Scientific Research Publishing

Journal of Applied Mathematics and Physics, 2023, 11, 2397-2413
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Asset Pricing and Simulation Analysis Based on the New Mixture Gaussian Processes

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How to cite this paper: Peng, B. (2023) Asset Pricing and Simulation Analysis Based on the New Mixture Gaussian Processes. *Journal of Applied Mathematics and Physics*, **11**, 2397-2413.
https://doi.org/10.4236/jamp.2023.118153

Received: July 24, 2023
Accepted: August 21, 2023
Published: August 24, 2023

Abstract

European compound option pricing model is established by using the mixed bifractional Brownian motion. Firstly, using the principle of risk-neutral pricing, the European option pricing formulas and the parity formulas are obtained. Secondly, with the Delta hedging strategy, the corresponding compound option pricing formulas and the parity formulas are got. Finally, using the daily closing price data of "Lingang B shares" and "Yitai B shares" respectively, the results show that the mixed model is closer to the true value than the previous model.

Keywords

Bifractional Brownian Motion, Compound Option, Option Pricing

1. Introduction

Options have become one of the most dynamic financial derivatives, and have been rapidly developed and widely used. Especially in 1973, literature [1] proposed the classic Black-Scholes (B-S) pricing model, which had an important impact on the history of financial mathematics. With continuous in-depth research on the classic Brownian motion model, it is found that some of the original assumptions are not in line with financial reality. The assumption of geometric Brownian motion cannot describe the self-similarity and long-term correlation of financial markets, so some scholars tried to use fractional Brownian motion to describe financial market prices [2] [3] [4] [5]. Option pricing driven by fractional Brownian motion had also once become one of the hot spots in financial mathematics research.

Although the fractional Brownian motion model can greatly describe the process of asset price changes in financial markets, it allows the existence of arbitrage opportunities [6] [7]. In order to solve the arbitrage problem in the financial market, a large number of scholars have proposed modified fractional Brownian motion models to describe the price changes in the financial market [8] [9] [10]. Xu studied bifractional Black-Scholes model for pricing European options and compound options [11]. But the latest research shows that there is still arbitrage [12]. Therefore, based on self-similarity, avoiding arbitrage, long-term correlation and other characteristics of financial markets, this paper constructs a Gaussian mixture process to characterize the price of financial assets.

The writing arrangement is as follows. The first part is preliminary knowledge, the second part is the main conclusion of establishing the European option and corresponding compound option pricing model under the mixed bifractional Brownian motion, the third part is numerical simulation, and the fourth part is the conclusion.

2. Pre-Knowledge

Mainly introduce the new mixed Brownian motion model and its related definitions.

Definition2.1 [13] Let (Ω, *F*, *P*) is a complete probability space, the linear combination of Brownian motion, sub-fractional Brownian motion and bifractional Brownian motion, then the mixed bifractional Brownian motion model is

$$\mathrm{d}S_t = \mu S_t \mathrm{d}t + \sigma S_t \mathrm{d}\left(\beta_t + \beta_t^H + \beta_t^{H,S}\right), \tag{2.1}$$

$$\mathrm{d}Z_t = rZ_t\mathrm{d}t,\ \ M_t^{H,S,a,b,c} = a\beta_t + b\beta_t^H + c\beta_t^{H,S}, \forall t \ge 0, \tag{2.2}$$

where

β_t^H is subfractional Brownian motion, β_t is Brownian motion, and $\beta_t^{H,S}$ is bifractional Brownian motion. β_t, β_t^H and $\beta_t^{H,S}$ are independent of each other. H is the Hurst index. a, b and c are constants. S is the parameter. Z_t is risk-free asset bond. S_t is risky asset stock.

3. Main Results

The mixed bifractional Brownian motion is used to replace the random part of the model, and the European option and corresponding compound option pricing formula under the mixed bifractional Brownian motion is obtained.

The assumptions for the financial market are as follows,

1) There are two assets in the financial market, such as risky asset stocks S_t, which satisfy Equation (2.1). The risk-free asset bonds Z_t, which satisfy Equation (2.2);

2) The risk-free interest rate r is a constant, the expected rate of return μ is a constant, the market is complete and the underlying asset volatility σ is a constant;

3) The option can only be exercised on the expiry date;

4) In market transactions, there are no transaction fees;

5) The transaction is infinitely divisible.

Under the assumptions 1) - 5), through the risk-neutral pricing principle, the European option and corresponding compound option pricing formula can be obtained.

Theorem 3.1 Assuming that the underlying asset price S_t satisfies the formula (1.1), $t\in[0,T]$, then at time T, the European call option price G driven by the mixed bifractional Brownian motion model satisfies the following equation

$$\begin{aligned}&\frac{1}{2}\sigma^2S_t^2\frac{\partial^2G}{\partial S_t^2}+H\sigma^2\left(2-2^{2H-1}\right)t^{2H-1}S_t^2\frac{\partial^2G}{\partial S_t^2}+\frac{\partial G}{\partial t}+rS_t\frac{\partial G}{\partial S_t}\\&+HS\sigma^2t^{2HS-1}S_t^2\frac{\partial^2G}{\partial S_t^2}=rG\end{aligned}\tag{3.1}$$

