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Application of Elzaki Transform Method to Market Volatility Using the Black-Scholes Model

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Abstract

Black-Scholes Model (B-SM) simulates the dynamics of financial market and contains instruments such as options and puts which are major indices requiring solution. B-SM is known to estimate the correct prices of European Stock options and establish the theoretical foundation for Option pricing. Therefore, this paper evaluates the Black-Schole model in simulating the European call in a cash flow in the dependent drift and focuses on obtaining analytic and then approximate solution for the model. The work also examines Fokker Planck Equation (FPE) and extracts the link between FPE and B-SM for non equilibrium systems. The B-SM is then solved via the Elzaki transform method (ETM). The computational procedures were obtained using MAPLE 18 with the solution provided in the form of convergent series.

Keywords

Elzaki Transform Method, European Call, Black-Scholes Model, Fokker-Planck Equation, Market Volatility

1. Introduction

Volatility has to do with the degree of variation in prices of financial instruments over a period of time. The market expectation of future price fluctuations can be measured and expressed as a financial model. The Black-Scholes model is a partial differential equation responsible for the mathematical analysis of price evolution in a European call ([1] [2]). Basically, the Black-Scholes Model (B-SM) as the foundation for option pricing can be used to explore a variety of options in a European call. It is efficient to compute the price of an option correctly [3]. It can also be used to reveal different observable factors such as underlying price options, determine the range of price movements, rates and other factors not simultaneously observable and underlying fear factors according to Hicks [4]. For a European call depending on stock paying with no dividends, the B-SM is given as [5]

$$\frac{\partial}{\partial t}V+\frac{1}{2}S^2\varphi^2\frac{\partial^2}{\partial S^2}V+rS\frac{\partial}{\partial S}V-rV=0 \tag{1}$$

with

$$V(S,0)=S-Ke^{rT}, V(S,t)=S-Ke^{r(T-t)} \text{ as } S\to\infty. \tag{2}$$

where;

- V is the price of the option;
- φ is the volatility of the stock;
- S is the stock price; and
- r is the rate-free interest rate.

Biazar *et al.* [6] carried out a detailed study of Equation (1) and suggested a straightforward version which is rewritten as

$$\frac{\partial}{\partial t}V(S,t)+\frac{1}{2}S^2\varphi^2\frac{\partial^2}{\partial S^2}V(S,t)=rV(S,t)-rS\frac{\partial}{\partial S}V(S,t), \tag{3}$$

for proper financial interpretation. Equation (3) is referred to as the standard B-SM.

FOKKER-PLANCK EQUATION ([7] [8])

Let us consider the general form of a nonlinear FPE

$$\frac{\partial u}{\partial t}=\left[-\frac{\partial A(x,t,u)u}{\partial x}+\frac{\partial^2 B(x,t,u)}{\partial x^2}u\right] \tag{4}$$

With initial conditions as $u(x,0)=f(x)$, $x\in R$. Such that $u(x,t)$ is the unknown distribution function in Equation (4). Also, a special case of equation (4) is the FPE derived from Plasma Physics having the form

$$\frac{\partial P}{\partial t}=\frac{1}{2}\frac{\partial^2}{\partial x^2}\left(\frac{1}{2}x^{1-2\epsilon}P(x,t)\right)-\frac{\partial}{\partial x}\left(\frac{1}{4}x^{-2\epsilon}P(x,t)\right) \tag{5}$$

$P(x,t)$ in Equation (5) is the probability density function. Expressing

$$P(x,t)=\sqrt{2}x^{(2\epsilon-1)/2}\omega(r,y), r=t, y=\frac{2\sqrt{2}}{2\epsilon+1}x^{2\epsilon+1/2}$$

and substituting for it, then Equation (5) is reduced to a linear Brownian motion of FPE given as

$$\frac{\partial}{\partial r}\omega(r,y)=-\frac{\partial}{\partial y}\left(\frac{\omega}{\partial y}\right)\frac{1}{2}\frac{\partial^2\omega}{\partial y^2}$$

with initial conditions

$$\omega(0,y)=\text{erf}\left[\frac{1}{2}\sqrt{2}y\right]e^{y^2/2}+e^{y^2/2}$$

Or as

$$\frac{\partial}{\partial t}C(S,t)=\frac{1}{2}V^2(S,t)s^2\frac{\partial^2}{\partial S^2}C(S,t)+rS\frac{\partial}{\partial S}C(S,t)-rC(S,t), \tag{6}$$

using the Mellin transformation [9].

