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Separating Space and Time for Dimensional Analysis and Euclidean Relational Modeling

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Abstract

The theory of relativity links space and time to account for observed events in four-dimensional space. In this article we describe an alternative static state causal discrete time modeling system using an omniscient viewpoint of dynamical systems that can express object relations in the moment(s) they are observed. To do this, three key components are required, including the introduction of independent object-relative dimensional metrics, a zero-dimensional frame of reference, and application of Euclidean geometry for modeling. Procedures separate planes of matter, extensions of space (relational distance) and time (duration) using object-oriented dimensional quantities. Quantities are converted into base units using symmetry for space (Dihedral_{360}), time (Dihedral_{12}), rotation (Dihedral_{24}), and scale (Dihedral_{10}). Geometric elements construct static state outputs in discrete time models rather than continuous time using calculus, thereby using dimensional and positional natural number numerals that can visually encode complex data instead of using abstraction and irrationals. Static state Euclidean geometric models of object relations are both measured and expressed in the state they are observed in zero-time as defined by a signal. The frame can include multiple observer frames of reference where each origin, point, is the location of a distinct privileged point of reference. Two broad and diverse applications are presented: a one-dimensional spatiotemporal orbital model, and a thought experiment related to a physical theory beyond Planck limits. We suggest that expanding methodologies and continued formalization, novel tools for physics can be considered along with applications for computational discrete geometric modeling.

Keywords

Spacetime, Relationalism, Quantum, Classical, Signal, Discrete Geometry

1. Introduction

Current approaches to model classical and quantum mechanics differ, and one of those key differences is the need to treat time and position differently [1] [2]. Einstein stated that his approach does not provide a physical description for all natural phenomena, particularly when considering electrodynamics and optics [3]. The main purpose of this article is to introduce an alternative approach to dimensional analysis to model matter (orthogonal planes), space (extensions of relative distance), time (extension of time, or duration, between cyclic signals), and discrete translational motion using dimensional quantities and spatiotemporal units. Using a causal system, we take inputs of what was (past) and what is (present), thus creating repositories of data to create models that can retrospectively test predictions, post-processing, of what will be (a recorded future) in select N-body dynamical systems. Measurements of defined dimensional quantities are taken by alternative methodologies.

In current approaches, there are descriptions of an apparent quantum defiance of Einsteinian spacetime is discussed by Brumfield, where quantum mechanics allows for another way to coordinate information [4]. Baclawski also noted that the observer effect "may be responsible for the widely held implicit assumption that 'real' observer effects are exhibited only by quantum objects and not by classical objects" [5]. Thus, for any proposed system to be of greater applicability, it should be inclusive for both classical and quantum mechanics.

A consistent assumption in physics became evident in the study of dynamical systems when an approach to modeling used discrete signals, the assumption is that quantities of matter are continuous in a quantum state. Models like Loop Quantum Gravity [6] alter the fabric of spacetime to accommodate discrete observations of matter given an assumption of a continuous state of matter. The aim of this article is to not only introduce new tools for dimensional analysis and modeling mechanics in classical and quantum physics, but also allow for new perspectives. Separating space and time, as well using discrete signals opens opportunities for thought experiments about the state of matter, or particles, in a quantum state, that extend beyond the requirements of existing principles and equations. An example being an ability to generate a zero-time frame of reference for constructing models, using time integrals of displacement, and object-relational dimensional qualities for Euclidean based geometric modeling.

In general, early civilization's concept of time has two prevailing notions: linear and cyclic time [7]. Linear, "a forward, straight sequence of steps or stages" and cyclic time, described as "pattern of moments or stages, at some definite interval, repeats itself" [7] (p. 84). It was noted Egyptians referred to cyclic time as *neheh* (recurrent cycles) and non-cyclic time as *djet* (immutable permanence, zero-time, $t = 0$) [8]. Before time was proposed to be a measure of motion, ancient civilizations used a cyclic signal system to measure time. It appears from the author's research, that metrologists assume historical civilizations regarded astronomical signals as being periodic [9], which would create intervals (or base units) of time that are inconsistent from one cycle to the next. Academic interpretations of ancient calendars, from Neolithic architecture to Mesoamerican missionary chronicles [10] [11], contextualize findings in reference to our modern Julian style calendar system. The author was unable to find mathematical research of calendar systems using a hypothesis signals were considered aperiodic. We support an argument that proposed timekeeping technologies of ancient civilizations may have been different, referred to as "our ignorance of the ancient astronomical methods" (p104) [12].

Aristotle's belief was that time flowed continuously and uniformly at a constant rate, an inherent property of the universe, resting largely on continuity of motion, and "it is the cessation of motion that divides a line" [13]. The approach opens the possibility to model continuous motion between any two points, A and B, without regard for mid-points. However, this article argues that by using defined and non-dimensional point signals and a function passing a vertical line test, mid-points are possible. Modeling today exclusively uses both continuous spatial and temporal quantities, inclusive to a point mass, which are linked for spacetime modeling.

Newton considered the concepts of absolute and common time as non-exclusive in his seminal work, the "Mathematical Principles of Natural Philosophy" published in 1687. Absolute time being a mathematical construct and common time was explained as perceptible, external, detailed, cyclic, relational, and also uniform [14] [15]. Relational physics introduced by Leibniz was unable to definitively describe relational time, but his theory suggested space only exist as a relational distance between two points, infinitely divisible [16]. Although relationalism can model kinematics, it is not directly capable of formulating laws of motion [17]. Leibniz was also not able to find a way to remove the application of velocity, including his conservations principle, mv^2, which excludes motion being modelled as purely relative [17].

Modern considerations of time link it with continuous motion and space; however, this creates issues at the quantum level. In quantum physics, the study of motion is more difficult with various theories to describe it including aspects related to hidden variable theory [18], many-worlds interpretation [19], stochastic interpretation [20], and a theory of discontinuous motion of particles [2] [21]. Time today is standardized internationally as a dimensional base unit, one of seven defined by the International Bureau of Weights and Measures (BIPM) [22]. In general, dimensional analysis is based on the concept of selecting a phenomena, assigning physical base quantities, and through formulas, generate base units [23]. The SI unit for time, a second, can be considered a "derived" dimensional based unit because it uses two or more measures, an agreed upon interval and a count to represent that interval [22] [23]. In dimensional analysis, the principle of homogeneity allows us to measure object specific dimensional properties in space and time with SI units and equate them.

Dimensional quantities and units are rigid standardized measures for a particular privileged reference frame, very useful for scientific research. In quantum physics, Planck time (t_P) is may be considered a dimensional quantity (1 t_P = $5.391247(60) \times 10^{-44}$ s) [24] [25]. Astronomy for example standardizes the astronomical unit of mass using the mass of the Sun, $M_\odot$ [26] (pp. 48-51), and even Earth's average orbital distances to the sun, 1 AU = 149,597,870,700 m (±3 m) [27]. Canonical references, especially in dynamical systems have quantities that change over time. The change can be very small from our perspective given a window in the life of the Universe; however, they do change, and adjustments are needed. For example, at one point the Sun did not exist so current approaches are limited to the moment the measurements are taken and the resolution of our measurement technology.

In addition to dimensional analysis and canonical references, symmetries from nature are applicable for foundational physics [28] and largely based on works of Hamilton and Einstein [29]. Mathematicians are introducing proofs [30] that support various symmetry invariances across a broad class of models. The author reviewed familiar ancient symmetry measures [31] [32] [33] [34] which opened a hypothesis that Dihedral symmetry groups have already been used for the current agreed upon symmetries in nature, including scale (D_{10}), rotation (D_{24}), time (D_{12}), and space (relative distances) (D_{360}) since the beginning of recorded history. The concept of ancient symmetry groups is applied to dimensional analysis in this article, aligning with historical divisional and subdivisional structures of dimensional quantities. In a novel approach, we stay consistent to these groups for conversion of object-oriented quantities into base units consistent to dimensional analysis [23].

Beyond time, various models have been proposed to characterize celestial motion. From geocentric models described by Ptolemy of Alexandria, heliocentric models put forth by Copernicus, and descriptions of planetary motion by Kepler. Each of these models give assumptions and partial information. For example, Earth's distance to the Sun changes at each recurrent aphelion position making a planetary orbit an elliptical-*like* orbit when the Sun is modelled as a fixed point within a two-dimensional (2D) plane. There has yet to be a one-dimensional (1D) orbital model proposal to display information about a single object and a single orbit, nor an application for this. Today, we understand the stars move in a galaxy creating a 3D helical model [35] and Einstein's theory of relativity is the primary methodology used to conceptualize, model, and accurately predict motion and events in the modeled 4-dimensional Minkowski space [36]. Einstein argued against truth of the principles and axioms of Euclidean geometry related to plane, point (zero-mass), and straight line [37] (Ch1), it should be considered Euclidean geometry is not possible using Einstein's' approach to model motion *through* spacetime. Einstein's theory is exceptionally powerful yet there are still noted challenges [38] including transferability to quantum physics.

It has been noted that ancients referred to time-frames, independent of space [7] but a description has yet to be offered that holds consistent against the theory of relativity. For Einsteinian relativity, time normalization across observer reference frames of reference can be calculated using a Lorentz factor that adjusts the measurement of a time intervals between events across four-dimensional spacetime, referred to as *proper* interval of time. However, a recent mathematical study demonstrates inconsistencies and mathematical contradictions in the equations, proposed by the study as disproving Special Relativity predictions [39].

Modeling movement and location of objects in space requires an appropriate frame of reference. Aristotle described an absolute time frame of reference. Absolute (continuous) time frame of reference has limitations as descriptions do not account for the effects of relativity [3] [40]. According to Galilean relativity, any inertial frame is as good as any other (so far as the laws of motion are concerned). Another consideration is that in continuous time, there is no privileged frame of reference: no specific frame that is uniquely correct about which objects are really at rest. Relationalists theories suggested an alternative privileged reference frame may be possible but a solution would only admit relational quantities of motion and not allow for inertial frames [41]. If a solution is provided, it would address problems Leibniz had to describe motion without using velocity since time-derivative of displacement requires individual reference to identify states of motion [41].

A non-inertial frame of reference is defined as a frame where Newton's first law is not applicable because it is accelerating or rotating, however the frame still required continuous state of normal matter, motion, and time. It is very unlikely there is a place in the universe where matter is stationary. Considerations in the past have been made for an absolute, or universal, frame of reference but this has been dismissed. However, decision about such a frame uses a perspective that is based continuous motion, states of matter, and time, notwithstanding modern methodologies to model these phenomena. In discrete time modeling, movement (spatiotemporal translations) can be geometrically expressed as discrete changes in relative locations of two independent privileged points in space. A choice was made in this article to use privileged "point" rather than privileged "frame" of reference since it seems a better fit considering the word frame may be misinterpreted as a 2D plane with breath and length quantities.

