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Heat Propagation of Eyring-Prandtl Double Reaction and Pressure Driven Hydromagnetic Viscous Heating Fluid in a Device

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Abstract

The effect toxic industrial discharge on the environment and ecosystem cannot be overlooked. This is owing to a partial combustion of hydrocarbon arising from industrial activities and human endeavours. As such, this investigation focuses on the pressure driven flow and heat propagation of combustible Prandtl-Eyring viscous heating fluid in a horizontal device. The combustion-reaction of the viscoplastic material is considered to be inspired by two-step exothermic reaction. With negligible reactant consumption, the flowing fluid is influenced by a chemical kinetic, activation energy and electromagnetic force. An invariant transformation of the partial derivative model to an ordinary derivative model is obtained through an applied dimensionless variable. The solutions to the unsteady thermal fluid flow model are obtained via a semi-implicit difference scheme, and the outputs of the solution are displayed in plots and tables. As revealed, an enhanced heat propagation is obtained that in turn encourages the combustion process of the system. Also, increasing material dilatant simulated fluid molecular bond and viscosity. Therefore, the outcomes of this study are treasured to the thermal and chemical engineering, and the environmental management.

Keywords

Viscous Heating, Exothermic Reaction, Two-Step Diffusion, Viscoelastic Fluid

1. Introduction

Recently, the main industrial base fluids are viscoplastic materials resulting from their unusual rheological characteristics and usefulness. Different dynamical tensor stress are considered in developing the individual fluid material, Prandtl-Eyring material is one the most used working fluids. A hyperbolic strain rate function is assumed for the topographical materialistic of the fluid model with linear stress slip interaction [1] [2] [3]. The fluid can be utilized in hosepipe, manufacturing of shoes, bulletproof jackets, PVC pipes and so on, Qureshi [4]. Owing to the usages of Prandtl-Eyring material induced by Lorentz force, Munjam *et al.* [5] discussed magneto-radiation of a Prandtl-Eyring flowing fluid through a convective heated elongated surface. As revealed, the flow velocity is reduced as the Lorentz force is stimulated to boost the material dilatant property. Salawu *et al.* [6] examined nonlinear radiation absorption and heat transfer maximization of hybridized Prandtl-Eyring nanofluids. High heat absorption and distribution is recorded with an increased viscoplastic material term. Hayat *et al.* [7] investigated gyrotatic microorganism nanofluid thermal melting of a Prandtl-Eyring heat transport. The outcomes show that the heat propagation is encouraged as the Prandtl and thermal dissipation parameter values are raised. Rehman *et al.* [8] studied invariant thermal and species distributions of Eyring-Prandtl diffusion-reaction fluid via a numerical scheme. A sensitivity analysis of the entrenched parameters depicted that strengthening the viscous heating and viscoelastic material enhanced the thermal and concentration fields along the flow region. Hence, the kind of materials considered determines the quantity of thermal distribution in a system.

The viscoelastic fluid thermal transport and species dispersion can be stimulated by kinetics exothermic reaction. As found in nature and industrial system, for a system in which exothermic reaction is involved, high quantity of heat is released. In such a diffusion-reaction system occurs a combustion process [9] [10]. In chemical sciences and engineering, combustion process is very significant, and it is useable material processing, propulsion rocket, pollution mitigation, fire and safety management and so on, Ajadi [11]. As a result, idealized single-step exothermic diffusion-reaction has been examined by several scientists that include Zhu *et al.* [12], their investigation focuses on the partially premixed self-absorption of effective combustible thermal radiation. It was reported that the premixed combustion flame is augmented by thermal radiation. Makinde [13] discussed third grade non-Newtonian thin film gravity driven flow and thermal criticality in an inclined adiabatic device. Internal thermal combustion is raised with a rise in the material term, as such large heat generation must be watched to prevent and low efficiency of thermal system. Salawu and Disu [14] examined temperature propagation of an Oldroyd 6-constant fluid for a pressure driven Couette flow and thermal criticality. The species initiation rate and Frank-Kamenestskii were presented separately to have influenced the thermal distribution of the reactive species. Hassan *et al.* [15] carried out study on the magneto-exothermic reaction fluid and thermal radiation in a porous flow medium. A chemical kinetics of $n = 0$ depicting the Arrhenius reaction was assumed for the single-step diffusion-reaction, and the heat transfer was found to have been raised. Several other observations on the exothermic reaction are reported by [16] [17] [18] [19] [20]. However, single reaction-diffusion is not enough to explain combustion process especially the burn of hydrocarbon.

For a heat propagating system, in which single reaction-combustion is not sufficient, double exothermic diffusion remains a good platform for a reactive species combustion of viscoelastic fluid, Salawu *et al.* [21]. As an example, hydrocarbon combustion in an engine required catalytic converter for a complete burn of hydrocarbon serves as a platform for double exothermic reaction as reported by Szabo [22]. To minimize the quantity of unwanted environmental toxic released from industrial activities, automobile engines and other human endeavours, double combustion system should be encouraged. This will reduced the volume of CO_2 discharged to surroundings that in turn affects the ecosystem, Makinde *et al.* [23]. As a result, Kareem and Gbadeyan [24] discussed entropy minimization and thermal ignition of an electrically induced flowing fluid along a Couette device. It was noticed from the investigation that the thermal propagation profile is boosted for an increased values of two-step reaction term, this resulted in an enhanced reaction-combustion. Likewise, Salawu *et al.* [25] theoretically investigated two-step reaction-diffusion of couple stress fluid and thermal ignition bifurcation in a convective medium with Reynolds viscosity and optical radiation. It was reported that that the fluid material inviscid and molecular bond is raised as the second step and non-Newtonian term values are increased. Thus, the usability and applications of two-step reacting species in combustion technology and thermal cannot be overlooked.

