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Impact of Melting Heat Transfer and Variable Characteristics on an MHD Non-Newtonian Shear-Thinning Fluid Flow with Gyrotactic Microorganisms over a Nonlinear Stretched Surface

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Abstract

The objective of this work is to examine how temperature-dependent thermal conductivity and concentration-dependent molecular diffusion affect Reiner-Philippoff nanofluid flow past a nonlinear stretching sheet. At the interface of the elongated surface zero-mass flux and melting heat condition are incorporated. The formulated mathematical problem is simplified by implementing suitable similarity transformations. For the numerical solution bvp4c is utilized. The parameters emerging in the model are discussed versus allied profiles through graphical illustrations. It is perceived that the velocity of the fluid decays on incrementing the Bingham number. The gyrotactic microorganism profile declines on amplifying the Peclet number. The validation of the proposed model is also added to this study.

Keywords

Reiner-Philippoff Nanofluid, Nonlinear Stretching Sheet, Melting Heat Transfer, Gyrotactic Micro-Organisms

1. Introduction

The topic of boundary layer flow past a stretching surface is of great curiosity for researchers owing to its numerous applications such as the production of rubber and plastic sheets, aerodynamics, glass blowing, metallurgical processes, and metal extrusion. A good number of investigations may be referred to in this regard highlighting various scenarios. The first step in this direction was taken by

Crane [1] who discussed fluid flow past a deformable surface. The behavior of activation energy combined with the convective condition on a magnetohydrodynamic third-grade nanofluid flow over a nonlinear deformable sheet is demonstrated by Hayat *et al.* [2]. Rasool *et al.* [3] examined nanofluid flow over a nonlinear elongated surface. The amalgamation of the Soret-Dufour effects on a silver-oxide water-based nanofluid flow on a nonlinear extended surface with slip condition is explored by Bhatti *et al.* [4]. Khan *et al.* [5] numerically investigated the impression of variable viscosity amalgamated with inclined Lorentz force on Pseudoplastic nanoliquid flow past a variable deformable surface. Seth *et al.* [6] discussed slip condition on a MHD nanoliquid flow past a nonlinearly deforming sheet.

In the field of engineering, Reiner-Philippoff (R-P) fluid has a vast utilization. Reiner-Philippoff is a non-Newtonian shear-thinning fluid. The association between shear stress and rate of strain in the R-P fluid model is given in [7] as follows:

$$\frac{\partial v}{\partial y}=\frac{\tau}{\mu_\infty+\dfrac{\left(\mu_0-\mu_\infty\right)}{1+\left(\dfrac{\tau}{\tau_s}\right)^2}},\tag{1}$$

where μ_0 is zero shear viscosity, τ_s is the reference shear stress, shear-stress is τ and μ_∞ is the limiting viscosity. The dimensionless form of Equation (1) is defined by Ahmad [8].

$$f\left(\sigma\right)=\frac{\sigma}{1+\left(\lambda-1\middle/1+\sigma^2\right)},\tag{2}$$

where $\lambda=\left(\mu_0/\mu_\infty\right)$ and $\sigma=\left(\tau/\tau_s\right)$. The R-P fluid is a non-Newtonian model which displays all three behaviors *i.e.,* dilatant fluid, pseudoplastic fluid, and Newtonian fluid. The R-P fluid exhibits features of dilatant fluid for $\lambda<1$, it behaves as pseudoplastic fluid for $\lambda>1$ and Newtonian fluid for $\lambda=1$. Recent studies featuring R-P fluid flow over varied geometries may be found in [9] [10] [11] [12] [13].

The phenomenon of melting heat transfer over a stretching surface has extensive applications which include fiber technology, the welding process, the freezing treatment of sewage, the preparation of semi conductors, oil extraction, and the melting of permafrost. On a dissipative flow, the effect of melting heat and Joule heating is explored by Hayat *et al.* [14] across an elongated surface. Mabood *et al.* [15] numerically discussed the impression of melting heat coalesced with nonlinear thermal radiation on hybrid nanoliquid flow across an elongated sheet. Mabood *et al.* [16] in another investigation discussed the flow of the Sisko nanoliquid over a deforming surface with melting heat and binary chemical reaction. It is noticed in this study that fluid temperature drops as the heat generation parameter escalates. Hayat *et al.* [17] numerically illustrated the impact of homogeneous-heterogeneous reactions and melting heat transfer on an extendable surface. Mabood *et al.* [18] analyzed the impression of melting heat on a radiative Casson fluid flow over a stretching sheet. It is comprehended in this study that by varying the magnetic and permeability parameters, the fluid temperature upsurges.

