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Relational and Euclidean Temporal Space

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Abstract

In mathematics, space encompasses various structured sets such as Euclidean, metric, or vector space. This article introduces temporal space—a novel concept independent of traditional spatial dimensions and frames of reference, accommodating multiple object-oriented durations in a dynamical system. The novelty of building temporal space using finite geometry is rooted in recent advancements in the theory of relationalism which utilizes Euclidean geometry, set theory, dimensional analysis, and a causal signal system. Multiple independent and co-existing cyclic durations are measurable as a network of finite one-dimensional timelines. The work aligns with Leibniz's comments on relational measures of duration with the addition of using discrete cyclic relational events that define these finite temporal spaces, applicable to quantum and classical physics. Ancient formulas have symmetry along with divisional and subdivisional orders of operations that create discrete and ordered temporal geometric elements. Elements have cyclically conserved symmetry but unique cyclic dimensional quantities applicable for anchoring temporal equivalence relations in linear time. We present both fixed equivalences and expanded periods of temporal space offering a non-Greek calendar methodology consistent with ancient global timekeeping descriptions. Novel applications of Euclid's division algorithm and Cantor's pairing function introduce a novel paired function equation. The mathematical description of finite temporal space within relationalism theory offers an alternative discrete geometric methodology for examining ancient timekeeping with new hypotheses for Egyptian calendars.

Keywords

Relationalism, Pairing Function Equation, Discrete Euclidean Timelines, Ancient Timekeeping

1. Introduction

We expand upon a recently introduced rishta (Rt) system that introduced new concepts for relationalism, capable of separating planes of matter, extensions of space (relational distance), and time (duration) for discrete Euclidean geometric modelling. [1] In this article, we describe independent and overlapping networks of object-oriented relational timelines, durations measurable by dimensional time that can be equated to a shared temporal space. Alternatively, relativity uses mathematical approaches that include continuums in time, or the speed of light as a measure that requires displacement through space to measure time. Nevertheless, discrete observations in quantum states, like discrete gravity and discontinuous motion, pose challenges for relativity-based modelling when describing quantum phenomena. The relational-based rishta (Rt) system is being developed as a potential alternative for studying classical and quantum physics as it uses discrete signals for geometric-based modelling.

This article is motivated to propel modeling of time that is detached from spatial constraints while also being relevant across quantum and classical scales. One of the objectives is to build an innovative dimensional-based geometric computational tool, particularly in the field of discrete event simulations. These simulations demand a tool capable of representing object-relational change at discrete points with varying durations separating those points. Addressing the intricacies of modeling temporal dynamics, especially within complex systems, presents inherent challenges. The introduction of a 1D geometric temporal space is proposed to simplify the representation of time, facilitating the development of mathematical models, algorithms, and computations for both analytical solutions and numerical simulations.

Throughout history, three primary methods have been employed to model space and time. These encompass Galilean relativity (Newtonian space-time) utilizing absolute time, Leibniz's descriptions based on relationalism [2]-[7] which stem from earlier work of Plato and Aristotle, and Einstein's relativistic Minkowski spacetime. Prior to the introduction of Rt relationalism [1], all systems relied on motion as a function of continuous time, encompassing inertial frames of reference.

Relativity successfully reconciled the discord between Newtonian mechanics and Maxwell's electromagnetic theory. Maxwell's equations and Leibniz's theories were challenged given two inertial frames of reference are needed. [8] Taking a departure from conventional paths, the rishta system introduces the possibility of a universal frame characterized by zero-dimensional attributes and uses discrete geometry to model motion. [1] Relational durations and dimensional time is added into the model by utilizing shifts in relational space expressed using geometry, Euclidean translations, and time integrals of displacement, like absement [L·T] applying object-relational spatiotemporal units. Several characteristics are still in the early stages of development as Rt relationalism has only been recently introduced, including its formalization in set, group, and measurement theories.

The discrete Rt approach eschews the use of inertial frames, calculus, infinitesimals, motion in continuous time, or derivatives related to time such as velocity or acceleration. Consequently, Newton's laws of motion are managed much differently in this mathematical approach as time derivatives of displacement are not applicable. Predictions into the future are computed using non-causal operations that leverage archival datasets from the past and present inputs from a causal system. [1] The causal-based system focuses on recording and communicating data from a dynamical system that can be used as inputs for modern functions, offering new tools for ongoing development.

Aristotle characterized time as an unbroken, linear continuum. Aristotle states "change is always faster or slower, whereas time is not." (p. 69) [9] Aristotle goes on to propose that "time is present equally everywhere and with all things", adding "Clearly then it [time] is not movement." Newton's describes time as "absolute, true, and mathematical time, of itself, and from its own natural flows equable without regard to anything external." (p. 77) [10] Newton followed by adding another premise that a "duration: relative, apparent, and common time… measure of duration by means of motion." Leibniz proposed time (generally speaking) as being based on relations yet was unable to provide a methodology that was consistent with his thesis. [7] [8] Leibniz considered time as being a duration, commenting "duration is a successive repetition" (p251) and that "duration is a multitude of something or the duration of something." (p179) [11]

The recent introduction of Rt relationalism, a duration was defined as being related to a period between discrete cyclic events from object-oriented signals, consistent with ancient timekeeping systems. [1] The work aligns with Leibniz' comments whereby "…extension is to space as duration is to time."(p261) [11] Relativity in context of Rt relationalism can be seen as the relativity of a network of independent durations, a methodology described as in more detail in this article. As opposed to Einstein's approach to relativity, Rt relationalism can include multiple privileged points of reference and does not use inertial frames.

Continuous time is used in modern equations for physics that use time derivatives of displacement, and time can only approach zero but never reach it, $t \neq 0$. The mathematical use of continuous time extends to infinity in the past and future, is mapped on a real number line, uses infinitesimals, and necessitates the application of calculus. In contrast, in Rt relationalism physical durations are taken from within continuous time, based on signals from normal matter, measured by dimensional time using fundamental constants from this universe in a scale relevant to us. Physical time is also applicable to discrete Euclidean geometry, includes time points of no parts, is separated by relational temporal line elements, has a bound beginning and end, and is mapped on a natural number line with either wholes cycles (W) or counted parts of a whole (k). [1] Physical time is applicable for computational and discrete geometric modelling where discrete functions describe dynamical systems. Physical time uses zero [$t = 0$] [1] which is not possible in continuous time models.

In mathematics, dimensions can be inclusive of Euclidean geometry with zero-dimensional points (zero-D), one-dimensional lines (1D), two-dimensional planes (2D), or three-dimensional (3D) objects. In the theory of relativity, time is added as a fourth dimension (4D) spacetime, where time is a coordinate in Minkowski's light cone. In quantum physics, the mathematics of string theory introduces many more dimensions. On the other hand, in metrology, dimensions are quantities like the dimension of time [T] and length [L]. Dimensional quantities are defined as fundamental constants in the universe, such as the speed of light, with precision limited by the technology to measure and not the definition. In dimensional analysis, base quantities are divided into base units using operations with dimensional quantities associated as parts of a whole. [12] [13] Russell's work from the early twentieth century is interpreted where base units are "individuals" from the perspective of a unit being the whole. [14] [15] Russell's definition is applicable to the familiar modern-day application of dimensional units for time, or seconds.

The concept of a network of overlapping dimensional relational time [T_r] was recently introduced as part of the rishta system. [1] Here, a duration has a finite-dimensional quantity (seconds) with a description of relational physical time (cyclic duration), and is applicable to geometric dimensions (1D) and dimensional analysis. From this unique cyclic duration, operations build an ordered set of units unique in dimensional quantities but having conserved cyclic symmetry. Spatial quantities, or the concept of metric space, is an independent quantity, added as necessary for the model's scale, from the limits of nanotechnology to a spatial scale of a fixed geometric model that spans Europe for example.

In Rt relationalism, time can be continuous using an infinite scale, but physical durations are bound to our physical reality. Relational time is dependent upon object-oriented signals, thus finite with a beginning and end. Unlike continuous infinitesimal time, physical durations are bound to Planck's limits. In contrast to using the speed of light as the measure of time through space, which requires displacement in space, in relationalism, time is fundamental and not linked to spacetime as there is no spatial information in the measure. The approach, which includes the application of a causal signal system, opens opportunities for using a fundamental measure of time whereby discrete functions can model dynamical systems that do not require a distortion of space or continuous states of matter [1], both of which are necessary for conventional mathematical formulas and assumed to be factually linked for models like quantum loop [16] and string theory [17].

In a sense, Rt relationalism takes a perspective as being an omniscient observer overlooking the observers defined by the theory of relativity. When considering the concept of an observation, in relativity an "observation" means the observed event is within the mathematical construct of spacetime. An observation of an event in relativity is the spacetime coordinates of where and when an event occurred, and not as the observer "sees" it. A privileged observer is at a privileged point of reference that can be interpreted as being equivalent to the origin of a light cone, or "null cone". Einstein's relativity is focused on events in the light cone of spacetime. In contrast, for object-oriented time, a privileged point received the signal (ρ) at the origin of the light cone and captures only the cyclic duration between one signal and the next as the light cone moves through continuous time (**Figure 1(a)**). A proposition is that the progression of dimensional quantities of time (e.g., seconds) at each privileged point in temporal space is the same, measured by standardized dimensional time that uses natural constants of the physical (normal matter) universe. Support of this is that an atomic clock is precise relative to itself in spacetime.