S is the the exact price that undergoes Brownian motion, $C(s,t)$ is the call option price, V is the volatility and r is the interest rate. Equation (6) is more useful if the problem is to determine the call option. However, B-SM when solved provides a strategy for determining not just the prices of underlying security, current time and time of expiration, and return of the free-risk assets but also eliminating the risk involved in the whole process.

Assumptions of the Black-Scholes Model

- The model B-SM computes the present value of a call option without dividends.
- The model requires the value of the standard deviation. It can be calculated from the variance by taking the square root.
- The hedge ratio is the ratio of the expected stock price at expiration to the current stock price at the onset.
- Free risk assets, underlying prices, strike prices and expiration can be used to hedge positions on the option.

Equation (1) can both be solved analytically and numerically. The few available analytic methods are complex and difficult to handle requiring linearization, perturbation, or weak assumptions. Therefore, several numerical methods have been developed and implemented over the years for seeking the solution of the Black-Scholes equation. The Finite Difference Method (FDM) has often been the most privileged numerical scheme used by many researchers in recent times to seek the approximate solutions of the B-SM. For instance, Andallah and Anwar [5] used the FDM for the numerical solution of B-SM. Dura and Mosneagu [10] also considered the numerical approximation of the B-SM via the FDM. Duffy [11] employed the FDM in financial engineering through the B-SM. A unitary transformation has been applied to the classical B-SM to obtain the quantum effects associated with the market fear factor especially that causing the increase in volatility rate [12]. Characterizing and determining volatility rate has also been the focus of many researchers [13]. There is also the non-commutative B-SM presented in Accardi and Boukas [14] that was transformed into an integral equation. Wei [15] listed the three benchmarks of a stochastic system for B-SM to test the accuracy of Fokker-Planck Equation (FPE) via time-dependent methods of FDM, Finite Volume and Finite Element. The B-SM of option pricing can also be regarded as a reaction-diffusion equation that is entirely based on stochastic analysis. Here, the B-SM is first transformed into a simple FPE and then analyzed. Several research has been carried out in the computational sense for the numerical solution of this derived FPE ([9] [15] [16]).

The empirical performance of B-SM has been examined in [17]. However, the link between B-SM and FPE has been investigated and analyzed by many researchers. These include [18]; they even introduced Fractional FPE in their work by deriving the fractional FPE order governing the dynamics of the equation and then determine the Black-Scholes differential equation that involves the stock asset and fair prices of the European option. Before this, others have used fractional to analyze stock exchange market dynamics ([19] [20] [21]). Consequently, once the market prices have been observed for the option, the B-SM can be inverted to determine the volatility. In addition, the B-SM when solved provides a strategy for eliminating the risk and determining the prices of underlying security ([22] [23]).

In recent years, the focus on B-SM has been on reviews in terms of redefinition ([24] [25]), Computations [26], Applications ([27] [28]) and solution methods [29] and sensitivity analysis [30].

This paper is motivated to seek the numerical solution of the B-SM using the Elzaki transform method (ETM). This method is necessitated by its simplicity, less computational rigor, and rapid rate of convergence as will be observed in section 3.

2. Elzaki Transform Method

The set [31]

$$\Omega=\left\{g(t):\exists N,\alpha_1 \text{ and } \alpha_2>0:\left|g(t)\right|<N\mathrm{e}^{|t|\alpha_j},\text{ if } t\in(-1)^j\times[0,\infty)\right\}, \tag{7}$$

define by

$$E\left[g(t)\right]=r\int_0^{\infty}g(t)\mathrm{e}^{-t/r}\mathrm{d}t=T(r),r\in(-\alpha_1,\alpha_2), \tag{8}$$

is called the Elzaki transform of the function $g(t)$.

2.1. Properties of Elzaki Transforms Method

1) $E\left[\dfrac{\partial u(x,t)}{\partial t}\right]=\dfrac{1}{q}T(x,q)-qu(x,0)$.

2) $E\left[\dfrac{\partial^2 u(x,t)}{\partial x^2}\right]=\dfrac{1}{q^2}T(x,q)-q\dfrac{\partial u(x,0)}{\partial t}-u(x,0)$.