The microscopic organisms (microorganisms) can be seen only through an optical microscope. Microorganisms are everywhere around us and these are the root cause of many diseases including citrus canker, cholera, and tuberculosis. The gyrotactic microorganisms live on the still water and their movement is opposite to gravity. The random motion of the microorganisms is translated as bioconvection. Researchers are motivated to discuss the bioconvection phenomenon in numerous circumstances. Waqas *et al.* [19] studied the rate of mass transfer combined with gyrotactic microorganisms on a non-Newtonian nanofluid flow over an elongated surface. The impacts of the homogeneous-heterogeneous reactions and gyrotactic microorganisms on a Maxwell nanofluid flow past a deformable surface are scrutinized by Sohail *et al.* [20]. Waqas *et al.* [21] explored the Jeffery nanofluid flow over an elongated surface with motile microorganisms and stratification. Naz *et al.* [22] deliberated a numerical solution of the Cross nanofluid flow containing gyrotactic motile microorganisms with irreversibility analysis.

It is inferred from the above-cited literature that abundant studies may be found that discuss the impression of gyrotactic microorganisms embedded with non-Newtonian fluid flows. But very little research can be quoted that analyzed the flow of the Reiner-Philippoff fluid over varied geometries. Nevertheless, on a magnetohydrodynamic Reiner-Philippoff nanoliquid flow, gyrotactic microorganisms amalgamated with zero mass flux at the surface and melting heat transfer conditions over a nonlinear extended surface is still not discussed in the literature. An inbuilt function of MATLAB (bvp4c) software is used to solve the ODEs. The parameters emerging in the model are discussed versus allied profiles through graphical illustrations. The outcomes are quite interesting.

2. Equations

We consider a two-dimensional Reiner-Philippoff nanofluid flow with a non-uniform magnetic field $B(x)=\left(B_0/x^{0.3}\right)$ (is magnetic field strength) perpendicular to the nonlinear extended surface with stretching velocity where d is a constant (**Figure 1**). The flow is assisted by the impact of gyrotactic microorganisms. To explore the heat transfer phenomenon, melting heat and zero mass flux conditions are analyzed.

The equations governing the mathematical problem are:

$$u_x + v_y = 0, \tag{3}$$

$$uu_x + vu_y = \frac{1}{\rho}\tau_y - \frac{\sigma B^2 u}{\rho}, \tag{4}$$

$$uT_x + vT_y - \tau\left[D_B C_y T_y + \frac{D_T}{T_\infty}\left(T_y\right)^2\right] = \frac{1}{\rho C_p}\frac{\partial}{\partial y}\left(\kappa(T)T_y\right), \tag{5}$$

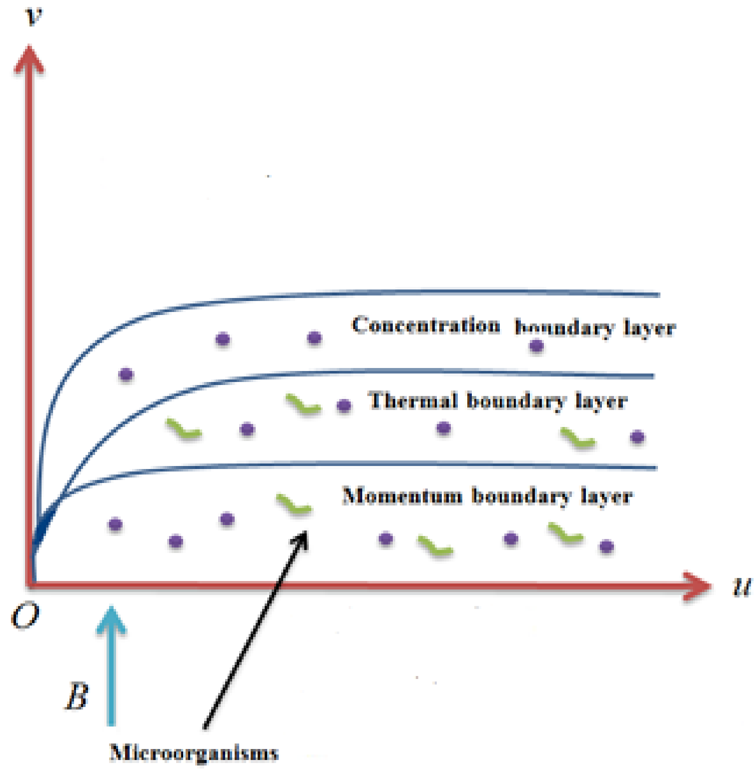

Figure 1. Schematic diagram of the Reiner-Philippoff nanofluid flow.