A privileged point may in many respects also be considered an inertial point of reference, however, there are notable differences. A privileged point is isolated, no forces act upon it, it has zero-velocity as time derivatives of displacement are not used in Rt relationalism, has zero-dimensional quantities (including mass) relative to itself in space and time, simply a location for downstream relational coordinates. A point is a location set upon a zero-D universal frame of reference. A model output of Rt relationalism is a physical static geometric model built upon a universal zero-dimensional frame, each zero-dimensional point is at rest relative to itself and other privileged points. [1] Motion is added to a 2D Euclidean lattice using Euclidean translations with discrete geometric-based dimensional relational functions applicable for relational-based integral kinematics (e.g., absement; $[L_r \cdot T_r]$ and absity; $L_r \cdot T_r^2$), including the example of a 1D orbital model (**Figure 1(b)**). [1]

Temporal space contains a network of overlapping object-oriented durations, however temporal space itself can also be considered a duration. Each duration can be organized into groups based on shared features, like a whole temporal space duration, or an object-oriented orbit (extrinsic duration) or rotation (intrinsic duration). Cyclic durations have a conserved symmetry yet are cyclically

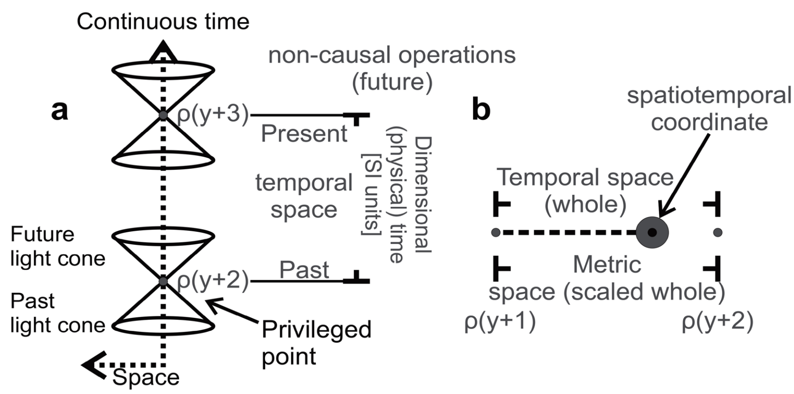

Figure 1. Temporal space and spatiotemporal coordinates. (a) 4D spacetime light cones in continuous time (infinitesimal) compared with a relational duration of a cyclic signal measured in dimensional time at a privileged point. (b) Discrete zero-D geometric object centroidal origin point within a cyclic duration for an object-specific spatiotemporal coordinate mapped in a 1D orbital model.

unique when using aperiodic signals creating unique dimensional quantities each cycle. The structure of ancient formulas provides tremendous inherent capabilities. Inherently, ancient formulas provide divisions of a whole element, a cycle or temporal element spanning temporal space, consistent with dihedral symmetry group classifications with ordered elements (radix groups). [1]

In Hindu timekeeping, the Rig Veda references a chakra, or a wheel of 360 spokes placed in the sky. (p. 184) [18] A 360-unit cycle divided into 12 equal units is also referenced in ancient Chinese (Shang Dynasty) bone carvings (p. 75) [19] [20], Babylonian and Persian calendars (p. 610) [21], pre-Roman Egyptian time units and calendars. [22] [23] [24] Greeks applied the formula $12(30^1)$ in a few different ways that including their lunisolar calendar where a count of 12 lunation signals with about 30 Earth days signals each lunation (an average of 29.530589 days). Evidence was also found supporting noted findings at Neolithic site of Göbekli Tepe [25], the formula is again used as a time unit system where time is divided into 12 zodiac signs with 30 degrees each. The most recent note is in an architectural study proposing 12(30) divisional structures in the design of Stonehenge. [26] In India, Earth's orbital period was divided into a greater number of units, resulting in $N = 12(30^2) = 10{,}800$ units, known as "muhūrta," within a year. The rishta system uses an aligned approach to ancient Vedic and Hindu timekeeping, and other ancient civilizations, an alternative to Greek defined applications. This article begins to expand the application of ancient formulas and the proposed cyclic and discrete symmetry of time, first considered to be related to D_{12}. [1]

As an application for mapping temporal space, we explore a well-defined calendar system from ancient Egypt. In 600 BCE, Thales of Miletus introduced the Egyptian year of 365 days to the Greeks. (p. 451) [22] The 365-day calendar was compared against Greek methodologies which led to interpretations that ancient timekeeping systems were vague, and accumulated errors, making the system "…further and further away from reality". (p. 24) [27] However, the authors of this article argue there have been unrecognized Eurocentric assumptions when interpreting ancient calendars using Greek methodologies. These potential researcher biases become apparent when an alternative methodology for ancient units and calendars is understood.

The first section of this article details temporal space and object-oriented cyclic durations that span space. We highlight how the whole temporal space can be equated to mathematical object-oriented sets of temporally equated elements using various principles consistent with Euclidean geometry and ancient formulas. The second section introduces novel applications of Euclid's division algorithm, Cantor's pairing function, and Gödel numbering for introducing a calendric methodology consistent with ancient timekeeping descriptions. The methodology is used in a preliminary re-interpretation of the ancient Egyptian calendar systems, the first non-Greek-based interpretation that can be found by the author at the time this article is written.

2. Methods

Relational durations begin at the first cyclic physical signal (alpha) and ends with the last signal (omega). In Rt relationism, time is measured by dimensional units of time (seconds) taken as a duration between discrete signals defined from a privileged point of reference. Thus, we can construct an overlapping network of object-oriented durations of time measurable by SI units, each independent of space.

Dihedral symmetry at this stage is considered a mathematical feature of the system that is preserved or remains unchanged. We present a methodology where symmetry groups are shared by similar mathematical objects such as orbits and rotation as well as dimensional time and metric space. Ancient divisional formulas were found to mathematically introduce the inherent group and numerical positions that include symmetry properties applicable to set and group theory. These are preserved in later geometric and algebraic outputs. Symmetries are inclusive of spatial symmetry (D_{360}), time symmetry (D_{12}), rotation symmetry (D_{24}), scale symmetry (D_{10}), and a proposed orbital symmetry (D_{18}). Given there is currently only one example discovered in the literature for D_{18} divisional references (e.g., Mesoamerica) that the author has identified, at this point this is a conjectured proposal. [1]

Euclid's definitions are exceptionally well suited to describe temporal space and object conditions since his universal approach allows mathematical objects to exist without prerequisite parts, properties, quantities, and so on. Applications create an ability to use geometry to map temporal space using layers of object-oriented dimensional quantities like time (relational duration); length (relational extension), mass represented in the scale of our dimensional reality. Euclid's work spanned 13 books, we will focus on Book I, V, VII, and X [28] to frame our application definitions for point, line, and unit, or part of a whole line.

2.1. Temporal Space with a Network of Object-Oriented Durations

Temporal space begins as a set with no elements, capable of spanning some portion of continuous time which is abstract and cannot be described by set theory. The limits for temporal space are bound by non-abstract signals creating dimensionally measurable durations within a scale of our dimensional reality based on fundamental constants of our universe.

To define a finite temporal space as an interval measurable by dimensional time, we introduce object-oriented cyclic signals that set the boundaries. If signals occur at the same point of dimensional time, there is no duration to measure. Therefore, quantum superposition is proposed as being at the dimensional limit [$t = 0$] for relational time. Such a concept of zero-time is not possible for abstract continuous time, which is without physical dimensional limits.

An object-oriented cyclic signal, like a New Moon conjunction, is marked through a relationship with a start/stop signal received at a privileged point of reference. These signals define the limits of a particular temporal space. The discrete non-dimensional (geometric) points [$\varnothing_t$] are zero-dimensional instants in discrete physical time [$t = 0$] mapped upon a natural number line. In conventional physics based on continuous time, there cannot be absolute zero ($t \neq 0$) on a real number line. The accuracy of the finite relational dimension of time, or base quantity, is limited by our ability to measure it and not by the definition. Following the definitions of a signal system, a single cycle is a sample period [T_s] and multiple recurrent sample periods are referred to as a frame period [T_f] and each is measurable using SI unit seconds.

Euclid's third definition describes a line segment with extremities of a line being points, in this application defined by cyclic signals marking. The temporal space between [$\varnothing_t$, $\varnothing_t$] begins undefined with no dimensional elements, {·}, and only until the interval is measured in SI seconds is a dimensional temporal line element created (**Figure 2**). This subscript notation for a point is needed for consistency to distinguish temporal zero-D points from those used in metric space which includes centroidal origins of matter. [1]

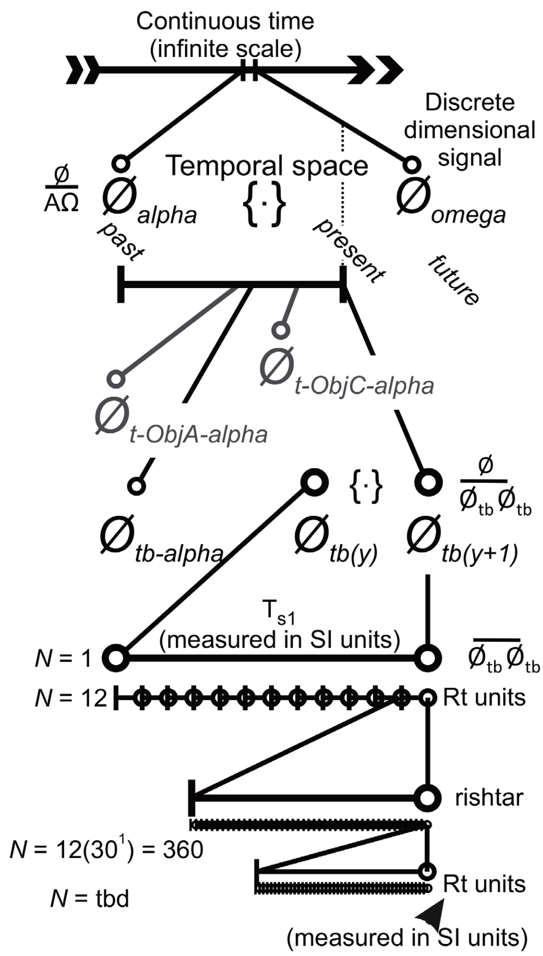

Figure 2. Co-existing network of overlapping object-oriented durations within temporal space and continuous time. $\overline{A\Omega}$ alpha-omega line segment, $\overline{\varnothing_t\varnothing_t}$ sample (T_s) period, and Rt (temporal rishtar) units of a unique whole cyclic period for a particular object-oriented relationship.

Within temporal space, the greatest object-oriented temporal line segment is a set that contains the first signal, each subsequent signal in the lifetime of the relationship from a privileged point, the final signal. This segment is termed the alpha-omega segment, or ($\overline{A\Omega}$) (**Figure 2**). For example, from the first solar rotation (day) signal of what will become Earth, to the last. This segment is a finite number line of temporal signal points from whole cycles, consistent with Euclid's fourth definition, where points [$\varnothing_t$] can lie on the straight-line. These points, define a set of temporal zero-D points on a natural number line, and the object-oriented relationship between points (temporal lines) creates segments that are lesser than the greater, defined as a period ($\overline{\varnothing_t \varnothing_t}$). A single sample period, between two recurrent points on the natural number line, is defined as a whole cycle. Aperiodic signals create periods that are not dimensionally identical, rather they are unique temporal anchors, unique line elements, that can be identified on a natural number line.

Each unique period is a whole cycle measurable in dimensional units, applicable to set theory where each whole can then be divided and subdivided into parts of a whole, creating unit multiples with a base quantity, referred to as temporal rishtars. The approach is consistent with Euclid's descriptions of units in reference to Book V on proportions and relationships between magnitudes as well as Book VII, elementary number theory. We can also compare a rishtar unit (element) with Russell's work from the early twentieth century which is interpreted where base units as "individuals" from the perspective of a unit being the whole. [14] [15]

2.2. Units; Dimension, Symmetry, Order, and Zero

In Book V, Proportions, Euclid explores the concept of proportional relationships between magnitudes which includes ratios. Consistent with definition six, each Rt_n unit (a magnitude) a subset of the object relative segment ($\overline{A\Omega}$), and the same lesser defined period, ($\overline{\varnothing_t \varnothing_t}$), is a ratio of a whole. As we understand astronomical cycles are dynamical with aperiodic signals, no two object-oriented ($\overline{A\Omega}$) will be the same, no two lesser periods ($\overline{\varnothing_t \varnothing_t}$) within the greater are the same, and finally, the proportionality of the lesser Rt_n units in totality are only proportional to the defined period of which these magnitudes are the lesser.