3) $T_m(x,q)=\dfrac{T(x,q)}{q^m}-\sum_{k=0}^{m-1}q^{2-m+k}\dfrac{\partial^k u(x,0)}{\partial t^k}$, m is the order of the highest derivative.

4) $E\left[t^n\right]=n!q^{n+2}$.

5) $E^{-1}\left[q^{n+2}\right]=\dfrac{t^n}{n!}$.

2.2. Elzaki Transform Method for Black-Scholes Model

Here, we apply the ETM to solve B-S equation of the form (1).

Applying the Elzaki transform on both sides of Equation (2), we have

$$E\left[\frac{\partial}{\partial t}V(S,t)\right]=E\left[rV(S,t)-rS\frac{\partial}{\partial S}V(S,t)-\frac{1}{2}S^2\varphi^2\frac{\partial^2}{\partial S^2}V(S,t)\right] \tag{9}$$

Using 2.1(iii), we have that

$$E\left[V(S,t)\right]=\sum_{k=0}^{m-1}q^{2-m+k}\frac{\partial^k V(S,t)}{\partial t^k} + qE\left[rV(S,t)-rS\frac{\partial}{\partial S}V(S,t)-\frac{1}{2}S^2\varphi^2\frac{\partial^2}{\partial S^2}V(S,t)\right] \tag{10}$$

Having the inverse on both sides if Equation (10), we have

$$V(S,t)=E^{-1}\left[\sum_{k=0}^{m-1}q^{2-m+k}\frac{\partial^k V(S,t)}{\partial t^k} + qE\left[rV(S,t)-rS\frac{\partial}{\partial S}V(S,t)-\frac{1}{2}S^2\varphi^2\frac{\partial^2}{\partial S^2}V(S,t)\right]\right] \tag{11}$$

Thus the approximate solution is given as

$$V(S,t)=\sum_{n=0}^{\infty}V_n(S,t)$$

Thus, Equation (11) becomes

$$\sum_{n=0}^{\infty}V_n(S,t)=E^{-1}\left[\sum_{k=0}^{m-1}q^{2-m+k}\frac{\partial^k V(S,t)}{\partial t^k} + qE\left[rV(S,t)-rS\frac{\partial}{\partial S}V(S,t)-\frac{1}{2}S^2\varphi^2\frac{\partial^2}{\partial S^2}V(S,t)\right]\right] \tag{12}$$

By comparing both sides of Equation (12), we obtain the recurrence relations

$$V_0(S,t)=E^{-1}\left[\sum_{k=0}^{m-1}q^{2-m+k}\frac{\partial^k c_n(s,v,0)}{\partial t^k}\right], \tag{13}$$

$$V_{n+1}(S,t)=E^{-1}\left[qE\left[rV_n(S,t)-rS\frac{\partial}{\partial S}V_n(S,t)-\frac{1}{2}S^2\varphi^2\frac{\partial^2}{\partial S^2}V_n(S,t)\right]\right] \tag{14}$$

Hence, $V_1(S,t),V_2(S,t),\cdots,V_n(S,t)$ for $n\geq 1$ are calculated using the Equations (13) and (14).

Finally, the required approximate solution of the Black-Scholes equation becomes

$$V(S,t)=\sum_{n=0}^{\infty}V_n(S,t) \tag{15}$$

Equation (15) is the determining index for cash flow, risk profile for buying and selling underlying asset and predicting future price movement.

3. Numerical Illustrations

Here, we implement the ETM on the B-SM using Maple 18 software. Results obtained is presented as a power series.

Thus, using the ETM scheme (13) and (14) on

$$\frac{\partial}{\partial t}V+\frac{1}{2}S^2\varphi^2\frac{\partial^2}{\partial S^2}V+rS\frac{\partial}{\partial S}V-rV=0,$$

with the parameters $r=0.12$, $T=1$, $K=100$ and $\varphi=0.10$, (Andallah and

Anwar, 2018), we obtain the following approximations:

$$V_0 = S - 112.7496852$$

$$V_1 = -\frac{2}{3}S^2t^2$$

$$V_2 = 0.5341666665S^2t^4$$

$$V_3 = -0.1599741666S^2t^5$$

$$V_4 = 0.04266786181S^2t^6$$

$$\vdots$$

Thus, the required computed solution is

$$\begin{aligned} V(S,t) = S - 112.7496852 - \frac{2}{3}S^2t^2 + 0.5341666665S^2t^4 \\ - 0.1599741666S^2t^5 + 0.04266786181S^2t^6 \end{aligned} \quad (16)$$

Using the values of the call option on Equation (16) over the range $0 \le S \le 100$ for $t = 0$ to 1, the striking price is illustrated in Figure 1 produced by MAPLE 18. The approximate values of the model $t = 0$ to 1

Example 2

Let

$$C(S,t) = \sum_{i=0}^{n} a_i \varphi_i(t) \quad (17)$$

be an approximate solution of (6) with $a_i's$ being constant parameters, and $\varphi_i's$, Mamadu-Njoseh polynomials.