$$uC_x + vC_y = \frac{\partial}{\partial y}\left(D_B\left(C\right)C_y\right) + \frac{D_T}{T_\infty}T_{yy}, \tag{6}$$

$$un_x + vn_y + \frac{bW_c}{C_\infty}\left[n_y C_y + nC_{yy}\right] = D_n n_{yy}. \tag{7}$$

In Equations (5) and (6) $\kappa\left(T\right), D_B\left(C\right)$ depicts temperature-dependent thermal conductivity and dependence of molecular diffusion on concentration. The mathematical form of $\kappa\left(T\right), D_B\left(C\right)$ is specified as:

$$\kappa\left(T\right) = \kappa_\infty\left(1 + h_1\theta\right), \tag{8}$$

where κ_∞ is thermal conductivity at the free stream and h_1 is the variable thermal conductivity parameter. Similarly,

$$D_B\left(C\right) = D_{B_\infty}\left(1 + h_2\phi\right), \tag{9}$$

where D_{B_∞} is molecular diffusivity at the free stream and h_2 is a variable molecular diffusivity parameter.

Boundary constraints are:

$$\begin{gathered} u(x,y) = U_w(x) = dx^{1/3},\ \kappa\left(T\right)T_y = \rho\left[\chi + c_s\left(T_m - T_0\right)\right]v,\ T = T_m, \\ \frac{D_T}{T_\infty}T_y + D_B C_y = 0,\ n = n_w,\ \text{at}\ y = 0, \\ u \to 0,\ T \to T_\infty, C \to C_\infty, n \to n_\infty,\ \text{as}\ y \to \infty. \end{gathered} \tag{10}$$

Here

$b, B_0, C, \sigma, c_s, c_p, W_c, D_B, T, D_T, (u,v), D_n, \kappa(T), \rho, T_m, T_\infty, \chi, n, n_w, U_w(x), (x,y)$ and n_∞ are chemotaxis constant, the magnitude of magnetic field strength, concentration of nanoparticles, the conductivity of electricity, heat capacity of solid surface, specific heat capacitance, maximum cell swimming speed, Brownian motion coefficient, the temperature of boundary layer fluid, thermophoresis coefficient, components of velocity, micro-organism diffusion coefficient, thermal conductivity, density, the temperature of the wall, temperature of the ambient fluid, latent heat, concentration of micro-organisms, the surface concentration of micro-organisms, stretching velocity, coordinate axes and ambient concentration of micro-organisms.

Using local similarity transformation defined in [8]:

$$\begin{gathered}\zeta = \sqrt{\frac{U_w}{vx}}y,\ \ \Psi = \sqrt{U_w x v} f(\zeta),\ \ \tau = \rho\sqrt{d^3 v} m(\zeta),\ \ C = C_\infty \phi(\zeta) + C_\infty, \\ T = (T_\infty - T_m)\theta(\zeta) + T_m,\ n = (n_w - n_\infty)R(\zeta) + n_\infty. \end{gathered} \tag{11}$$

Equation (3) is satisfied. However, Equations (4)-(6), (9), and (10) are transmuted as:

$$m' = \frac{1}{3}(f')^2 - \frac{2}{3}ff'' + Haf', \quad \text{where} \quad m = f''\frac{m^2 + \lambda\gamma^2}{m^2 + \gamma^2}, \tag{12}$$

$$(1 + h_1\theta)\theta'' + h_1\theta'^2 + N_b\theta'\phi' + \frac{2}{3}\Pr f\theta' + N_t(\theta')^2 = 0, \tag{13}$$

$$(1 + h_2\phi)\phi'' + h_2\phi'^2 + \frac{2}{3}S_c f\phi' + N_t\frac{\theta''}{N_b} = 0, \tag{14}$$

$$R'' + \frac{2}{3}fR'Lb - Pe\left[\phi'R' + (R + \omega)\phi''\right] = 0. \tag{15}$$

boundary constraints

$$\begin{gathered} f'(\zeta) = 1,\ M(1 + h_1\theta)\theta'(\zeta) + \Pr f(\zeta) = 0,\ \theta(\zeta) = 0,\ N_t\theta'(\zeta) + N_b\phi'(\zeta) = 0, \\ R(\zeta) = 1, \quad \text{at} \ \zeta = 0, \\ f(\zeta) \to 0,\ \theta(\zeta) \to 1,\ \phi(\zeta) \to 0,\ R(\zeta) \to 0, \quad \text{as} \ \zeta \to \infty. \end{gathered} \tag{16}$$