Physics uses key fundamentals that include modelling displacement of objects using time derivatives of displacement (speed or velocity; m/s), assuming a state of continuous observable (normal) matter, and a requirement for continuous time ($t \neq 0$). Various mathematics have developed solutions around an inability to divide by zero because it creates an indeterminate form, a type of singularity. To maintain consistency to the approach, infinitesimal calculus creates infinitely small time intervals that can approach but never reach indivisible zero. At the time of this publication, a methodology to remove time from a frame of reference [$t = 0$] has not yet been proposed. Nor has an ability to add time back into a model using object-oriented dimensional spatiotemporal metrics, absement [$L \cdot T$] or absity [$L \cdot T^2$]. Thus far, applications for time integrals of displacement (integral kinematics) have been limited to mostly the study of fluid flow and Lagrangian modeling of electrical circuits.

This article introduces the Rishta (Rt) system, derived from the Sanskrit word for relation, applying a signal system-based methodology for deriving independent object-oriented dimensional quantities for space (relative distance) and time. We describe how non-dimensional quantities, relational point location, can be defined in continuous time within a linear time-invariant system separating discrete time without loss of information. The approach uses relational recurrent durations of cyclic and discrete time within a linear map unified with a shift-invariant system in continuous time (independent variable). The property permits the initial value of the independent variable of each cycle to arbitrarily set to zero, $[t = 0]$, during the analysis of such a system (Ch. 2.16.2) [42].

By removal of dimensional quantities for relational point space modeling, a non-dimensional signal marks a relational location in a zero-Dimensional (zero-D) point of time. We walk though how to use Euclidean space to place relational points that can be used to add object-oriented dimensional quantities using geometric elements on 2D planes. Object-oriented metrics for space and time (intrinsic and extrinsic) are separated from spacetime whereby time-integrals of displacement and spatiotemporal units model motion using absement and absity rather than velocity or acceleration.

Emphasizing the point, it's crucial to note that a noncausal discrete-time system is generally not feasible for real-time implementation. This is due to the requirement of calculating the output value at a specific time, which would involve using future values of the input sequence. Such an implementation would only be viable by taking into account pre-recorded discrete-time input sequences [43].

2. Methodology

The universe is classified into three different components: normal matter, dark matter, and dark energy. This article focuses on signals and a casual system approach applicable for normal matter which is inclusive to both discrete and continuous phenomena. Signals and systems are typically associated with the fields of electrical and computer engineering, yet the principles are transferable to our applications in classical (non-relativistic) and quantum mechanics. We collectively refer to our signal system as the Rishta (Rt) system, inclusive to object-oriented dimensional metrics, a zero-time dimensional frame of reference, and the use of Euclidean geometry to express metrics and construct static models of dynamical systems.

Each of the system's components work together in harmony, similar to how Einstein's theory of relativity required unison between principles such as an inertial frame of reference, an observer frame of reference, spacetime metrics (rigid rod and light clock), as well as the linked fabric of spacetime. Continuous motion, continuous time, and continuous state of matter are well suited for contemporary equations and theories.

Using a causal signal system, measurements of real-world dimensional quantities create geometric canonical models defined in an instant of time(spatial) or a period of linear time (duration). A requirement for using dimensional analysis is that object-oriented dimensional quantities are taken from a modelled static state universe. A causal system can be used because known dimensional quantities are needed, predicted measures from the future cannot be used. A real-world system model cannot be noncausal as each instant allows for a swarm of possibilities in the future. Separating object-relative dimensional quantities of space and time work together with applications of object-relative spatiotemporal units. Finite object-relative dimensional quantities are infinitely divisible, and scaling units with formulas consistent with geometric symmetry groups and radix numerals inherently layer several properties into the scaled units, particularly important for downstream geometric modeling.

Using a zero-time frame and layering object-relative time periods opens opportunities for subgroup datasets to be associated with different (specified) time intervals, tremendous flexibility for a grid-based system. For N-body modeling, an omniscient perspective and a zero-D frame models can display multiple privileged zero-D points, each the origin of object-specific geometric constructs of dimensional quantities. The Rt system components, including unit symmetries and ordered numeral positions, work together to open new opportunities for Euclidean modeling using point, line, and plane with metrics based on object-relative dimensional quantities. The approach opens a consideration for using object-relative dimensional based equations. Each of the tools that include dimensional metrics (with selected ancient formulas), zero-time dimensional frame of reference, and geometric modeling work together harmoniously, the significance becoming increasingly apparent as applications develop and mature.

There are two classifications for the properties of objects, intrinsic and extrinsic. Intrinsic properties are object specific, these include rotational symmetry, relative axial tilt (relative to orbital plane), plane specific semi-major or semi-minor axis lengths (spheroid), and so on. Extrinsic properties can be subdivided into either cyclic or non-cyclic. Extrinsic cyclic properties would be consistent with orbital properties involving a parent body or a wave amplitude along an x-axis. Examples of non-cyclic extrinsic properties would be related to distances between two stars (objects) in the same or different galaxies in the universe and is beyond the scope of this article. Both classifications can be referred to as *object-oriented*, different from *object-condition*. Object-conditions ($\text{Object}_{\text{condition}}$) are defined as being descriptive of the object-oriented classification. Two familiar examples for extrinsic cycles would be $\text{Earth}_{\text{orbit}}$ or $\text{Moon}_{\text{lunation}}$. Object oriented analysis is a class of user defined data type which contains data members and member functions to operate.

The Rt system uses discrete (periodic or aperiodic) signals from normal matter. Signal counts are bound, or non-infinite. For example, Earth at one point did not exist, and in the future, it is predicted it will cease to exist as well, so any cyclic signals like a rotation or precession defined by the Earth are finite. Each object contains data that is unique to a particular moment in time and space. Matter is consumed by blackholes which themselves are also postulated to end [44]. Some theories hypothesize the universe as being cyclic [45]. The Rt system can encapsulate modeling a cyclic universe, consistent with an Oscillating universe theory [46], with a beginning and end.

2.1. Snapshot Function

We begin with the unification of linear continuous mathematical time with object-oriented cyclic time for canonical modeling and embrace the omniscient observer perspective. A novel snapshot function uses object-oriented recurrent signals to define independent dimensional quantities for each cyclic interval. Using this function, one discrete signal ρ [n], does not overlap with continuous time from one cycle to the next, modelled as a non-dimensional and non-divisible time point, $\varnothing_t$ (**Figure 1**). Singularity and other special signal types used in modeling are commonly used but solely based on continuous time and dimensional properties, even a photon of light has dimensional properties. Instead we use Euclid definitions of zero-D points, with no parts and indivisible. Mathematically, without this function a signal, even a singularity signal, would overlap linear continuous time from one natural cycle to the next. Unlike Newton and other works, this zero-D point can contain no mass as we consider this a dimensional property which is divisible. The function passes the vertical line test at [t = 0] as there can only be one zero-D instant in continuous time with a change in direction or cycle. Object-relative geometric centroids represent the origin for both a privileged point and observer frame of reference, a zero-D object/observer centroidal origin (OCO) point. This zero-D point in Special Relativity correlates to the light cone's zero-D origin, without time or space on either axis.

The real cyclic signal for a snapshot function is a zero-D location, an alignment of zero-D points to a defined position relative to one another. In zero-time, a point is in a stationary state with no dimensional attributes of matter (mass, motion, force, etc.) so the observed state does not change when it is measured. By current definitions and using exclusive use of linear time, zero-time cannot exist in continuous linear continuous time. This is addressed however by unifying continuous and cyclic time, whereby a cycle offers a natural signal as a moment of change in a modelled state where an object must stop in order to change direction as there is no such thing as infinite acceleration. In a modelled state, direction in the model can change, for example from up to down (**Figure 1(a)**) or toward to away (**Figure 1(b)**).

Signals in continuous time, including a unit impulse function, occur over an interval of linear time, no matter how infinitesimally small of interval. In contrast, we argue that in a relational zero-D model of recurrent cyclic time (periodic or aperiodic) there is a defined start and stop, an indivisible instant in time [t = 0], $\varnothing_t$. This is mathematically shown in the equivalence of a cycle start/stop in time [1 (0)/(12) = (12)/(12)].

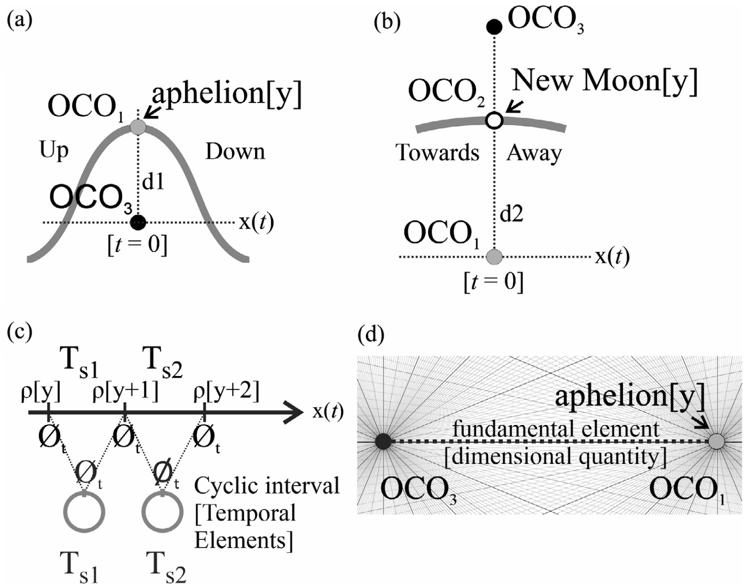

Figure 1. Snap function. (a) 2D orbital wavefunction with cresting zero-D OCO_1 point at [t = 0], change of direction from up to down. (b) Alignment signal (New Moon) of three zero-D points on a dimensionless line setting point at [t = 0]. (c) Synchronization of linear and equal cyclic invariant time periods. (d) Two-point perspective of relational distance for cyclic signal setting dimensional quantity of distance [L]. Earth centroidal origin: OCO_1. Moon centroidal origin: OCO_2. Sun centroidal origin: OCO_3.

2.2. Independent Time and Space Metrics

To measure dimensional quantities and later identify unique temporal intervals integrated into a model to represent a cycle, three things are required. First is a defined object whose cycle interval is being measured. Second, identification of the object-oriented cycle being measured, these can be intrinsic (rotation) or extrinsic (object-relationships like orbit). Lastly, identification of the signal [ρ] defining the start/stop position for the cycle (e.g., aphelion, fixed star, etc.). For aperiodic signals, intervals are unique from one cycle to another. Time measures have a fundamental symmetry (Dihedral$(D)_{12}$) associated with the quantity being measured.

Thus far there are three types of temporal intervals that can be defined by cycles in a linear time invariant (LTI) system. These include sample periods, frame periods, and intervals from pairing functions. There are numerous cycles, each needs to be defined given it is unique in space and time. For modeling, the user selects the data relevant to the goal of the model output and an interval of cyclic time is a transformation output of the system, incorporated into the 2D model with a canonical temporal period of reference using a process that will be described.