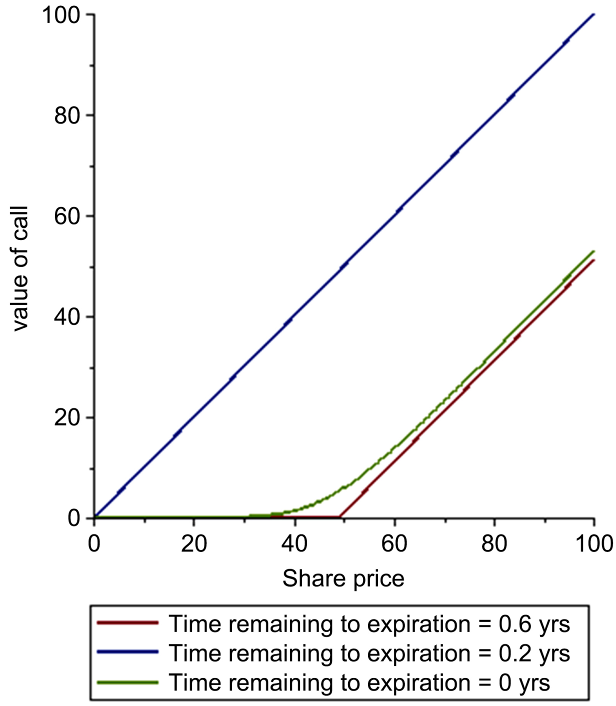

Figure 1. Numerical simulation for European call at different time steps with $r = 0.12$, $T = 1$, $K = 100$ and $\varphi = 0.10$.

Now, substituting (17) into (6), we get

$$C(S,T)=a_1+\frac{10}{3}a_2t+a_3\left(\frac{42}{5}t^2-\frac{3}{5}\right)-\frac{1}{2}V^2S^2\left(\frac{10}{3}a_2+\frac{84}{5}a_3\right)$$
$$+rs\left(a_1+\frac{10}{3}a_2s+a_3\left(\frac{42}{5}s^2-\frac{3}{5}\right)\right) \quad (18)$$
$$-r\left(a_0+a_1t+a_2\left(\frac{5}{3}t^2-\frac{2}{5}\right)+a_3\left(\frac{14}{5}t^3-\frac{3}{5}t\right)\right)$$

Evaluating (18) at the conditions

$$C(S,T)=\max(S-K),\ 0\le S<\infty,$$
$$C(0,t)=0,\ 0\le t<T,$$
$$C(s,T)=S-\mathrm{e}^{-r(T-t)}\ \text{ as } S\to\infty,$$

we obtain the following system of equations:

$$a_1+\frac{10}{3}a_2t+a_3\left(\frac{42}{5}t^2-\frac{3}{5}\right)-\frac{1}{2}V^2S^2\left(\frac{10}{3}a_2+\frac{84}{5}a_3\right)$$
$$+rs\left(a_1+\frac{10}{3}a_2s+a_3\left(\frac{42}{5}s^2-\frac{3}{5}\right)\right)$$
$$-r\left(a_0+a_1t+a_2\left(\frac{5}{3}t^2-\frac{2}{5}\right)+a_3\left(\frac{14}{5}t^3-\frac{3}{5}t\right)\right)=S-K\mathrm{e}^{0.3676425560r}$$

$$a_1+\frac{10}{3}a_2t+a_3\left(\frac{42}{5}t^2-\frac{3}{5}\right)-\frac{1}{2}V^2S^2\left(\frac{10}{3}a_2+\frac{84}{5}a_3\right)$$
$$+rs\left(a_1+\frac{10}{3}a_2s+a_3\left(\frac{42}{5}s^2-\frac{3}{5}\right)\right)$$
$$-r\left(a_0+a_1t+a_2\left(\frac{5}{3}t^2-\frac{2}{5}\right)+a_3\left(\frac{14}{5}t^3-\frac{3}{5}t\right)\right)=S-K\mathrm{e}^{-0.3676425560r}$$