A diffusion-reaction of Eyring-Prandtl reacting species influenced by Lorentz force and pressure gradient in a channel gained low or no scientists attention. These research gaps and its usefulness motivated the present theoretical study. The considered analysis is very significant and essential in enhancing jet and rocket propulsion, pollution control, lubricant oil and others. Therefore, the focus of the investigation is on the Eyring-Prandtl two-step diffusion-reaction of viscous hydromagnetic exothermic reaction with Joule heating in a Couette non-isothermal medium. The flowing fluid rate and heat dispersion field for different parametric sensitivities of a double fluid reaction is examined. The thermal science quantities of attention, the fluid wall drag and temperature gradient are considered and the computed outputs are displayed in tables. Also, the impact of entrenched dependent parameters is plotted in graphs. The finite semi-implicit method is used to provide all the solutions and the discussion of results is performed comprehensively.

2. Formulation Mathematical Model

Examine a hydromagnetic Eyring-Prandtl Couttte generalized flow of double-step species reactant combustion. In the absence of reactant absorption, an Ohmic dissipation of the viscoelastic matrial is assumed for an exothermic species generalized kinetic model. The Eyring-Prandtl flow material is exposed to two-step diffusion-reaction in a non-isothermal horizontal channel. The pressure driven fluid under the impact of Lorentz force flows through a Couette medium is affected by pre-exponential index n and activation energy. Neglecting the z-flow direction, the viscoplastic fluid moves in the x-direction and y-direction is taken to be normal to it, as geometrically displayed in **Figure 1**.

The exothermic two-step irreversible reaction for the k-fluid oxidation reactant species is expressed according to Williams [26]

$$C_2H_5OH + 2\left(94/25\,N_2 + O_2\right) \Rightarrow 188/25\,N_2 + 2CO + 3H_2O$$

$$CO + 1/2\left(95/25\,N_2 + O_2\right) \Rightarrow 47/25\,N_2 + CO_2$$

The dynamical Eyring-Prandtl stress tensor compatible model is considered inline with [27] [28]

$$\tau = \partial w_y \frac{\Lambda \sin^{-1}\left(\frac{1}{\alpha}\left[\left(\partial w_y\right)^2 + \left(\partial w_y\right)^2\right]^{1/2}\right)}{\left(\partial w_y\right)^2 + \left(\partial w_y\right)^2}. \tag{1}$$

The velocity curving mechanisms is depicted by $\boldsymbol{W} = \left[\boldsymbol{w}_1(x,y,0), \boldsymbol{w}_2(x,y,0), 0\right]$, α and Λ denote fluid viscous parameters. The double-step heat reaction balance for the magneto-exothermic reactant is applied according to Salawu *et al.* [21] [25]

$$\rho \partial T_t = -div\boldsymbol{q} + \Lambda\left(\nabla^2 \boldsymbol{W}\right)^2 + A_1 H_1 C_1 + A_2 H_2 C_2. \tag{2}$$

Here H_1 and H_2 represent the reaction heat, C_1 and C_2 connote reactant species, T is temperature of the fluid, $\boldsymbol{q}$ is the thermal flux satisfying Fourier law and defined as $\boldsymbol{q} = -k\nabla T$ representing the heat conduction, ρ illustrates density of the fluid. Following [29] [30], the temperature diffusion-reaction rate A_1 and A_2 are expressed for the Arrhenius reaction as,

$$A_1 = R_1\left(\frac{BT}{v\hbar}\right)^n e^{-E_1/RT},\ A_2 = R_2\left(\frac{BT}{v\hbar}\right)^n e^{-E_2/RT}. \tag{3}$$

Here, B is Boltzmann constant, $\hbar$ is Planck's constant, E_1 and E_2 connote activation energy, R_1 and R_2 present the reacting species order, v is frequency of vibration, and n is the reaction ($n \in \{-2, 0, 1/2\}$) for the reaction kinetics. Taken that low reactant absorption coefficient occur, subject to the mentioned assumptions, the unsteady reactive Eyring-Powell flowing fluid momentum and the reaction heat distribution balance gives:

$$\partial w_{\bar{t}} = -\partial \bar{p}_{\bar{x}} + \frac{\Lambda}{\rho\alpha}\partial^2 w_{\bar{y}\bar{y}} - \frac{\Lambda}{2\rho\alpha^3}\left(\partial w_{\bar{y}}\right)^2 \partial^2 w_{\bar{y}\bar{y}} - \frac{\sigma B_0^2}{\rho} w, \tag{4}$$

$$\rho C_p \partial T_{\bar{t}} = k\partial^2 T_{\bar{y}\bar{y}} + \frac{\Lambda}{\alpha}\left(\partial w_{\bar{y}}\right)^2\left(1 - \frac{1}{6\alpha^2}\left(\partial w_{\bar{y}}\right)^2\right) + \sigma B_0^2 w^2 + H_1 C_1 A_1 + H_2 C_2 A_2, \tag{5}$$

the suitable initial and boundary conditions takes the form:

$$\begin{aligned} &w(\bar{y},0)=0,\ T(\bar{y},0)=0,\ w(0,\bar{t})=0,\ T(0,\bar{t})=T_a, \\ &\partial w_{\bar{y}}(b,\bar{t})=0,\ T(b,\bar{t})=T_0,\ \text{for } \bar{t}>0. \end{aligned} \tag{6}$$

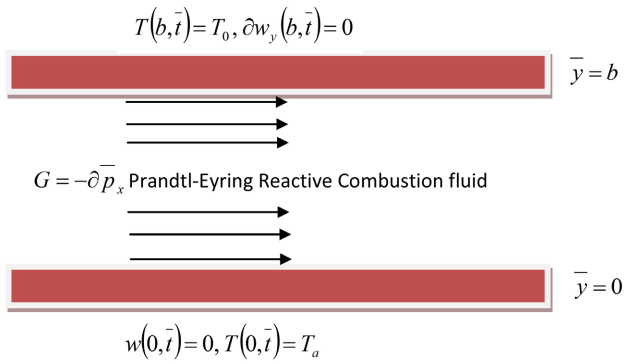

Figure 1. Flow schematic configuration.