One universal frame of reference in an extensive model can have many sub-models, each independent of each other as a whole. Within each sub-model, a duration of time can be represented by spatiotemporal units and geometric translations. A methodology is needed to geometrically, and algebraically, differentiate each object-oriented cycle and to do this, temporal and spatial metrics need to be unique, differentiable, and object-oriented dimensional quantities. The period itself also needs to be identified as being unique in the life cycle for the object, a modeling process beyond the scope of this article.

For a specified cycle, the temporal interval is measured in continuous time between discrete signals and is a divisible dimensional measure of time. Signals can be either periodic or aperiodic, creating a sample period T_s measured between the first $\rho\,[y]$ and next recurrent signal, $\rho\,[y+1]$ (**Figure 2**). Because a signal occurs using zero-D properties, it does not affect the measure of continuous time. For $\rho\,[y]$, ρ is the description of the parameter of the signal and y is the count in the sample or frame. Like a window function in signal processing, the summation of two or more recurrent periods from the same cycle is being termed a frame period. A frame input signal $\rho\,[y]$ and frame output signal $\rho\,[y+\mathbb{N}\,]$ create a sum of continuous linear time measured by dimensional units, SI units, and named the frame period, T_f. The example, Equation (1) below, presents a summation formula and illustration of two recurrent cycles.

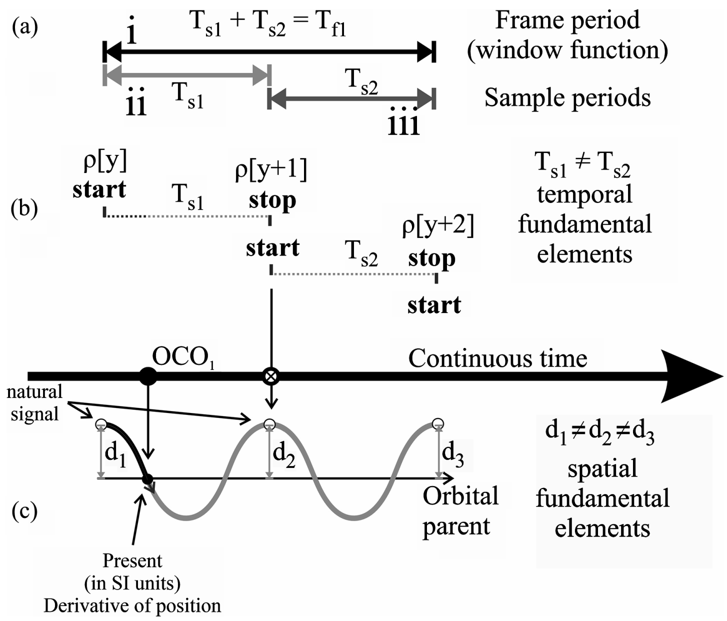

Figure 2. Discrete time signals in linear continuous time. (a) A frame period (i) composed of two recurrent sample periods ((i) and (ii)). (b) Two unique intervals, temporal elements, of linear time invariance between aperiodic signals. (c) Synchronized 2D wave function of cycle with different spatial distances to orbital parent, or amplitude. Present instant in non-causal system shown as derivative of position.

$$\sum\nolimits_{y=0}^{2} T_f = T_{s1} + T_{s2} = x \text{ seconds} \tag{1}$$

Given each object-oriented cycle is bound there is a beginning and an end, the lifecycle of object-oriented signals. A point of reference used to define the first cyclic signal is defined as alpha, α (also shown as y_α), a zero-dimensional point in time and space, $\varnothing_\alpha$, with the first completed cycle being $y = y_\alpha + 1$. Omega, ω (also shown as y_ω), is the last signal count for the defined signal. Within the life cycle of an object, from alpha to omega, W (designated after a window function in signaling systems) is a natural number count of whole recurrent cycle(s) measured for a designated period, written as Equation (2):

$$\sum_{y=0}^{2}\left[T_f\right]=T_{s1}+T_{s2}=2W \tag{2}$$

Designation of one unique cycle in the entire life cycle of the object is theoretically possible to geometrically model in a relational system and should ultimately be interpretable to an analyst. Modelling rules to build this capability are beyond the scope of this article. A bound sequence of cycles in a lifetime is counted using natural numbers and notated as $\mathcal{S}$, in which the y^{th} number is a complete cycle natural number count y in the sequence is denoted $\mathcal{S}[y]$ and formally written in Equation (3) as,

$$\mathcal{S}=\left(\mathcal{S}[y]\right),\ \alpha\le y\le\omega \tag{3}$$

where the y^{th} number is an ordered natural number count of a defined signal taken.

Related to geometric modeling, independent metrics for spacial dimensions are also assigned a symmetry group property, D_{360} symmetry this is separate from rotational symmetry (D_{24}) and time symmetry (D_{12}). The temporal sample period for Earth to make one axial rotation, a cycle specific dimensional quantity, can be converted into time-based units using a familiar formula $N = 12(30^1) = 360$. To make this sample period equitable to SI units, a relational spatial reference signal is required (solar or fixed star). Associating this interval of time with a distance, or length interval between [a, b] using spatial symmetry, creates very specific object-oriented spatiotemporal units, a process that will be discussed in step. It is important that spatial and temporal symmetries are maintained, and attributes of dimensional analysis are adhered to.

Clarification is needed between symmetry D_{360}, and $N = 360$ base units, or parts of a whole, and 360 degrees. The familiar 360 degrees, followed by arcminutes and arcseconds, of a circle are spatial units of a circumference derived using a symmetry group (D_{360}) and formula, $N = 360(60^2) = 1{,}296{,}000$. In this application, the same spatial division is applied to a straight line as well, a dimensional length quantity. For example, given a base quantity length (semiminor axis of Earth), it can be divided into 360 base units with D_{360} symmetry. In this scenario, one unit circle has a radius equal to one base unit and the unit circumference equal to one degree of the whole circumference. In both, the measure is a dimensional measure, one is straight, and one is curved. When considering 360 degrees to represent a time quantity, this is an incorrect interpretation using dimensional analysis as temporal quantities (days or hours) are not equitable to spatial quantities (degrees) as per the principle of homogeneity of terms.

Numerically, it is easy to confuse the number 360 in various interpretations, an example of this can be seen in a clock. When a temporal cycle quantity is converted to 360-unit counts, perhaps one rotation of the Earth measured from Sun at zenith-to-zenith signals at a particular point on the planet. The temporal period [$N = 12(30^1) = 360$] is assigned to a circle with a 360-degree circumference on the 24-hour clockface, this also aligns with rotation symmetry, D_{24}. The hour hand moves in 360 discrete steps around the circle. In this scenario, each step, or arc, contains both spatial (one degree) and temporal unit properties (one part of whole, 1/360) and is therefore defined as a spatiotemporal unit. This is an application of time-integrals of displacement, formalized as absement $[L \cdot T]$, or the first time integral of displacement. A small but important consideration when geometric modeling based on the principles of dimensional analysis and homogeneity of symmetry.

Maintaining symmetry group homogeneity and dimensional like quantities highlight how D_{12} time symmetry is not equal to D_{24} rotation symmetry, notwithstanding modern definitions for SI units of time. Any cyclic movement however, including an object's rotation can be used as a measure of time (e.g., 24 hours in a solar day) however it requires an extrinsic reference to start and stop the time interval. On the other hand, an object rotation is an intrinsic reference, a property of the object itself. To take a dimensional measure of rotational time, the observer (OCO) needs to be on the surface of the rotating object so sample or frame period(s) can be measured based on the interval between recurrent signal(s) from a designated reference point (e.g., Sun or fixed star). Historically, humans have associated an aperiodic signal of a solar reference to define a sample period (solar day) with Earth's rotational symmetry base quantities and units ($N = 24\ [60^2] = 86{,}400$). Now we measure discrete counts of atomic transitions to define a SI unit *second* [22], thus decoupling dimensional time with Earth's rotational symmetry and a dynamical temporal sample period.

A temporal pairing function uses time intervals from two or more independent object-oriented cycles and synchronises them. In a paired cycle, potential for slight temporal differences require longer term considerations, like leap years. For this application, we assume dimensional time, measurable by SI unit seconds at a privileged point frame of reference, OCO can measure linear and continuous dimensional time relative to itself only at the origin. We can use a pairing function to synchronize temporal periods from many OCO points taken from anywhere in the universe in one of two different ways. The first is to mathematically align the start signals for each comparative cycle in the model (termed a cyclic synchronization; **Figure 3**), and second is to use an ad hoc signal observed from both observer frames of reference, like a super nova, consistent with relativity of simultaneity (termed observer synchronization). Because the temporal duration of each cycle is measured from a privileged observer frame of reference, there is no relativistic proper time interval to calculate. However, to stay true to the approach of the model, superimposing signals from two or more OCOs is recommended as being naturally occurring synchronizations rather than simulated alignments from two different instants in time relative to a shared linear continuous time. There are various ways to do this, including the use of solar eclipse shadow cone on the surface of Earth, giving a fairly accurate longitude and latitude position that can designate a naturally occurring cyclic synchronization point, a start position in space and time for Earth's OCO.

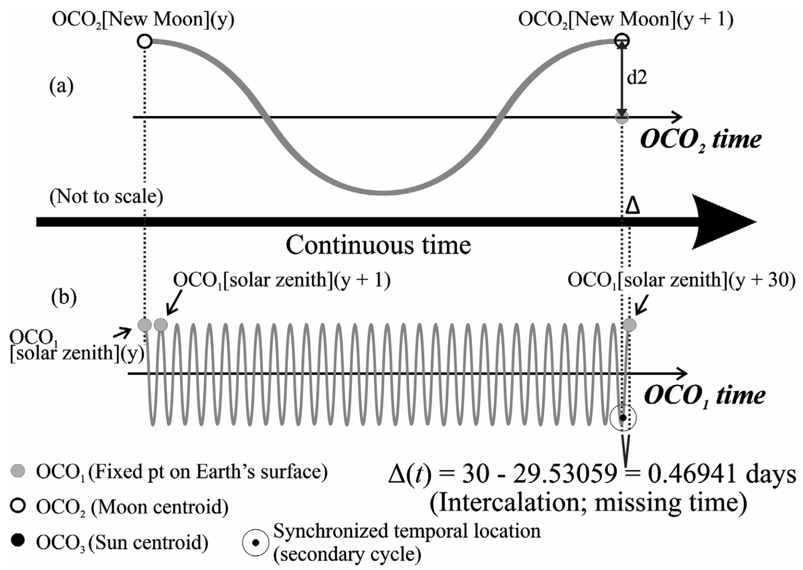

Figure 3. Pairing synodic month with Earth solar rotation. (a) 2D wavefunction of lunation, New Moon to New Moon signal through linear time invariant system counted by OCO_2. (b) Synchronized start signal for OCO_1, a specific point position of surface of Earth when Sun at Zenith, with rotational frame period counted (whole natural number count) in linear invariant time for OCO_1. d2 = distance from Earth (OCO_1) and Moon's (OCO_2) centroids at signal primary period of model, New Moon [y + 1].