In studying ancient timekeeping formulas to create units from a whole period (parts of a whole). The approach aligns with the advanced complexity of Vedic and Hindu time from ancient India which has been simplified by European based translations to fit within familiar definitions of days, hours, seconds. [29] We recognized universal mathematical patterns and conjecture applications of symmetry groups (dihedral) and positional numerals (radices), properties that can be mathematically applied and expanded upon for applications that extend far beyond current known uses. Formulas introduced orders of operations, thus far the first division is used to assign uniform dihedral symmetry with properties of rotation and reflection where examples include D_{360}, D_{24}, D_{12}, D_{18}, and so on.

When studying time in isolation as a 1D line, applications for dihedral properties of rotation and reflection are not overly apparent. However, downstream discrete dimensional geometric modelling (beyond this article) using discrete steps in independent time coupled with independent space, rotation, and so on, unlocks tremendous possibilities. Therefore, these groups share properties that create an "individual class of objects", to use Euclid's translation. The symmetry group can also be referred to as a class like that defined by Russell (1919), "the number of a class is the class of all those classes that are similar to it."

In general, once symmetry is assigned, a second operation of an ancient formula can integrate a positional numeral system, radix (e.g., base-10, base-20, base-30, base-60) which creates an order to the set of Rt elements, or units. Ordered elements are important for many applications, including Cantor pairing functions used in mathematically modelling ancient calendars as will be described.

Temporal space as a whole is measured by dimensional time, creating a timeline divisible by the ancient $12(30^x)$ formula which creates temporal base units that are parts of a whole. Measuring temporal space in seconds defines the period as singleton element for temporal space to build time dimensional base unit multiples of the period, or a set with multiplicity of $N = 12(30^x)$. This contrasts Greek applications for counts of lunations or vague alignment of twelve months with about 30 days each for 365.24 days in one orbit for example.

In Rt system notation, k/N represents (ordered part in whole)/(all parts of whole) and not (divided) ÷ (divisor). Consistent with a unit fraction, the k counts are a ratio, and conserves ancient methodologies for unit fractions. Finding a path for reduction to the simplest form is more complicated given the complexity of the formula, understanding symmetry alone is different than symmetry with dimensional quantities, and awareness of dimensional limits for normal matter. For example, each cycle shares the same symmetry each cycle, but the dimensional quantities will shift cycle to cycle.

In a mixed-number Rt unit fraction count, k, represents an ordered part of one complete count of ordered units in a cycle, with the whole number (W) in the mixed fraction representing the count of completed cycles. [1] There are an overwhelming finite number of ordered sample periods, frame periods, object-relations, and so on mapped over the duration of the universe and each can be uniquely identifiable by notation, and eventually geometric models. To provide initial notation (term 1), and expanding upon previous work, the rishta cyclic term includes the properties so far introduced, including count (k), $\text{object}_{\text{condition}}$ (description of a whole period, e.g., $\text{Earth}_{\text{rotation}}$ or Luniterranean year 13-lunations), multiplicity of the multiset (N), symmetry (n), and signal [ρ], with the whole number (W) in the mixed fraction representing the count of completed cycles, which is used where cyclic terms in temporal Rt_n units are shown as,

Cyclic term

$$W\frac{k}{N}\text{Object}_{\text{condition}}\text{Rt}_{n}\left[\rho\right] \tag{1}$$

Book VII of Euclid helps define a unit as where a unit is a part of temporal space and said to be one and the number is a multitude composed of units which are natural numbers not including zero. Therefore, in this application, 1 unit is a ratio of 1:360 and 2 units is a ratio of 2:360 using $N = 12(30^1) = 360$ where the full ratio, or 360:360 (k:N), is temporally congruent to the period ($\overline{\varnothing_t \varnothing_t}$). Following Euclid's third definition, the number (unit count, or k) is part of another number (the period count), the lesser of the greater ($\overline{\mathrm{A}\Omega}$) when it (the unit) measures the greater (period).

Euclid's definitions from Book X, Incommensurable Magnitudes, opens an important clarification of the definition of irrational numbers from today verses how Euclid used them. Today ratios of numbers are known as rational numbers while other real numbers (specifically, infinite, non-repeating decimals) are known as irrational numbers. Combining Euclid's first three definitions, Euclid defines magnitudes (temporal segments) measured by the same measure (object-oriented temporal measure for the same period) as being commensurable, but those which no (magnitude) admits being a common measure are said to be incommensurable. Therefore, elements $r \in E$ are commensurable and elements that are not a member of set E (e.g., a different object-oriented set) are incommensurable. Euclid continues, straight-lines can be commensurable meaning rational, and incommensurable being called irrational.

Units are members of a set where a whole period element is divided into base unit multiples with an ordered count of N which is a natural number ($\mathbb{N}$). In mathematical terms, this is often represented using a set and the concept of partitions. Let set E be the whole element divided into unit multiples with a multiplicity of N. Note, $k = 0$ is not a unit alone, but a point element of a unit that is at the terminal of the unit's line element. The notation can be expressed as:

$$E = \left\{\left(Rt_k, Rt_k, Rt_k\right) \mid 1 \le k \le N, Rt_k \in UnitMultiples, k \in N\right\} \tag{2}$$

In this notation:

- E is the whole element.
- Each Rt_k represents a unit multiple within the ordered sequence.
- The ordered sequences are within distinct subsets, (Rt_{k1}, Rt_{k2}, ···, Rt_{kj}).
- $1 < k < N$ indicates that there are N distinct subsets.

Notation conveys that set E is a set where the unit multiples are ordered within each subset, and where there are N such subsets and N is a natural number, $\mathbb{N}$.

Discrete-time point elements are separated by non-abstract geometric line elements, both components to a rishtar element which is also a part of a whole and used as a base unit once the multiplicity of the set (N) is finalized by an operation. Let us represent the temporal element (Rt) containing both a line element (⊦) and a point element (∘), with the line element being a unit multiple counted with natural numbers where $1 < k < N$ using mathematical notation as follows:

$$Rt = \left\{ \left(\overline{k}, \circ \right) \mid \overline{k} \in \mathbb{N}, 1 \le \overline{k} \le N \right\} \tag{3}$$

This notation communicates that a count k of a temporal rishtar contains ordered pairs (⊦, ∘), where each ordered pair consists of a line element $\overline{k}$ (an ordered unit multiple counted with natural numbers where $1 < \overline{k} < N$) and the corresponding terminal point element∘. Line elements contain object-condition-specific information which describes a relationship and mathematical features consistent between the discrete points, and where a discrete temporal point is at the terminus of the line. [1]

The approach shares consistencies with ideographic scripts beginning with Shang oracle bone form, coin form, and later bronze form. [30] Interpretations of the discussed ancient cycles agree in general that cycles begin at 10 out of 10, this was concluded by the authors since the successive count from 1 to 9 has an additional place-value component in these examples. Using these examples in the context of the proposed temporal rishtar, the 10th symbol in this example ends one cycle, using the line element, and overlaps the next using the point element.

Let us consider two recurrent and unique cycles with conserved symmetry, each with ten ordered rishtars as the parts of the whole (where D_{10} is either a potential scale symmetry (cycle-like) or base-10 radix group). Using the described counting notation, the first cycle completes at the tenth line element (⊦), or 10/10, which is also equal to 1 [(0)/(10)] being consistent with the point element (∘) of the tenth rishtar of that period, or cycle. In the next cyclic set of 1 to 9, this can be notated as 1 [(1 to 9)/10] where $W = 1$ can become a novel and descriptive symbol or representation. Whereas at the tenth line element of the second cycle, we now have 1 [(10)/(10)] which is also equal in mathematics (given the described notation) and Rt description to 2 [(0)/(10)]. The approach is consistent with Babylonian sexagesimal place-value notation, 0, 01, ⋯, 58, and 59, where both the first and last digits are non-null, [31] [32] as we interpret these findings to be consistent with the rishtar approach as discussed. Also consistent with the Mesoamerican vigesimal base-20 counting system (0, 01, ⋯, 18, 19). [33] [34]

An axiom in Rt relationalism relevant to this article is that physical timelines can be divided and subdivided to intervals that reach quantum limits, where an interval in time cannot be reduced any further. Therefore, there is zero of time in physical dimensional reality as defined by signals with no dimensional duration between them. Zero-time is a natural point value as it falls upon a natural number line, it separates line elements. These points are where relational and dimensional time for our universe cannot be divided further, doing so would not only require the addition of a signal system that included non-normal matter signals, but also an ability to observe (directly or indirectly) or measure such signals.

Physical, or dimensional, zero excludes both temporal measures in symmetry and/or dimensional quantities which addresses the definition for a proper system of measure. Temporal zero is a natural point value, applicable to zero from the start/stop of a whole period (sample or frame), the start/stop of a whole object-oriented start/stop cycle, and from an object-oriented alpha to omega signal. In these cases, zero has no dimensional quantities and corresponds to a matter-based zero-dimensional signal alignment captured from a privileged point of reference. Zero is also a point element, shared between two unique line elements, for example:

$$\left[\varnothing_{t(y)}\right],\left[\overline{\varnothing_{t(y)}\varnothing_{t(y+1)}}\right],\left[\varnothing_{t(y+1)}\right],\left[\overline{\varnothing_{t(y+1)}\varnothing_{t(y+2)}}\right],\left[\varnothing_{t(y+2)}\right],\cdots$$

Zero-point time is consistent to the point-values on the number line for the rishtar point elements (∘), or ordered parts (k) of the whole period set multiplicity (N), and y is the first primary signal that marks a start point on a zero-D time point on a natural number line. It is postulated that there are a countable finite number of natural point values for object-oriented cyclic units (N) given dimensional Planck limits and findings of superposition phenomena.

2.3. Temporal Equivalences

Temporal spaces contain a network of overlapping and independent object-oriented cyclic durations defined using cyclic events, or signals, however only one is a primary (1˚) cycle. Additional object-oriented cycles are considered secondary (2˚) cycle(s) in relation to the primary cycle. A primary period is defined by one or more recurrent primary cycles, either a sample or frame period respectively.

A primary cycle's signal(s) mark the start/stop of either a sample or frame period duration for temporal space, using a particular object-oriented signal of interest. An example would be a signal from a New Moon conjunction, a technology would be to measure it using eyewitness observation on a privileged point on Earth or modern lasers, the methodology is the same only the technology differs. A temporal space duration, between signals, can be defined by either a sample period (e.g., one-lunation cycle) or a frame period (e.g., >1 recurrent sample period, or perhaps 13-lunation cycles) but both use a primary cycle signal (New Moon in this example) to mark the start/stop of the temporal space duration. In this example, the primary cycle (1˚) is a New Moon cycle (lunation), and a secondary cycle (2˚) could be Earth rotations equated to the temporal space.