Here, $Ha, M, P_e, \lambda, N_b, \gamma, \Pr, \omega, N_t, L_b$ and S_c denote the Hartmann number, melting parameter, Peclet number, fluid parameter, Brownian motion parameter, Bingham number, Prandtl number, bioconvection concentration difference parameter, Thermophoresis parameter, Lewis number, and Schmidt number. These are demarcated as under:

$$\begin{gathered} Ha = \frac{\sigma B_0^2}{\rho U_0}, M = \frac{c_f(T_\infty - T_m)}{\chi + c_s(T_m - T_0)}, P_e = \frac{bW_c}{v}, \lambda = \frac{\mu_0}{\mu_\infty}, N_b = \frac{\tau D_B C_\infty}{v}, \gamma = \frac{\tau_s}{\rho\sqrt{d^3 v}}, \\ \Pr = \frac{\mu c_p}{k}, \omega = \frac{n_\infty}{n_w - n_\infty}, N_t = \frac{\tau D_B(T_\infty - T_m)}{T_\infty v}, L_b = \frac{v}{D_n}, S_c = \frac{v}{D_B}. \end{gathered} \tag{17}$$

The mathematical forms of heat and mass flux are defined as:

$$Nu_x = \frac{xQ_w}{\kappa_\infty(T_\infty - T_m)}, \quad Q_w = -\kappa(T)T_y\big|_{y=0}, \tag{18}$$

$$Sh_x = \frac{xQ_m}{D_{B_\infty} C_\infty}, \quad Q_m = -D_B(C) C_y\big|_{y=0}. \tag{19}$$

The non-dimensional form of (18) and (19) are given as:

$$Nu_x \operatorname{Re}_x^{-0.5} = -\left(1 + h_1 \theta(0)\right)\theta'(0), \tag{20}$$

$$Sh_x \operatorname{Re}_x^{-0.5} = -\left(1 + h_2 \phi(0)\right)\phi'(0). \tag{21}$$

The rate of heat transfer for numeric values of Pr is presented in **Table 1**. A fabulous association is observed between the numeric outcomes of the present study with the published work of Sajid *et al.* [9] and Reddy *et al.* [11].

3. Figures

The outcomes of dimensionless parameters are graphically inspected through **Figures 2-9**. **Figure 2** illustrates the upshot of the Bingham number γ versus the fluid velocity profile $f'(\zeta)$. It is comprehended that for growing values of γ, fluid velocity deteriorates. For higher values of shear rate, apparent viscosity increases which result in depreciation in $f'(\zeta)$. **Figure 3** portrays the outcome of varying the Reiner Philippoff fluid parameter λ on $f'(\zeta)$. The velocity is enhanced for numerous values of λ As λ is the quotient of shear viscosity to

Table 1. Comparison of rate of heat transfer for numeric values of Pr by considering $Ha = 1, \lambda = 1, \gamma = 0.7$ and $N_t = S_c = P_e = \omega = L_b = 0$ with Sajid *et al.* [9] and Reddy *et al.* [11].

Pr	$Nu_x \operatorname{Re}_x^{-0.5}$		
	Sajid *et al.* [9]	Reddy *et al.* [11]	Present
1	0.556065	0.559879	0.556879
1.5	0.727928	0.727497	0.727227
2	0.873992	0.886106	0.889140

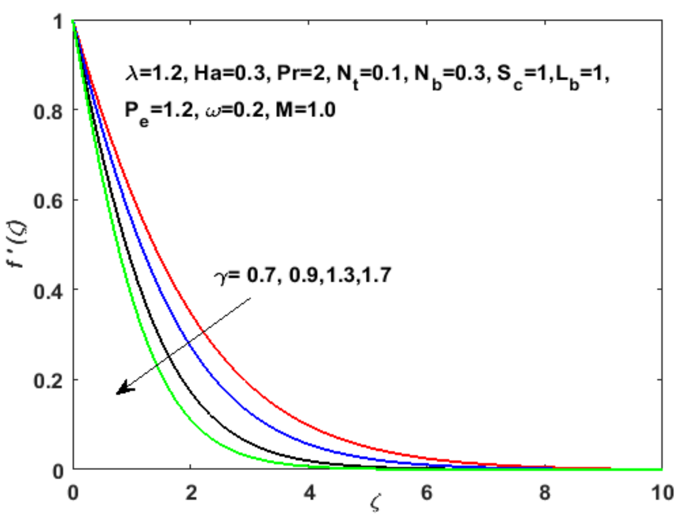

Figure 2. Upshot of Bingham number on velocity profile.