$$a_1+\frac{10}{3}a_2t+a_3\left(\frac{42}{5}t^2-\frac{3}{5}\right)-\frac{1}{2}V^2S^2\left(\frac{10}{3}a_2+\frac{84}{5}a_3\right)$$
$$+rs\left(a_1+\frac{10}{3}a_2s+a_3\left(\frac{42}{5}s^2-\frac{3}{5}\right)\right)$$
$$-r\left(a_0+a_1t+a_2\left(\frac{5}{3}t^2-\frac{2}{5}\right)+a_3\left(\frac{14}{5}t^3-\frac{3}{5}t\right)\right)=S-K\mathrm{e}^{0.8756710201r}$$

$$a_1+\frac{10}{3}a_2t+a_3\left(\frac{42}{5}t^2-\frac{3}{5}\right)-\frac{1}{2}V^2S^2\left(\frac{10}{3}a_2+\frac{84}{5}a_3\right)$$
$$+rs\left(a_1+\frac{10}{3}a_2s+a_3\left(\frac{42}{5}s^2-\frac{3}{5}\right)\right) \quad (19)$$
$$-r\left(a_0+a_1t+a_2\left(\frac{5}{3}t^2-\frac{2}{5}\right)+a_3\left(\frac{14}{5}t^3-\frac{3}{5}t\right)\right)=S-K\mathrm{e}^{-0.8756710201r}$$

Thus, solving the above equations for $a_i, i=0,1,2,3$, using the estimates $V=0.2$, $S=60$, $r=0.01$, $t=1$, $K=100$, we obtain **Figure 2** below.

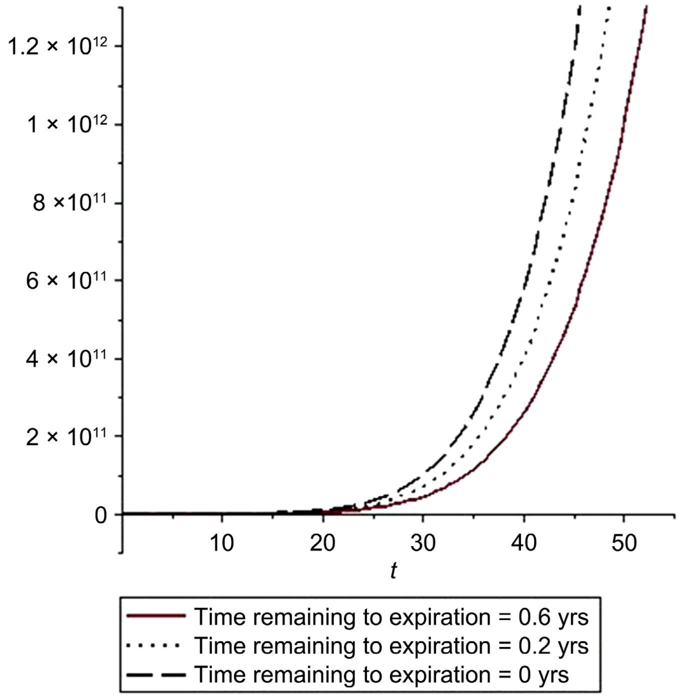

Figure 2. Numerical simulation for Equation (6) at different time steps with $V = 0.2$, $T = 1$, $K = 100$, $S = 60$ and $r = 0.01$.

$$a_0 = 41888.37454, a_1 = 303.5807685,$$
$$a_2 = 0.6107413174, a_3 = 0.0006165491976$$

Substituting the above into (17) yields the approximate solution to (6). Further simulation gives.

4. Conclusion

We have considered the numerical solution of the B-SM using the Elzaki transform method. Results obtained showed that the ETM is very reliable for simulating the call depending on stock paying with no dividends. The numerical illustration shows that the method is accurate for solving the FPE, hence for B-S equation. Also, the results agreed with those found in the literature ([5] [31]) for comparison. B-SM in its numerous modifications and expansion [31] can be used for hedging and risk mitigation, put and call options, Forex options, cash call, assets with continuous yielding dividends, stock options etc (DF, 2022) [32].

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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