Proof. There is a portfolio Π_t, then Π_t satisfies following equation

$$\begin{aligned}\mathrm{d}\Pi_t&=\mathrm{d}G-\Delta\mathrm{d}S_t\\&=\left(\frac{1}{2}\sigma^2S_t^2\frac{\partial^2G}{\partial S_t^2}+H\sigma^2\left(2-2^{2H-1}\right)t^{2H-1}S_t^2\frac{\partial^2G}{\partial S_t^2}+\frac{\partial G}{\partial t}+\mu S_t\frac{\partial G}{\partial S_t}\right.\\&\quad\left.+HS\sigma^2t^{2HS-1}S_t^2\frac{\partial G^2}{\partial S_t^2}\right)\mathrm{d}t+\sigma S_t\frac{\partial G}{\partial S_t}\left(\mathrm{d}\beta_t+\mathrm{d}\beta_t^H+\mathrm{d}\beta_t^{H,S}\right)\\&\quad-\Delta S_t\mu\mathrm{d}t-\Delta S_t\sigma\left(\mathrm{d}\beta_t+\mathrm{d}\beta_t^H+\mathrm{d}\beta_t^{H,S}\right)\\&=\left(\frac{\partial G}{\partial t}+\mu S_t\left(\frac{\partial G}{\partial S_t}-\Delta\right)+HS\sigma^2t^{2HS-1}S_t^2\frac{\partial G^2}{\partial S_t^2}\right.\\&\quad\left.+\frac{1}{2}\sigma^2S_t^2\frac{\partial^2G}{\partial S_t^2}+H\sigma^2\left(2-2^{2H-1}\right)t^{2H-1}S_t^2\frac{\partial^2G}{\partial S_t^2}\right)\mathrm{d}t\\&\quad+\left(\frac{\partial G}{\partial S_t}-\Delta\right)S_t\sigma\left(\mathrm{d}\beta_t+\mathrm{d}\beta_t^H+\mathrm{d}\beta_t^{H,S}\right).\end{aligned}\tag{3.2}$$

Let $\frac{\partial G}{\partial S_t}=\Delta$, $\mathrm{d}\Pi_t=r\Pi_t\mathrm{d}t$, we have

$$\begin{aligned}&r\left(G-S_t\frac{\partial G}{\partial S_t}\right)\mathrm{d}t\\&=\left(\frac{1}{2}\sigma^2S_t^2\frac{\partial^2G}{\partial S_t^2}+H\sigma^2\left(2-2^{2H-1}\right)t^{2H-1}S_t^2\frac{\partial^2G}{\partial S_t^2}+\frac{\partial G}{\partial t}+HS\sigma^2t^{2HS-1}S_t^2\frac{\partial G^2}{\partial S_t^2}\right)\mathrm{d}t.\end{aligned}\tag{3.3}$$

The proof is completed.

Theorem 3.2 Assuming that the underlying asset price S_t satisfies the formula (2.1), $t\in[0,T]$, and the strike price is K. Then at time T, the European call option price G driven by the mixed bifractional Brownian motion model is

$$G=S_tN(d_1)-\mathrm{e}^{-r(T-t)}KN(d_2),\tag{3.4}$$

where

$$d_1=\frac{\ln\frac{S_t}{K}+\frac{1}{2}\sigma^2\left(\left(2-2^{2H-1}\right)\left(T^{2H}-t^{2H}\right)+(T-t)\right)+r(T-t)+\frac{1}{2}\sigma^2\left(T^{2HS}-t^{2HS}\right)}{\sigma\sqrt{\left(T^{2HS}-t^{2HS}\right)+\left(2-2^{2H-1}\right)\left(T^{2H}-t^{2H}\right)+(T-t)}},$$

$$d_2=d_1-\sigma\sqrt{\left(T^{2HS}-t^{2HS}\right)+\left(2-2^{2H-1}\right)\left(T^{2H}-t^{2H}\right)+(T-t)}.$$

Proof. Let $S_t = \mathrm{e}^x$, $G = V(x,t)$, then we have

$$\ln S_t = x,\ \frac{\partial G}{\partial t} = \frac{\partial V}{\partial t},\ \frac{\partial G}{\partial S_t} = \frac{1}{S_t}\frac{\partial V}{\partial x},\ \text{and}\ \frac{\partial^2 G}{\partial S_t^2} = \frac{1}{S_t^2}\left(\frac{\partial^2 V}{\partial x^2} - \frac{\partial V}{\partial x}\right). \quad (3.5)$$

Substituting (3.5) into (3.1), we have

$$\begin{aligned}&\frac{\partial V}{\partial t} + r\frac{\partial V}{\partial x} - \left(HS\sigma^2 t^{2HS-1} + \frac{1}{2}\sigma^2 + H\sigma^2\left(2-2^{2H-1}\right)t^{2H-1}\right)\frac{\partial V}{\partial x} \\ &+\left(HS\sigma^2 t^{2HS-1} + \frac{1}{2}\sigma^2 + H\sigma^2\left(2-2^{2H-1}\right)t^{2H-1}\right)\frac{\partial^2 V}{\partial x^2} = rV,\end{aligned} \quad (3.6)$$

and we get $V(T,x) = \left(\mathrm{e}^x - K\right)^+$.