Pairing functions have application for geometric spatial and temporal modeling but can also include more familiar calendric purposes when applied to astronomical cycles. For a pairing function, see pairing of Equations (4) and (5) for delta of 0.46941 days, there is a primary canonical reference that can be a sample or frame period. An example of a frame period would be the ancient Egyptian civilian calendar using 365 Earth rotations (360 days plus five days to honor the gods). This frame period can also be described by the Rt system as a base quantity that can be divided into twelve equal time symmetry base units, or D_{12}, shown as $N = 12\ [30^0] = 12$, or $N = 12\ [30^1] = 360$. Both approaches can take a perspective of either 360 plus 5 days (described in the Egyptian calendar year) or 360 units in a whole cycle, and are not mutually exclusive and stay true to the Rt system methodologies. Secondary period(s) are synchronized to the primary period with a delta (Δ) that accumulates over subsequent cycles that is managed using temporal intercalation events.

$$\sum\nolimits_{y=0}^{1} OCO_2\left[T_f\right] = T_{s1} = \text{Lunation primary cycle}\left(\textit{seconds}\right);\ \text{where } \rho = \text{New Moon} \quad (4)$$

$$\sum\nolimits_{y=0}^{30} OCO_1\left[T_f\right] = T_{s1} + T_{s2} + \cdots + T_{s30} = \text{Solar days, secondary cycle}\left(\text{seconds}\right);\ \text{where } \rho = \text{solar zenith} \quad (5)$$

Δ = Primary cycle-secondary cycle = temporal delta for paired function. Δ ≈ 0.46941 day

Beyond time, an object-oriented signal provides a unique instant for a spatial dimensional length measure for each cycle. Length measures are taken using zero-dimensional OCO points, which can include intrinsic and extrinsic distances. Intrinsic, or object-specific, lengths can include orthogonal planes with using the OCO of the object (**Figure 4**). For example, for rotating spheroids there are orthogonal 1D length measures taken from both the polar (p) plane and the equator (eq) plane, semiminor and semimajor respectively. As the quantities need to differentiate a plane from which they are derived, notation is needed where [γ] needs to be defined as either polar or equatorial for intrinsic object-oriented length measures. In an unusual scenario for a rotating (perfect) sphere, a distant reference, or perhaps the relativistic jet of a blackhole, other properties can help define Euclidean planes. The approach provides intrinsic object-oriented and independent length measures taken from orthogonal planes.

Having independent length measures for each plane provides applications in modeling whereby based units can differentiate plane specific quantities of matter on a shared 2D universal frame of reference. Euclid also defines a plane as a surface which lies evenly with the straight-lines on itself. So, here planes can rotate yet the lines remain consistent to the length definition, a property useful for discrete geometric modeling on a shared, or orthogonal, 2D plane(s).

Dimensional length measurements use a memoryless system, meaning the measure is only taken in the present as defined by a snapshot function as the final signal used to define a temporal interval for the model's canonical frame. Given the measure is a spatial dimensional quantity [length], symmetry is associated with D_{360}. At each snapshot [$t = 0$] spatial measures are specific to that instant, thus just like time intervals change each cycle, length measures can also vary. Examples of changing intrinsic length measures include Earth's expansion over time [47] [48].

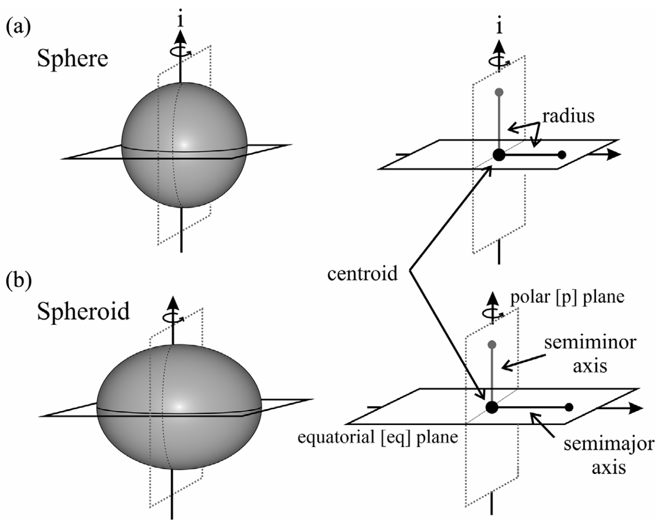

Figure 4. Intrinsic spherical and spheroidal object-oriented orthogonal length measures. (a) Sphere with axis of rotation used to differentiate two planes for intrinsic and orthogonal length dimensional quantities originating from OCO and terminating at the surface. (b) Spheroid with semi-major and semi-minor axis used for natural reference of dimensional quantities for length originating from OCO and on independent orthogonal planes. Both measures are taken at a designated signal output of a model, $t = 0$.

For generating length dimensional quantities, the methodology is consistent to principles of Euclidean geometry [49] for point and line. In contrast to Newton's point mass, this article considers mass a dimensional property and therefore by Euclid's first definition in Book I, not included in a zero-D point. A zero-D point is assigned to a centroid, geometric center, of an object, therefore in a relational model, represents only a location. Given properties of mathematical zero-D points, they can be overlapped and set upon the same plane. To emphasize, this is not true for matter, even electrons have dimensional characteristics. Relational distances being discussed in this article are described as a line or a length, with no breadth, consistent to Euclid's second definition in Book 1.

2.3. Dimensional Quantities and Rt Units

Dimensional analysis can be used for many different applications. This article introduces some examples of how cyclically unique base quantities of time and length can be both scaled and converted into base units consistent to their symmetry group. The approach uses ancient and many modern concepts, however, application of these attributes creates tremendous possibilities for downstream geometric modeling applications. Although the base unit's dimensional quantities change from one cycle to the next, they are real quantities of a fixed state of the universe at the instant they were measured. These units, and their various properties, are then used to geometrically model the state of that universe. The approach taken will set a path to unify the three major types of symmetry rolled into one more extensive symmetry for geometric modeling, these include translation symmetry (shifting objects consistent with Euclidean geometry), rotational symmetry (axial rotation), and scale symmetry.

We use ancient formulas with Dihedral symmetry (properties of rotation and reflection) and radix groups (*base-10*, *20*, *30*, *60*, etc.; positional numeral system) for our formulas that convert cyclic dimensional quantities measurable in SI units. In this article, symmetries include scale (D_{10}) [not to be confused with radix *base-10* used in a positional numerical system], rotation (D_{24}), space (D_{360}), and time (D_{12}). In Rt unit nomenclature, the Dihedral n symmetry of the quantity or base unit is denoted as n, shown as Rt_n. Each dimensional quantity is applied a formula that converts it into base Rt units, where the multiplicity (N) of the multiset of Rt units is also defined. Each Rt element, a rishtar, has object-oriented specificity as well as properties of symmetry, direction, time and/or length, and order. The natural number Rt base units are parts of a whole element, which is unlike SI unit base units which today are continuous and without order, are scaled smaller by factors of 10^x, decimal notation (consistent with scale symmetry; D_{10}) and create decimal notation with no whole. Also applying familiar decimal rounding rules for real numbers of individual Rt units will inherently remove accuracy for larger measures (scalar lengths) and cumulative errors will result with larger magnitudes.

Given there are multiple object-relative quantities a methodology was developed to capture the various properties of each unit or magnitude (k count) (**Figure 5**). For Rt units we used Euclid's description of units and ratios [49] as well as Egyptian unit fractions [50]. In Euclid's book V [49] [51], "greater magnitude [length]" can be set to be the equivalent of a fundamental element and the "lesser" of the greater. Interpreting Euclid's ratio presented in Book V for this application, a unit count (k) can extend up to, and including, multiples of lesser elements (N; multiplicity). Thus, a count (k) in Rt units can be considered a unit ratio count (k: N), in this case $k \geq 1$ in metric space. Magnitude can then be considered a length count of k of the whole N that can be converted back into SI units (or any dimensional units). When $k = 0$, this can be considered a point with no line elements, or characterized geometrically as an object specific, centroidal zero-D location, $<\varnothing_l>$, or point set location for a designated object in space. The Formula (6) below highlights the symmetry of the dimensional quantity (n) and most commonly, ordered elements from a positional numeral system, radix or base.

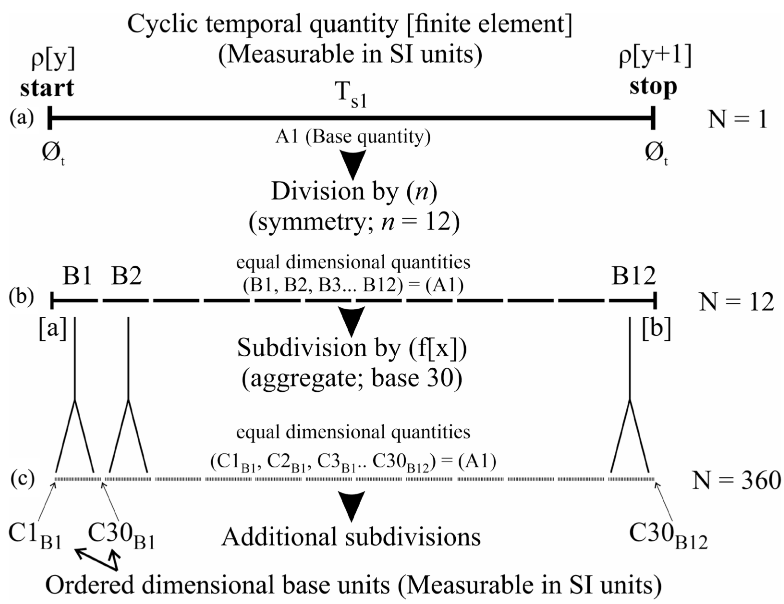

Figure 5. Uniform scaling a temporal dimensional quantity into base units using D_{12} symmetry. Line (a): ($N = 1$) Temporal dimensional base quantity for an object-oriented cycle. Multiset (b): ($N = 12$) ordered base units with temporal symmetry (D12). Multiset (c): ($N = 12$ [301] = 360) Rt12 base units with symmetry and positional numerical system used for order of time elements. Union of line elements from B equals base quantity of A, union of line elements from C equals base quantity of A.

$$N = n\left[f(b)\right] \tag{6}$$

For temporal Rt units, we present a modified approach influenced by Egyptian unit fraction technology. In a mixed-number Rt unit fraction count, k, represents an ordered part of one complete count of incremental units in a cycle, with the whole number (W) in the mixed fraction representing the count of completed cycles in a frame period. A numerator (counts of a cycle) splits unity as part of the procedure (expressing unity as $N/N = 1/N + 1/N + \cdots$), where N is finite and defined by certain rules and $1/N$ is a unit fraction, but in this case each unit fraction is in a unique sequence. The inclusion of a null multiset, Ø, can be shown in counts as ($k/N = 0/N + 1/N + 1/N + \cdots$), where k can equal 0 [$\mathbb{N}_0$] and k = 1 represents the first of N ordered geometric elements in the metric space of cycle fractions. Rt unit natural-number counts are consistent with Babylonian sexagesimal place-value notation, a 60-base system using finite sequences of sexagesimal digits 00, 01, …, 58, and 59, where both the first and last digits are non-null [52] [53], and the Mesoamerican vigesimal base-20 counting system (00, 01, …, 18, 19) [54]. In comparison to the Rt system, 00 corresponds to both 0 and 60 in a cycle, 1 0/60 = 60/60, or 1 0/20 = 20/20. In contrast to a linear system, for a cyclic system there is a risk of complicating interpretations considering the first point may be designated as one rather than a position (node) of 00, the beginning and end of a cycle. Natural-number counts represent addition of dimensional geometric elements that are part of a whole; the numerator and denominator in this application are not a fraction in the conventional sense and cannot be presented in a decimal notation or as a percentage.