To include descriptions from above, a geometric object of a single cycle, either primary or secondary cycle, is a whole cycle period segment ($\overline{\varnothing_t\varnothing_t}$) which is lesser of the greater for each object-oriented alpha-omega relationship. Object-oriented whole cycles can be grouped into symmetry classes using operations from ancient formulas, excluding D_{12} symmetry. The dihedral symmetry of D_{12} is uniquely used with temporal space defined as a standalone dependent duration from the independent primary cycle(s), which can correspond with a measure of dimensional time (e.g., seconds).

A whole temporal space by Euclidean definitions is different than a whole object-oriented cycle. A temporal space duration can be measured by dimensional units of time (seconds) creating a singleton with dimensional quantities. It is this temporal space segment, or singleton, that uses operations involving D_{12} symmetry, or $12(30^x)$, to define dimensional time units that are unique to that single temporal period.

Each temporal space is described using a primary cycle and one or more secondary cycles. The methodology allows for tremendous capability for layering and expanding temporal spaces from short or longer durations. It opens up for the division and subdivision of a whole cycle into units of a whole on one side, as well as addition of whole cycles on the other. For example, one temporal space may be defined by a frame period of 13 primary recurrent cycles (13-lunations). Also, any given primary cycle, or period, for one modelled temporal space may become of secondary cycle for a much larger temporal space within an expanded network of durations.

There are two applications for equivalences described in this article, first is a fixed equivalence applicable for static geometric canonical data modelling, an example previously shows how Earth and Mars can be equated [1] which will be expanded upon in this article to include exoplanet equivalence modelling. The second is calendar equivalences capable of predicting cyclic alignments into the future through non-causal operations from archival datasets from a causal system. Each fall within the principles of homogeneity of terms for dimensional analysis however, beyond dimensional time, we also have different classes of symmetry to accommodate and again, ancient applications of timekeeping offered inspiration for a universal methodology.

We propose heterogeneity of symmetry for cyclic mathematical objects that define durations. Today, equivalence maintain homogeneity of terms for dimensional quantities, but not necessarily classes of symmetry. This can also be described by saying the characteristics of the relation R of two terms, *a* and *b*, is dimensional, not necessarily sharing the geometric symmetry class (S_n).

For equivalence, if two independent object-oriented durations and subsets, *a* [1° cycle] and *b* [2° cycle(s)], are taken from two (or more) independent object-oriented cycles from a privileged point and equal to a dimensional base quantity, they are dimensionally congruent to each other. Reflexivity can be shown as ($a \sim a$), symmetry (as defined by binary relations) shown as ($a \sim b$), and transitivity with the addition of a dimensional quantity (c) using seconds shown as $a \sim b$ and $b \sim c$ then $a \sim c$. Where c is a dimensional quantity measure, a is the primary cycle reference element, b is the secondary cycle. Introduction

$$1\frac{0}{360}\text{Moon}_{\text{lunation}}\text{Rt}_{18}\left(\text{NM}\right) \cong 29\frac{45843}{86400}\text{Earth}_{\text{rotation}}\text{Rt}_{24} \tag{4}$$

$$1\frac{0}{360}\text{Moon}_{\text{lunation}}\text{Rt}_{18}\left(\text{NM}\right) \approx \text{measured in 2551443 seconds} \tag{5}$$

$$29\frac{45843}{86400}\text{Earth}_{\text{rotation}}\text{Rt}_{24}\left[\text{solar}\right] \approx 2551443\ \text{seconds} \tag{6}$$

of SI units to measure time adds dimensional base quantities to the temporal space which adds a property to Euclidean properties for transitivity. However, the Rt_n units, rishtars, for each side of the equivalence relationship are unique in dimensional time, and incommensurable. Using an example (Equations (4)-(6)), Let *a* be the Earth's rotation, *b* the Moon's orbit, and *c* is the measure in seconds of one lunation cycle from New Moon to New Moon.

For temporal equivalence, we introduce notation applicable to this article. Using "approximately equal to" or ≅, to signify both terms in the equation (for lack of a better term) are not equal because they can use different symmetries (Equation (4)) but they are equated to the temporal space defined by dimensional time. As temporal space begins as an empty set, measurement in SI units create a dimensional element of time dependent upon the defined cycle period, assigned the symmetry of time D_{12} (Equation (7)).

$$\begin{aligned} & W\frac{0}{N}1^{\circ}\,\text{Object}_{\text{condition}}\text{Rt}_n\left[\rho\right] \\ & =1\frac{0}{N}\text{Temporal space}_{\overline{\varnothing_t\varnothing_t}}\text{Rt}_{12}\left[\rho\right]\text{, measured in x seconds} \end{aligned} \tag{7}$$

Single primary period with primary cyclic object equivalent to the dimension and symmetry of time

2.3.1. Application of Euclid Division Algorithm

Each temporal space has a unique duration and a methodology to equate the primary period with counts of secondary cycles. As such, we introduce a remainder system consistent with set theory, object-oriented rishtar units, and Euclid's division algorithm. In this application, the divisibility is not one of abstract numbers, but actual dimensional values. In contrast to modern calendars, a cycle is not an average, but the record of a period in the lifespan of recurrent and cyclic object-oriented relationships, such as one unique Moon lunation (sample period) in the lifespan of Moon orbits, or even 13 unique recurrent lunations (frame period) in a luniterranean year.

Euclid division algorithm described in Book VII (propositions 1 - 2) and Book X (Propositions 2 - 3) [28] is useful for finding the greatest common divisor of two positive integers, but it is also applicable to the proposed calendar methodology and remainder system. Euclidean division algorithm uses two integers a and b with $a > b$ and with $b \neq 0$. When the remainder equals zero, the algorithm stops and the final non-zero remainder is then the greatest common divisor of the original a and b. There exists unique integers q (quotient) and r (remainder) such that $a = bq + r$, where $0 < r < |b|$ and where $q \neq 0$. In this article, we introduce an application of the division algorithm for various uses, including equating two object-oriented cycles (**Table 1**).

The primary cycle has no remainders because the signals from this cycle define the finite primary period segment, 1° ($\overline{\varnothing_t\varnothing_t}$). Possible remainders for secondary cycles are associated with the natural number of parts of a whole, or Rt_n unit multiples represented by natural number k count ratio, k/N. Applying the Rt system methodology, discrete points in time can have smaller and smaller line

Table 1. Describing variables used in the application of Euclid's division algorithm for equating an undefined duration with a period using object-oriented cycles.

a (divided)	b (divisor) Whole period	q (quotient) Count of whole (W)	r (remainder) Part of a whole
Temporal space Measurable with dimensional time units	Object-oriented cycle period $\overline{\varnothing_t \varnothing_t}$ (period segment)	Natural number count of object-oriented cycle whole period (divisor)	Natural number k part of a whole W Ratio $k{:}N$

element separations by increasing the total of unit multiples (N) for the cycle. [1] This approach can build a natural number line defining temporal space, suggestive of a possibility where two independent cycle timelines can share a discrete point in time, a temporal node. This precision extends beyond modern technology. Theoretically, a zero-time point element can be an intersection of two sets, a temporal node, whereby a point element from each respective independent cycle could be shared across each object-oriented natural number line.

Equivalence relations and division algorithms in the necessary precision are possible using the methodology of ancient formulas from Vedic texts. Exceptionally precise discrete points in time can be equated to a common temporal measure. For example, an Earth orbit divided into $12(30^x)$ where $x = 6$, $12(30^6)$, $N = 8{,}748{,}000{,}000$ Rt_{12} units for intervals that can create equivalence relations to within 3.6 milliseconds, or even further by increasing x which tests the limits of modern technology.

Euclid's division algorithm uses an integer quotient and a remainder required to be smaller than the absolute value of the divisor. So, since a natural number ratio is part of a whole ($k < N$) the remainder is smaller than the absolute value of the divisor value (whole). For example, if we are using Earth's solar day (rotation) count for the secondary cycle's (quotient), the remainder cannot be equal to a day (as there would then be no remainder) or be larger than a day.

2.3.2. Fixed (Finite) Temporal Space

A previous example for a fixed temporal space compared the orbital periods for Earth and Mars using a static fixed geometric model. [1] In this article, we expand the application to present how Earth's orbital period can be equated to exoplanets using an example from the Tau Ceti star system expressed as an algebraic equation. Tau Ceti's fourth orbiting mass (inclusive of either a planet or planet/satellites), termed Tau Ceti f, and we show Earth's orbital period as the secondary cycle (Equation (8)). Our postulate states that at each privileged point, we can use dimensional time to measure the duration of a cycle. We can use existing technologies to make predictions as the orbital period of Tau Ceti f shown in Equation (8) is an equivalence relation within the error bars of modern-day measurements [35], we extended the precision to demonstrate the capabilities of the methodology.

$$1\frac{0}{8748000000}\text{Tau Ceti } f_{\text{oribit}}\text{Rt}_{12}\left[\text{alphelion}\right] \approx 2\frac{5402663539}{8748000000}\text{Earth}_{\text{orbit}}\text{Rt}_{12}\left[\text{aphelion}\right] \tag{8}$$

Today's technology is not able to create a precision of Tau Ceti *f* orbital period to millisecond precision. However, it does allow us to create a geometric diagram of this precision if one day it is known. It also makes it possible to decipher the information from a geometric diagram constructed by someone with this knowledge if a known comparator [e.g., Earth's 1D orbit(s) models] is included in the overarching model.

To begin to contextualize key differences in methodologies for the calendar system, we look at the remainder in Equation (8) as a duration beyond Earth's aphelion, approximately 225.57 days. The date on a modern Gregorian calendar will vary for an aphelion position, from July 2nd to 6th. From the Rt signal system methodology for calendars, there is an interesting occurrence where the remainder falls upon an ancient Pagan holiday, later known as Lupercalia from Roman times which occurred between February 13 and 15th. Today the celebration is referred to as St. Valentine's Day, fixed to February 14th. In space and time, by fixing this remainder to end on February 14th, the remainder consequentially fixes aphelion timing to July 4th in the previous Gregorian year.

For conditions of the fixed temporal space, let X be a non-empty set, and let $d{:}X = \left[\overline{\varnothing_t\varnothing_t}\right] \to$ seconds be a given dimensional quantity mapping of temporal space. Let d be the summation of a dimensional interval of time on X (condition 9). Independent primary (condition 10) and secondary object$_{\text{condition}}$ cycles (condition 11), each with their own independent set, can be equitable to the domain if the following conditions are satisfied. Let k/N represent a ratio of a whole cycle, being the remainder of the division algorithm for a secondary cycle. Where time symmetry is Rt_{12}, let this be the symmetry of time for the dimensional interval of time on X (see condition 12) with N derived from using the formula $N = 12(30^x)$ where $x \in (1,2,3,\cdots)$, a countable limit to x has not yet been determined.

$$\text{segment of } d\left[\overline{\varnothing_t\varnothing_t}\right] = \text{seconds}\left(\text{or any dimensional unit for time}\right) \tag{9}$$

$$d\left[\overline{\varnothing_t\varnothing_t}\right] = W\frac{k}{N}1^\circ\ \text{Object}_{\text{condition}}\text{Rt}_n\left[\rho 1\right], \text{ where } W \in \left\{1,2,3,\cdots\right\} \text{ and } k = 0 \tag{10}$$

$$d\left[\overline{\varnothing_t\varnothing_t}\right] = W\frac{k}{N}2^\circ\ \text{Object}_{\text{condition}}\text{Rt}_n\left[\rho 2\right], \text{ where } W \in \left\{null\right\} \cup \left(1,2,3,\cdots\right) \text{ and } 0 \le k \le N \tag{11}$$

$$d\left[\overline{\varnothing_t\varnothing_t}\right] = W\frac{k}{N}1^\circ\ \text{Object}_{\text{condition}}\text{Rt}_{12}\left[\rho 1\right], \text{ where } W = 1 \text{ and } k = 0 \tag{12}$$

where W, k, N are all natural numbers, with only k being a natural number including zero as a point, and n is a dihedral symmetry group, with n = 12 being the dihedral symmetry group for time.