Let $u(f,z) = V(t,x)\mathrm{e}^{r(T-t)}$,

$$f = \frac{1}{2}\sigma^2\left(\left(T^{2HS} - t^{2HS}\right) + \left(2-2^{2H-1}\right)\left(T^{2H} - t^{2H}\right) + (T-t)\right), \text{ and}$$

$$z = x + r(T-t) - \frac{1}{2}\sigma^2\left(\left(T^{2HS} - t^{2HS}\right) + \left(2-2^{2H-1}\right)\left(T^{2H} - t^{2H}\right) + (T-t)\right),$$

then we get

$$\begin{aligned}\frac{\partial V}{\partial t} = r\mathrm{e}^{-r(T-t)}u &- \mathrm{e}^{-r(T-t)}\left(HS\sigma^2 t^{2HS-1} + \frac{1}{2}\sigma^2 + H\sigma^2\left(2-2^{2H-1}\right)t^{2H-1}\right)\frac{\partial u}{\partial f} \\ &+ \mathrm{e}^{-r(T-t)}\left(-r + HS\sigma^2 t^{2HS-1} + \frac{1}{2}\sigma^2 + H\sigma^2\left(2-2^{2H-1}\right)t^{2H-1}\right)\frac{\partial u}{\partial z},\end{aligned}$$

$$\frac{\partial V}{\partial x} = \mathrm{e}^{-r(T-t)}\frac{\partial u}{\partial z},\ \text{and}\ \frac{\partial^2 V}{\partial x^2} = \mathrm{e}^{-r(T-t)}\frac{\partial^2 u}{\partial z^2}. \quad (3.7)$$

Substituting (3.7) into (3.6), we get

$$\frac{\partial u}{\partial f} = \frac{\partial^2 u}{\partial z^2}, \quad (3.8)$$

where the boundary value condition is

$$u(0,z) = \left(\mathrm{e}^z - K\right)^+.$$

Then (3.8) has a unique strong solution, which is described by the equation

$$u(f,z) = \frac{1}{2\sqrt{\pi f}}\int_{-\infty}^{+\infty}\left(\mathrm{e}^\eta - K\right)^+ \mathrm{e}^{-\frac{(\eta-z)^2}{4f}}\,\mathrm{d}\eta, \quad (3.9)$$

substituting the boundary value condition into (3.9), we get

$$u(f,z) = \mathrm{e}^{f+z}N\left(\frac{z+2f-\ln K}{\sqrt{2f}}\right) - KN\left(\frac{z-\ln K}{\sqrt{2f}}\right). \quad (3.10)$$

By the inverse transformation, we can get the European call option pricing formulas.

Similarly, the price P of the European put option driven by the mixed bifractional Brownian motion model is

$$P = \mathrm{e}^{-r(T-t)}KN(-d_2) - S_tN(-d_1), \quad (3.11)$$

in the formula, d_1, d_2 and $N(\cdot)$ are the same as the above. Similarly, the parity formula of the European call and put option driven by the mixed bifractional Brownian motion model is

$$P - G = e^{-r(T-t)} K - S_t . \quad (3.12)$$

The proof is completed.

For further promotion, we consider the compound option pricing formula driven by the mixed bifractional Brownian motion model.

Theorem 3.3 The price GG of the compound option (a call on a call) driven by the mixed bifractional Brownian motion model is

$$GG = S_t N_1(d_3 + m, d_2 + n; \rho) - K e^{-r(T-t)} N_1(d_3, d_2; \rho) - K_1 e^{-r(T_*-t)} N(d_3), \quad (3.13)$$

where

$$d_3 = \frac{\ln \frac{S_t}{X} + r(T_* - t) - \frac{1}{2}\sigma^2 (T_*^{2HS} - t^{2HS}) - \frac{1}{2}\sigma^2 \left(\left(2 - 2^{2H-1}\right)\left(T^{2H} - t^{2H}\right) + (T - t) \right)}{\sigma \sqrt{\left(T_*^{2HS} - t^{2HS}\right) + \left(2 - 2^{2H-1}\right)\left(T^{2H} - t^{2H}\right) + (T - t)}},$$

$$m = \sigma \sqrt{\left(T_*^{2HS} - t^{2HS}\right) + \left(2 - 2^{2H-1}\right)\left(T^{2H} - t^{2H}\right) + (T - t)},$$

$$n = \sigma \sqrt{\left(T^{2HS} - t^{2HS}\right) + \left(2 - 2^{2H-1}\right)\left(T^{2H} - t^{2H}\right) + (T - t)},$$

and $$\rho = \frac{\sqrt{\left(T_*^{2HS} - t^{2HS}\right) + \left(2 - 2^{2H-1}\right)\left(T^{2H} - t^{2H}\right) + (T - t)}}{\sqrt{\left(T^{2HS} - t^{2HS}\right) + \left(2 - 2^{2H-1}\right)\left(T^{2H} - t^{2H}\right) + (T - t)}}.$$