The individual parameters are the initial temperature T_0, channel width b, ambient heat energy T_a, flow rate w, pressure p, magnetic strength B_0, specific heat C_p, time $\bar{t}$, electrical conduction σ and heat conduction k. Together with the applicable subsequent variables, the invariant equations are offered

$$G = -\frac{\partial \bar{p}}{\partial \bar{x}}, t = \frac{\bar{t}\mu}{\rho b^2}, x = \frac{\bar{x}}{b}, u = \frac{w}{W_0}, \theta = \frac{E_1(T - T_0)}{T_0^2 R}, p = \frac{b\bar{p}}{W_0 \mu}, y = \frac{\bar{y}}{b},$$
$$Br = \frac{E_1 \mu W_0^2}{T_0^2 Rk}, a = \frac{E_2}{E_1}, \gamma = \frac{C_1 H_1 E_1 R_1 b^2}{k T_0^2 R}\left(\frac{T_0 B}{\hbar v}\right)^n e^{-\frac{1}{\lambda}}, \lambda = \frac{T_0 R}{E_1}, \beta = \frac{\Lambda}{\mu \alpha}, \tag{7}$$
$$\delta = \frac{W_0}{2\alpha^2 b^2}, \chi = \frac{C_2 H_2 R_2}{R_1 C_1 H_1}, \theta_a = \frac{E_1(T_a - T_0)}{T_w^2 R}, Pr = \frac{C_p \mu}{k}, H = \frac{\sigma B_0^2 b^2}{\mu}.$$

Using the above variables (7) on the partial derivative thermofluidic equations and on the boundary constraint, the invariant physical equations are gotten:

$$\partial u_t = G - \delta\beta\left(\partial u_y\right)^2 \partial^2 u_{yy} + \beta \partial^2 u_{yy} - Hu, \tag{8}$$

$$Pr \partial \theta_t = \partial^2 \theta_{yy} + \gamma\left\{(1+\lambda\theta)^n \left(e^{\frac{\theta}{1+\lambda\theta}} + \chi e^{\frac{a\theta}{1+\lambda\theta}}\right)\right\} + Br\left[Hu^2 + \beta\left(\partial u_y\right)^2 - \frac{\delta\beta}{3}\left(\partial u_y\right)^4\right], \tag{9}$$

the invariant initial and boundary conditions take the form:

$$u(0,t) = 0, u(y,0) = 0, \theta(y,0) = 0, \theta(0,t) = \theta_c, \partial u_y(1,t) = 0, \theta(1,t) = 0, \text{ for } t > 0. \tag{10}$$

The separate embedded parameters H, β, δ, G, γ, λ, Pr, Br, a and χ are magnetic term, Eyring-Prandtl materials, pressure gradient, activation energy, Frank-Kamenetskii, Prandtl number, Brinkman number, activation energy ratio and reaction second step. The invariant important variables of thermal science interest are the wall drag (Sr) and temperature gradient (Hr) described as:

$$Sr = \left(\beta \partial u_y - \frac{1}{3}\beta\delta\left[\partial u_y\right]^3\right)\Bigg|_{y=0}, \quad Hr = -\partial \theta_y\big|_{y=0} \tag{11}$$

The detailed solution procedures of the invariant derivative Equations (8) to (11) are computationally done for the parameters dependent sensitivity.

3. Computational Method and Procedures

The computational method is formulated on a consistent and convergent finite semi-implicit method as demonstrated by [30] [31] [32] for a viscoelastic non-isothermal flow constraints. According to Makinde [33], the computation procedures is used on the double-step diffusion-reaction Eyring-Prandtl reacting species. An implicit time $(m+h)$ intermediate level is utilized in the h range of [0, 1]. For the numerical analysis, $h=1$ is taken for large a computational step times, as employed by Chinyoka [34]. A discretization uniform finite differences in a cartesian linear grid and mesh for the invariant derivative equations is carried out. A spatial approximated differentiation is done via a central difference of second order. Thus, the resulted equations and boundary conditions are integrated in grid point for the computation. Hence, the finite semi-implicit method for the flow velocity module is expressed as

$$\frac{u^{(m+1)}-u^{(m)}}{\Delta t}=G-\delta\beta\partial u_y^{2(m)}\partial^2 u_{yy}^{(m+h)}+\beta\partial^2 u_{yy}^{(m+h)}-Hu^{(m+h)}, \tag{12}$$

with $u^{(m+h)}$ described as

$$\begin{aligned}&-\xi_1 u_{j-1}^{(m+1)}+(2\xi_1+1)u_j^{(m+1)}-\xi_1 u_{j+1}^{(m+1)}\\&=u^{(m)}+\Delta t(1-\xi)\beta u_{yy}^{(m)}-\Delta t\beta\delta(u_y)^{2(m)}u_{yy}^{(m)}+\Delta tG-\Delta tHu^{(m)},\end{aligned} \tag{13}$$

where, $\xi_1=\frac{\Delta t}{\Delta y^2}$. The method of solution for $u^{(m+1)}$ invariants for the tri-diagonal inverse matrix, this resulted in a suitable outcomes than using the complete implicit method. Hence, the combustible fluid temperature field reaction in a semi-implicit discretization form is taken after the flow rate module. The second order partial differentiation thermal module is illustrated as:

$$\begin{aligned}Pr\frac{\theta^{(m+1)}-\theta^{(m)}}{\Delta t}&=\theta_{yy}^{(m+h)}+\gamma(1+\lambda\theta)^n\left(e^{\frac{\theta}{1+\lambda\theta}}+\chi e^{\frac{a\theta}{1+\lambda\theta}}\right)^{(m)}\\&\quad+Br\left(Hu^2+\beta(\partial u_y)^2-\frac{\delta\beta}{3}(\partial u_y)^4\right)^{(m)},\end{aligned} \tag{14}$$

where $\theta^{(m+1)}$ is described as

$$\begin{aligned}&-\xi_2\theta_{j-1}^{(m+1)}+(2\xi_2+Pr)\theta_j^{(m+1)}-\xi_2\theta_{j+1}^{(m+1)}\\&=\Delta tBr\left(Hu^2+\beta(\partial u_y)^2-\frac{\delta\beta}{3}(\partial u_y)^4\right)^{(m)}\\&\quad+\Delta t\gamma(1+\lambda\theta)^n\left(e^{\frac{\theta}{1+\lambda\theta}}+\chi e^{\frac{a\theta}{1+\lambda\theta}}\right)^{(m)}+\theta^{(m)}+\Delta t\theta_{yy}^{(m)},\end{aligned} \tag{15}$$

here, $\xi_2=\frac{\Delta t}{\Delta y^2}$. The numerical procedures for $\theta^{(m+1)}$ resulted to a tri-diagonal inverse matrix. The solution method is established for consistency, convergence and exactness when $h=1$ for the first and second order individually in space and time. Thus, the solution scheme is offered for a spatial and temporal convergence, which is presented not to depend on the step and mesh length.