2.3.3. Expanding Temporal Space (Calendars)

Timekeeping involves both units as well as calendar systems that can maintain cyclic alignments over long intervals, an ever-expanding temporal space. We present a universal non-Greek-based calendric methodology thus far consistent with both the rishta system and mathematical descriptions of ancient calendar systems. Using this methodology, an observational calendar system of ancient times can accurately maintain astronomical-based cyclic time with harmony and temporal equilibrium to both observational signals and mathematically equated durations with a temporal space.

Currently, three differing scenarios build flexibility in a calendar system; paired durations, N-body layering, and shift of primary cycles. The former and latter applications will be discussed in more detail. Layering will require further study and formalization. In short, a layering approach opens the possibly to create a multi-celestial, or N-body, cyclic calendar system that can begin with Earth's rotation, layered to include the Moon's orbit, Earth's orbit, and inclusion of the planets and satellites in this star system. N-body calendars can even extend in theory to include exoplanet orbits, as well as stars orbiting a Galaxy's centre, precision being limited to modern measurement technologies.

Signals create natural number counts relative to the independent cycles that can be either observed (new Crescent Moon) or mathematically calculated (e.g., aphelion for Earth or exoplanets) which together are used to define the unique count of a dependent (paired) cycle. Using the rishta nomenclature, a primary cycle count (1° W) is paired with a secondary cycle count (2° W) $\rightarrow$ to form a dependent calendar cycle (cc) with a unique new cycle count cc W'. For ancient observation calendars, a pairing function, $\mathbb{N}$ [x] $\mathbb{N} \rightarrow \mathbb{N}$, captures the natural number counts for the two (or more) independent cycles. The creation of the unique natural can be applied to the principles of Gödel numbering where a pairing function's unique single natural number can be a representative symbol, such as a glyph.

However, as seen in the fixed temporal space, from the perspective of an equation, it may appear an observation-based calendar creates disequilibrium in temporal space. However, rules can be implemented whereby temporal equivalence between independent cycles for the calendar in terms of an equation is maintained independent of observer inaccuracy. The approach leads us to introduce a so-called pairing function equation that maintains natural number counts of observed signals, temporal space, and temporal equivalence (Equation (13)).

$$\mathbb{N}\left(1^\circ W\right)[\times]\mathbb{N}\left(2^\circ W\right)+(\pm\text{paired remainder})\rightarrow\mathbb{N}\left(ccW'\right)+(\pm\text{paired remainder}) \tag{13}$$

To maintain temporal equilibrium for a pairing function equation, we add or subtract temporal mathematical objects from both sides as needed. Building from fixed temporal space with a remainder of the secondary cycle ($k < N$), counts (k) are added (or subtracted) to a term so $k = N$, creating only natural number whole cycle counts in pairing functions. To maintain temporal equilibrium in the equation, the addition (or possible subtraction pending further study) occurs on both sides of the equation. This operation is considered outside of the temporal space quantity being extended and tracked over long periods, a mathematical object that is unique to the calendar system. As this operation creates a unique temporal value for the unique paired cycle, we term this as a paired remainder (pr). A resulting paired remainder can accumulate with each recurrent paired cycle count, so when $k \geq N$ used to equilibrate the pairing function equation, it would trigger an intercalary event to maintain calendric equilibrium with the temporal space being extended by the primary cycle. When a rolling calendar cycle accumulates a paired remainder to where $k \geq N$, it triggers an intercalary event for an independent cycle in the pairing function used to derive N. In a causal system, inputs include past and present, so to be consistent, any secondary cycle(s) completes to a natural number W count for accurate measurements in the calendar cycles (**Figure 3**). However, these durations can also be predicted using previous non-causal operations from data archives. A situation can occur where the secondary cycle has $W < 1$ (**Figure 3(b)**), this raises questions if such a cycle can be used for pairing function equations related to calendric extensions of temporal cycles, proposed better suited for a non-rolling calendar where the remainder does not accumulate the same way as a rolling calendar. A question raised from later findings related to the study of a stellar year (e.g., rise of Sirius in Egypt) using 365 days as a primary cycle.

In calendric expansions of temporal space, at intercalary events there are new counts, typically of a secondary cycle W, either an addition (leap) or removal (skip) event. These new recurring W counts offer an opportunity to create a new pairing function, another dependent calendar cycle of W^{*} counts. To describe this, we use the signal-based McKenna-Meyer luniterranean calendar that has a primary cycle count (1˚ W) of lunations and a secondary count (2˚ W) of Earth's rotations.

For the luniterranean calendar, after 13 lunations, a pairing function creates a unique number output, the luniterranean year (384 Earth days [×] 13 lunations → 1 luniterranean year), or calendar cycle 1 (cc1, referred to in Equation (14)). The resulting mathematical paired remainder (pr) accumulates each successive year. Accumulation requires an intercalary 385-day leap-year (385 Earth days [x]13 lunations→ 1 luniterranean decade) every tenth luniterranean year, or decade (cc2) (Equation (15)). Again, the resulting unique negative paired remainder for this new paired cycle, in this case, accumulates towards a whole complete secondary cycle, balanced by another intercalary event (383-day skip-year) (cc3). Cc3 again would associate with another unique nature number remainder (equation 16). The luniterranean calendar system uses a 13-lunation cycle as the primary period, where 13 = 13-lunations, 384 = 384 Earth rotations, 385 = 385

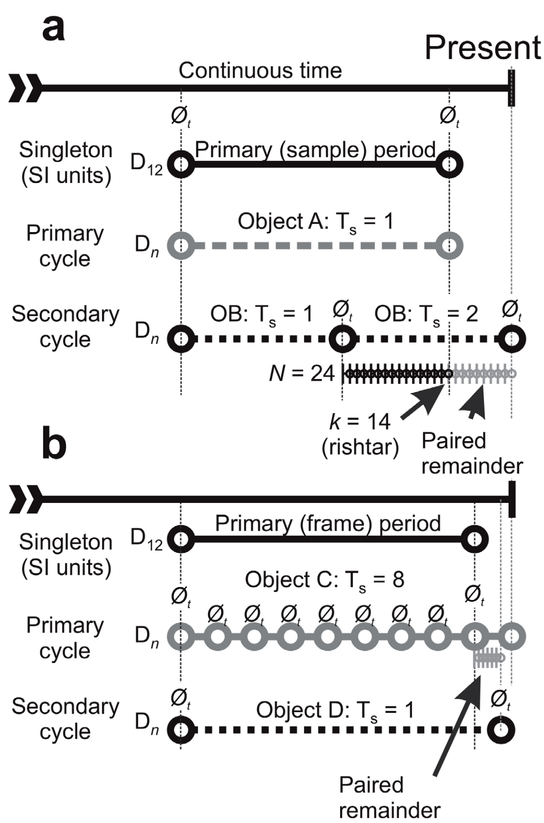

Figure 3. Rishtar calendar methodology for a causal system. Primary periods are equated to primary and secondary object-oriented cycles. (a) Fixed temporal remainder shown in ordered Object B (OB) Rtn unit multiples (black) with a paired remainder (grey). (b) Temporal remainder for Object D using Object C Rtn units (grey).

Earth rotations, and 383 = 383 Earth rotations [36], full notation is removed for ease of display (Equations (15)-(16)). A temporal period from a rolling calendar expands with each primary cycle period, again becoming a growing temporal space that can be measured by dimensional time.

Pairing function equations with intercalary durations

$$1^{\circ}13[\times]2^{\circ}384+2^{\circ}\,\mathrm{pr}_{\mathrm{cc1}}\rightarrow 1\,\mathrm{cc1}+2^{\circ}\,\mathrm{pr}_{\mathrm{cc1}} \quad (14)$$

$$9\left[1_{\mathrm{cc1}}+2^{\circ}\,\mathrm{pr}_{\mathrm{cc1}}\right][\times]1\left[1^{\circ}13[\times]2^{\circ}385+2^{\circ}\,\mathrm{pr}_{\mathrm{cc2}}\right]\rightarrow 1\,\mathrm{cc2}+\mathrm{pr}_{\mathrm{cc2}} \quad (15)$$

$$399\left[1_{\mathrm{cc1}}+2^{\circ}\,\mathrm{pr}_{\mathrm{cc1}}\right][\times]1\left[1^{\circ}13[\times]2^{\circ}383+2^{\circ}\,\mathrm{pr}_{\mathrm{cc3}}\right]\rightarrow 1\,\mathrm{cc3}+\mathrm{pr}_{\mathrm{cc3}} \quad (16)$$

Signal timings and counts for cc3 rely on approximations due to the unavailability of information on the temporal space duration as measured in the dimension of time for equivalence. The exact number of lunations or Earth rotations requiring a 383-skip year is dependent on precise measurements taken from recurrent periods of Earth' rotation and the recurrent periods of lunation cycles. As shown in Table 2 using averages, the approximate timing of this 383-day skip year may vary, shown from 420 to approximately 400 (13-lunation) years, giving 383-day intercalary event to maintain consistency with the temporal space and both the lunation cycle and Earth's rotation cycle.

Table 2. Rolling luniterranean years with remainders after necessary intercalary events.

Luniterranean (13-Lunations) years	Total lunations (New Moon)	Earth solar days per luniterranean	Average solar rotations of Earth per luniterranean year	remainder (over total days) in seconds
1	13	384 (regular)		−8843.558400 sec. (over 384 days)
9	117	384 (regular year)		−79592.025600 sec. (over 3456 days)
10	130	385 (leap year)	383.897644	+2035.584000 sec. (over 3841 days)
420*	5460	383 (skip year)		−905.471307 sec. (over 161,320 days)
400*	5200	383 (skip year)		−0.010288 sec. (over 153,638 days)

SI unit delta calculations used one solar day equal to 86,400 seconds (as of 1967) and 1 lunation cycle equals the average of 29.530588 days. (https://www.gpo.gov/). *Due to limitations in modern timekeeping methods, using a fixed mean for Earth rotations/lunation and fixed rotational time, inaccuracies (or unknown measures) can accumulate. **exploratory assignment of 1 lunation cycle variable to an average of 29.5305769231 days/lunation.

