}$, then the decision problem in question is a more complicated one and we surmount it and any other similar problems through Step 6.

Alternatively, we can solve such problems through Definition 5.1 and Theorem 2 [13].

Definition 5.1. Let α and β be two alternatives represented sequentially by the evidence matrices *M*, *N* which in turn are induced by the evidence couples $\langle a,b \rangle$, $\langle x,y \rangle$ respectively. Then, β dominates over α, denoted by $\alpha \preceq \beta$ if 1) $M \le N$ or 2) $M \not\le\not\ge N$ but $T\left(M^{\perp} \oplus N\right) > T\left(N^{\perp} \oplus M\right)$. Specifically, if $M = N$, then α, β are equally good and it is denoted by $\alpha \equiv \beta$. If $M \not\le\not\ge N$, but $T\left(M^{\perp} \oplus N\right) = T\left(N^{\perp} \oplus M\right)$, then α, β are weakly equally good and it is denoted by $\alpha \equiv_w \beta$ [13].

Theorem 2. The relation $\equiv$ is an equivalence relation on the set of alternatives while that of $\equiv_w$ is not. The relation $\preceq$ defines a quasi-order on the set of alternatives [13].

Proof. See [13]. ☐

To illustrate the process of ranking via Step 6 or via Definition 5.1 and Theorem 2, we take a look at the following two examples.

Example 5.1. Let us assume that the aggregated evidence matrix for alternative A_1 is M_{AG_1} and that of A_2 is M_{AG_2}. If matrix M_{AG_1} is generated by the evidence couple $\langle 0.6, 0.3 \rangle$ and matrix M_{AG_2} is generated by the couple $\langle 0.9, 0.2 \rangle$, what is the dominant alternative between A_1 and A_2?

Solution 5.1. If the evidence couple for M_{AG_1} is $\langle 0.6, 0.3 \rangle$, then the evidence couple for $M_{AG_1}^{\perp}$ is $\langle 0.6^*, 0.3^* \rangle = \langle 0.4, 0.7 \rangle$ and the evidence couple for the evidence matrix $M_{AG_1}^{\perp} \oplus M_{AG_2}$ is $\langle 0.4 \oplus 0.9, 0.7 \odot 0.2 \rangle = \langle 1, 0 \rangle$. So, in terms of matrices,

$$M_{AG_1}^{\perp} \oplus M_{AG_2} = \begin{bmatrix} 1^* \wedge 0 & 1 \odot 0 \\ 1^* \odot 0^* & 1 \wedge 0^* \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

By Definition 4.1 and Theorem 4.1, $T\left(M_{AG_1}^{\perp} \oplus M_{AG_2}\right) = 1$, thus $M_{AG_1} \le M_{AG_2}$ and so alternative A_2 dominates A_1.

Example 5.2. Let us assume that the aggregated evidence matrix for alternative A_1 is M_{AG_1} and that of A_2 is M_{AG_2}. If matrix M_{AG_1} is induced by the couple $\langle 0.7, 0.2 \rangle$ and M_{AG_2} by the couple $\langle 0.9, 0.3 \rangle$, what is the dominant alternative between A_1 and A_2?

Solution 5.2. Given that the evidence couple for M_{AG_1} is $\langle 0.7, 0.2 \rangle$, the evidence couple for $M_{AG_1}^{\perp}$ is $\langle 0.3, 0.8 \rangle$ and the evidence couple for the matrix

$M_{AG_1}^{\perp} \oplus M_{AG_2}$ is $\langle 0.3 \oplus 0.9, 0.8 \odot 0.3 \rangle = \langle 1, 0.1 \rangle$. So,

$$M_{AG_1}^{\perp} \oplus M_{AG_2} = \begin{bmatrix} 1^* \wedge 0.1 & 1 \odot 0.1 \\ 1^* \odot 0.1^* & 1 \wedge 0.1^* \end{bmatrix} = \begin{bmatrix} 0 & 0.1 \\ 0 & 0.9 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},$$

hence, $M_{AG_1} \not\leq M_{AG_2}$. Conversely, the matrix $M_{AG_2}^{\perp}$ is obtained from the couple $\langle 0.9^*, 0.3^* \rangle = \langle 0.1, 0.7 \rangle$. Therefore, the evidence couple for the matrix $M_{AG_2}^{\perp} \oplus M_{AG_1}$ is $\langle 0.1 \oplus 0.7, 0.7 \odot 0.2 \rangle = \langle 0.8, 0 \rangle$ and

$$M_{AG_2}^{\perp} \oplus M_{AG_1} = \begin{bmatrix} 0.8^* \wedge 0 & 0.8 \odot 0 \\ 0.8^* \odot 0^* & 0.8 \wedge 0^* \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0.2 & 0.8 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Thus, $M_{AG_2} \not\leq M_{AG_1}$. This means M_{AG_1} and M_{AG_2} are incomparable ($M_{AG_1} \not\leq\not\geq M_{AG_2}$). However, $T\left(M_{AG_1}^{\perp} \oplus M_{AG_2}\right) = 0.9 > 0.8 = T\left(M_{AG_2}^{\perp} \oplus M_{AG_1}\right)$. Hence, A_2 dominates A_1.

6. A Case Study: Ranking Energy Production Methods in Ghana

The group decision-making model introduced in this paper has been applied to the selection of the optimal energy mix from eight energy sources for Ghana. The eight available energy sources for the country are expressed in the set *X*:

$$X = \{\text{hydro}, \text{wind}, \text{solar}, \text{naturalgas}, \text{nuclear}, \text{biomass}, \text{oil}, \text{coal}\}.$$

In this particular group decision-making problem, four energy experts from the ministry of energy have evaluated separately the power generation potentials of each one of the eight sources of energy vis-à-vis 26 criteria—11 pros criteria, and 15 cons criteria. So, in all, we have 4 different data sets one from each of the four experts. In addition, to be able to determine the relative weight of each of the four experts, three supra chief executive officers who have worked with these four experts at various time periods were tasked to rank the four experts ordinally based on every expert's level of experience, knowledge, skills, perception, judgement and other relevant apptitudes in connection with this field. Furthermore, to determine and factor into the model the relative importance or weights of the given 26 criteria, the four experts were tasked to rank the 26 criteria ordinally.

The ordinal rankings of these experts by the 3 CEOS are in Table 5; the rankings of the pros criteria and the cons criteria inline with their relative importance by the four experts are shown in Table 6, and Table 7 respectively. The corresponding Borda's score for the Table 5, Table 6 and Table 7 are Table 8, Table 9 and Table 10. Also, the short forms, HYD, WIN, SOL, GAS, NUC, BIO, OIL and COA have been sequentially used to represent the 8 energy alternatives in *X*. The symbol α_i or α too denotes members of *X*.