4. Results and Discussion

The two-step hydromagnetic Eyring-Prandtl fluid combustion-reaction occurs in a non-isothermal device, this is numerically analyzed via finite semi-implicit finite method. This scheme is utilized due to its consistence, exactness, convergent and stability. Following earlier published results in [5] [12] [13], the parameters $\gamma = 0.1$, $\theta_c = 0$, $\lambda = 0.3$, $H = 0.7$, $a = 1$, $\chi = 0.5$, $\delta = 0.3$, $n = 0.5$, $\beta = 0.2$, $Pr = 3$, $Br = 0.3$ and $G = 0.3$ are set as default values, otherwise each plot will indicate. **Table 1** establishes thermal ignition numerical outputs for various reaction kinetics, thus, $n = -2$ for sensitized kinetic, $n = 0$ for Arrhenius kinetic and $n = 0.5$ for Bimolecular kinetic. The heat reaction-diffusion parameters *Br*, χ and λ are examined for the γ_{cr} and $\theta_{\max}$. As observed, the sensitized kinetic gives larger computed outcomes for the γ_{cr} and $\theta_{\max}$, than Arrhenius kinetic follow then by Bimolecular kinetic. The boundary film constraints and fluid material influenced the solution outputs. Meanwhile, in **Table 2**, the plate drag force and thermal propagation gradient outcomes are established. As noted, some parameters caused a rising impact while some declined the flow fluid near the channel wall. This is traceable to the electromagnetic force strength, material viscosity and Joule heating.

Table 1. Numerical outputs for thermal ignition under diverse reaction kinetics.

λ	χ	Br	Kinetic $n = 0.5$		Kinetic $n = 0$		Kinetic $n = -2$	
			γ_{cr}	$\theta_{\max}$	γ_{cr}	$\theta_{\max}$	γ_{cr}	$\theta_{\max}$
0.1	0.0	0.0	3.307231	1.447599	3.823559	1.851517	7.644849	2.068259
0.1	0.0	0.2	3.210989	1.396427	3.666767	1.640554	9.063334	4.314019
0.2	0.7	0.5	1.786553	1.202721	2.009753	1.390478	3.868611	2.570552
0.2	0.5	0.0	2.204408	1.412590	2.548717	1.835072	5.077140	2.053736
0.3	0.5	0.2	2.140021	1.379997	2.444390	1.640554	6.042482	4.319192
0.3	1.0	0.5	1.443763	1.146831	1.607958	1.258885	2.809475	1.991306

Table 2. Numerical outputs for the wall friction (*Sr*) and plate heat gradient (*Hr*).

H	λ	χ	γ	G	Sr	Hr
0.7	0.3	0.5	0.1	0.3	0.8064272086	0.0864398473
1.0					0.0680149172	0.0840103029
1.5					0.5563276261	0.0818279427
	0.1				0.0862222878	0.8064272086
	0.2				0.0863309014	-
		0.7			0.0968217967	-
		1.0			0.1124839370	-
			0.15		0.1256185998	-
			0.20		0.1654853598	-
				0.05	0.0765329919	0.1276016933
				0.20	0.0807744238	0.5208829695

The thermfluidic flow dimension field reaction to rising viscoelastic dilatant material (β) and magnetic (H) parameters are individually displayed in **Figure 2** and **Figure 3**. As observed from the **Figure 2**, the Eyring-Prandtl parameter inspired the viscous fluid reaction-diffusion that arises from an increasing fluid shear thickness and shear strain. This resulted in the propagation of the fluid particles in the non-Newtonian material chemistry of the surface suspended reactant. Therefore, the flowing fluid rate declined steadily along the stream regime. Likewise, in **Figure 3**, increasing values of the magnetic parameter discouraged the Eyring-Prandtl flowing fluid field in the bounded non-isothermal medium. The stimulated Lorentz force induced via magnetic field boosted the fluid molecular bond, which resulted in opposing the flow velocity profile as illustrated in

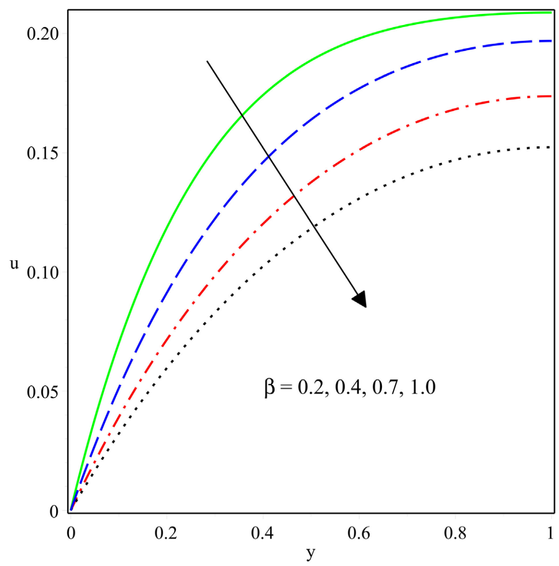

Figure 2. Flow rate field with diverse β.