Shifting primary cycles also gives flexibility when needed for a desired outcome of a model. Shifting is where the calendar's independent cycles remain constant, but the primary cycle shifts to become the secondary cycle (Equations (17) and (18)). An example would be to shift a 13-lunation primary period to a 384-day primary period in a luniterranean calendar as discussed. These features create added flexibility for layering calendars, or adding secondary cycles, where the system shares a primary cycle for the primary period, thus shared signals that define the temporal space. Beyond this, we can also use any one of the cycles (cc1, cc2, or cc3) as a primary cycle or secondary cycle for another layered calendar. Where W is a natural number ($\mathbb{N}$).

Shifting primary/ secondary of Calendar cycle ($1^\circ\ W[X]\ 2^\circ\ W$)

$$W\frac{0}{N}1^{\circ}\,\mathrm{ObjectA}_{\mathrm{condition}}\mathrm{Rt}_{\mathrm{n}}\left[\rho\right]\left[\times\right]W\frac{k}{N}2^{\circ}\,\mathrm{ObjectB}_{\mathrm{condition}}\mathrm{Rt}_{\mathrm{n}}\left[\rho\right] \tag{17}$$

$$W\frac{0}{N}1^{\circ}\,\mathrm{ObjectB}_{\mathrm{condition}}\mathrm{Rt}_{\mathrm{n}}\left[\rho\right]\left[\times\right]W\frac{k}{N}2^{\circ}\,\mathrm{ObjectA}_{\mathrm{condition}}\mathrm{Rt}_{\mathrm{n}}\left[\rho\right] \tag{18}$$

3. Results: Pre-Greek Egyptian Calendar Systems

In this study, we apply the described methodologies and present findings from study of the observation-based ancient Egyptian calendars, including the lunar and civil systems as well as the solar calendar aligning to regional seasons. There are compounding inaccuracies when these calendars are studied in comparison to the Julian-style methodology. The rise of Sirius reappears on average of 365.25 days but this shifts from one cycle to the next. This difference, in combination with aperiodic signaling, leads to interpretations of inaccuracies for a 365-day civil calendar when using Julian-style comparisons using Earth's orbital period as the primary period in the calendar. In contrast, when studied using the rishta system methodology, either the rise of Sirius (Equation (19)) or a frame of 365 days (Equation (20)) can be tested as the primary period of the calendar.

$$1\frac{0}{N}1^{\circ}\,\text{Sirius}_{\text{Egypt}}\text{Rt}_{12}\left[\text{helical rise}\right]\cong 365\frac{21600}{86400}2^{\circ}\,\text{Earth}_{\text{rotation}}\text{Rt}_{24}\left[\text{sunrise}\right] \tag{19}$$

$$365\frac{0}{N}1^{\circ}\,\text{Earth}_{\text{rotation}}\text{Rt}_{24}\left[\text{sunrise}\right]\cong\frac{359}{360}2^{\circ}\,\text{Sirius}_{\text{Egypt}}\text{Rt}_{12}\left[\text{helical rise}\right] \tag{20}$$

The pre-Greek Egyptian civil calendar features a 365-day cycle associated with the helical rise of Sirius, or re-appearance on the horizon on the 365th day at variable times in any given year. In about 600 B.C., Thale (Seler), "Thales of Miletus introduced the Egyptian year of 365 days to the Greeks, without hint of any correction being required." (p 451) [22] Parker continued to comment there is also no hint in the whole four centuries and a half covered by the classical literature that the Egyptians had any memory of ever having used a Greek-style fixed year or ever having recognized its as being a desirable system to use. The high priests of Egypt even put a statement in the oath of kings that the 365-day count would not be modified. (p. 463) [22] Aligning the primary period to 365 Earth rotations (frame period) the secondary cycle with a remainder would be the helical rise of Sirius. The rise is still consistent to a signal beginning a New Year on day 365, without fail over hundreds if not thousands of years. The helical rise occurs after 70 days of absence at a latitude shared by Egypt, 30.1˚ to 29.2˚. (pp. 63, 66) [27] As an observer's latitude changes, the count of sunrises begins to shift from 365 days. These reports support a test scenario for using Earth's rotation (Equation (20)) as the primary cycle which was previously suggested [1], but this provides a unique scenario.

When considering the rise of Sirius as a primary cycle, mathematically it creates a zero quotient, inconsistent with Euclid's divisional algorithm whereby the quotient cannot be less than zero. Thus $W = 365$ counts is more appropriate. However, with <1 stellar signal in 365 Earth rotations, it creates a secondary cycle count where $W < 1$, and $k < N$. This suggests application of rolling calendar as seen with the luniterranean example is not mathematically consistent to the proposed methodology, opening an alternative possibility of a non-rolling calendar, a recurrent cyclic addition of a single duration without an accumulation of a remainder.

Currently, lunar interpretations work around the Greek 12-lunation lunisolar year model with alternating between 29-day, and 30-day months to approximate the 12(30) = 360 formulae. This methodology creates an inconsistent 354 days in a regular lunar year but was adjusted with intercalations to synchronize the calendars and stay true for long term synchronization. Parker contended the Egyptian lunar calendar ran parallel with the civil calendar, yet in his approach was based on using 12 lunation signals to construct a 12 lunar month year with an occasional thirteenth lunar month intercalation in a given year. [27] Prior to Parker, Borchardt was the first to propose the Egyptians around 1850 BCE adopted a lunar calendar of altering 29- and 30-day months with a "now and again" month of 31 days completely unrelated to the civil calendar. (p 27) [27] Borchardt's hypothesis is still debated even today. These 29-, 30-, and occasional 31-day months are also consistent with the luniterranean calendar and McKenna-Meyer's interpretation [36], yet a mathematical study of the 13-lunation calendar has yet to be tested against the Egyptian calendar system.

Parker proposed the lunar and civil calendars could be aligned and used in parallel. [27] This proposal has consisted with descriptions from Australia and Americas, where other ancient calendars used a unique stellar signal relevant to their region to initiate a lunar calendar system. [27] [37] In this study of the Carlsberg 9 papyrus cycle, we align with conventional discussion that proposes the cycle is a description of two calendars synchronized, the Egyptian civil calendar, and a lunar based calendar system. From the papyrus, there are notes of a 25-year civil year synchronization with 309 lunations. However current approaches to mathematically model synchronization of these ancient calendars use Eurocentric interpretations which have not been overly successful and gaps remain. [22]

In contrast to other studies, we begin exploring applicability of the luniterranean calendar system, an untested comparator for the lunar-based system in Egypt.

Findings from this study support proposing a hypothesis that the Carlsberg 9 cycle describes a novel triple calendar system (Equation (21) and (22)). The proposed primary period is a frame of 9125 Earth rotations and the two secondary cycles are the frame period of twenty-five sample period rises of Sirius in Egypt (non-rolling; no accumulating remainder) and a frame period of 309 lunations with necessary object relative Rt_n unit paired remainders (Equation (22); simplified notation). (**Table 3**) As previously reported, 309 lunations are approximately equal to 9124.95 days using current averages, a difference of 0.0480 days or approximately 69 minutes. [38] [39] The precise difference cannot be reported because the International Celestial Reference System does not currently include an aperiodic measure of sequential lunation durations. In addition, the twenty-five Egyptian civil calendar "years" of a 365-day frame period, equals 9125 days. To note, we propose a novel symmetry group that is needed to describe mathematical objects that characterize helical stellar rises on the horizon.

$$\mathbb{N}\,1^{\circ}[\times]^{\mathbb{N}\,2a^{\circ}+2a^{\circ}\,\mathrm{pr}}_{\mathbb{N}\,2b^{\circ}+2b^{\circ}\,\mathrm{pr}} \rightarrow \mathbb{N}^{+2a^{\circ}\,\mathrm{pr}}_{+2b^{\circ}\,\mathrm{pr}} \tag{21}$$

$$9125\text{ days}[\times]^{25[\text{Sirius rises}]+25[2a^{\circ}\,\mathrm{pr}]}_{309\text{lunations}+2b^{\circ}\,\mathrm{pr}} \rightarrow 1\text{ Carlsberg 9 cycle}^{+25[2a^{\circ}\,\mathrm{pr}]}_{+2b^{\circ}\,\mathrm{pr}} \tag{22}$$

Table 3. Two dual calendar systems connected using a triple calendar system by shifting primary cycle references.

Cycle Name	1˚ cycle $N = 12(30^x)$	2˚ cycle	Remainder	Calendar type
Egyptian civil "year"	365 days	1 Rise of Sirius in Egypt	+0.25 days	Dual calendar system
Luniterranean year	13 Moon orbit	384 days (regular year) + pr	−0.10236 days	Dual calendar system
Luniterraneane 309-lunation cycle	9125 days	309 lunations + pr	−0.048 days	Triple calendar system (Carlsberg 9 cycle)
Egyptian 365-day 25-"year" cycle		25 stellar rises + 25 pr	25 (+0.25 days)	

a. Calculations using: Tropical year average = 365.2421897. Lunation average = 29.5305888531 Sample of a Table footnote (*Table footnote is dispensable*).

The Carlsberg 9 cycle is therefore proposed to be equivalent to a finite temporal space that occurred in the past, an anchored and unique primary frame period of 9125 recurrent Earth rotations, logically beginning upon a New Moon signal at a designated privileged point on Earth in the region of Egypt. The duration between a unique and defined sequence of 9125 recurrent Earth rotations can be measured in SI units (Equation (23)) which can be equated (shown using definitions, or 86,400 seconds/rotation, not actual recurrent summation from aperiodic cycle signals). One whole Carlsberg cycle can be assigned and divided into the symmetry of time, D_{12}. In addition, the dependent Carlsberg cycle is a novel ordered natural number count of signals which is consistent with pairing functions and Gödel numbering applications.

$$9125\ 1^{\circ}\mathrm{Earth}_{\mathrm{rotation}}\mathrm{Rt}_{24}\left[\mathrm{sunrise}\right] \approx 788400000\ \mathrm{seconds} \tag{23}$$

We continue to test the luniterranean calendar for Egypt by shifting the primary cycle and studying alignments with descriptions in the ancient texts. The Papyrus includes a discussion of nine "Great" years. An additional consideration is around a note of 16 "small" years, each with 12 divisions. As discussed, any temporal space can be measured by SI units and divided into the symmetry of time using the formula $N = 12(30^x)$. When considering what these 16 "small" years represent, it opens different avenues to explore. For example, when using the luniterranean calendar as shown in the Carlsberg cycle where Earth rotation is the primary cycle, small years of 383-day skip year occur every 10th luniterranean year. We open an option to explore whereby the sixteen recurrent "small" years representing 160 luniterranean years in total (1˚ Earth rotation). In contrast, when the lunation signal is the primary cycle for the 13-lunation year calendar, a 383-day skip year could be the small year every ~400th luniterranean year, making sixteen such small year cycles occur after ~6400 luniterranean years, or approximately 6726.89 Earth orbits. Suggestion of alternatives for the 16 "small" years, or cycles, is also not excluded at this point.