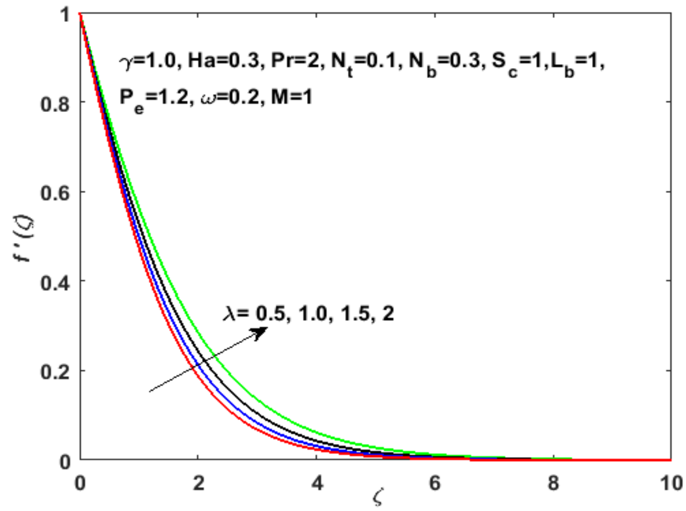

Figure 3. Upshot of Reiner Philippoff fluid parameter on velocity profile.

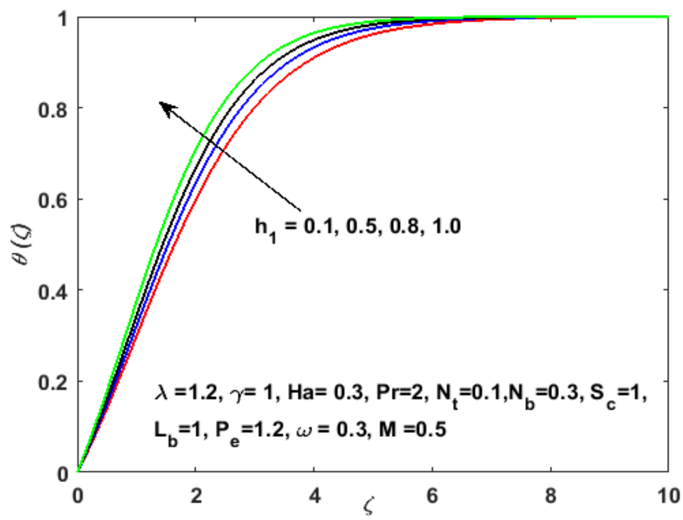

Figure 4. Upshot of varying thermal conductivity parameter on temperature profile.

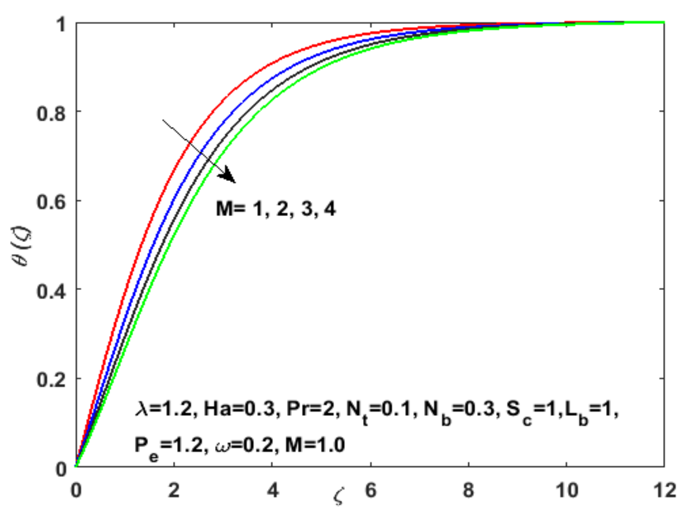

Figure 5. Upshot of melting parameter on temperature profile.

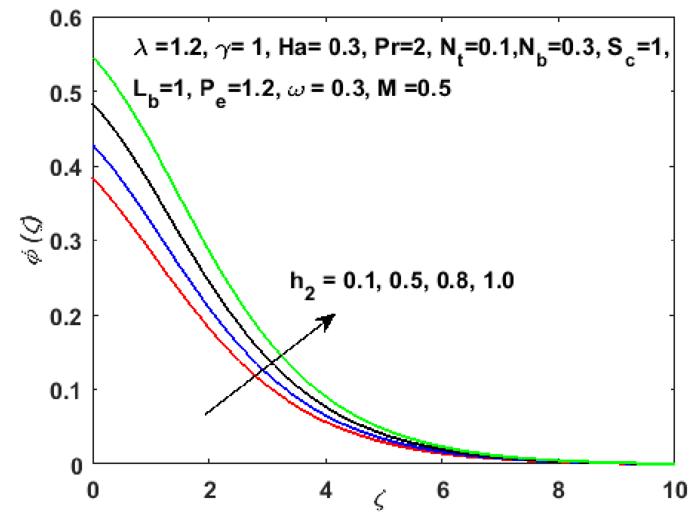

Figure 6. Upshot of varying molecular diffusivity on solutal profile.