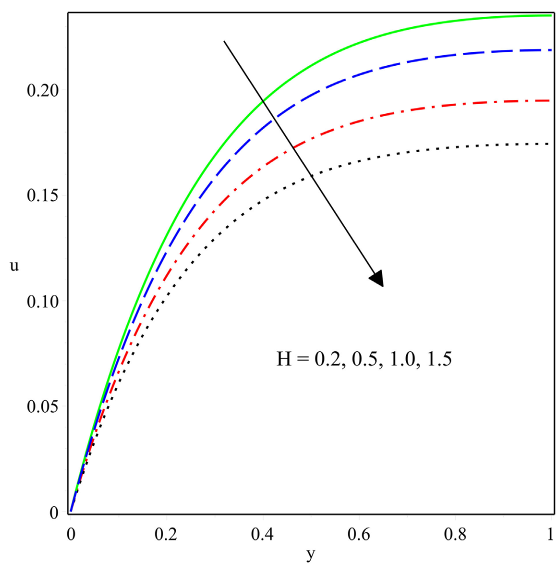

Figure 3. Impact of H on velocity profile.

the diagram. The created electromagnetic force resists free particles collision to damp the flow rate profile. **Figure 4** depicts the pressure gradient (G) impact on the reactive non-Newtonian fluid velocity distribution. As noted, a progressively rise in the velocity field is obtained as the exact pressure on the fluid is raised rapidly along the horizontal flow medium. The fluid diffusion-reaction rate is steadily boosted to inspired internal heat generation and Joule heating, this in turn opposes the viscous Eyring-Prandtl fluid bond. An overall increased in the fluid particle propagation and interaction is noticed, and this thereby enhanced the velocity of the flow profile. Also, the pressure gradient (G) and the magnetic parameter (H) impact are separately demonstrated on the **Figure 5** and **Figure 6**. Both terms (G) and (H) encourages exothermic reaction and thermal energy

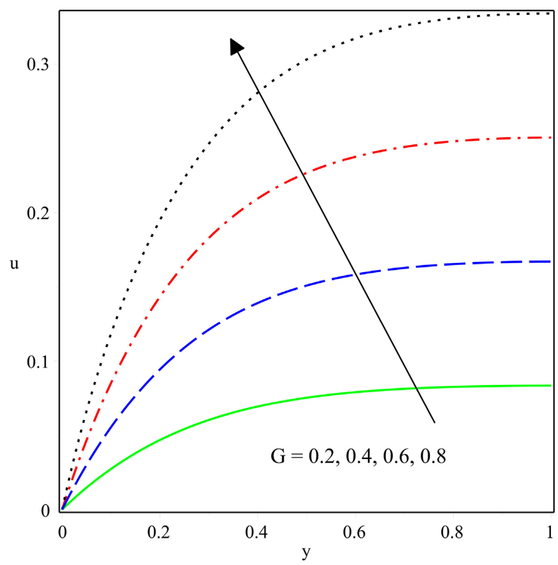

Figure 4. Velocity profile with enhancing G.

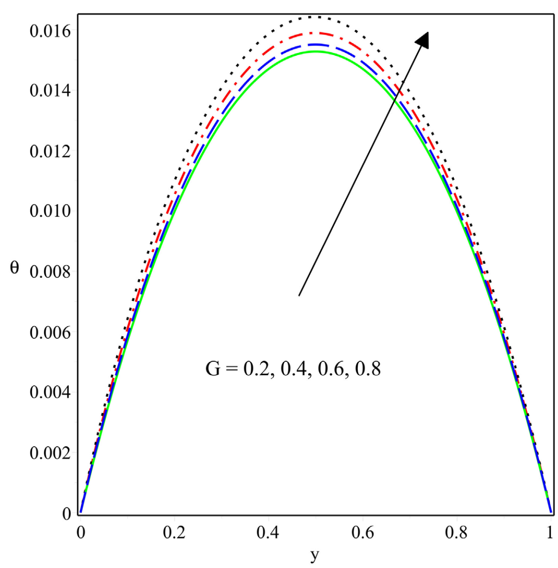

Figure 5. Temperature field for rising G.

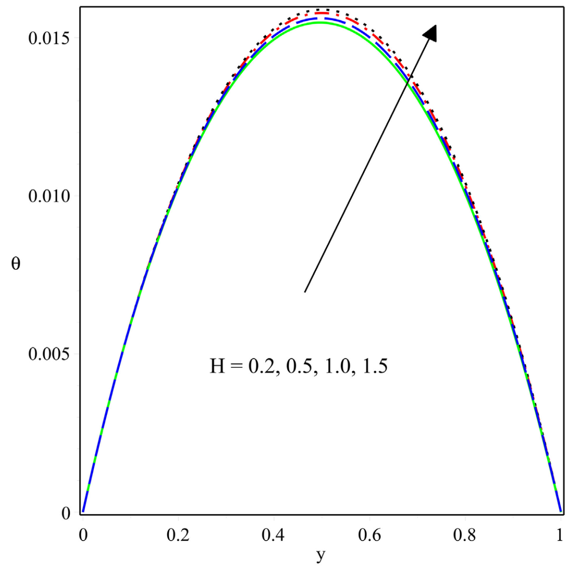

Figure 6. Thermal profile for increasing *H*.

source terms of the heat Equation (9), that in turn decreased the viscoelastic fluid particles bonding. Hence, fluid particles collision is boosted to stimulate Joule heating, which causes an enhancing heat dispersion in the thermal reaction device. Additionally, an electromagnetic force is created by the magnetic field to encourage the Eyring-Prandtl viscous property, thus the diffusion-reaction is stimulated to propel the combustible species reaction. Therefore, the thermal distribution is increased.