Recall that the row total gives the weight of each expert.

Eeach criterion induces a fuzzy subset of *X*. Hence, we have 26 fuzzy subsets which consist of 11 pros and 15 cons. Tables 11-18 show the degree to which every option $\alpha \in X$ satisfies the criteria per the opinion of each of the four experts.

Table 5. Odering of 4 energy experts by 3 CEOS.

Experts	CEO 1	CEO 2	CEO 3
Expert 1	1st	2nd	1st
Expert 2	3rd	4th	4th
Expert 3	4th	1st	3rd
Expert 4	2nd	3rd	2nd

Table 6. The rankings of the Pros criteria by the 4 experts.

Criteria	Expert 1	Expert 2	Expert 3	Expert 4
1. Availability	1st	2nd	3rd	2nd
2. Energy storage versatility	5th	5th	2nd	7th
3. Energy storage capability	5th	4th	2nd	5th
4. Self-sufficiency and reliability	2nd	2nd	2nd	2nd
5. Energy yield	1st	1st	2nd	1st
6. Renewable	1st	1st	4th	2nd
7. Job creation	3rd	2nd	2nd	6th
8. Other benefits (including those from by-products)	6th	4th	2nd	7th
9. Plant's versatility/flexibility	7th	6th	1st	5th
10. Life span	4th	3rd	1st	4th
11. Technological impact	3rd	4th	1st	3rd

Table 7. The rankings of the cons criteria by the 4 experts.

Criteria	Expert 1	Expert 2	Expert 3	Expert 4
1. Energy outsourcing	6th	4th	4th	3rd
2. Green house gases emission	1st	1st	5th	1st
3. Rainfall fluctuation	3rd	2nd	3rd	2nd
4. Ecosystem and livelihood	2nd	1st	3rd	4th
5. Pollution	2nd	2nd	4th	3rd
6. Waste management	3rd	3rd	1st	4th
7. Capital cost	2nd	1st	2nd	1st
8. Operational cost	2nd	2nd	1st	2nd
9. Price volatility	4th	3rd	1st	4th
10. Human consequence	3rd	2nd	3rd	2nd
11. Inter and/or intra boundary disputes	5th	5th	5th	4th
12. Civil unrest or social disorder	5th	6th	3rd	5th
13. Location	6th	7th	4th	7th
14. Land mass or space consumption	6th	8th	5th	6th
15. Political interference	7th	6th	1st	7th

Table 8. Corresponding borda's score of experts.

Experts	CEO 1	CEO 2	CEO 3	F_B (weight)
Expert A	3	2	3	8
Expert B	1	0	0	1
Expert C	0	3	1	4
Expert D	2	1	2	5
				Sum of F_B (weights): 18

Table 9. Corresponding borda's score.

Criteria	Expert 1	Expert 2	Expert 3	Expert 4	F_B (weight)
1. Availability	9	7	1	8	25
2. Energy storage versatility	2.5	1	4.5	0.5	8.5
3. Energy storage capability	2.5	3	4.5	3.5	13.5
4. Self-sufficiency and reliability	7	7	4.5	8	26.5
5. Energy yield	9	9.5	4.5	10	33
6. Renewable	9	9.5	0	8	26.5
7. Job creation	5.5	7	4.5	2	19
8. Other benefits	1	3	4.5	0.5	9
9. Plant's versatility/flexibility	0	0	9	3.5	12.5
10. Life span	4	5	9	5	23
11. Technological impact	5.5	3	9	6	23.5
					Sum of weights: 220

Note: the row total gives the weight of each criterion.

Table 10. Borda's Score of the cons criteria.

Criteria	Expert 1	Expert 2	Expert 3	Expert 4	F_B (weight)
1. Energy outsourcing	2	5	4	8.5	19.5
2. Green house gases emission	14	13	1	13.5	41.5
3. Rainfall fluctuation	8	9.5	7.5	11	36
4. Ecosystem and livelihood	11.5	13	7.5	5.5	37.5
5. Pollution	11.5	9.5	4	8.5	33.5
6. Waste management	8	6.5	12.5	5.5	32.5
7. Capital cost	11.5	13	10	13.5	48
8. Operational cost	11.5	9.5	12.5	11	44.5

Continued

9. Price volatility	6	6.5	12.5	5.5	30.5
10. Human consequence	8	9.5	7.5	11	36
11. Inter and/or intra boundary disp.	4.5	4	1	5.5	15
12. Civil unrest or social disorder	4.5	2.5	7.5	3	17.5
13. Location	2	1	4	0.5	7.5
14. Land mass or space consumption	2	0	1	2	5
15. Political interference	0	2.5	12.5	0.5	15.5
					weight sum: 420

Table 11. Pros: Membership functions determined by expert A.

	HYD	WIN	SOL	GAS	NUC	BIO	OIL	COA
μ_{P_1}	1	0.5	0.75	0.75	0.25	0.75	0.75	0.25
μ_{P_2}	1	0.25	0.5	0.75	0.25	0.5	0.5	0.25
μ_{P_3}	1	0.25	0.5	0.75	0.25	0.75	0.75	0.5
μ_{P_4}	0.5	0.75	0.5	0.75	0.25	0.75	0.75	0.25
μ_{P_5}	1	0.5	0.25	0.5	0.75	0.5	0.5	1
μ_{P_6}	1	1	1	0	0.25	1	0	0
μ_{P_7}	1	0.75	0.5	0.75	0.75	0.75	1	1
μ_{P_8}	1	0.5	0.5	0.75	0.25	0.75	1	1
μ_{P_9}	0.75	0.75	0.75	0.75	0.25	0.75	0.5	0.5
$\mu_{P_{10}}$	1	0.75	0.5	0.75	0.75	0.75	0.5	1
$\mu_{P_{11}}$	0.75	0.75	0.75	0.75	0.5	0.75	0.75	0.75

Table 12. Cons: Membership functions determined by expert A.

	HYD	WIN	SOL	GAS	NUC	BIO	OIL	COA
μ_{C_1}	0.75	0.5	0.5	0.75	0.5	0.5	0.5	0.5
μ_{C_2}	0	0	0	0.25	0.75	0.25	0.75	0.75
μ_{C_3}	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5
μ_{C_4}	0.75	0.5	0.75	0.75	1	0.5	0.75	0.75
μ_{C_5}	0	0.25	0.25	0.5	0.75	0.5	0.75	0.75
μ_{C_6}	0	0.25	0.25	0.25	0.75	0.5	0.25	0.75
μ_{C_7}	0.75	0.75	0.75	0.75	0.75	0.25	0.75	0.75

Continued

μ_{C_8}	0	0	0	0.5	0.25	0.5	0.5	0.5
μ_{C_9}	0	0	0	0.75	0.25	0.5	0.75	0.5
$\mu_{C_{10}}$	0.75	0.5	0.5	0.75	1	0.5	0.75	0.5
$\mu_{C_{11}}$	0.75	0.5	0.5	0.75	0.5	0.5	0.75	0.5
$\mu_{C_{12}}$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$\mu_{C_{13}}$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$\mu_{C_{14}}$	0.75	0.5	0.75	0.5	0.25	0.5	0.5	0.5
$\mu_{C_{15}}$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Table 13. Pros: Membership functions determined by expert B.