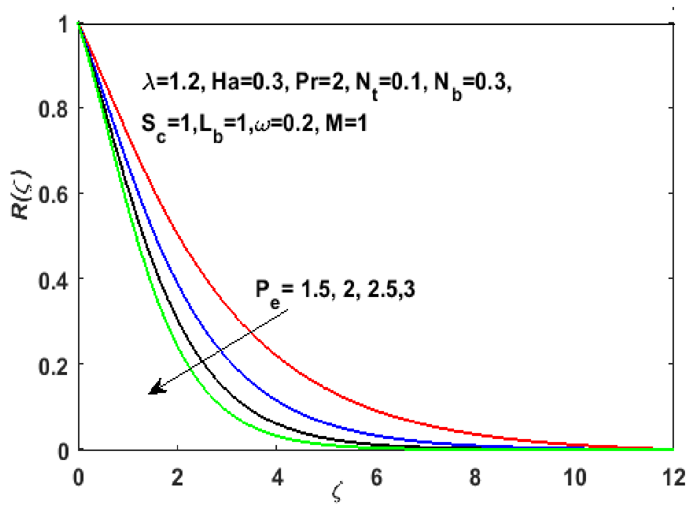

Figure 7. Upshot of Peclet number on gyrotactic microorganisms profile.

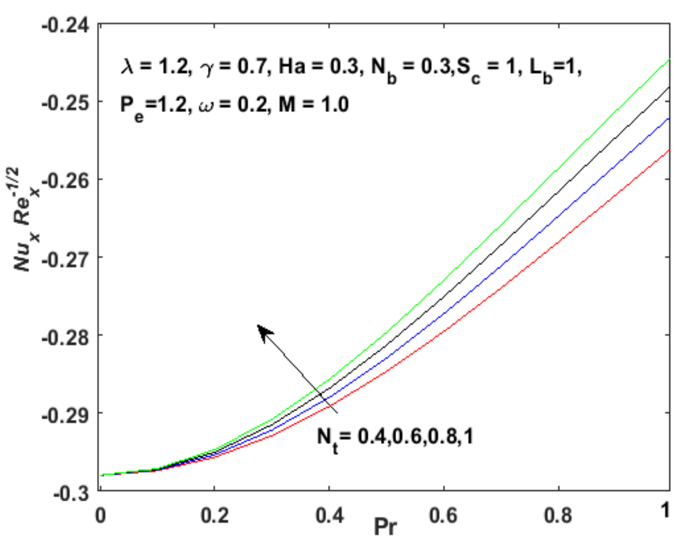

Figure 8. Upshot of Prandtl number and thermophoresis parameter on heat transfer rate.

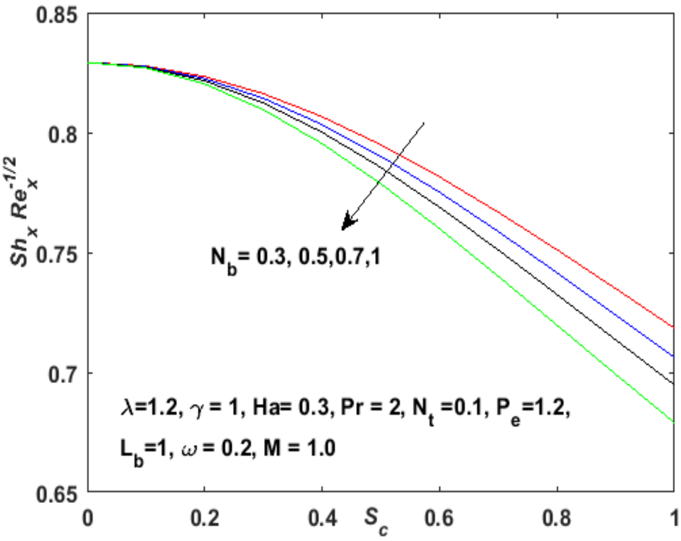

Figure 9. Upshot of Brownian motion parameter and Schmidt number on rate of mass transfer.

the limiting viscosity. Higher values of λ result in a decrement in the viscosity due to which the fluid moves freely, and $f'(\zeta)$ augments.