In addition to the Carlsberg 9 cycle, a hypothesis commented on by Parker, is the Egyptian lunar calendar runs in parallel to the civil calendar system. Our findings support this hypothesis, showing consistency with timing of the Egyptian feasts/festivals: Wagy (interpreted to be associated with lunar cycle), Thoth, and Tekh. [27] Parker believes Egyptians lunar feasts were marked by the civil calendar system and argues the basis of the names of the lunar months and transference to the civil calendar. (p. 249) [27] We find that the timing of Egyptian festivals shows consistencies with the luniterranean regular cycle and related leap/skip days. The Wagy festival occurs on the first month of a civil new year on day +18 (365 + 18 = 383), Thoth festival on day +19 (365 + 19 = 384), and the Tekh festival on day + 20 (365 + 20 = 385). If we assume both the civil and luniterranean calendar systems started with a synchronization of events, then we would anticipate the calendar at a (privileged) point in Egypt started on a sunrise (Earth rotation) during a New Moon/new crescent (lunation stop/start) as Sirius rises on the horizon (2˚ cycle, stellar year). From this triple cycle calendar beginning point, the civil calendar year would end on day 365 (signified by the rise of Sirius) but the luniterranean calendar system would continue to day 384 (regular year). This would align only on the first cyclic alignment, after which may only be symbolic of the alignment until the next cycle resynchronizes.

The final calendar to discuss is the 360-day calendar with the addition of 5 extra days (epagomenal). [40] This system comprised of three seasons, each with four months with 30 days each, plus an additional 5 days. The Epagomenal (5 days), or intercalary month, was used by Egyptian, Coptic, and Ethiopian calendars where 360 counts of days in a solar orbital year was then added with five extra days to honor the gods. [23] The numerical count of (360 + 5) has consistency with the product of the universal formula $N = 12(30^1) = 360$ as well as the civil calendar of 365 days. However, the 360 + 5-day calendar does not build upon signals unique to space and time, but rather variant seasonal changes like the flooding of the Nile that associates with a solar orbital period and eventually resets at the rise of Sirius. We deduce the seasonal calendar could have also been a purposeful simplification for the largely illiterate public. [41]

4. Discussion

In the historical exploration of mathematical spaces, numerous meticulous descriptions have emerged, yet the characterization of temporal space has presented unique challenges. We introduced a methodology to mathematically map temporal space using discrete and independent object-oriented cyclic durations equitable to a finite period of physical time measured by modern dimensional time and applicable to set theory. These durations are recorded using cyclic signals from normal matter in our dimensional reality, marked from a privileged point of reference.

The concept of temporal space introduces a novel paradigm by employing geometric principles, a causal signal system, and dimensional analysis to define a discrete and finite framework for cyclic durations. This involves delineating multiple cyclic zero-D temporal points that bind object-oriented durations, or 1D lines definable by SI unit time. Traditional spatial dimensions include points, line, and plane that represent the physical extent of objects. In contrast, temporal space provided geometric temporal elements of object-oriented durations taken from a privileged point of reference. The approach enables visualization of temporal events in a manner similar to spatial positions. This offers a paradigm shift in expressing discrete time, opening applications for various fields.

Temporal space, as elucidated in this article, served to unify both discrete and continuous temporal intervals, applicable to the scale of both quantum and classical physics given discrete event signals from each can be identified. This approach marks a departure from conventional applications that solely use a single continuous and linear progression of time. This network of multiple overlapping object-oriented durations is, however, linkable to a single uniform progression of a timeline within our dimensional reality relevant to a privileged point of reference.

In comparison, the conventional use of a single continuous nature of time in physics is a fundamental approach for various theories, such as classical mechanics, quantum mechanics, and relativity. In classical mechanics, time is an absolute, independent parameter progressing uniformly and separate from space. Einstein's theory of relativity, particularly in special relativity, recognizes time as part of a 4D spacetime continuum, where the rate of time passage can vary based on observer motion and gravitational fields, leading to phenomena like time dilation. At the quantum level, time is often a parameter in equations governing quantum state evolution, as seen in the Schrödinger equation. While time is generally treated as continuous in most theories. Quantum temporal space in essence uses quantum discrete signals to define a duration at or near Planck's limits.

Another key characteristic of temporal space is its departure from the use of infinitesimal time where $t \neq 0$ and singularities are used. Instead, temporal space is based on physical durations where there is a limit, where time can be zero [$t = 0$]. This temporal limit aligns with Planck's limits and the Mohist definition of an "atomic," representing a line indivisible into smaller parts. For instance, consider the phenomenon of superposition, where two states can simultaneously occur. Here the simultaneous occurrence of these two states occurs precisely when $t = \varnothing_t = 0$, which is independent of space but measurable using separate Rt relational spatial metrics when observed. In contrast, infinitesimal intervals of continuous time, proposed to exist beyond our dimensional reality, can occur between these superposition discrete event signals. The definition of temporal space is confined to our observable and dimensional reality, measured on a scale relevant to our observed universe of normal matter. In contrast to relativistic modeling, where time and space are intertwined, Rt relationalism allows for the separation of space from time.

The recent advancements for Rt relationalism demonstrated how to separate space (an extension in a memoryless system) and time (a duration in a memory system). [1] One key advancement was to define cyclic durations that are object-relational, separated by zero-D points [$Ø_t$], and measurable from a privileged point of reference using dimensional time (seconds). Integration principles for signal systems enabled us to set zero-D time points, separated by a duration that can be measured by SI units of time. This theoretical advancement forms the basis for incorporating finite geometry into temporal space. Euclidean geometry can now be used to model temporal space similar to how we model spatial space. The causal based system focuses on recording and presenting inputs that can be used for functions in casual system operations that can make predictions, creating new tools for consideration. Using alternative methodologies, we underscored the complementary roles of relativity and relationalism, each framework serving distinct purposes and generating unique model outputs.

We employed inductive reasoning and assimilated mathematical approaches seen in such ancient timekeeping systems as Vedic, Egyptians, Sumerians, Babylonians, and Chinese civilizations. Mathematical research also extended to include the study of Neolithic architecture, with a focus that included alignment to aperiodic astronomical signals and other features. [1] [26] [42] [43] [44] From a more modern standpoint, the study also addressed viewpoints from Newton and Einstein, which deviated from the cyclic and signal-centric descriptions prevalent in ancient models. Leibniz proposed a relational concept for time and space, but his theses still utilized both inertial frames and time derivatives of displacement, both of which are not used in Rt relationalism. This article advanced Rt relationalism by unifying concepts of infinitesimal continuous and object-oriented dimensional durations (physical) and provided a non-conflicting application for both within temporal space, inclusive to a network of multiple overlapping object-oriented durations. Gaps in relationalism are being filled by integrating modern technologies, mathematical theories, and ancient timekeeping descriptions.

The capability of generating overlapping cyclic durations, or temporal extensions, is based upon privileged observers, located at privileged points of reference, or the origin of multiple light cones. Privileged observers measure the duration between signals received at that point. A much different approach than that taken by the theory of relativity where time is one coordinate in 4D spacetime. Networks of durations were shown to be layered, each with a conserved cyclic symmetry groups and unique recurrent cyclic dimensional quantities (assuming aperiodic signals) and if cyclic whole duration is measured. Independent object-oriented time/durations can be layered and span the same defined temporal space. These independent durations can be equated using dimensional time and principles related to a causal (signal) system. We gave a novel application of Euclid's divisional algorithm with applications that include fixed temporal space geometric models as well as calendared methodologies with the inclusion of an introduced paired function equation.

In this article we proposed dimensional relational time is fundamental, an independent measurable quantity without time dilation. Dimensional relative time [T_r] is argued to be fact, not subjective, given it is a finite duration bound by signals from normal matter. For a whole cycle, we consider there is a finite number of units, or parts of a whole cycle, on the temporal number line. Dimensional relative time is not a mathematical construct as it is in relativity and is like ordinary time, measured by the invention of seconds which is based on fundamental constants of our dimensional reality using a relevant scale. Today we use Cs^{133} counts as a uniform measure of dimensional time, but a primary element like Hydrogen could be used to measure dimensional time before Cs^{133} was created in the universe. We entered these descriptions to contextualize the axioms that dimensional relative time is a real count, emergent from the first signal from normal matter and stopping with the last in this dimension. As it is not possible to have a privileged time reference for every measure, modern theories can provide workable estimates, again highlighting the complementary roles of Rt relationalism and the theory of relativity.

Beyond time and durations of object-oriented cycles observed from privileged points, discrete symmetry becomes integrated into the various temporal units applying inherent features of ancient formulas. In an extensive mathematical interrogation of various ancient timekeeping systems by the author, excluding Eurocentric interpretations, we conjectured a universal methodology where the temporal units (with both point and line elements) can be divided and subdivided into units with symmetry, order, and cyclically unique dimensional time quantities (if measured). There are two different types of discrete symmetry, mathematical objects that share the same attributes, like rotational symmetry (D_{24}) and the proposed symmetry of time, D_{12}. Thus far each appears to be associated with dihedral geometric symmetry in ancient formulas. Given all matter was theoretically connected at the big bang, it is reasonable to suggest all matter shares the same symmetry of time. This arguably requires a postulate that temporal symmetry for a universe is conserved, but not critical to this methodology.

We applied a familiar ancient formula for temporal spaces, $N = 12(30^x)$. This interpretation of the formula deviates from the various historical Eurocentric interpretations. Historical use of the formula is linked to timekeeping but has been used and interpreted for various best-fit inconsistencies, applied in various applications that create vague approximations of their own. For example, we do not see $N = 360$, or even 360 + 5, as a vague measure of ~365.2422 days in a tropical year, which is a typical interpretation in the literature. Instead, we argue the methodology of ancient timekeeping was unique from that of today, more universally applicable, and possibly more similar to the introduced methodology of this paper where accuracy can be maintained over long periods of time.

A pending step in a description of universality of ancient time is related to mathematically defining the framework for the symmetry of temporal cyclic forms with divisional units of a whole paired with accumulative whole cycles.

Relational space and time are independent, but both are physically based and bound the by limits of our universe, supported in principle by Planck limits. However, we maintain openness that Rt_{12} may be infinitely divisible to a countable infinity as it can be seen as a unique symmetry, independent of object-oriented mathematical objects such as rotation which is managed differently. To support a finite count of units as part of a whole cycle, we can consider the quantum phenomena for superposition. We logically speculate quantum superposition of states can span an interval of infinitesimal continuous time. However, in quantum superposition, there is no measurable duration between signals, no duration of dimensional relative time, thus a dimensional temporal limit, [$t = 0$]. A particle's location of possibilities can include two positions at the same instant of dimensional relative time. Since the quantum particle only appears in one possible position when observed, we hypothesize that when we observe it in this dimension, the particle is giving a spatial coordinate in addition to the temporal coordinate.