	HYD	WIN	SOL	GAS	NUC	BIO	OIL	COA
μ_{P_1}	0.75	0.25	1	0.75	0.75	0.75	0.75	0.25
μ_{P_2}	0.75	0.25	0.25	0.75	0.5	0.75	0.25	0.25
μ_{P_3}	0.75	0.25	0.25	0.75	0.75	0.5	0.75	0.5
μ_{P_4}	0.5	0.5	0.75	0.75	0.5	0.75	0.75	0
μ_{P_5}	1	0.5	0.5	0.75	0.75	0.5	0.75	0.75
μ_{P_6}	1	1	1	0	0	1	0	0
μ_{P_7}	0.75	1	0.5	0.5	0.25	0.5	0.5	0.25
μ_{P_8}	1	0.25	0.25	0.5	0.25	0.5	0.25	0.25
μ_{P_9}	0.75	0.25	0.25	0.25	0.25	0.25	0.25	0.25
$\mu_{P_{10}}$	0.75	0.75	0.5	0.75	0.25	0.75	0.5	0.25
$\mu_{P_{11}}$	1	0.5	0.5	0.25	0.5	0.5	0.5	0.25

Table 14. Cons: Membership functions determined by expert B.

	HYD	WIN	SOL	GAS	NUC	BIO	OIL	COA
μ_{C_1}	0.75	0.25	0.25	0.5	0.25	0.25	0.5	1
μ_{C_2}	0	0	0	0.75	0.25	0.25	0.75	0.75
μ_{C_3}	0.75	0.5	0.75	0	0	0.5	0	0
μ_{C_4}	0.75	0.5	0.25	0.75	0.75	0.25	0.75	0.5
μ_{C_5}	0	0.25	0.25	0.75	1	0.25	0.75	0.75
μ_{C_6}	0	0.25	0.25	0.5	0.75	0.75	0.5	0.5

Continued

μ_{C_7}	0.5	0.75	0.75	0.75	0.5	0.75	0.75	0.75
μ_{C_8}	0.25	0.5	0.75	0.75	0.5	0.5	0.75	0.75
μ_{C_9}	0	0.25	0.5	0.75	0.75	0.25	0.75	0.75
$\mu_{C_{10}}$	0.75	0.25	0.5	0.5	0.75	0.25	0.5	0.5
$\mu_{C_{11}}$	0.5	0.25	0.5	0.75	0.5	0.25	0.75	0.25
$\mu_{C_{12}}$	0.25	0	0.25	0.75	0.5	0.25	0.75	0.5
$\mu_{C_{13}}$	0.25	0.75	0.5	0.5	0.5	0.5	0.5	0.25
$\mu_{C_{14}}$	0.25	0.5	0.5	0.5	0.5	0.75	0.5	0.5
$\mu_{C_{15}}$	0.25	0.25	0.25	0.75	0.5	0.5	0.75	0.25

Table 15. Pros: Membership functions determined by expert C.

	HYD	WIN	SOL	GAS	NUC	BIO	OIL	COA
μ_{P_1}	1	0.5	1	0.75	0.5	1	0.75	0
μ_{P_2}	0.75	0.5	0.25	0.25	0.5	0.75	0.75	0.25
μ_{P_3}	1	0.5	0.5	0.75	0.5	0.75	0.75	0.25
μ_{P_4}	1	0.5	0.75	0.75	0.25	0.75	0.75	0.25
μ_{P_5}	0.75	0.5	0.5	0.75	0.5	0.75	0.5	0.5
μ_{P_6}	1	1	1	0	0	1	0	0
μ_{P_7}	0.75	0.25	0.5	0.5	0.25	0.5	0.5	0.25
μ_{P_8}	0.75	0.25	0.25	0.25	0.25	0.5	0.5	0.25
μ_{P_9}	0.5	0.25	0.25	0.25	0.25	0.5	0.25	0.25
$\mu_{P_{10}}$	0.75	0.5	0.5	0.25	0.5	0.75	0.25	0.25
$\mu_{P_{11}}$	0.5	0.25	0.5	0.25	0.25	0.5	0.25	0.25

Table 16. Cons: Membership functions determined by C.

	HYD	WIN	SOL	GAS	NUC	BIO	OIL	COA
μ_{C_1}	0.25	0.25	0.25	0.5	0.25	0	0.25	1
μ_{C_2}	0	0	0	1	0.5	0.5	0.75	0.75
μ_{C_3}	0.75	0.5	0.25	0.25	0.25	0.75	0.25	0
μ_{C_4}	0.5	0.5	0.25	0.5	0.5	0	0.75	0.25
μ_{C_5}	0.25	0.25	0	0.5	0.75	0.25	0.75	0.75

Continued

μ_{C_6}	0.25	0.25	0.75	0.5	1	0.25	0.5	0.75
μ_{C_7}	0.75	0.75	0.5	0.75	0.75	0.75	0.75	1
μ_{C_8}	0.5	0.75	0.25	0.5	0.75	0.25	0.5	0.5
μ_{C_9}	0.25	0.25	0.25	0.75	0.5	0.25	0.75	0.5
$\mu_{C_{10}}$	0.5	0.25	0.25	0.5	1	0	0.5	0.5
$\mu_{C_{11}}$	0.5	0.25	0.25	0.75	0.25	0.25	0.75	0.25
$\mu_{C_{12}}$	0.25	0.25	0.5	0.75	0.75	0.25	0.75	0.25
$\mu_{C_{13}}$	0.5	0.5	0.75	0.25	0.5	0.25	0.25	0.25
$\mu_{C_{14}}$	0.25	0.5	0.5	0.5	0.25	0.75	0.25	0.25
$\mu_{C_{15}}$	0.25	0.5	0.5	0.75	0.75	0.25	1	0.75

Table 17. Pros: Membership functions determined by expert D.