Rt spatial term includes the properties so far introduced, including count (k), object or $\text{object}_{\text{condition}}$, multiplicity of the multiset (N), dihedral symmetry (n), and signal [ρ] or plane [γ], with the whole number (W) in the mixed fraction representing the count of completed cycles, which is used where Rt units are shown as,

Spatial terms

$$k\ \text{Object}_{\text{condition}}\text{Rt}_n^N\left[\rho\right] \ \rightarrow \text{extrinsic (object-relation)}$$

$$k\ \text{Object}_{\text{condition}}\text{Rt}_n^N\left[\gamma\right] \ \rightarrow \text{intrinsic (object-specific)}$$

Temporal term

$$W\ (k)/(N)\ \text{Object}_{\text{condition}}\text{Rt}_n\left[\rho\right]$$

An important consideration when interpreting a model is that a fundamental element is a base quantity and different formulas can convert the quantity into different Rt units. This creates scenarios where Rt units with different symmetries can have common geometric magnitudes. Euclid would describe this as having an equal construct measure (geometric magnitude) by two unique [Rt] units, but incommensurable (different common [unit] measures) [49]. For example, compare a spatial length $30\,\text{ObjectRt}_{360}^{1296000}\left(\text{eq}\right)$ measure to a rotational symmetry magnitude $2\,\text{ObjectRt}_{24}^{86400}\left(\text{eq}\right)$ [axial rotational symmetry]; both convert to a scale of 1:43,200 of the base quantity.

The proposed terms and notation offer flexibility. It is possible to modify the terms to where temporal symmetry is noted as spatial terms, spatiotemporal Rt units by using (and showing) time symmetry (n = 12) in a spatial term structure, $k\ \text{Object}_{\text{condition}}\text{Rt}_{12}^N\left[\rho\right]$. For logarithmic scaling using D_{10} symmetry there are differences to decimal notation, we scale up or down using base-10 multiples to maintain properties of the quantity or units but still allows for further scaling in natural numbers. An example is where $N = 360(60^2) = 1,296,000$ becomes $N = 360(60^2) \times 10^1 = 12,960,000$. This logarithmic utility enables a model output to be 10 times smaller, conserving the natural number unit k count (0.1 Object $\mathrm{Rt}_{360}^{1296000}$ = 1 Object $\mathrm{Rt}_{360}^{12960000}$).

Maintaining natural numbers and order of metrics that are object relative, allows for infinite dimensional scaling of the entire N-body model without rounding errors. In a single physical model, constructs use multiple units of measure, each particular to a symmetry group, object, temporal instant, and so on. Using Euclidean superposition "proofs", such as a circumference being a visual geometric expression rather than a numerical output calculated by π. If there were no overarching rules to guide the unit formulas, fractions, and removal of irrationals there are risks of complexity and generates large demands in computer processing. The units and relative proportions of geometric modeled distances themselves become visually interpretable to understand the properties of the object/relationships being modelled.

Other benefits for a natural number-based formula which divide and subdivide parts of a whole, include applications related to scaling. Scaling with decimal notation opens opportunities for rounding errors. By only changing a variable in a formula, a new multiplicity of a multiset for the base unit is created. Thus, an entire model with various object-relative datasets can be simultaneously scaled while maintaining integrity of symmetry and data. The same geometric dimensional model can be expressed across kilometers or down to nanometers by only changing one variable in the related formulas. The capability of the technology is therefore not limited by definition, but only the tools at hand for measurement and creation of physical models.

Although symmetry of time is consistent with D_{12} in modeling, as discussed it can also be considered for use with rotational symmetry of D_{24} as well, largely for historical familiarity. However, care needs to be taken so there is no confusion between time and intrinsic rotational properties. For example, a useful property of D_{24} is in modeling discrete step(s) for an object's dimensional rotation in space and time (like a spiral seashell, rather than rotational motion), applicable to spiral functions using an OCO as an origin, or fixed rotation point. Also, when designing or interpreting models a temporal period requires a signal reference to equate with SI units of time. So, without understanding if $[\rho]$ represents a solar or a sidereal signal the time measure the model could be misinterpreted or designed incorrectly.

3. Discrete Geometric Modeling

Spacetime geometric modeling cannot be uniform and isotropic. In contrast, the Rishta system can model object relational space and time in geometric models that are discrete, uniform, and isotropic. The Rt system can therefore utilize well defined Euclidean geometric modeling concepts but also use metrics that are object-specific, thus the expressed geometric shapes originating from a fixed point (OCO) are dimensionally accurate scales of the object itself. Motion is geometrically expressed using absement and Euclidean translations using object-relational displacements with unique dimensional spatiotemporal units. Discreteness uses properties of pointsets, in this case, object relative OCO's and Rt units (geometric elements with a beginning and end point). On the same model, both object-oriented intrinsic (e.g., rotation) and extrinsic (e.g., distance between to bodies) properties can be expressed. This opens up the ability to express an output with rotational properties of an object at the same time as distance to a parent star using different scales and Rt units.

Today, non-Euclidean three-dimensional (3D) and four-dimensional (4D models) are used to model objects and events in spacetime using an observer frame of reference. The Rt system discrete geometric outputs are static and physical Euclidean geometric models that do not use an observer frame of reference, only object-specific privileged points references, OCO, set upon a universal frame of reference that does not include time or space. Application of object-relational discrete geometric modeling will need to be built upon and formalized considering the diverse possibilities.

This section introduces some basic geometric model output models and theoretical applications. We present a novel 1D spatiotemporal orbital model and other 2D geometric outputs including lattice spacing and Archimedean spirals that can express object-rotational properties in discrete spatiotemporal steps. A slightly more advanced astrophysics model compares Earth and Mars intrinsic and extrinsic properties. The geometric outputs can also be used to study ancient calendar systems and quantum phenomena. Modeling with independent space and time metrics on a universal frame opens thought experiments around a continuous physical theory that can address discrete non-continuous motion of quantum particles.

3.1. One-Dimensional Orbital Model and Two-Dimensional Geometry

As we consider how to model an orbit, layers of information can be selectively layered for each N-body. These data can be visually represented in scaled geometric models using many attributes we are familiar with, including grids, and even angles relative to cardinal directions. Take for instance the orbit of a planet and its spatial and temporal data. Using the Rt system, several layers of information can be independently modelled enabling multiple orbital details to be simultaneously modelled, even orbits from independent star systems. The approach can simultaneously include each planet's intrinsic dimensional quantities like semi-minor/semi-major axis, tilt, and so on, and built upon a zero-D frame. In this section we describe a visual 1D dimensional canonical data model to express the time extension (duration) it takes for one unique orbit for an object that can be identified by the unique spatial distance to the orbital parent. Using the same approach, two planets (Earth and Mars) are shown on the same data model, each expressing their unique dataset in a relational model.

To express a 1D dimensional relational distance on a geometric model, interval [a, b], there are several ways to start. A basic approach is by applying scaling symmetry (D_{10}) to the dimensional length quantity, but there are several other approaches that can use more elaborate procedures. A second approach is to consider time integrals of displacement using dimensional units and natural constants. For example, a "temporospatial" displacement is where time is fixed, and length is variable. This is already done using a fundamental constant, the absement [L · T] of light to define 1 meter length. Consider the meter as a "temporospatial" displacement, for example, c^a = [L · 1/299,792,458th of a second], where c^a = absement of light in a vacuum (299,792,458 m·s) and L equates to one meter by SI unit definition [22]. Changing the time interval to 1296/299,792,458th of a second would then provide a distance of 1296 meters. Used in modeling, 1D intervals can be defined as geometric measures of light using dimensional analysis and time integrals of displacement that are object specific. For example, one Earth second, based on a temporal measure of rotational symmetry, may be longer than one exoplanet second if that planet is rotating faster than Earth, thus creating a shorter dimensional length [L] measure for that planet.

In contrast to temporospatial lengths, spatiotemporal lengths begin with a fixed length, interval [a, b], perhaps a scaled dimensional quantity. A dimensional quantity of time can then be equated to [a, b], which now becomes the representation of a whole temporal cycle that can be divided and subdivided into spatiotemporal Rt units (e.g., N = 12 [30^x]). Important to note, in an actual model, averages are not used (as shown for examples), rather the measures would come from a unique and specific instant in space and time recorded at the signal ρ [y + 1]. Both the dimensional length, Equation (7), and dimensional temporal interval, Equation (8), for the model would share the same stop signal marker.

$$\begin{aligned}[a,b] &= 360\ \text{Earth}_{\text{orbit}}\text{Rt}_{360}^{360}[\text{alpheion}]\cdot\left(1/10^9\right) \\ &= \sim 147,098,925,000\ \text{m}/10^9 = \sim 147.098\ \text{m}\end{aligned} \tag{7}$$

$$\begin{aligned}&(12)/(12)\ \text{Earth}_{\text{orbit}}\text{Rt}_{12}[\text{aphelion}]\cdot 1/([a,b]) \\ &= (\sim 365.259636\ \text{days})/(147.098\ \text{m}) = \sim 2.483\ \text{days/m}\end{aligned} \tag{8}$$

For canonical geometric dimensional analysis, maintaining the principle of homogeneity of terms and symmetry group homogeneity adds capabilities for modeling, either algebraic or geometric, see Equations (9) and (10). Time can be measured from any privileged point of reference and equated using very small intervals of time, for example, using Vedic style formulas N = 12(30^6) = 8,748,000,000, a level of precision limited only by today's ability to measure the interval.