Consistent with the Rt system, Euclidean modeling can be considered for dimensional relative time, mappable on a straight natural number line using zero-D temporal points separated by object-oriented line elements that can be divided and subdivided to a physical limit of the universe. We therefore consider a possibility zero-D points on independent lines may intersect across co-existing, yet independent, object-oriented 1D natural number temporal lines, or for simplicity, "timelines". A theoretically shared discrete point across timelines is termed a temporal node [$\varnothing_{tn}$]. A temporal node is not considered possible in continuous time because time cannot equal zero. If the network-independent timelines are parallel, we consider if temporal nodes can intersect in space and time.

To consider if a temporal node located on two independent timelines can intersect, we consider Euclid's fifth postulate in Book I. Euclid comments that two straight lines with internal angles less than 90 degrees can meet, thus creating an intersects of lines. If we assume temporal 1D lines to be straight, curvature of some sort, may create a hypothetical intersect of timelines. A path yet to be explored is curvature in spherical geometry, inclusive of the interconnection of three points of a spherical triangle using straight lines on a curved surface. This exploratory thought is a natural progression taken from Euclidean principles and the methodology presented in this article.

At this moment, future theoretical work needs to evaluate if Rt relationalism can be considered to have absolute zero time given there is no $\Delta t1/\Delta t2$ as used in continuous time. The system also allows for addition, a type of ordered subtraction as well as Euclidean 1D temporal space, and it is open to n-dimensional metric space, each of which is consistent with a strong measurement system.

[13] It would be prudent to embark on a comprehensive assessment of the suggested temporal space measurement system to determine if it is a weak or strong form of measurement on a set of objects, taking into account the criteria outlined by Brazilai [45] or other relevant sources.

To enable the combination of measures from modern dimensional quantities with cyclic symmetries of ancient systems, we presented a unique pairing function equation that is inclusive of new considerations for combining geometric dimensional quantities with Euclid's division algorithm, Cantor pairing functions, and Gödel numbering. The universal methodology described in this paper is consistent with ancient cyclic durations, conserved cyclic symmetry, signal-based observations, mathematics, and modern measurements and units of time. Simultaneously, the methodology is capable of absolute precision that challenges even modern technologies.

The author is not aware of a recognized Greek-based calendar system that precisely uses the description of ancient formulas, including $N = 12(30^1) = 360$, and there are various unresolved issues, including intercalary events. [46] [47] Greek methodologies included the application of $N = 12(30^x)$ as twelve counts of a moon orbit with a vague fit of 30 days each lunation as well as in the design of Julian-style calendars with twelve months with +/− 30 days each. Greek calendars were diverse, and different city-states often had their own systems. One notable Greek calendar was the Attic calendar, used in Athens. It was a lunar calendar based on the phases of the moon. The months alternated between 29 and 30 days, however Greek interpretations of the ancient formula 12(30) = 360 led to creating a 354-day year with an additional month periodically added as an intercalary event. A non-Greek calendar methodology is proposed to use the ancient formulas, including $N = 12(30^x)$ with precision, not vagueness, which are most closely aligned with ancient Hindu timekeeping descriptions. The introduced non-Greek based calendar methodology that uses independent object-relational durations defined by discrete cyclic events, and non-vague applications of ancient formulas, such as $12(30^x)$, highlights gaps, limitations, and the non-universality of a Julian-style calendar methodology. The Julian calendar, introduced by Julius Caesar, is a "solar" calendar with 12 varying-length months, totaling 365 days plus an extra day in leap years. In the context of this article's methodology, the approach can be seen as taking two independent cycles, Earth's orbit and Earth's rotation, and merging them with a vague alignment of the universal formula, $N = 12(30^x)$. The Greek method of combining cycles introduces inaccuracies in timekeeping that deviate from the consistency found in ancient timekeeping systems based on natural number counts of signal events and various distinct formulas. This proposition implies that employing Greek methodologies in present-day interpretations of ancient time-keeping systems may have led to inconsistencies and misinterpretations, overlooking a simpler application yet more advanced mathematical approach.

With few alternative methodologies to study ancient astronomical timekeeping systems, Eurocentric research has exclusively used the well-preserved and mass published documents that characterize ancient Greece concepts of time and calendars, including the familiar Julian-style calendar systems. [33] [48]-[53] In general, conventional literature about ancient calendar systems often contains discussion that ancient civilizations adhered to relatively unsophisticated observation-based calendar systems compared to the accuracy of more modern measures used by the Julian-style calendar, including Pope Gregory XIII's Gregorian calendar. Modern reinterpretations of observational-based calendar systems highlight their inherent inaccuracies from Eurocentric perspectives.

Methodologies seen in Egypt for example are not unique. Using the rise of a star, like Sirius in Egypt, is not uncommon as a marker of a New Year. Different stars are used in different regions (differing privileged observers and privileged points) across the world by ancient civilizations. In many places which includes tribes in Mexico, as well as Australia, they used the morning rising of Pleiades with the best time to witness it is during a New Moon. [54] In the traditional Māori Maramataka, or lunar calendar, the new year begins with the first new moon following the appearance of Matariki (also known as Pleiades). In New Zealand the lunar calendar started with the rise of the winter star of Puangu (also known as Rigel). [55] This suggests the proposed Rt methodology could be studied in a global context.

It is noteworthy to also consider the historical backdrop of timekeeping when considering the plethora of existing literature given actions through history may have created potential for inadvertent selection and researcher biases in the historical studies of ancient timekeeping.

Our global transition to a shared Roman/Greek derived Julian (45 BCE) and later Gregorian calendar system (1582 by Pope Gregory XIII) has been rooted in aggressive, and in many cases, brutal replacement of indigenous timekeeping methodologies. Examples can be seen in Mesoamerica. [33] [56] It was in 1524 that the first 12 Franciscan missionaries, also referred to as the "Twelve Apostles of Mexico" lead by Fray Martin de Valecia, arrived in Mesoamerica and began their "spiritual conquest" of New Spain. [57] During the Christian missionary visits, documents from the indigenous populations were largely burnt and destroyed and then chronicles, now used as source documents, were written to largely replaced the destroyed native written records.

Through history, attempts were made to convert the Egyptian system to the Greek system, including Ptolemy III's Canopus Decree. These early attempts were repelled by the Egyptian priests until Augustus from Rome imposed upon Egypt the Julian-style system on August 1st, 30 BCE, under the name Alexandrian year. (p. 452) [22] [58] The introduction of Greek year of 365.24 days in a solar year was "…regarded by the Egyptian people as an abhorrent innovation with which they would have absolutely nothing to do with and interpreted as 'obnoxiously foreign' and seen as undesirable in Egypt. (p. 452) [22] The calendar was referred to the Egyptians the 'Greek Year' to distinguish it from the year 'according to the ancients' [22] [58] and was not used by the natives until they had given up their own religion and had adopted Christianity." (p. 452) [22] Until the broad adoption of Alexandrian year with the spread of Christianity, for three thousand years, the 365-day year was used as the civil calendar.

Written explanation of methodologies for an Egyptian lunar calendar system have not well survived the ages or reaches of public publication, creating speculation and various hypotheses that are still being debated. Lunar calendar proposals from Borchardt and Parker have been disputed however using conventional comparisons, no clear explanation related to differing findings can be made. To highlight the challenges, Meyer [58] is stated to concluded that the 365-day civil year as being artificial "since neither month nor season nor even year corresponded to any natural period." (p. 30) [27] We propose that this interpretation of an "artificial" calendar as a good example of a published Eurocentric interpretation of ancient calendars.

To study the application of the Rt calendar methodology, we provided an initial mathematical study of pre-Greek Egyptian calendar systems. Tests included using a luniterranean calendar system originally derived and published from a study originating from ancient Neolithic China. [36] We demonstrated consistency with an accurate primary 365-day calendar system that is not only accurate using more advanced mathematics and physics than Julian-style calendars, but one that contains several advantages over Greek based timekeeping alternatives. For example, Rt calendar methodology can take any primary period in a temporal space and divide it into 12 equal elements of time requiring more context to appreciate what temporal space is being represented.

Initial study and findings of the Carlsberg 9 cycle support a new hypothesis for a triple cycle calendar system whereby Earth's primary cycle creates a primary period of 9125 days, equated with 25 stellar rises of Sirius and 309 lunation cycles, where each secondary has a pairing remainder added to both sides of the pairing function equation to maintain temporal equality. There are limitations as references from the papyrus like 16-small years have not yet been fully modeled, requiring more in-depth assessment of numerous astronomical cycles and calendar variables. However, the study continued to support Parker's proposal that a parallel lunar calendar system ran along with the civil system. Supporting Parker's hypothesis there was a synchronized start of both a civil and luniterranean calendars, it is noted in this article that the timing of Egyptian festivals falls upon the 383-day (365 + 18), 384-day (365 + 19), and 385-days (365 + 20) which are also associated with actual (first year of cyclic synchronization) and later potentially symbolic luniterranean calendar's skip, regular, and leap years respectively. As with any new hypothesis, these will require continued mathematical study from existing artifacts, both written and geometric.

Potential applications for this mathematical tool can be considered when exploring challenges in quantum computing as well as use for discrete event simulations. For example, temporal space has a time point equal to zero, where the start/stop cycle is synchronized with discrete events in our physical reality. The approach is an alternative to abstract and infinitesimal continuous time where time cannot equal zero. We propose this as an alternative perspective for approaching obstacles that have been described in relation to designing a quantum clock. [59] Practical applications can be extended for consideration in discrete event simulation modelling, opening opportunities for building discrete state models of relational data whereby the geometric model itself is constructed using object-relational geometric elements frozen in a point of zero-time. The approach opens considerations for developing an automaton application for Euclidean translations for the Rt system. We also consider an opportunity to construct a concentric polynomial clock with many millions of discrete states presented as a sequence of states inclusive of temporal space. Such a clock could be managed using a primary cyclic signal and with a single discrete input event, simultaneously providing two or more state outputs using a polar coordinate system. Such a clock is beyond the scope of this article.

This novel methodology provides precision, universality, and timekeeping technology with potential applications in computational geometric modeling as well as modern physics. These capabilities underscore limitations in today's existing technologies. For example, the need for additional data repositories essential for implementing an N-body causal calendar system made possible by this methodology.

5. Conclusion

Temporal space combines cyclic signals, discrete events, object-oriented relationships, and the interconnectedness of multiple timelines within a finite and one-dimensional geometric framework. Unlike traditional uses of time and metric space, temporal space is finite and modelled using ordered discrete elements equated to dimensional time. Its applicability extends to a natural number line instead of an infinitesimal real number line. Temporal elements possess diverse properties, including object-relational durational data and dimensional quantities of time. Notably, temporal space allows for the intriguing possibility of a network of multiple timelines that can be equated to a single shared linear timeline, introducing a novel and dynamic dimension to our understanding of time and space.

The concept of temporal space emphasizes cyclic signals, discrete events, and divisional/subdivisional structures with consistent application of ancient formulas to create ordered temporal elements. The approach not only offers a new view on durations and time but also introduces innovative applications in studying ancient timekeeping and calendar systems.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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