Figure 4 portrays the impression of varying thermal conductivity parameter h_1 on the thermal field $\theta(\zeta)$. An upsurge is seen for higher values of h_1 as collision among the fluid particles escalates. Thus, more heat is transmitted through the fluid. Hence, $\theta(\zeta)$ elevates. **Figure 5** depicts the behavior of the augmenting M versus fluid temperature $\theta(\zeta)$. The heat is transferred more promptly due to the difference in temperature between the surface and the ambient fluid. Hence, $\theta(\zeta)$ deteriorates. **Figure 6** illustrates how varying molecular diffusivity parameter h_2 affects the solutal field $\phi(\zeta)$. On amplifying h_2, an upsurge is noticed in $\phi(\zeta)$. The upshot of the bioconvection Peclet number P_e on the gyrotactic micro-organisms profile $R(\zeta)$ is inspected in **Figure 7**. A decline in $R(\zeta)$ is witnessed. On escalating P_e, it is observed that the diffusion of microorganisms reduces. Consequently, $R(\zeta)$ declines. The behavior of Pr and N_t on heat transfer rate is depicted in **Figure 8**. On augmenting Pr and N_t heat flux upsurges. The influence of N_b and S_c on the rate of mass transfer $Sh_x \mathrm{Re}_x^{-0.5}$ is revealed in **Figure 9**. Here, $Sh_x \mathrm{Re}_x^{-0.5}$ is a decreasing function of N_b.

4. Conclusions

Magnetohydrodynamic Reiner-Philippoff nanoliquid flow is inspected over a nonlinear extended surface with the impression of gyrotactic microorganisms. The uniqueness of the envisioned model is upgraded considering the zero-mass flux and melting heat condition. The problem is handled numerically. The following are the most notable findings of this investigation:

- The velocity profile is elevated by augmenting the fluid parameter, whereas, an opposite behavior is depicted by the Bingham number.

- An inverse behavior is witnessed in velocity and fluid temperature for the melting heat parameter.
- Gyrotactic microorganisms profile diminishes on augmenting the Peclet number.
- The rate of heat and mass transfer depicts an opposite behavior for Brownian motion and thermophoresis parameter.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Crane, L.J. (1970) Flow Past a Stretching Plate. *Zeitschrift für angewandte Mathematik und Physik ZAMP*, **21**, 645-647. https://doi.org/10.1007/BF01587695

[2] Hayat, T., Riaz, R., Aziz, A. and Alsaedi, A. (2020) Influence of Arrhenius Activation Energy in MHD Flow of Third Grade Nanofluid over a Nonlinear Stretching Surface with Convective Heat and Mass Conditions. *Physica A*: *Statistical Mechanics and Its Applications*, 124006. https://doi.org/10.1016/j.physa.2019.124006

[3] Rasool, G., Shafiq, A., Khalique, C. M. and Zhang, T. (2019) Magnetohydrodynamic Darcy-Forchheimer Nanofluid Flow over a Nonlinear Stretching Sheet. *Physica Scripta*, **94**, 105221. https://doi.org/10.1088/1402-4896/ab18c8

[4] Bhatti, M.M., Khalique, C.M., Bég, T.A., Bég, O.A. and Kadir, A. (2020) Numerical Study of Slip and Radiative Effects on Magnetic Fe_3 O_4-Water-Based Nanofluid Flow from a Nonlinear Stretching Sheet in Porous Media with Soret and Dufour Diffusion. *Modern Physics Letters B*, **34**, 2050026. https://doi.org/10.1142/S0217984920500268

[5] Khan, M., Malik, M.Y., Salahuddin, T. and Hussian, A. (2018) Heat and Mass Transfer of Williamson Nanofluid Flow Yield by an Inclined Lorentz Force over a Nonlinear Stretching Sheet. *Results in Physics*, **8**, 862-868. https://doi.org/10.1016/j.rinp.2018.01.005

[6] Seth, G.S. and Mishra, M.K. (2017) Analysis of Transient Flow of MHD Nanofluid Past a Non-Linear Stretching Sheet Considering Navier's Slip Boundary Condition. *Advanced Powder Technology*, **28**, 375-384. https://doi.org/10.1016/j.apt.2016.10.008

[7] Ahmad, A., Qasim, M. and Ahmed, S. (2017) Flow of Reiner-Philippoff Fluid over a Stretching Sheet with Variable Thickness. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, **39**, 4469-4473. https://doi.org/10.1007/s40430-017-0840-7

[8] Ahmad, A. (2016) Flow of Reiner Philippoff Based Nano-Fluid Past a Stretching Sheet. *Journal of Molecular Liquids*, **219**, 643-646. https://doi.org/10.1016/j.molliq.2016.03.068

[9] Sajid, T., Sagheer, M. and Hussain, S. (2020) Impact of Temperature-Dependent Heat Source/Sink and Variable Species Diffusivity on Radiative Reiner-Philippoff Fluid. *Mathematical Problems in Engineering*, 2020. https://doi.org/10.1155/2020/9701860