$$1(0)/(360)\ \text{Earth}_{\text{orbit}}\text{Rt}_{12}[\text{perihelion}] \approx 365.259636\ \text{Earth days} \tag{9}$$

$$\begin{aligned}&1(0)/(8748000000)\text{Mars}_{\text{orbit}}\text{Rt}_{12}[\text{perihelin}] \\ &\approx 1(7265380125)/(8748000000)\text{Earth}_{\text{orbit}}\text{Rt}_{12}[\text{perihelion}]\end{aligned} \tag{10}$$

Extending the application further, geometric equivalence models can also be generated as transformation outputs. We geometrically express a cyclic synchronous model using the same algebraic information using the Mars orbital period shown above as the canonical reference for comparing Mars to Earth. In the output, interval [*a*, *b*] in this case uses a scaled aphelion orbital distance for both relative Mars and Earth distances (**Figure 6(a)**). Intrinsic and extrinsic properties can be extensively modelled using only geometry, including axial tilts relative to an orbital plane on a universal frame of reference aligned to North as well as orbital distances. The Rt system does not use abstract numbers that are not associated with any quantity. Nor does the Rt system require irrational numbers like Pi(π) as the geometric model is physical, so Euclidean superposition modeling can visually take a circle, or arc, to find a center point which would provide the measure a radius, a radius constructed using the natural number Rt unit particular for that object-specific zero-D point. A more extensive model incorporating several layers of data is shown expressing groups of datasets, each independently decipherable if one of the object-oriented datasets are known (**Figure 6(b)**). The relative scale of the planets on the polar plane uses spatial geometric measures, e.g., $15\,\text{MarsRt}_{360}^{1296000}[\text{polar}]$ and $15\,\text{EarthRt}_{360}^{1296000}[\text{polar}]$, as a radius for a circle construct, useful to locate the zero-D OCO point for Earth on a 1D orbital line and identify what plane is being modelled.

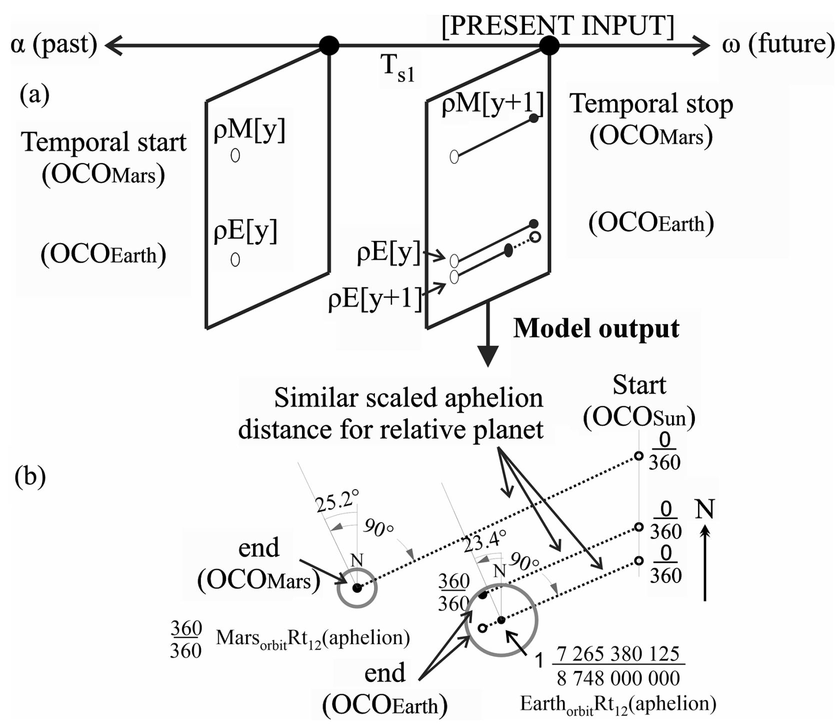

Figure 6. One-dimensional orbital models with relational properties. (a) 1D orbital model of selected sample period (T_{s1}) of Mars(M) orbit (aphelion [*y*] to aphelion [*y* + 1] signal). Cycle stop at $t = 0$ at [*y* + 1]. Synchronized start of Earth(E) orbit at aphelion signal [*y*]. Orbital temporal count by 1D orbital model for Earth equal to T_{s1}. (b) Geometric output operations at discrete time signal express object-oriented orbital distances, axial tilt, relative polar semi-major axes lengths, and relative orbital time for T_{s1}.

The simplest 2D symmetrical object created by plane rotation of a 1D line (Rt_n unit) anchored by a fixed point is a circle (**Figure 6(b)**). Euclid described a circle as being a series of lines of the same length with a fixed origin. In the Rt system, an object-relative Rt unit circle, or arc, is defined using the first ordered element (k = 1) extended from a zero-D point acting as an object-relative null set, $1\,\text{ObjectRt}_n^N(\gamma)$. The construct is denoted as $1\,\text{Object} \odot \text{Rt}_n^N[\gamma]$ with a radius of $1\,\text{Object\_rRt}_n^N[\gamma]$, where the operator $\odot$ designates a circle construct for the Rt term and r = radius. The resulting plane rotation creates a single natural-number unit circle, for example $1\,\text{Object} \odot \text{Rt}_{360}^{1296000}[p]$, where [$p$] refers to semi-minor axis. Applying intuitive geometric superposition principles, the function simultaneously defines three linked-unit geometric superposition functional outputs that are visually discernible and also measurable:

$$1\,\text{Object\_rRt}_{360}^{1296000}[p],\ 1\,\text{Object\_dRt}_{360}^{1296000}[p], \text{ and } 1\,\text{Object\_cRt}_{360}^{1296000}[p],$$

where c = circumference and d = diameter and the count of one is a quality-defined measure of a familiar abstract arcsecond of 1,296,000 arcseconds/circle. In Rt modeling, there are no abstract numbers so 1 arcsecond is always associated with an object quantity measured at a unique moment in space and time.

There are several functions that can be developed for this system. Establishing orders of operations with system variables like plane rotation, symmetric rotation, lattice rotations, as well as temporal, spatial, and spatiotemporal Rt units, opens various options (**Figure 7**). A plane-specific discrete spiral (**Figures 7(c)-(e)**) can be constructed using a function that adds one spatial (or spatiotemporal) Rt unit at each step (denoted concisely as $k + 1$) with one perpendicular plane axis rotation. A discrete planar Archimedean spiral [55] maintains consistency with natural-number counts, Rt unit symmetry, planar centroidal coordinate systems, actions of a discrete group, zero-D fixed points, rotation, and so on. Building an Archimedean spiral integrates properties of a polar coordinate system and axial rotation perpendicular to the plane of the spiral, with the origin at the zero-D point. Because both θ and k are positive, the spiral grows counterclockwise from the central fixed (zero-D) point.

The number of rays of a classical polar coordinate system range from 2 to 360, in the adapted approach for the Rt system, the number of rays can be either set using the symmetry group (n) or multiplicity (N). For example, with Rt_{24} there are 24 rays with 3600 (or 60^2) rotations for 86,400 discrete steps, equal to 86,400 counts within Earth's rotation. When N is used, there are N rays, and exactly one spiral rotation creating the same full scale. In either option, when $k = N$, the full-scale plane-specific construct is complete.

3.2. Quantum Biphasic Matter-Energy Equivalence

Since the Rt system is based on discrete signals, where space and time can be separated, it suggests the approach is well suited for the discrete qualities of quantum physics. Similar concepts have already been used, including descriptions for Planck limits such as length and time. One Planck time unit (t_P) is similar to the

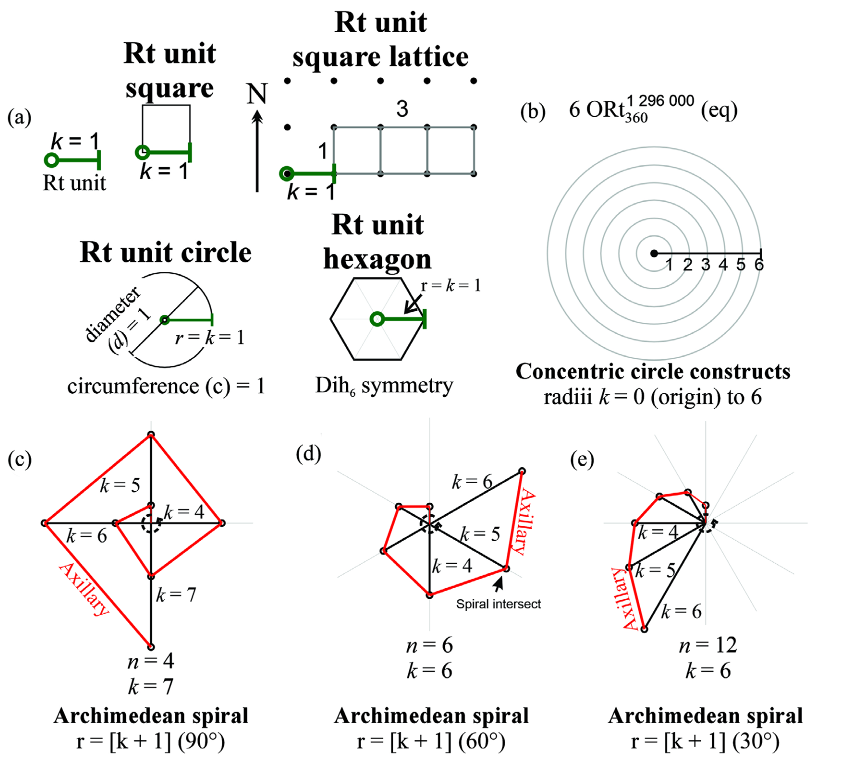

Figure 7. Two-dimensional geometric structures using Rt symmetry properties. (a) Rt unit square, unit circle, unit lattice group for universal frame of reference, and unit hexagon shown using OCO plane rotation and unit symmetry. (b) Radius units (1 to 6) with concentric circle constructs using shared fixed point. ((c)-(e)) Outward planar discrete Archimedean spirals (counterclockwise) using Rt element radials intersects ("*n*=" rays). Red line (auxiliary) connects each sequence of terminal $k + 1$ radii.

methodology of the Rt system defined as the duration between photon signals is equated to one temporal base quantity, or sample period. No current physical theory, especially one based in spacetime, can describe timescales shorter than the Planck time. Since the Rt system separates time and space, independent shorter timescales can be mathematically proposed which aligns with the definition of continuous time being indivisible, with possibly infinite intervals, see Equation (11).

$$1\,t_P = (360)/(360)\,\text{particle}_{\text{condition}}\text{Rt}_{12}[\rho] \approx 5.39116\times10^{-44}\ \text{s}, \quad \text{where } N = 12\left(30^1\right) = 360 \tag{11}$$

Quantum particles are typically imagined as traveling in a straight line in discontinuous motion through continuous space, typically modelled in the center of a wave function. The wave properties of light has been well studied in context of rotating 2D waves with circular polarization first described by Fresnel in 1821. At the time of this publication, the authors are unable to find a proposal for particles to be following a helical orbit, similar to planets orbiting a moving star (**Figure 8(a)**). In a thought experiment based on an understanding of the Rt system, we initially explore what may be possible if at the quantum level, matter and energy are in a biphasic transition.