Proof. According to Theorem 3.2, we can get

$$G = S_{T_*} N(y_1) + K e^{-r(T-t)} N\left(y_1 - \sigma \sqrt{\left(T^{2HS} - T_*^{2HS}\right) + \left(2 - 2^{2H-1}\right)\left(T^{2H} - T_*^{2H}\right) + (T - T_*)} \right),$$

where

$$y_1 = \frac{\ln \frac{S_{T_*}}{K} + r(T - T_*) + \frac{1}{2}\sigma^2 \left(T^{2HS} - T_*^{2HS}\right) - \frac{1}{2}\sigma^2 \left(\left(2 - 2^{2H-1}\right)\left(T^{2H} - T_*^{2H}\right) + (T - T_*) \right)}{\sigma \sqrt{\left(T^{2HS} - T_*^{2HS}\right) + \left(2 - 2^{2H-1}\right)\left(T^{2H} - T_*^{2H}\right) + (T - T_*)}}, \quad (3.14)$$

Let $G = K_*$, then X satisfies the following equation

$$X N(y_1^*) - K e^{-r(T-t)} N\left(y_1^* - \sigma \sqrt{\left(T^{2HS} - T_*^{2HS}\right) + \left(2 - 2^{2H-1}\right)\left(T^{2H} - T_*^{2H}\right) + (T - T_*)} \right) = K_*,$$

where

$$y_1^* = \frac{\ln \frac{X}{K} + r(T - T_*) + \frac{1}{2}\sigma^2 \left(T^{2HS} - T_*^{2HS}\right) + \frac{1}{2}\sigma^2 \left(\left(2 - 2^{2H-1}\right)\left(T^{2H} - T_*^{2H}\right) + (T - T_*) \right)}{\sigma \sqrt{\left(T^{2HS} - T_*^{2HS}\right) + \left(2 - 2^{2H-1}\right)\left(T^{2H} - T_*^{2H}\right) + (T - T_*)}}. \quad (3.15)$$

According to the above, we can get

$$GG = I_1 - I_2, \quad (3.16)$$

where $I_1 = e^{-r(T_*-t)} \tilde{E}[G \mathbf{1}_A]$, $I_2 = K_* e^{-r(T_*-t)} \tilde{E}[\mathbf{1}_A]$, $A = \{S_{T_*} \mid S_{T_*} > X\}$, I_1 and I_2 denote the indicator function.

Noted that

$$S_T = S_t \exp\left\{ r(T-t) - \frac{1}{2}\sigma^2 \left(T^{2HS} - t^{2HS}\right) - \frac{1}{2}\sigma^2 \left(\left(2-2^{2H-1}\right)\left(T^{2H} - t^{2H}\right) + (T-t)\right) + \sigma\left(\beta_T^{H,S} - \beta_t^{H,S} + \beta_T^H - \beta_t^H + \beta_T - \beta_t\right) \right\}, \tag{3.17}$$

where

$$A = \left\{ s_{T_*} \mid s_{T_*} > X \right\} = \left\{ -\frac{\beta_{T_*}^{H,S} - \beta_t^{H,S} + \beta_{T_*}^H - \beta_t^H + \beta_{T_*} - \beta_t}{\sqrt{\left(T_*^{2HS} - t^{2HS}\right) + \left(2-2^{2H-1}\right)\left(T_*^{2H} - t^{2H}\right) + \left(T_* - t\right)}} \mid -\frac{\beta_{T_*}^{H,S} - \beta_t^{H,S} + \beta_{T_*}^H - \beta_t^H + \beta_{T_*} - \beta_t}{\sqrt{\left(T_*^{2HS} - t^{2HS}\right) + \left(2-2^{2H-1}\right)\left(T_*^{2H} - t^{2H}\right) + \left(T_* - t\right)}} < d_3 \right\}.$$

Due to

$$G = \mathrm{e}^{-r(T-T_*)} \tilde{E}_{T_*}\left[\left(S_T - K\right) \mathbf{1}_{\{S_T > K\}} \right], \tag{3.18}$$

where

$$S_T = S_{T_*} \exp\left\{ r(T-T_*) - \frac{1}{2}\sigma^2 \left(T^{2HS} - T_*^{2HS}\right) - \frac{1}{2}\sigma^2 \left(\left(2-2^{2H-1}\right)\left(T^{2H} - T_*^{2H}\right) + (T-T_*)\right) + \sigma\left(\beta_T^{H,S} - \beta_{T_*}^{H,S} + \beta_T^H - \beta_{T_*}^H + \beta_T - \beta_{T_*}\right) \right\},$$

so we can get

$$G = \mathrm{e}^{-r(T-T_*)} \tilde{E}_{T_*}\left[S_T \mathbf{1}_B \right] - K\mathrm{e}^{-r(T-T_*)} \tilde{E}_{T_*}\left[\mathbf{1}_B \right], \tag{3.19}$$

where

$$B = \left\{ \left(S_{T_*}, S_T\right) \mid S_{T_*} > X, S_T > K \right\} = \left\{ -\frac{\beta_{T_*}^{H,S} - \beta_t^{H,S} + \beta_T^H - \beta_t^H + \beta_T - \beta_t}{\sqrt{\left(T_*^{2HS} - t^{2HS}\right) + \left(2-2^{2H-1}\right)\left(T_*^{2H} - t^{2H}\right) + \left(T_* - t\right)}} < d_3, -\frac{\beta_T^{H,S} - \beta_t^{H,S} + \beta_T^H - \beta_t^H + \beta_T - \beta_t}{\sqrt{\left(T^{2HS} - t^{2HS}\right) + \left(2-2^{2H-1}\right)\left(T^{2H} - t^{2H}\right) + \left(T - t\right)}} < d_2 \right\},$$