[10] Najiyah, I.W., Khashi'ie, S., Kasim, A.R.M., Zainal, N.A., Ishak, A. and Pop, I. (2022) Radiative Heat Transfer of Reiner-Philippoff Fluid Flow Past a Nonlinearly Shrinking Sheet: Dual Solutions and Stability Analysis. *Chinese Journal of Physics*. https://doi.org/10.1016/j.cjph.2021.11.037

[11] Reddy, M.G., Rani, S., Kumar, K.G., Seikh, A.H., Rahimi-Gorji, M. and Sherif, E.S.M. (2019) Transverse Magnetic Flow over a Reiner-Philippoff Nanofluid by Considering Solar Radiation. *Modern Physics Letters B*, **33**, 1950449. https://doi.org/10.1142/S0217984919504499

[12] Kumar, K.G., Reddy, M.G., Khan, M.I., Alzahrani, F., Khan, M.I. and El-Zahar, E.R. (2021). Heat Transfer and Melting Flow of a Reiner-Philippoff Fluid over a Surface with Darcy-Forchheimer Medium. *Case Studies in Thermal Engineering*, **28**, 101649. https://doi.org/10.1016/j.csite.2021.101649

[13] Waini, I., Khashi'ie, N.S., Kasim, A.R.M., Zainal, N.A., Ishak, A. and Pop, I. (2022) Nonlinear Radiative Heat Transfer of Magnetohydrodynamic Non-Newtonian Fluid Flow Past a Shrinking Sheet: Reiner-Philippoff Model. *Waves in Random and Complex Media*, 1-22. https://doi.org/10.1080/17455030.2022.2038815

[14] Hayat, T., Shah, F., Alsaedi, A. and Ahmad, B. (2020) Entropy Optimized Dissipative Flow of Effective Prandtl Number with Melting Heat Transport and Joule Heating. *International Communications in Heat and Mass Transfer*, **111**, 104454. https://doi.org/10.1016/j.icheatmasstransfer.2019.104454

[15] Mabood, F., Yusuf, T.A. and Khan, W.A. (2020) Cu-Al_2O_3-H_2O Hybrid Nanofluid Flow with Melting Heat Transfer, Irreversibility Analysis and Nonlinear Thermal Radiation. *Journal of Thermal Analysis and Calorimetry*, 1-12. https://doi.org/10.1007/s10973-020-09720-w

[16] Mabood, F., Ibrahim, S.M. and Khan, W.A. (2019) Effect of Melting and Heat Generation/Absorption on Sisko Nanofluid over a Stretching Surface with Nonlinear Radiation. *Physica Scripta*, **94**, 065701. https://doi.org/10.1088/1402-4896/ab1164

[17] Hayat, T., Muhammad, K., Khan, M.I. and Alsaedi, A. (2019) Theoretical Investigation of Chemically Reactive Flow of Water-Based Carbon Nanotubes (Single-Walled and Multiple Walled) with Melting Heat Transfer. *Pramana*, **92**, 57. https://doi.org/10.1007/s12043-019-1722-6

[18] Mabood, F. and Das, K. (2019) Outlining the Impact of Melting on MHD Casson Fluid Flow Past a Stretching Sheet in a Porous Medium with Radiation. *Heliyon*, **5**, e01216 https://doi.org/10.1016/j.heliyon.2019.e01216

[19] Waqas, H., Khan, S.U., Hassan, M., Bhatti, M.M. and Imran, M. (2019) Analysis on the Bioconvection Flow of Modified Second-Grade Nanofluid Containing Gyrotactic Microorganisms and Nanoparticles. *Journal of Molecular Liquids*, **291**, 111231. https://doi.org/10.1016/j.molliq.2019.111231

[20] Sohail, M., Naz, R. and Abdelsalam, S.I. (2020) On the Onset of Entropy Generation for a Nanofluid with Thermal Radiation and Gyrotactic Microorganisms through 3D Flows. *Physica Scripta*, **95**, 045206. https://doi.org/10.1088/1402-4896/ab3c3f

[21] Waqas, M., Khan, M.I., Hayat, T., Farooq, S. and Alsaedi, A. (2019) Interaction of Thermal Radiation in Hydromagnetic Viscoelastic Nanomaterial Subject to Gyrotactic Microorganisms. *Applied Nanoscience*, **9**, 1193-1204. https://doi.org/10.1007/s13204-018-00938-7

[22] Naz, R., Noor, M., Hayat, T., Javed, M. and Alsaedi, A. (2020) Dynamism of Magnetohydrodynamic Cross Nanofluid with Particulars of Entropy Generation and Gyrotactic Motile Microorganisms. *International Communications in Heat and Mass Transfer*, **110**, 104431. https://doi.org/10.1016/j.icheatmasstransfer.2019.104431