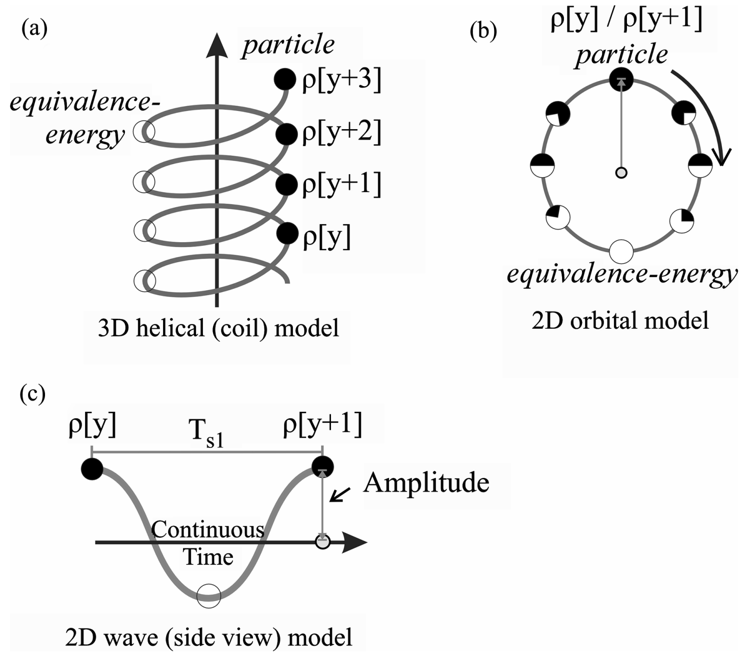

Figure 8. Quantum orbital model with biphasic dualism. (a) Helical particle orbit during a biphasic transition, consistent with particle following a circular polarization. (b) 2D circular/elliptical-like orbital model of a linear transition in biphasic composition of matter and energy. (c) 2D wave model with cyclic unique amplitude and temporal sample period, Ts1. LTI = Linear time invariant system.

Modeling a photon's OCO in a helical orbit and introducing a quantum biphasic matter-equivalence proposal, we begin our thought experiment (**Figure 8**). Following the photon's observer centroidal origin, it is possible to maintain a continuum in the quantum state, yet an observer witnesses' particles and properties as discrete observations. Given the proposed dimensional biphasic limit and superposition principle, any P_1 would be observed as normal matter speculatively consistent with observed signals traveling a straight line. Questions are raised around if the model would be consistent with quantum rotational momentum, spin angular momentum, and orbital angular momentum as well as upholding the particle-wave duality phenomenon.

If you increase the mass at a given force the rate of acceleration slows. Therefore, mass is inversely proportional to acceleration of matter, but what if acceleration approaches the speed of light. We propose that as vibrations speed up or matter approaches the speed of light, a barrier is reached where matter enters a phasic transition with matter-energy equivalence. If the vibration, or matter reaches the speed of light, the state of matter as we know it in our reality no longer exists, only an energy-equivalence. In the energy-equivalent state we propose there are no dimensional properties including mass and gravity. When the speed (or vibration) decreases, energy enters a biphasic transition again and eventually passes a limit and enters a continuous state of normal matter. The proposal is consistent with existing hypotheses that energy can be converted into particles [56]. Therefore, a biphasic state suggests that observations and descriptions of quantum particles, energies, collisions, gravity and so on, are the study of entities in this proposed biphasic state. The model supports that continuous time is independent of relational time which is based on direct observations of dimensional properties.

When we consider quantum gravity, Newton's units for the gravitational constant raises a question about if gravity may be associated somehow to acceleration of phasic matter through time. Units for the Gravitational constant, $6.674 \times 10^{-11}\ m^3 \cdot kg^{-1} \cdot s^{-2}$ and when used in the formula for gravitational force between two bodies, $F = Gm_2m_2/r^2$, the units remaining are kg·[m·s^{-2}] kg·[m/s^2], equal to mass times acceleration using time differentials, or [kg·m·s^{-2}]. When using time integrals, it can be shown in dimensional units as [$M^1 \cdot L^1 \cdot T^2$]. We consider if the biphasic transition of matter *through* time occurs with an exponential shift, or acceleration of phasic matter (**Figure 9**) which can be shown geometrically. We speculate if acceleration *through* time is associated with the shift in normal matter densities if this may be related to signals of discrete gravity. We also speculate if energy-equivalence state can also accelerate (or change direction) the dimensional signal would appear to change velocity and perhaps shift wavelength without an observed dimensional acceleration. This would require a type of forces able to act upon energy-equivalent state, a potential for electromagnetic forces which can be tested. If an electromagnetic force, without dimensional quantities like mass, can alter the path of a photon we propose this would support the hypothesis. Proposing a dimensional limit to matter suggests that within the quantum state, changes in either a quantum wave amplitude or wavelength will not change the temporal period between the periodic signals.

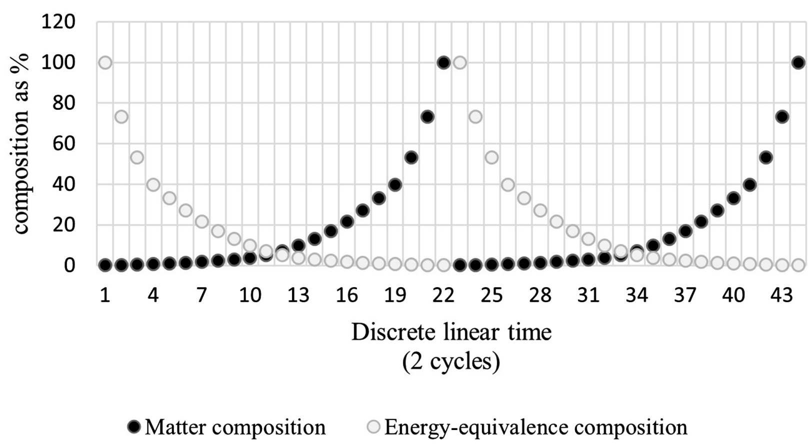

Figure 9. Exponential shift in biphasic of matter and energy-equivalent composition. D_{22} symmetry selected for graphing cyclic and exponential biphasic transition between matter and energy-equivalence in quantum state. Two recurrent cycles plotted in discrete and linear time with shift in composition between biphasic states.

We graphically represent discrete linear time (x-axis) in a cycle with exponential increase in matter and energy-equivalence at passing discrete linear points in time using a 22-gon symmetry, inspired by Hebrew letter position and value associations. Graphing the summation creates an exponential curve (**Figure 9**). When modeled as a circle, the design is similar to the Chinese yin-yang dualism symbol (not shown).

4. Discussion

Efforts to reconcile classical and quantum physics have been based on fundamental concepts of time, continuous zero-D point mass, and an inertial frame of reference, including principles and postulates associated with relativity. Quantum Logic and Probability Theory, as well as advancements in the standard model each have varying degrees of success. In this article, we have introduced an alternative methodology to model matter, space, time, and motion using dimensional quantities of objects and fundamental symmetries of nature. The approach combined numerous principles from dimensional analysis, classical and quantum physics, as well as ancient signal timekeeping systems and Euclidean geometry. The approach uses a dimensional matter-based signal system applicable to continuous or discontinuous dimensional properties in a recurrent cycle.

Potential implications of adopting object-relational dimensional analysis and the proposed modeling approach for the field of physics and computational discrete geometric modeling may open up new possibilities for both research and applications. One application already in development is a multi-celestial, or N-body, timekeeping device. Methods are capable of relating both orbital and solar day times from various planets, including moons, on a single customized display, either digital or analogue. Using multiple privileged points of reference and methodologies described, challenges related to time dilation and proper time intervals can be addressed.

In research, this alternative approach to measure and model matter (orthogonal planes), space (extensions of relative distance), time (extension of time, or duration, between cyclic signals), and discrete translational motion offers new tools to explore with. Including how to use object-oriented dimensional quantities and spatiotemporal units for canonical data modeling and equations. The approach opens new applications for time integrals of displacement and offers suggestions to explore complex equations using object-oriented dimensional time $[T_r]$, where r represents a relational dimension, unique from SI units which are based instead to natural constants like the speed of light. This opens an ability to utilize real-world dimensional data, like an entire temporal cycle as $[T_r]$ for example, and visually express complex datasets in a static model with tremendous precision. Extending the possibility further, an object-relative dimensional unit system may be worth exploring, including not only time $[T_r]$ and length $[L_r]$, but also mass $[M_r]$ and other object-relative quantities that can be scaled, linked to a unique instant in space and time. Such an approach would be exceptionally helpful for computational geometric applications and canonical data modeling dynamical systems, including perhaps stelar cartography.

Relational physics in the Rt system does not require an inertial frame of reference and is able to separate time and space which does not require a continuous state of matter. Considered impossible using time-derivatives of displacement, continuous zero-D point mass, and other principles of relativity, we described a universal frame of reference *in* zero-D time and space. By separating space and time into object-oriented metrics, new possibilities for modeling were presented, including a 1D spatiotemporal orbital model for canonical modeling, ancient calendar interpretations, and exploration of a biphasic natural continuum with discrete signals of matter in quantum states.

Introduction of the Rishta system focused on three key topics (1) object-oriented dimensional quantities and base units for spatial distance (D_{360}), time (D_{12}), rotation (D_{24}), and scale (D_{10}), (2) a universal frame on which to model, and (3) basic concepts for combining Rt units with discrete Euclidean geometric ($\mathbb{N}^n$) modeling. The ability to use object-relational dimensional units opens up the ability to physically model relational and scaled states of objects, or discrete signals from objects when observed, and creates an output of the state of the universe when it was observed. The approach opens up a new direction for computational discrete geometric modeling space and time using discrete spatiotemporal cells.

The Rt system used Euclidean geometric definitions for zero-D points, taken as location signals to set periods of cyclic time that preserved linear continuous time by using a proposed snapshot function. Object-relative zero-D privileged points of reference create a zero-D point in space and time, with no divisible dimensional qualities of matter or time. The snapshot function passes a zero-D vertical line test using natural cycles with points and lines. If the approach tested using object-relative zero-D point mass (a divisible quantity) the snapshot function would fail. Using familiar zero-D mass points would be more applicable to a singularity function.