	HYD	WIN	SOL	GAS	NUC	BIO	OIL	COA
μ_{P_1}	0.75	0.5	1	0.75	0.75	1	0.75	0
μ_{P_2}	0.75	0.5	0.5	0.5	0.25	0.5	0.5	0.25
μ_{P_3}	0.75	0.5	0.75	0.75	0.75	0.75	0.75	0.25
μ_{P_4}	0.75	0.25	0.75	0.75	0.5	0.75	0.75	0.25
μ_{P_5}	0.75	0.5	0.5	0.75	0.75	0.75	0.5	0.75
μ_{P_6}	1	1	1	0	0	1	0	0
μ_{P_7}	0.75	0.5	0.25	0.5	0.25	0.75	0.5	0.25
μ_{P_8}	0.75	0.25	0.25	0.5	0.25	0.5	0.25	0.25
μ_{P_9}	0.5	0.25	0.25	0.25	0.25	0.5	0.25	0.25
$\mu_{P_{10}}$	0.5	0.25	0.25	0.5	0.75	0.5	0.5	0.25
$\mu_{P_{11}}$	0.5	0.25	0.5	0.5	0.25	0.5	0.5	0.25

Table 18. Cons: Membership functions determined by expert D.

	HYD	WIN	SOL	GAS	NUC	BIO	OIL	COA
μ_{C_1}	0.5	0.25	0.25	0.5	0.5	0	0.25	1
μ_{C_2}	0	0	0	1	0.5	0.5	0.75	0.5
μ_{C_3}	0.75	0.25	0	0	0.25	0.75	0	0
μ_{C_4}	0.25	0.25	0.25	0.5	0.5	0.25	0.5	0.25
μ_{C_5}	0	0.25	0.25	0.5	0.5	0.25	0.5	0.75

Continued

μ_{C_6}	0.25	0.25	0.25	0.5	0.75	0.5	0.5	0.5
μ_{C_7}	0.5	1	1	0.75	0.75	0.5	0.75	1
μ_{C_8}	0.25	0.5	0.5	0.5	0.5	0.25	0.5	0.75
μ_{C_9}	0.25	0.5	0.25	0.75	0.5	0.25	0.75	0.5
$\mu_{C_{10}}$	0.25	0.25	0.25	0.25	0.75	0.25	0.5	0.25
$\mu_{C_{11}}$	0.25	0.25	0.25	0.5	0.25	0.25	0.5	0
$\mu_{C_{12}}$	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
$\mu_{C_{13}}$	0.25	0.5	0.25	0.25	0.25	0.25	0.25	0.25
$\mu_{C_{14}}$	0.25	0.5	0.25	0.25	0.25	0.75	0.25	0.25
$\mu_{C_{15}}$	0.5	0.5	0.5	1	0.75	0.5	0.75	1

1) Application of the extended method.

We apply the novel method to the energy data as follows.

The evidence couple $\langle a_i, b_i \rangle$ of each of the eight power sources $\alpha \in X$ have been determined from each expert's data via the weighted mean formulas, Equations (4.1) and (4.2), where the former is used to calculate the global strength a_i and the latter formula is used to derive the global weakness b_i. In this process, recall that, we incorporate into (4.1) and (4.2) the weights of the pros criteria as presented in the last column of **Table 9** [F_B (weight)] and the weights of the cons criteria are expressed in the last column of **Table 10** [F_B (weight)], respectively. The evidence couples for the four experts are shown in **Tables 19-22**.

Furthermore, using the weights of experts as established in column 4 [F_B (weight)] of **Table 8** in the weighted mean Equations (5.3) and (5.4), the aggregated evidence couples for the group of four experts are presented in **Table 23**.

The corresponding evidence matrices to the evidence couples in **Table 23** are as follows:

$$H = \begin{bmatrix} 0.1759 & 0.1954 \\ 0 & 0.6287 \end{bmatrix},\ W = \begin{bmatrix} 0.3764 & 0 \\ 0.0682 & 0.5554 \end{bmatrix},\ S = \begin{bmatrix} 0.3587 & 0 \\ 0.0448 & 0.5965 \end{bmatrix}$$

$$G = \begin{bmatrix} 0.4408 & 0.1267 \\ 0 & 0.4325 \end{bmatrix},\ N = \begin{bmatrix} 0.5723 & 0.0265 \\ 0 & 0.4012 \end{bmatrix},\ B = \begin{bmatrix} 0.2747 & 0.1213 \\ 0 & 0.6040 \end{bmatrix}$$

$$O = \begin{bmatrix} 0.4655 & 0.1189 \\ 0 & 0.4156 \end{bmatrix},\ C = \begin{bmatrix} 0.5753 & 0 \\ 0.0224 & 0.4023 \end{bmatrix},$$

where *H*, *W*, *S*, *G*, *N*, *B*, *O* and *C* represent HYD, WIN, SOL, GAS, NUC, BIO, OIL, and COA respectively.

Now, as detailed in [13] and as illustrated in Examples 4.1 and 4.2 in this manuscript, a pair of alternatives β_1, β_2 can be compared in terms of their evidence matrices denoted by M_1, M_2 respectively and if $M_1 \le M_2$ then, we conclude that the alternative β_2 has dominated β_1. Algebraically, we say that

Table 19. Evidence couples for expert A.

Option a	Evidence for a	Evidence against a
HYD	0.8989	0.4000
WIN	0.6540	0.3720
SOL	0.5920	0.3973
GAS	0.6222	0.5664
NUC	0.4472	0.6324
BIO	0.7330	0.4467
OIL	0.6040	0.6247
COA	0.5892	0.6149

Table 20. Evidence couples for expert B.

Option a	Evidence for a	Evidence against a
HYD	0.8244	0.3589
WIN	0.5517	0.3542
SOL	0.5977	0.4342
GAS	0.5460	0.6259
NUC	0.4487	0.5420
BIO	0.6403	0.4134
OIL	0.5170	0.6259
COA	0.2801	0.5756

Table 21. Evidence couples for expert C.

Option a	Evidence for a	Evidence against a
HYD	0.8131	0.4089
WIN	0.4875	0.3958
SOL	0.6131	0.3042
GAS	0.4642	0.5988
NUC	0.3369	0.6313
BIO	0.7358	0.3238
OIL	0.4563	0.5887
COA	0.2290	0.5616

Table 22. Evidence couples for expert D.

Option a	Evidence for a	Evidence against a
HYD	0.7131	0.2976

Continued

WIN	0.4528	0.3723
SOL	0.5903	0.3253
GAS	0.5369	0.5324
NUC	0.4648	0.5304
BIO	0.7216	0.3690
OIL	0.4892	0.5083
COA	0.2665	0.5229

Table 23. Aggregated evidence couples.