then

$$I_1 = \tilde{E}\left[S_T \mathbf{1}_B \right] - K\mathrm{e}^{-r(T-t)} N_1\left(d_3, d_2; \rho\right) = S_t N_1\left(d_3 + m, d_2 + \sigma\sqrt{\left(T^{2HS} - t^{2HS}\right) + \left(2-2^{2H-1}\right)\left(T^{2H} - t^{2H}\right) + \left(T-t\right)}; \rho \right) - K\mathrm{e}^{-r(T-t)} N_1\left(d_3, d_2; \rho\right). \tag{3.20}$$

Similarly, the price PG of the compound option (a put on a call) driven by the mixed bifractional Brownian motion model is

$$\mathrm{PG} = -S_t N_1\left(d_3+m, d_2+n; \rho\right) + K\mathrm{e}^{-r(T-t)} N_1\left(-d_3, d_2; \rho\right) + K_* \mathrm{e}^{-r(T_*-t)} N\left(-d_3\right). \quad (3.21)$$

The price GP of the compound option (a call on a put) driven by the mixed bifractional Brownian motion model is

$$\mathrm{GP} = -S_t N_1\left(-d_3-m, d_2+n; -\rho\right) - K\mathrm{e}^{-r(T-t)} N_1\left(d_3, d_2; \rho\right) + K_* \mathrm{e}^{-r(T_*-t)} N\left(-d_3\right). \quad (3.22)$$

And the price PP of the compound option (a put on a put) driven by the mixed bifractional Brownian motion model is

$$\begin{aligned}\mathrm{PP} = {} & S_t N_1\left(d_3 + \sigma\sqrt{\left(T_*^{2HS} - t^{2HS}\right) + \left(2-2^{2H-1}\right)\left(T_*^{2H} - t^{2H}\right) + \left(T_* - t\right)}, -d_2 - n; -\rho\right) \\ & - K\mathrm{e}^{-r(T-t)} N_1\left(d_3, d_2; \rho\right) + K_* \mathrm{e}^{-r(T_*-t)} N\left(d_3\right).\end{aligned} \quad (3.23)$$

The proof is completed.

4. Numerical Simulation

The stocks of “Lingang B shares” from March 7, 2022, to March 18, 2022, and principles [14] [15]. Taking the “ Lingang B shares” stock as the object, using the rescaled range (R/S) analysis method, the estimated value of the parameter H is calculated to be 0.7251.

Supposing the stock price is $S_0, S_1, \cdots, S_n$, and the rate of return is $\frac{S_1-S_0}{S_0}, \frac{S_2-S_1}{S_1}, \cdots, \frac{S_n-S_{n-1}}{S_{n-1}}$. Calculating the variance of the logarithmic return, and the parameter sigma is 0.01114215.

Calculating the average of the logarithmic return, and the parameter μ is 0.0007258801.

Substituting the specific parameter values into equation (2.1), and assuming that $S=1$. Taking the closing price of “ Lingang B shares” on March 7, 2022, at 1.068 as the initial price S_0, and getting S_{0+dt}, S_{0+2dt}, $\cdots$, until $S_{0+ndt} = S_T$. The simulated value of the mixed bifractional Brownian model,the classic B-S model and the true value of the stock are shown in **Figure 1**. The comparison between the model simulation value and the true value in the next 4 days is shown in **Table 1**.

It can be obtained from **Table 1** that the simulation effect of the mixed bifractional Brownian model is better in the next 4 days.

The stocks of “Lingang B shares” from April 29, 1994, to March 18, 2022, and principles [14] [15]. Taking the “Lingang B shares” stock as the object, using the rescaled range (R/S) analysis method, the estimated value of the parameter H is calculated to be 0.2332.

Supposing the stock price is $S_0, S_1, \cdots, S_n$, and the rate of return is $\frac{S_1-S_0}{S_0}, \frac{S_2-S_1}{S_1}, \cdots, \frac{S_n-S_{n-1}}{S_{n-1}}$. Calculating the variance of the logarithmic return, and the parameter sigma is 0.03012762.

Calculating the average of the logarithmic return, and the parameter μ is 0.000228982.