In **Figure 7**, the second step parameter (χ) impact on the heat transfer of reactive combustion, Eyring-Prandtl fluid is investigated. It was noted that the heat transport field is highly raised as the parameter (χ) values is increased. The rising heat propagation is momentous in propelling complete exothermic reaction, which reduces the toxic discharge that affects the ecosystem and the environment. As such, two-step diffusion-combustion of Prandtl-Eyring fluid species can be promoted to mitigate the quantity of carbon II oxide released to surroundings. The response of the viscoelastic fluid thermal distribution to an enhancing Brinkman (Br) and Frank-Kamenetskii (γ) parameter values are presented in **Figure 8** and **Figure 9** separately. **Figure 8** displays that the flow channel wall thermal conductivity to the non-Newtonian viscous species is strengthened past the flow medium. Therefore, the particle thermal conductivity of the Joule heating is stimulated to incline the heat transfer field. Likewise, in **Figure 9**, increasing thermal propagation is observed for the homogenous reactant mixtures in the non-isothermal flow channel. Hence, Frank-Kamenetskii parameter (γ) dominated thermal explosion process of the chemical species diffusion-reaction, thus species reactant absorption is considered very low and insignificant for the bimolecular kinetic. As a result, in the exothermic double-step diffusion-reaction system, Frank-Kamenetskii parameter (γ) needs to be consciously monitored to prevent system thermal explosion. In **Figure 10**, the ratio of activation energy parameter (a) on the thermal reaction distribution is presented. An increased in the temperature field is observed as the activation energy ratio is boosted. The quantity of energy needed to propel a chemical reaction defined activation energy, hence the Eyring-Prandtl molecular reactant undergoes species transformation. Thermal energy is stimulated by the activation energy, which turned to inspire exothermic diffusion thermal propagation across the Couette device. Raising the Prandtl number (*Pr*) causes a reduction in the temperature distribution as showed in **Figure 11**. The parameter (*Pr*) defined the ratio of the product of the density and heat capacity to the thermal conductivity. As obtained in the plot, the kinematic viscous diffusivity governs the exothermic two-step reaction-diffusion, which leads to a declined momentum diffusivity. Thus, internal and viscous heating are discouraged, this thereby damps the heat profile as obtained from the figure.

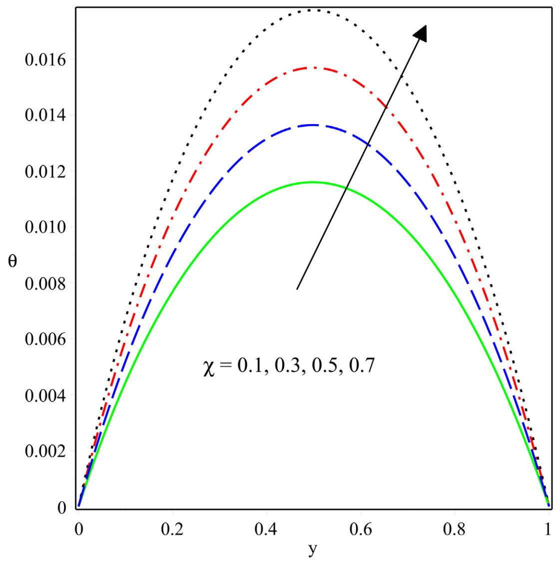

Figure 7. Heat field with boosting χ.

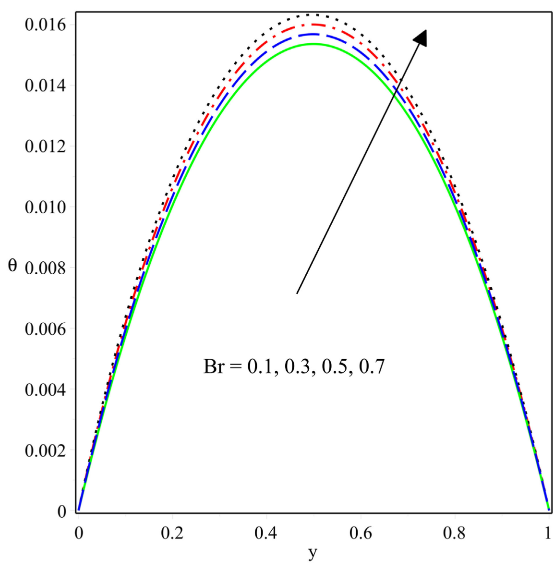

Figure 8. Heat diffusion for different *Br*.

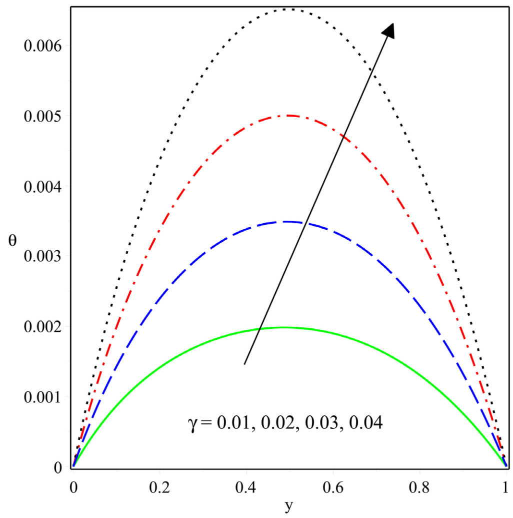

Figure 9. Temperature field for changing γ.

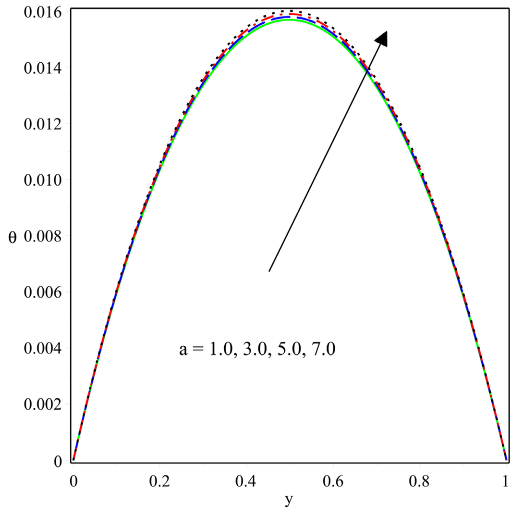

Figure 10. Thermal profile Heat with diverse a.

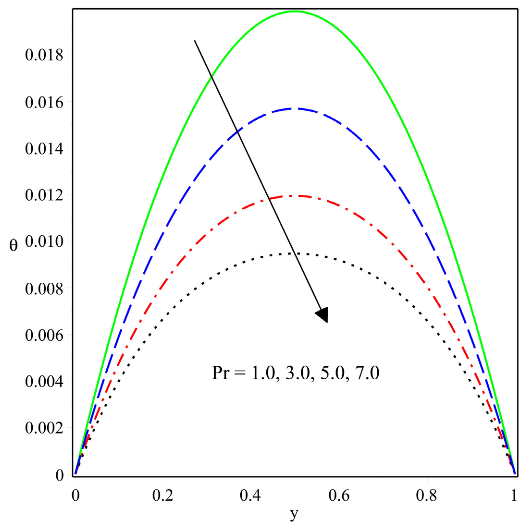

Figure 11. Heat field for rising Pr.