Option α	Evidence for α	Evidence against α
HYD	0.8241	0.3713
WIN	0.5554	0.3764
SOL	0.5965	0.3587
GAS	0.5592	0.5675
NUC	0.4277	0.5988
BIO	0.7253	0.3960
OIL	0.5345	0.5844
COA	0.4023	0.5753

$$M_1 \le M_2 \text{ iff } M_1^{\perp} \oplus M_2 = 1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}. \tag{6.1}$$

Moreover, via the evidence couples in Table 23, we generate the following evidence couples and their corresponding evidence matrices.

From Table 23, the evidence couple for $H = (0.8241, 0.3713)$. Hence, $H^{\perp} = (0.8241^*, 0.3713^*) = (0.1759, 0.6287)$. Also, $W = (0.5554, 0.3764)$ and $W^{\perp} = (0.5554^*, 0.3764^*) = (0.4446, 0.6236)$.

So, $H^{\perp} \oplus W = (0.1759 \oplus 0.5554, 0.6287 \odot 0.3764) = (0.7313, 0.0051)$. In terms of matrices,

$$\begin{aligned} H^{\perp} \oplus W &= \begin{bmatrix} 0.7313^* \wedge 0.0051 & 0.7313 \odot 0.0051 \\ 0.7313^* \odot 0.0051^* & 0.7313 \wedge 0.0051^* \end{bmatrix} \\ &= \begin{bmatrix} 0.2687 \wedge 0.0051 & 0.7313 \odot 0.0051 \\ 0.2687 \odot 0.9949 & 0.7313 \wedge 0.9949 \end{bmatrix} \\ &= \begin{bmatrix} 0.0051 & 0 \\ 0.2636 & 0.7313 \end{bmatrix} \ne \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \end{aligned}$$

Hence, $H \not\le W$.

Similarly, $W^{\perp} \oplus H = (0.4446 \oplus 0.8241, 0.6236 \odot 0.3713) = (1, 0)$. This implies

$$W^{\perp} \oplus H = \begin{bmatrix} 1^* \wedge 0 & 1 \odot 0 \\ 1^* \odot 0^* & 1 \wedge 0^* \end{bmatrix} = \begin{bmatrix} 0 \wedge 0 & 1 \odot 0 \\ 0 \odot 1 & 1 \wedge 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

so, $W \le H$. Thus, H dominates W. This means the group of experts preferred H to W. By using this same evaluation process, the analysis of the remaining pairs are as follows: $H^{\perp} \oplus S = (0.7724, 0)$, $S^{\perp} \oplus H = (1, 0.0126)$. Hence,

$$H^{\perp} \oplus S = \begin{bmatrix} 0 & 0 \\ 0.2276 & 0.7724 \end{bmatrix},\ S^{\perp} \oplus H = \begin{bmatrix} 0 & 0.0126 \\ 0 & 0.9874 \end{bmatrix}.$$

Thus, $H \not\le S$ and $S \not\le H$. These indicate that H and S are incomparable. But, $T(S^{\perp} \oplus H) = 0.9874 > 0.7724 = T(H^{\perp} \oplus S)$, hence, H dominates S.

$H^{\perp} \oplus G = (0.7351, 0.1962)$, $G^{\perp} \oplus H = (1, 0)$. In terms of matrices,

$$H^{\perp} \oplus G = \begin{bmatrix} 0.1962 & 0 \\ 0.0687 & 0.7351 \end{bmatrix},\ G^{\perp} \oplus H = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Thus, $H \not\le G$ but $G \le H$.

$H^{\perp} \oplus N = (0.6036, 0.2275)$, and $N^{\perp} \oplus H = (1, 0)$. In evidence matrices,

$$H^{\perp} \oplus N = \begin{bmatrix} 0.2275 & 0 \\ 0.1689 & 0.6036 \end{bmatrix},\ N^{\perp} \oplus H = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

So, $H \not\le N$, but $N \le H$.

$$H^{\perp} \oplus B = (0.9012, 0.0247),\ B^{\perp} \oplus H = (1, 0).$$

$$H^{\perp} \oplus B = \begin{bmatrix} 0.0247 & 0 \\ 0.0741 & 0.9012 \end{bmatrix},\ B^{\perp} \oplus H = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

This means $H \not\le B$, whereas $B \le H$.

$H^{\perp} \oplus O = (0.7104, 0.2131)$, $O^{\perp} \oplus H = (1, 0)$. So,

$$H^{\perp} \oplus O = \begin{bmatrix} 0.2131 & 0 \\ 0.0765 & 0.7104 \end{bmatrix},\ O^{\perp} \oplus H = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Therefore, $H \not\le O$, but $O \le H$.

$$H^{\perp} \oplus C = (0.5782, 0.2040),\ C^{\perp} \oplus H = (1, 0).$$

$$H^{\perp} \oplus C = \begin{bmatrix} 0.2040 & 0 \\ 0.2178 & 0.5782 \end{bmatrix},\ C^{\perp} \oplus H = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Thus, $H \not\le C$, but $C \le H$.

$$W^{\perp} \oplus S = (1, 0),\ S^{\perp} \oplus W = (0.9589, 0.0177).$$

$$W^{\perp} \oplus S = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},\ S^{\perp} \oplus W = \begin{bmatrix} 0.0177 & 0 \\ 0.0234 & 0.9589 \end{bmatrix}.$$

Hence, $W \le S$ and $S \not\le W$.

$$W^{\perp} \oplus G = (1, 0.1911),\ G^{\perp} \oplus W = (0.9962, 0).$$

$$W^{\perp} \oplus G = \begin{bmatrix} 0 & 0.1911 \\ 0 & 0.8089 \end{bmatrix},\ G^{\perp} \oplus W = \begin{bmatrix} 0 & 0 \\ 0.0038 & 0.9962 \end{bmatrix}$$

This implies $W \not\le G$ and $G \not\le W$. Thus, G and W are incomparable. However, $T(G^{\perp} \oplus W) = 0.9962 > 0.8089 = T(W^{\perp} \oplus G)$. So, W dominates G.

$W^{\perp} \oplus N = (0.8723, 0.2224)$, and $N^{\perp} \oplus W = (1, 0)$. By matrices, we have

$$W^{\perp} \oplus N = \begin{bmatrix} 0.1277 & 0.0947 \\ 0 & 0.7776 \end{bmatrix},\ N^{\perp} \oplus W = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

This means $W \not\leq N$ and $N \leq W$. Thus, *W* dominates *N*.