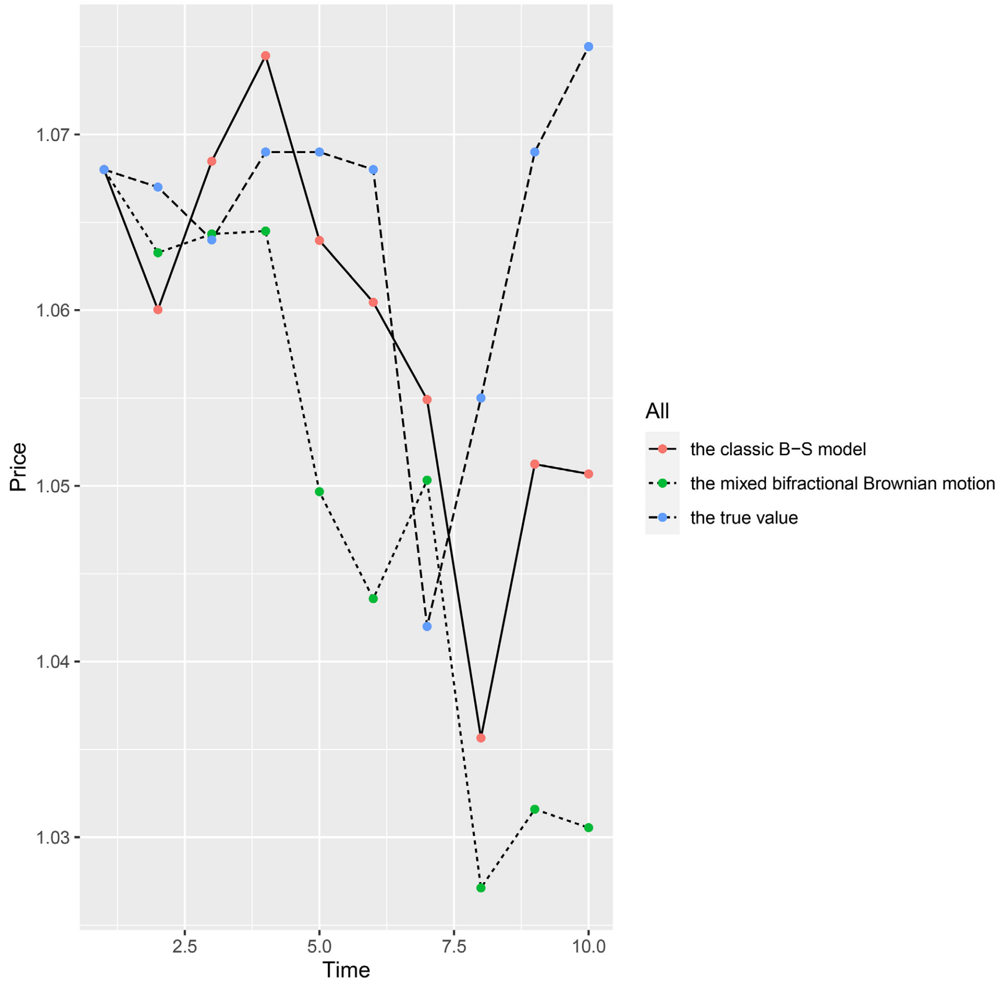

Figure 1. The comparison of mixed bifractional Brownian model, classic B-S model and true value.

Table 1. Comparison of model simulation value and true value in the next 4 days.

	stock price simulation of mixed bifractional Brownian model	stock price simulation of classic B-S model	Lingang B shares
March 8, 2022	1.063271346	1.06002429068931	1.067
March 9, 2022	1.064331773	1.06847507304147	1.064
March 10, 2022	1.06449992	1.07448110768571	1.069
March 15, 2022	1.050323999	1.05490899119777	1.042

Substituting the specific parameter values into Equation (2.1), and assuming that $S = 1$. Taking the closing price of " Lingang B shares" on April 29, 1994, at 0.25 as the initial price S_0, and getting S_{0+dt}, S_{0+2dt}, $\cdots$, until $S_{0+ndt} = S_T$. The simulated value of the mixed bifractional Brownian model, the classic B-S model and the true value of the stock are shown in **Figure 2**. The specific statistical analysis of the simulation results is shown in **Table 2**, and the comparison between the model simulation value and the true value in the next 12 days is shown in Table 3.

It can be seen from Table 2 that the mixed bifractional Brownian model simulates 1st qu., median, mean, 3rd qu., and variance of the stock price are closer to the real value of the stock than the classic B-S model. It can be obtained from Table 3 that the simulation effect of the mixed bifractional Brownian model is better in the next 12 days.