By separating divisible cyclic time periods with zero-D points in time, dimensional quantities can be measured in either a memoryless system (length) or a memory system (time). Leveraging ancient methodologies and unifying these with dimensional analysis, we introduced object-oriented dimensional quantities and base units. Unlike an approach to measure and model using a single standardized measure, a single standardized measure is used to measure natural quantities converted into object-relative base units for constructing model outputs for scaled representation of that quantity. Length measures are taken from a single snapshot and time measures between snapshots, units embrace symmetries of nature to develop elements with properties of symmetry for downstream object-relative geometric dimensional modeling. These quantities create object-oriented symmetry specific metrics for cyclic relational distance (D_{360}), time (D_{12}), rotation (D_{24}), and the symmetry of scale (D_{10}). Symmetries are well characterized in ancient texts and still used to this day in similar applications, yet with reduced functionality as proposed in this article. Two new symmetries are touched on, including D_{18} and D_{22} which require more exploration before assigning specific attributes. The importance of ordered Rt units is difficult to contextualize with the limited applications introduced in this article. As more detailed and complicated geometric modeling applications are formalized and introduced, the importance of symmetries and positional numerical values will become more evident.

Object-oriented dimensional metrics for space and time create tremendous potential for constructing physical discrete Euclidean geometric shapes for the state of the universe when observed. Consistent with Euclidean definitions, the points have no properties other than location, yet can contain information of the object with which is modelled from this fixed point on the Universal frame. Extensions between zero-D points do not move or change in zero time, the distance between marked by a 1D line (length and no breadth) and can be posited on a 2D plane (length and breadth only), without requiring a 3D coordinate system. Various geometric functions can be developed to describe motion and forces using object-oriented dimensional metrics. The approach build upon geometric modeling to include object-oriented dimensional metrics as discrete steps of rotation, time, and so on.

To patiently begin a path to open the potential of the Rt system, two very different applications presented include the first 1D spatiotemporal orbital model. The 1D orbital model opens up use of canonical geometric modeling that can express orbital periods from any orbiting object in the same class, for example a planet in our solar system against an exoplanet in a neighboring star system. The Rt system presents opportunities to advance geometric data modeling independent of any standardized unit system for time or length. In the 1970s, astronomers Frank Drake and Carl Sagan assisted NASA's missions related to attaching a message about who humans are and where we live in the galaxy, including what is called the pulsar map [57]. Our proposed canonical geometric data modeling system should provide opportunities to create greater detailed interstellar models using universal geometric expressions, a type of geometric communication instead of language as we currently understand it.

The system can also be used for considerations in applications for quantum physics. Through described methodologies, the Rt system separates space and time using discrete zero-dimensional signals. Quantum discrete phenomena like gravity or discontinuous particle motion can be used by the Rt system. As a result, the model opened up potential to explore quantum physics with a new tool. The Rt system has three key attributes useful for this application. The first is the system uses periodic or aperiodic discrete signals from normal matter, similar to discrete signals seen in quantum physics, meaning a continuous zero-D point mass is not necessary. In contrast, the Rt system uses a Euclidean defined zero-D point, a centroid that has no parts, that is without mass, normal matter, or energy. Whereas using a zero-D point mass assumes a continuous state of matter moving with continuous motion through continuous time (time-derivatives of displacement), discrete or continuous movement of an object relative zero-D point, OCO, does not require these assumptions. For modeling, the point contains all information in a non-dimensional state required to model the object.

The biphasic hypothesis combined several existing concepts that include wave functions, quantum interactions, and units for the gravitational constant. For *x* number of particles, a function of the systems momentum is all the individual particles P_1, P_2, P_3, … creating a superposition of all possible final states wave function. Using the energy-matter phasic equivalence proposal, a system with *x* particles sharing the same wave function can be imagined as a twirling tunnel of energy and matter, with each *x* particle shifting between equivalent-energy and an observed particle. It also offers a concept for describing probabilities and strength of interactions in quantum physics. A dynamical system is in flux and continually changing so particle, or object, has unique wave functions, angular momentum, and coordinates each cycle. Strength of interactions are hypothesized as being based on the phased state of matter-energy equivalence. The stronger the phase of matter then the stronger the interaction and higher collision probability. Therefore, matter (particles) should exist where energy is weakest implying collisions have a greater probability in a low energy state. The model also supports asymptotic freedom [58] [59] in that interactions between particles become weaker as energy scales increases and dimensional length scale decreases.

The quantum state is hypothesized to be at the dimensional limit of matter in our observational reality. Therefore, a change in the wavelength for light would not change the temporal sample period [T_s] for a photon signal. In this model, it proposes changes in wavelength would be possible by acceleration/deceleration of energy-equivalence. Although we study a photon as always being at *c*, meaning it does not accelerate, this model proposes that in an energy-equivalence state, the OCO could accelerate/decelerate in an energy-equivalent state creating a shift in a wavelength yet maintain a constant speed of light. Consistent with this model would be to consider an origin of a photon as the result of a slowing vibration of a stable energy-equivalent state. As a photon's energy-equivalence releases from an origin, the vibration slows, and the matter equivalent state (photon particle) becomes observed as a discrete particle of light in a biphasic quantum state near the speed of light.

Einstein described matter and energy as the same thing, matter being energy condensed to a slow vibration. A vibration in the context of this article is speculates as being unique from frequency as the unit of frequency (Hz) passes *through* continuous time, instead the term vibration is proposed as being consistent to a point *in* time. Consider Einstein's quote, "match the frequency of the reality you want, and you cannot help but get that reality." In this application, we consider the quote in a context where our observed reality of matter cannot extend beyond the vibration in which we exist to observe that matter. The proposed biphasic dualism hypothesis is difficult for modern physics to test since observational based findings are largely used in experimental research. However, given the plethora of double-slit experimental observations, a retrospective analysis of data interrogated against this hypothesis may provide some initial considerations.

When considering black hole evaporation, the phasic dualism hypothesis is proposed as being consistent to this phenomenon. In the biphasic hypothesis, matter reaches a quantum biphasic state near the speed of light. Once it reaches the speed of light the model proposes the state would be purely energy-equivalent. Given the proposal that acceleration of phasic matter is tied to gravity, when in a purely energy-equivalent state this logically assumes there would be no gravity. Schwarzschild proposed acceleration to the speed of light at the event horizon, and considering a state of matter cannot travel faster than light, we suggests that at the event horizon, matter completes a transition to an energy-equivalent state. Thus, the black center of a black hole is not because light cannot escape, but rather there are no photons of light, only their energy-equivalence. Given no dimensionality of matter would exist in the black central sphere, there would be no gravity, this contrasts with the mathematically proposed gravitational singularity from relativity. Energy-equivalence may not feel the effects of gravity thus may be able to pass through the event horizon, proposed as being consistent with Hawking's radiation.

The Rt system does not contradict relativity but rather creates an alternative way to model descriptions of space, time, and motion that are applicable to discrete signals from both classical of quantum physics. The modeling approach more closely aligns with qualities of Leibniz relational physics rather than Einsteinian relativity. A few initial comparisons between the Rt system and Leibniz physics include causality, extensions, time, and frame of reference.

We propose that Leibniz physics may be better suited if reinterpreted as a causal system, similar to the Rt system. Both systems can model kinematics but cannot be directly used to formulate laws of motion, make predictions into the future, or describe forces. The Rt system uses past and present inputs to model the static universe in the moment it is observed. For distance, Leibniz focused on relational distances between two points, so called extensions, that are infinitely divisible and do not move [60]. Aligned with Leibniz, points are positioned in place with no continuity, nor can they stand by themselves in a model as they are relational. Each point being a position of "an individual substance is a point with a form, not a quantity with a form, otherwise it could be divided into many substances" (p. 168) [16]. The Rt system measures object-oriented lengths as dimensional quantities that can be uniformly scaled into dimensional Rt_{360} units. Resulting base Rt units, or geometric elements, can include properties of length, plane (intrinsic), object condition (extrinsic), direction, symmetry, order, and a snapshot quantity within linear time.

Leibniz was unable to definitively describe relational time [16]. In contrast, the Rt system described a methodology to measure object-oriented dimensional quantities of time using signals and a memory system inspired by antient timekeeping methodologies. From privileged points in the universal frame of reference, measures of a duration of cyclic time were taken from a linear time invariance system, dimensional quantities of linear time. Object-oriented dimensional time as described in the Rt system is hypothesized as a solution for this missing component to Leibniz's thesis.

A noted problem with Leibniz physics was related to a proposed reference frame where only relational quantities of motion can be modelled, each with individual states of motion [41]. Leibniz was not able to find a way to remove the application of velocity, including his conservations principle, mv^2, which excludes motion being modelled as purely relative [17]. The hurdle has yet to be overcome considering a context of a inertial frame of reference which uses continuous velocity, mass, and only continuous time. Slowik noted that if inertial frames are removed, it solves one of the problems of Leibniz's thesis (p. 620) [41]. Slowik also proposed a type of privileged frame of reference, may provide a potential solution, which was supported by Huggett [61]. The authors of this article propose the Rt system's universal frame set in zero-time which uses multiple privileged reference points is a hypothesized solution to Leibniz's frame of reference issue.

A well-defined zero-D point with mass used by Newton, Leibniz, Einstein, Schrödinger, and so on is a component of an inertial frame of reference designed in continuous time ($t \neq 0$). The position-space Schrödinger equation includes and divides by mass, so mass cannot be zero, and the time-dependent Schrödinger equation requires continuous time ($t \neq 0$). The introduction of a zero-D point, without mass or time, is a component of a universal frame of reference in zero-time [$t = 0$] and starts a path for new possibilities to explore dimensional quantities and non-dimensional quantities differently.

Discrete geometric dimensional modeling layering object-oriented quantities creates new opportunities for computational modeling. The applications for object-relational discrete geometric modeling are far reaching and can embrace many well developed Euclidean based geometric modeling technologies. With this being a novel system that includes many straightforward concepts from numerous fields, understanding of the potential may need more examples and applications to absorb and process the new direction properly. Current, the system lacks an overarching formalization that should include set theory, symmetry notation, signal systems, and more. Overlapping terminologies from various fields has potential to leads to misinterpretation by individual fields of study.

There are several components still required to fully establish discrete geometric Rt dimensional modeling that are not included in this article. Some of these important components including a 2D lattice and coordinate system(s). The lattice system will require a mechanism to describe coordinates for a finite universal frame of reference used in a model on either a flat 2D plane or a plane superimposed upon a curved surface. The rudimentary geometric functions introduced will require additional formalization as well as contextualization for how they are applicable to a particular model that can express in an output the state of the universe being modelled. A geometric function missing is one that can express the distance between objects not in a direct cyclic relationship, an example being the distance between two stars in a galaxy, unique from the modern parallax function which is used to measure this distance.

A unifying theory applicable to both quantum and classical physics requires novel proposals to be presented, discussed, and tested for continual evolution. We present the Rishta system, a discrete signal system that can take dimensional quantities and zero-time points for both classical and quantum states for building scalable discrete geometric dimensional outputs expressing the universe when it was observed. Grounded in discrete Euclidean geometry, the Rishta system is a proposed tool for applied mathematics and physics.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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