$W^{\perp} \oplus B = (1, 0.0196)$, and $B^{\perp} \oplus W = (0.8301, 0)$. These in terms matrices give

$$W^{\perp} \oplus B = \begin{bmatrix} 0 & 0.0196 \\ 0 & 0.9804 \end{bmatrix} \text{ and } B^{\perp} \oplus W = \begin{bmatrix} 0 & 0 \\ 0.1699 & 0.8301 \end{bmatrix}$$

Thus, $W \not\leq B$ and $B \not\leq W$. But $T\left(W^{\perp} \oplus B\right) = 0.9804 > 0.8301 = T\left(B^{\perp} \oplus W\right)$. So, *B* dominates *W*.

$$W^{\perp} \oplus O = (0.9791, 0.208),\ O^{\perp} \oplus W = (1, 0).$$

$$W^{\perp} \oplus O = \begin{bmatrix} 0.0209 & 0.1871 \\ 0 & 0.7920 \end{bmatrix},\ O^{\perp} \oplus W = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

This means $W \not\leq O$ and $O \leq W$. So, *W* dominates *O*.

$$W^{\perp} \oplus C = (0.8469, 0.1989),\ C^{\perp} \oplus W = (1, 0).$$

$$W^{\perp} \oplus C = \begin{bmatrix} 0.1531 & 0.0458 \\ 0 & 0.8011 \end{bmatrix},\ C^{\perp} \oplus W = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Thus, $W \not\leq C$ but $C \leq W$. Hence, *W* dominates *C*.

$$S^{\perp} \oplus G = (0.9627, 0.2088),\ G^{\perp} \oplus S = (1, 0).$$

$$S^{\perp} \oplus G = \begin{bmatrix} 0.0373 & 0.1715 \\ 0 & 0.7912 \end{bmatrix},\ G^{\perp} \oplus S = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

This implies $S \not\leq G$ but $G \leq S$. Hence, *S* dominates *G*.

$S^{\perp} \oplus N = (0.8312, 0.2401)$, $N^{\perp} \oplus S = (1, 0)$. Thus,

$$S^{\perp} \oplus N = \begin{bmatrix} 0.1688 & 0.0713 \\ 0 & 0.7599 \end{bmatrix},\ N^{\perp} \oplus S = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Here too, $S \not\leq N$ but $N \leq S$. So, *S* dominates *G*.

$$S^{\perp} \oplus B = (1, 0.0373),\ B^{\perp} \oplus S = (0.8712, 0).$$

$$S^{\perp} \oplus B = \begin{bmatrix} 0 & 0.0373 \\ 0 & 0.9627 \end{bmatrix},\ B^{\perp} \oplus S = \begin{bmatrix} 0 & 0 \\ 0.1288 & 0.8712 \end{bmatrix}$$

This implies $S \not\leq B$ and $B \not\leq S$. However, $T\left(S^{\perp} \oplus B\right) = 0.9627 > 0.8712 = T\left(B^{\perp} \oplus S\right)$. So, *B* dominates *S*.

$$S^{\perp} \oplus O = (0.9380, 0.2257),\ O^{\perp} \oplus S = (1, 0).$$

$$S^{\perp} \oplus O = \begin{bmatrix} 0.0620 & 0.1637 \\ 0 & 0.7743 \end{bmatrix},\ O^{\perp} \oplus S = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Thus, $S \not\leq O$ but $O \leq S$. So, *S* dominates *O*.

$$S^{\perp} \oplus C = (0.8058, 0.2166),\ C^{\perp} \oplus S = (1, 0).$$

$$S^{\perp} \oplus C = \begin{bmatrix} 0.1942 & 0.0224 \\ 0 & 0.7834 \end{bmatrix},\ C^{\perp} \oplus S = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence, $S \nleq C$ but $C \leq S$. So, S dominates C.

$$G^{\perp} \oplus N = (0.8685, 0.0313),\quad N^{\perp} \oplus G = (1,0).$$

$$G^{\perp} \oplus N = \begin{bmatrix} 0.0313 & 0 \\ 0.1002 & 0.8685 \end{bmatrix},\quad N^{\perp} \oplus G = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence, $G \nleq N$ but $N \leq G$. So, G dominates N.

$$G^{\perp} \oplus B = (1,0),\quad B^{\perp} \oplus G = (0.8339, 0.1715).$$

$$G^{\perp} \oplus B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},\quad B^{\perp} \oplus G = \begin{bmatrix} 0.1661 & 0.0054 \\ 0 & 0.8285 \end{bmatrix}$$

Thus, $G \leq B$ but $B \nleq G$. So, B dominates G.

$G^{\perp} \oplus O = (0.9753, 0.0169)$, and $O^{\perp} \oplus G = (1,0)$. This means

$$G^{\perp} \oplus O = \begin{bmatrix} 0.0169 & 0 \\ 0.0078 & 0.9753 \end{bmatrix},\quad O^{\perp} \oplus G = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

This means $G \nleq O$ but $O \leq G$. So, G dominates O.

$G^{\perp} \oplus C = (0.8431, 0.0078)$, $C^{\perp} \oplus G = (1,0)$. In matrices, we have

$$G^{\perp} \oplus C = \begin{bmatrix} 0.0078 & 0 \\ 0.1491 & 0.8431 \end{bmatrix},\quad C^{\perp} \oplus G = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

So, $G \nleq C$ and $C \leq G$. Hence, G dominates C.

$N^{\perp} \oplus B = (1,0)$, and $B^{\perp} \oplus N = (0.7024, 0.2028)$. Thus,

$$N^{\perp} \oplus B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } B^{\perp} \oplus N = \begin{bmatrix} 0.2028 & 0 \\ 0.0948 & 0.7024 \end{bmatrix}$$

So, $N \leq B$ but $B \nleq N$. So, B dominates N.

$$N^{\perp} \oplus O = (1,0),\quad O^{\perp} \oplus N = (0.8932, 0.0144).$$

$$N^{\perp} \oplus O = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},\quad O^{\perp} \oplus N = \begin{bmatrix} 0.0144 & 0 \\ 0.0924 & 0.8932 \end{bmatrix}$$

So, $N \leq O$ and $O \nleq N$. Therefore, O dominates N.

$$N^{\perp} \oplus C = (0.9746, 0),\quad C^{\perp} \oplus N = (1, 0.0235).$$

$$N^{\perp} \oplus C = \begin{bmatrix} 0 & 0 \\ 0.0254 & 0.9746 \end{bmatrix} \text{ and } C^{\perp} \oplus N = \begin{bmatrix} 0 & 0.0235 \\ 0 & 0.9765 \end{bmatrix}$$

So, since $N \nleq C$ and $C \nleq N$ but

$T(C^{\perp} \oplus N) = 0.9765 > 0.9746 = T(N^{\perp} \oplus C)$, N dominates C.

$$B^{\perp} \oplus O = (0.8092, 0.1884),\quad O^{\perp} \oplus B = (1,0).$$

$$B^{\perp} \oplus O = \begin{bmatrix} 0.1884 & 0 \\ 0.0024 & 0.8092 \end{bmatrix},\quad O^{\perp} \oplus B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence, since $B \nleq O$ and $O \leq B$, we conclude that B dominates O.

$$B^{\perp} \oplus C = (0.6770, 0.1793),\quad C^{\perp} \oplus B = (1,0).$$

$$B^{\perp} \oplus C = \begin{bmatrix} 0.1793 & 0 \\ 0.1437 & 0.6770 \end{bmatrix},\quad C^{\perp} \oplus B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Here too, since $B \nleq C$ and $C \leq B$, we conclude that B dominates C.