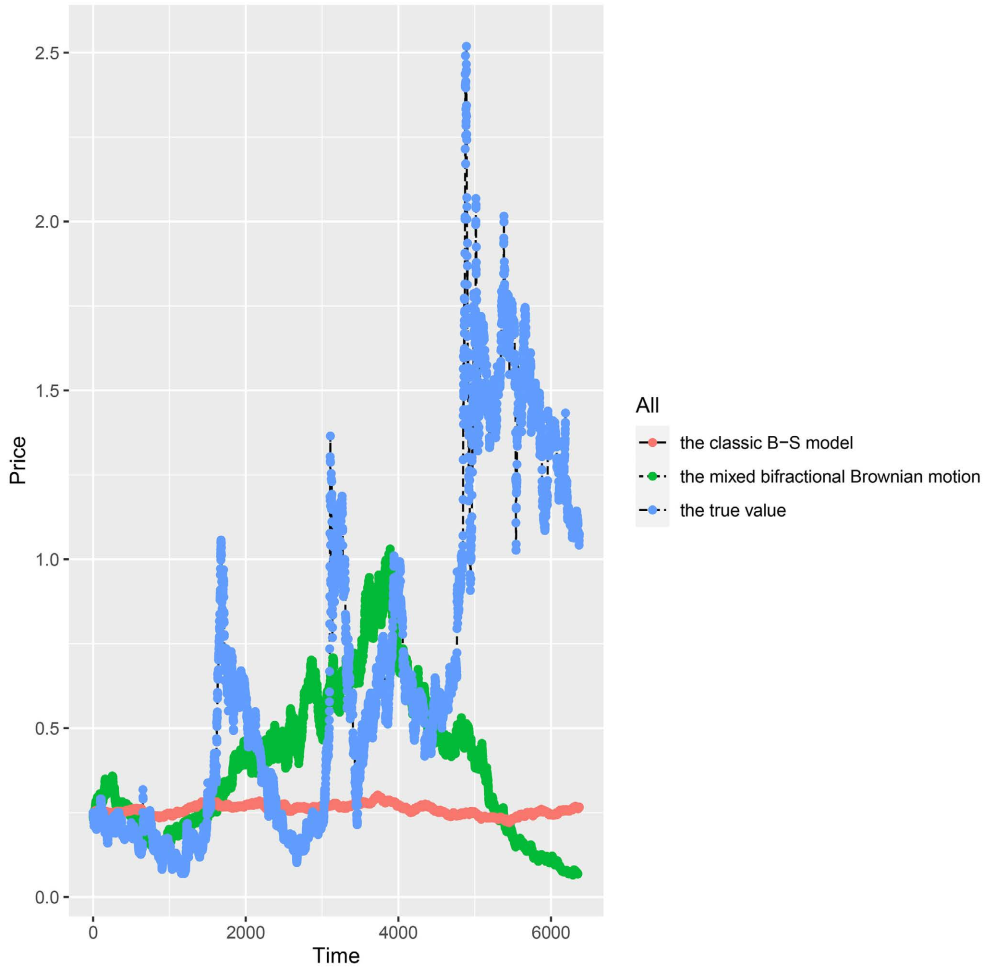

Figure 2. The comparison of mixed bifractional Brownian model, classic B-S model and true value.

Table 2. Statistics of simulation and true values.

	Mixed bifractional Brownian model simulation value	Classic B-S model simulation value	True value
1st qu.	0.2150	0.2479	0.2100
median	0.3931	0.2594	0.5460
mean	0.3939	0.2589	0.6643
3rd qu.	0.5228	0.2702	1.0420
variance	0.04721107	0.0002223041	0.2579141

Table 3. Comparison of model simulation value and true value in the next 12 days.

	stock price simulation of mixed bifractional Brownian model	stock price simulation of classic B-S model	Lingang B shares
March 12, 2015	0.476745781464515	0.247037583207719	1
March 13, 2015	0.470871594147794	0.24612100303794	1.01
March 16, 2015	0.469303942071588	0.246641436102686	1.013
March 17, 2015	0.467253422398352	0.247539276790604	1.01
March 18, 2015	0.47762562124569	0.247658372094785	1.018
March 19, 2015	0.475649656001741	0.249230366298891	1.007
March 20, 2015	0.467664339827485	0.249163120217517	0.993
March 23, 2015	0.484552440867171	0.249511346624663	0.977
March 24, 2015	0.491368306763706	0.248219479411458	0.986
March 25, 2015	0.490997401314635	0.247618858202052	0.986
March 26, 2015	0.489448509757798	0.24647079515958	0.97
March 27, 2015	0.498026160967964	0.247123219657576	0.992

The stocks of "Yitai B shares" from August 8, 1997, to March 18, 2022, and principles [14] [15]. Taking the "Yitai B shares" stock as the object, using the rescaled range (R/S) analysis method, the estimated value of the parameter H is calculated to be 0.5019.

Supposing the stock price is $S_0, S_1, \cdots, S_n$, and the rate of return is $\frac{S_1 - S_0}{S_0}, \frac{S_2 - S_1}{S_1}, \cdots, \frac{S_n - S_{n-1}}{S_{n-1}}$. Calculating the variance of the logarithmic return, and the parameter sigma is 0.03168232.

Calculating the average of the logarithmic return, and the parameter μ is 0.0001202555.

Substituting the specific parameter values into Equation (2.1), and assuming that $S = 1$. Taking the closing price of "Yitai B shares" on August 8, 1997, at 0.47 as the initial price S_0, and getting S_{0+dt}, S_{0+2dt}, $\cdots$, until $S_{0+ndt} = S_T$. The simulated value of the mixed bifractional Brownian model, the classic B-S model and the true value of the stock are shown in **Figure 3**. The specific statistical analysis of the simulation results is shown in **Table 4**, and the comparison between the model simulation value and the true value in the next 7 days is shown in **Table 5**.

It can be seen from **Table 4** that the mixed bifractional Brownian model simulates 1st qu., median, mean, 3rd qu., and variance of the stock price are closer to the real value of the stock than the classic B-S model. It can be obtained from **Table 5** that the simulation effect of the mixed bifractional Brownian model is better in the next 7 days.

The stocks of "Yitai B shares" from January 4, 2022, to March 18, 2022, and principles [14] [15]. Taking the "Yitai B shares" stock as the object, using the rescaled range (R/S) analysis method, the estimated value of the parameter H is calculated to be 0.9032.

Supposing the stock price is $S_0, S_1, \cdots, S_n$, and the rate of return is $\frac{S_1 - S_0}{S_0}, \frac{S_2 - S_1}{S_1}, \cdots, \frac{S_n - S_{n-1}}{S_{n-1}}$. Calculating the variance of the logarithmic return, and the parameter sigma is 0.0205726.