5. Conclusions

The investigation of exothermic diffusion-reaction Eyring-Prandtl hydromagnetic Joule heating reactive species in a Couette medium is carried out. With suitable variables, the invariant derivative model equations are presented for the flowing thermofluid physical properties. The computed solutions are represented in plots to depict the fluid flow rate and thermal distribution. A parametric sensitivity of the entrenched thermodynamical terms is demonstrated. The outputs summary is taken as:

- A monotonically increased in velocity field past the flow regime y declined as the magnetic and material dilatant parameters increased.
- The velocity distribution and the thermal profile strengthened over flow channel middle for a rise in the pressure gradient.
- The Eyring-Pradtl reaction-diffusion is stimulated as the values of Frank Kamenetskii, activation energy ratio and Brinkman number are raised.
- The second step parameter inspired the thermal distribution to assist in the complete combustion process.

This investigation is applicable in industrial processes and in nature that depends on the diffusion-reaction for their endaviour, such as, basic flow reacting species system, pollution management, fire mitigation, rocket and jet propulsion, power production and others. Therefore, extending the study to other viscoelastic fluids in concentric pipe should not be overlooked.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Darji, R.M. and Timol, M.G. (2011) Similarity Solutions of Leminar Incompressible Boundary Layer Equations of Non-Newtonian Viscoinelastic Fluids. *International Journal of Mathematical Archive*, **2**, 1395-1404.

[2] Hayat, T., Bibi, S., Alsaadi, F. and Rafiq, M. (2016) Peristaltic Transport of Prandtl-Eyring Liquid in a Convectively Heated Curved Channel. *PLOS ONE*, **11**, e0156995. https://doi.org/10.1371/journal.pone.0156995

[3] Salawu, S.O., Obalalu, A.M. and Shamshuddin, M.D. (2022) Nonlinear Solar Thermal Radiation Efficiency and Energy Optimization for Magnetized Hybrid Prandtl-Eyring Nanoliquid in Aircraft. *Arabian Journal for Science and Engineering*, **48**, 3061-3072. https://doi.org/10.1007/s13369-022-07080-1

[4] Qureshi, M.A. (2021) A Case Study of MHD Driven Prandtl-Eyring Hybrid Nanofluid Flow over a Stretching Sheet with Thermal Jump Conditions. *Case Studies in Thermal Engineering*, **28**, Article ID: 101581. https://doi.org/10.1016/j.csite.2021.101581

[5] Munjam, S.R., Gangadhar, K., Seshadri, R. and Rajeswar, M. (2021) Novel Technique MDDIM Solutions of MHD Flow and Radiative Prandtl-Eyring Fluid over a Stretching Sheet with Convective Heating. *International Journal of Ambient Energy*, **43**, 4850-4859. https://doi.org/10.1080/01430750.2021.1922498

[6] Salawu, S.O., Obalalu, A.M., Fatunmbi, E.O. and Oderinu, R.A. (2022) Thermal

Prandtl-Eyring Hybridized MoS_2-SiO_2/$C_3H_8O_2$ and SiO_2-$C_3H_8O_2$ Nanofluids for Effective Solar Energy Absorber and Entropy Optimization: A Solar Water Pump Implementation. *Journal of Molecular Liquids*, **361**, Article ID: 119608.

[7] Hayat, T., Ullah, I., Muhammad, K. and Alsaedi, A. (2021) Gyrotactic Microorganism and Bio-Convection during Flow of Prandtl-Eyring Nanomaterial. *Nonlinear Engineering*, **10**, 201-212. https://doi.org/10.1515/nleng-2021-0015

[8] Rehman, K.U., Malik, A.A., Malik, M.Y., Tahir, M. and Zehra, I. (2018) On New Scaling Group of Transformation for Prandtl-Eyring Fluid Model with Both Heat and Mass Transfer. *Results in Physics*, **8**, 552-558. https://doi.org/10.1016/j.rinp.2017.12.071

[9] Okoya, S.S. (2007) Criticality and Transition for a Steady Reactive Plane Couette Flow of a Viscous Fluid. *Mechanics Research Communications*, **34**, 130-135. https://doi.org/10.1016/j.mechrescom.2006.09.006

[10] Salawu, S.O. and Okoya, S.S. (2022) On Criticality for a Branched-Chain Thermal Reactive-Diffusion in a Cylinder. *Combustion Science and Technology*, **194**, 1815-1829. https://doi.org/10.1080/00102202.2020.1837120

[11] Ajadi, S.O. (2011) Approximate Analytic Solution for Critical Parameters in Thermal Explosion Problem. *Applied Mathematics and Computation*, **218**, 2005-2010. https://doi.org/10.1016/j.amc.2011.07.012

[12] Zhu, X.L., Gore, J.P., Karpetis, A.N. and Barlow, R.S. (2002) The Effects of Self-Absorption of Radiation on an Opposed Flow Partially Premixed Flame. *Combustion and Flame*, **129**, 342-345. https://doi.org/10.1016/S0010-2180(02)00341-3

[13] Makinde, O.D. (2009) Thermal Criticality for a Reactive Gravity Driven Thin Film Flow of a Third Grade Fluid with Adiabatic Free Surface Down an Inclined Plane. *Applied Mathematics and Mechanics*, **30**, 373-380. https://doi.org/10.1007/s10483-009-0311-6

[14] Salawu, S.O. and Disu, A.B. (2020) Branch-Chain Criticality and Thermal Explosion of Oldroyd 6-Constant Fluid for a Generalized Couette Reactive Flow. *South African Journal of Chemical Engineering*, **34**, 90-96. https://doi.org/10.1016/j.sajce.2020.06.004