$O^{\perp} \oplus C = (0.8678, 0)$, and $C^{\perp} \oplus O = (1, 0.0091)$. In terms of matrices, we have

$$O^{\perp} \oplus C = \begin{bmatrix} 0 & 0 \\ 0.1322 & 0.8678 \end{bmatrix} \text{ and } C^{\perp} \oplus O = \begin{bmatrix} 0 & 0.0091 \\ 0 & 0.9909 \end{bmatrix}$$

From these two matrices, it is clear that $O \npreceq C$ and $C \npreceq O$. However, $T(C^{\perp} \oplus O) = 0.9909 > 0.8678 = T(O^{\perp} \oplus C)$, hence O dominates C.

From the above evaluation, the dominance of each of the eight energy sources over the others is summarised as follows:

$$W, S, G, N, B, O, C \preceq H;$$
$$W, S, G, N, O, C \preceq B;$$
$$W, G, N, O, C \preceq S;$$
$$G, N, O, C \preceq W;$$
$$N, O, C \preceq G;$$
$$N, C \preceq O;$$
$$C \preceq N.$$

Therefore, the complete ranking of the eight sources in ascending order as given in the definition is

coal $\preceq$ nuclear $\preceq$ oil $\preceq$ natural gas $\preceq$ wind $\preceq$ solar $\preceq$ biomass $\preceq$ hydro.

2) Application of the fuzzy TOPSIS group decision-making method.

We now apply the fuzzy TOPSIS group decision-making method to the energy data to be able to compare the ranking of this traditional method with that of our novel approach. So, for want of space, we have summarised the fuzzy TOPSIS group decision-making procedure and results as follows.

The aggregated collective matrix D for the pros and the cons criteria are presented in Table 24 and Table 25 respectively.

From Table 24 and Table 25 combined, we have the ideal solution A^{+} and the negative-ideal solution A^{-} as

$$A^{+} = \{0.0261, 0.0083, 0.0755, 0.0226, 0.0328, 0.0301, 0.0186, 0.0090, 0.0089, 0.0207, 0.0269, 0.0019, 0.0000, 0.0048, 0.0068, 0.0011, 0.0024, 0.0143, 0.0052, 0.0023, 0.0065, 0.0026, 0.0036, 0.0016, 0.0008, 0.0040\}.$$

$$A^{-} = \{0.0036, 0.0024, 0.0056, 0.0071, 0.0146, 0.0000, 0.0093, 0.0026, 0.0036, 0.0116, 0.0100, 0.0090, 0.0168, 0.0161, 0.0164, 0.0150, 0.0156, 0.0250, 0.0155, 0.0136, 0.0197, 0.0061, 0.0052, 0.0023, 0.0019, 0.0066\}.$$

Table 26 is a summary of the separation measures, and the relative closeness to the ideal solution, C_i^{*}.

So, by the fuzzy TOPSIS group decision-making method, the ranking order of the eight energy sources is

nuclear $\preceq$ coal $\preceq$ oil $\preceq$ natural gas $\preceq$ wind $\preceq$ solar $\preceq$ biomass $\preceq$ hydro.

The rankings of the two methods are presented in Table 27.

Table 24. Aggregated collective matrix D for the pros criteria.

	HYD	WIN	SOL	GAS	NUC	BIO	OIL	COA
μ_{P_1}	0.0261	0.0138	0.0253	0.0213	0.0134	0.0249	0.0213	0.0036
μ_{P_2}	0.0083	0.0036	0.0042	0.0055	0.0031	0.0055	0.0052	0.0024
μ_{P_3}	0.0755	0.0056	0.0085	0.0115	0.0073	0.0113	0.0115	0.0058
μ_{P_4}	0.0205	0.0163	0.0193	0.0226	0.0100	0.0226	0.0226	0.0071
μ_{P_5}	0.0328	0.0188	0.0146	0.0240	0.0261	0.0234	0.0193	0.0302
μ_{P_6}	0.0301	0.0301	0.0301	0.0000	0.0034	0.0301	0.0000	0.0000
μ_{P_7}	0.0186	0.0126	0.0093	0.0132	0.0102	0.0147	0.0156	0.0126
μ_{P_8}	0.0090	0.0037	0.0037	0.0057	0.0026	0.0062	0.0065	0.0060
μ_{P_9}	0.0089	0.0067	0.0067	0.0067	0.0036	0.0085	0.0051	0.0051
$\mu_{P_{10}}$	0.0207	0.0145	0.0142	0.0149	0.0174	0.0178	0.0116	0.0153
$\mu_{P_{11}}$	0.0171	0.0130	0.0163	0.0269	0.0100	0.0163	0.0148	0.0126

Table 25. Aggregated collective matrix D for the cons criteria.

	HYD	WIN	SOL	GAS	NUC	BIO	OIL	COA
μ_{C_1}	0.0066	0.0042	0.0042	0.0071	0.0050	0.0027	0.0019	0.0090
μ_{C_2}	0.0000	0.0000	0.0000	0.0164	0.0148	0.0095	0.0185	0.0168
μ_{C_3}	0.0161	0.0116	0.0093	0.0060	0.0074	0.0134	0.0060	0.0048
μ_{C_4}	0.0124	0.0096	0.0106	0.0164	0.0115	0.0068	0.0152	0.0109
μ_{C_5}	0.0011	0.0050	0.0039	0.0102	0.0139	0.0072	0.0136	0.0150
μ_{C_6}	0.0024	0.0049	0.0070	0.0075	0.0156	0.0089	0.0075	0.0129
μ_{C_7}	0.0191	0.0234	0.0218	0.0215	0.0211	0.0143	0.0215	0.0250
μ_{C_8}	0.0052	0.0088	0.0063	0.0136	0.0118	0.0099	0.0136	0.0155
μ_{C_9}	0.0023	0.0038	0.0028	0.0136	0.0073	0.0065	0.0136	0.0093
$\mu_{C_{10}}$	0.0119	0.0077	0.0080	0.0116	0.0197	0.0065	0.0131	0.0092
$\mu_{C_{11}}$	0.0049	0.0032	0.0034	0.0061	0.0034	0.0032	0.0061	0.0026
$\mu_{C_{12}}$	0.0038	0.0036	0.0044	0.0052	0.0051	0.0038	0.0052	0.0039
$\mu_{C_{13}}$	0.0019	0.0023	0.0022	0.0017	0.0019	0.0017	0.0017	0.0016
$\mu_{C_{14}}$	0.0014	0.0015	0.0016	0.0013	0.0008	0.0019	0.0011	0.0011
$\mu_{C_{15}}$	0.0040	0.0045	0.0045	0.0067	0.0058	0.0041	0.0064	0.0063

Table 26. Separation measures and ideal solution.

Option	S^+	S^-	$S^+ + S^-$	$C^* = \frac{S^-}{S^+ + S^-}$
HYD	0.0141	0.0883	0.1024	0.8623
WIN	0.0755	0.0387	0.1142	0.3389
SOL	0.0714	0.0447	0.1161	0.3850
GAS	0.0768	0.0346	0.1114	0.3106
NUC	0.0837	0.0173	0.1010	0.1713
BIO	0.0671	0.0490	0.1161	0.4220
OIL	0.0794	0.0265	0.1059	0.2502
COA	0.0872	0.0200	0.1072	0.1866

Table 27. Ranking the eight energy sources in ascending order from the worst to the best by each of the two outranking methods.

Method	Ranking
Extended PMVS	coal $\preceq$ nuclear $\preceq$ oil $\preceq$ natural gas $\preceq$ wind $\preceq$ solar $\preceq$ biomass $\preceq$ hydro
Fuzzy TOPSIS group method	nuclear $\preceq$ coal $\preceq$ oil $\preceq$ natural gas $\preceq$ wind $\preceq$ solar $\preceq$ biomass $\preceq$ hydro

7. Discussion

Both the TOPSIS and PMVS approaches have provided complete rankings for the eight energy sources for Ghana. From their rankings as shown in Table 27, both methods have selected hydro power as the optimal energy source for the country. They have also unanimously agreed on biomass, solar, wind, natural gas, and oil as the second, third, fourth, fifth and sixth best respectively. However, their rankings differ between the last two sources. Whereas the extended PMVS settled on nuclear as the better option than coal, the fuzzy TOPSIS method says otherwise as seen on Table 27. Therefore, the six energy sources identified by both techniques in unison to be the optimal energy basket for Ghana are hydro power, biomass, solar energy, wind energy, natural gas, and oil. The worst source, however, according to the extended PMVS is coal, whereas by the fuzzy TOPSIS, the worst energy source is nuclear.

In terms of effort, the extended PMVS is less laborious, easier, and simpler than the fuzzy TOPSIS.

Moreover, the proposed method gives more insight into how options in each pair outperform each other. That is to say, any two options have comparative advantage over each other on some or all the set of criteria. So, the PMVS method can determine the overall value of the comparative advantage of one alternative over the other and vice versa. Put another way, the novel method determines the pair of values that measures the extent to which two alternatives are better than each other. For example, if we take solar and wind, it is obvious that on some criteria solar performs better than wind, while on the other criteria wind performs better than solar. So, the proposed method can establish the performance values in this specific case to enable users identify the better performer. Hence, from the PMVS calculation, the overall value of the comparative advantage of solar over wind is 1, whereas the overall value of the comparative advantage of wind over solar is 0.9589. Hence, solar is better than wind. Similarly, the value of the superiority of biomass over solar is 0.9627, while the value of the superiority of solar over biomass is 0.8712. This shows that biomass is better than solar. The TOPSIS method, however, does not give such details. It only uses the ultimate fixed values of relative closeness to the ideal solution to compare all options and rank them.

Furthermore, the extended PMVS approach can handle more efficiently large size group decision-making problems than the fuzzy TOPSIS method. Although critiques may argue that there are now powerful computers that can be used to analyse large size data sets, but nonetheless, easy methods are still useful especially in parts of the world where these powerful computers are not readily available. If even they are available, some decision-makers may not still be able to use them due to lack of technical know-how.

Also, one distinctive feature of the PMVS approach that gives it the edge over TOPSIS method, and some other outranking methods is the paraconsistent logic. This logic enables the PMVS to enjoy a much wider scope than the TOPSIS. Through this logic, the PMVS method can solve all decision problems that the TOPSIS is able to solve, but not all decision-making problems solvable by the PMVS approach are solvable by the TOPSIS approach. For instance, decision problems involving contradictory data can be handled by the novel method but cannot be solved by the TOPSIS method. In fact, classical logic upon which the TOPSIS method depends and other logics such as intuitionistic logic conform totally to the principle of non-contradiction, and as a result reject any information that appears contradictory. Paraconsistent logic and for that matter the PMVS approach, on the other hand, posits that a piece of information can be simultaneously true and false, and that there are true contradictions.

8. Conclusions and Suggestions

In this paper, the Paraconsistent Many-Valued Similarity (PMVS) method has been extended to deal with multipersonal decision-making problems in which a group of decision-makers evaluates a finite set of options with respect to the same set of criteria. In this extended version, the Borda rule is employed to determine the weights of criteria and the weights of members of the decision-making group. By means of the aggregated weighted mean, the individual global strengths and global weaknesses otherwise called the evidence couples are synthesised into aggregated global strengths and aggregated global weaknesses or better still, aggregated evidence couples. The introduced approach treads cautiously along lines of inconsistencies and only approves inferences that do not lead to explosion. An inference explodes if it leads to anything. In this regard, after deriving the evidence couple for every proposition, we split such an evidence couple into its four components parts, namely falsehood, contradiction, unknown, and truth. We, then, calculate the values of these components to establish the corresponding evidence matrix. Thereafter, we discard three components of the evidence matrix: falsehood, contradiction, and unknown. The value of the remaining component (the truth component) is what we use to advance our analysis of the decision options vis-à-vis the set of criteria to arrive at a synthesized or conglomerate decision.

However, the only drawback of the proposed method is rank reversal issue. That is, the addition of one or more decision alternatives to the set of decision alternatives that has been ranked may affect the initial ranking.

To illustrate the application of the novel approach to group decision-making challenges, it has been used to analyse data sets from seven energy experts in the Ministry of Energy in Ghana to unearth the optimal energy mix for the country. Moreover, to show the efficiency of the new method, its findings on the energy data have been compared with that of fuzzy TOPSIS group decision-making approach and it has emerged that their findings are almost identical. Eventually, the study discovered hydro power, biomass, solar energy, wind energy, natural gas and oil as the best energy basket for Ghana.

Acknowledgements

This research duly acknowledges the cooperation and contribution of Ghana's Ministry of Energy in terms of information sharing to the success of this work.

Sincere thanks to the members of JAMP for their professional performance, and special thanks to managing editor *Hellen XU* for a rare attitude of high quality.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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