[15] Hassan, A.R., Maritz, R. and Gbadeyan, J.A. (2017) A Reactive Hydromagnetic Heat Generating Fluid Flow with Thermal Radiation within Porous Channel with Symmetrical Convective Cooling. *International Journal of Thermal Sciences*, **122**, 248-256. https://doi.org/10.1016/j.ijthermalsci.2017.08.022

[16] Adesanya, S.O., Falade, J.A., Jangili, S. and Anwar Beg, O. (2017) Irreversibility Analysis for Reactive Third-Grade Fluid Flow and Heat Transfer with Convective Wall Cooling. *Alexandria Engineering Journal*, **56**, 153-160. https://doi.org/10.1016/j.aej.2016.09.017

[17] Frank, J.H., Barlow, R.S. and Lundquist, C. (2000) Radiation and Nitric Oxide Formation in Turbulent Non-Premixed Jet Flame. *Proceedings of the Combustion Institute*, **28**, 447-454. https://doi.org/10.1016/S0082-0784(00)80242-8

[18] Salawu, S.O., Disu, A.B. and Dada, M.S. (2020) On Criticality for a Generalized Couette Flow of a Branch-Chain Thermal Reactive Third-Grade Fluid with Reynold's Viscosity Model. *The Scientific World Journal*, **2020**, Article ID: 7915954. https://doi.org/10.1155/2020/7915954

[19] Grosshandler, W.L. (1993) Radical: A Narrow-Band Model for Radiation Calculations in a Combustion Environment. NIST Technical Note 1402. https://doi.org/10.6028/NIST.TN.1402

[20] Okoya, S.S. (2019) Computational Study of Thermal Influence in Axial Annular

Flow of a Reactive Third Grade Fluid with Nonlinear Viscosity. *Alexandria Engineering Journal*, **58**, 401-411. https://doi.org/10.1016/j.aej.2019.01.001

[21] Salawu, S.O., Abolarinwa, A. and Fenuga, O.J. (2020) Transient Analysis of Radiative Hydromagnetic Poiseuille Fluid Flow of Two-Step Exothermic Chemical Reaction through a Porous Channel with Convective Cooling. *Journal of Computational & Applied Research in Mechanical Engineering*, **10**, 51-62.

[22] Szabo, Z.G. (1964) Advances in Kinetics of Homogeneous Gas Reactions. Methuen & Co Limited, London.

[23] Makinde, O.D., Olanrewaju, P.O., Titiloye, E.O. and Ogunsola, A.W. (2013) On Thermal Stability of a Two-Step Exothermic Chemical Reaction in a Slab. *Journal of Mathematical Sciences*, **13**, 1-15.

[24] Kareem, R.A. and Gbadeyan, J.A. (2020) Entropy Generation and Thermal Criticality of Generalized Couette Hydromagnetic Flow of Two-Step Exothermic Chemical Reaction in a Channel. *International Journal of Thermofluids*, **5-6**, Article ID: 100037. https://doi.org/10.1016/j.ijft.2020.100037

[25] Salawu, S.O., Ogunseye, H.A., Shamshuddin, M.D. and Disu, A.B. (2022) Reaction-Diffusion of Double Exothermic Couple Stress Fluid and Thermal critiCality with Reynolds Viscosity and Optical Radiation. *Chemical Physics*, **561**, Article ID: 111601. https://doi.org/10.1016/j.chemphys.2022.111601

[26] Williams, F.A. (1985) Combustion Theory. 2nd Edition, Benjamin & Cuminy Publishing Inc., Menlo Park.

[27] Salawu, S.O. (2023) Two-Step Exothermic Reaction-Diffussion of Hydromagnetic Prandtl Eying Viscous Heating Fluid in a Channel. *International Journal of Thermofluids*, **17**, 100300. https://doi.org/10.1016/j.ijft.2023.100300

[28] Mekheimer, K.S. and Ramadan, S.F. (2020) New Insight into Gyrotactic Microorganisms for Bio-Thermal Convection of Prandtl Nanofluid over a Stretching/Shrinking Permeable Sheet. *SN Applied Sciences*, **2**, Article No. 450. https://doi.org/10.1007/s42452-020-2105-9

[29] Okoya, S.S. (2011) Disappearance of Criticality for Reactive Third-Grade Fluid with Reynold's Model Viscosity in a Flat Channel. *International Journal of Non-Linear Mechanics*, **46**, 1110-1115. https://doi.org/10.1016/j.ijnonlinmec.2011.04.008

[30] Salawu, S.O., Kareem, R.A., Shamshuddin, M.D. and Khan, S.U. (2020) Double Exothermic Reaction of Viscous Dissipative Oldroyd 8-Constant Fluid and Thermal Ignition in a Channel. *Chemical Physics Letters*, **760**, Article ID: 138011. https://doi.org/10.1016/j.cplett.2020.138011

[31] Makinde, O.D. and Chinyoka, T. (2010) Numerical Investigation of Transient Heat Transfer to Hydromagnetic Channel Flow with Radiative Heat and Convective Cooling. *Communications in Nonlinear Science and Numerical Simulation*, **15**, 3919-3930. https://doi.org/10.1016/j.cnsns.2010.01.013

[32] Makinde, O.D. and Chinyoka, T. (2011) Numerical Study of Unsteady Hydromagnetic Generalized Couette Flow of a Reactive Third-Grade Fluid with Asymmetric Convective Cooling. *Computer and Mathematics with Applications*, **61**, 1167-1179. https://doi.org/10.1016/j.camwa.2010.12.066

[33] Makinde, O.D. (2009) Thermal Stability of a Reactive Viscous Flow through a Porous-Saturated Channel with Convective Boundary Conditions. *Applied Thermal Engineering*, **29**, 1773-1777. https://doi.org/10.1016/j.applthermaleng.2008.08.012

[34] Chinyoka, T. (2008) Computational Dynamics of a Thermally Decomposable Viscoelastic Lubricant under Shear. *Journal of Fluids Engineering*, **130**, Article ID: 121201. https://doi.org/10.1115/1.2978993