Calculating the average of the logarithmic return, and the parameter μ is 0.001535545.

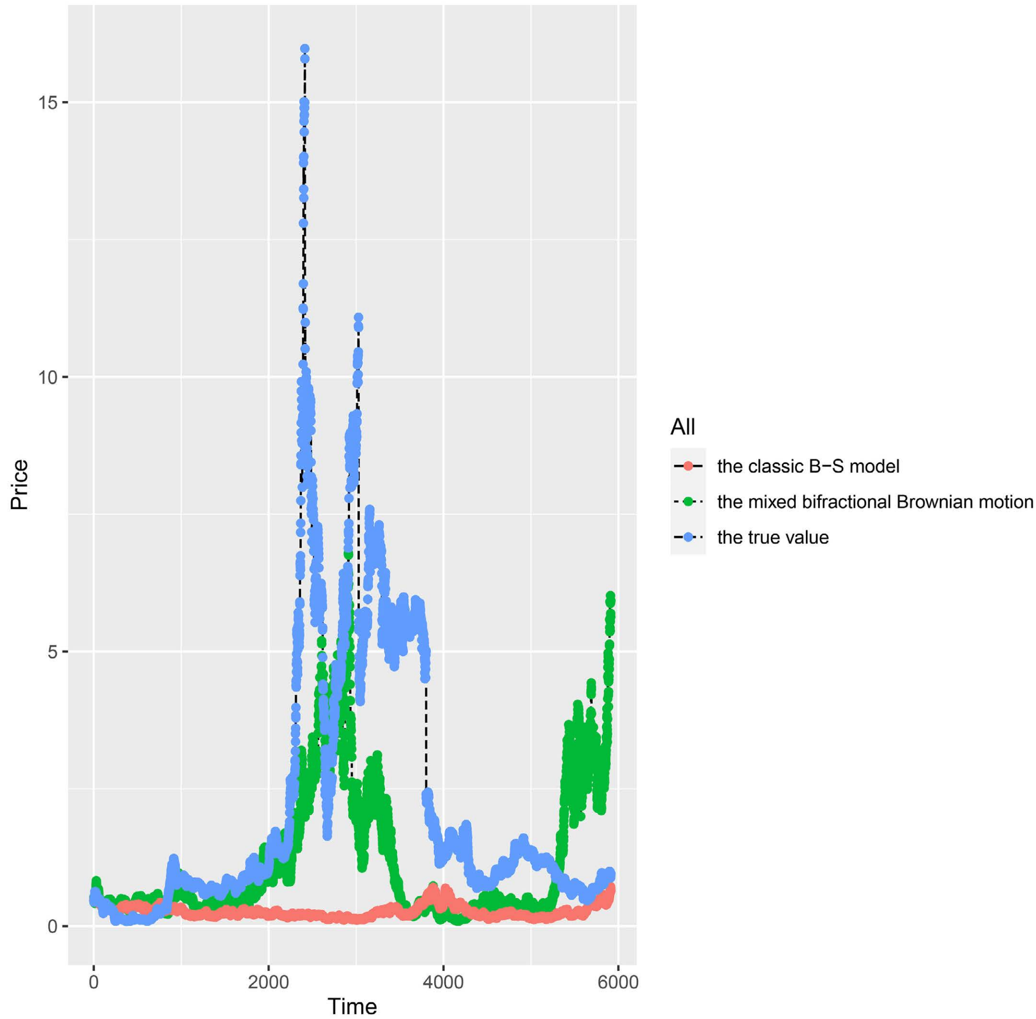

Figure 3. The comparison of mixed bifractional Brownian model, classic B-S model and true value.

Table 4. Statistics of simulation and true values.

	Mixed bifractional Brownian model simulation value	Classic B-S model simulation value	True value
1st qu.	0.35044	0.1909	0.708
median	0.50542	0.2336	1.042
mean	1.16469	0.2620	2.204
3rd qu.	1.80613	0.3131	2.540
variance	1.502611	0.01084814	6.210699

Table 5. Comparison of model simulation value and true value in the next 7 days.

	stock price simulation of mixed bifractional Brownian model	stock price simulation of classic B-S model	Yitai B shares
January 18, 2018	0.368691059441825	0.187861193583035	1.495
January 19, 2018	0.342805896793955	0.185059204986863	1.491
January 22, 2018	0.348823653279815	0.195691060109479	1.512
January 23, 2018	0.36820954133242	0.197193216245556	1.507
January 24, 2018	0.369445692945008	0.202073187553101	1.515
January 25, 2018	0.365511167771182	0.193943179312515	1.55
January 26, 2018	0.372890699299323	0.194888822159231	1.552

Substituting the specific parameter values into Equation (1.1), and assuming that $S=1$. Taking the closing price of "Yitai B shares" on January 4, 2022, at 0.889 as the initial price S_0, and getting S_{0+dt}, S_{0+2dt}, $\cdots$, until $S_{0+ndt}=S_T$. The simulated value of the mixed bifractional Brownian model, the classic B-S model and the true value of the stock are shown in **Figure 4**. The specific statistical analysis of the simulation results is shown in **Table 6**, and the comparison between the model simulation value and the true value in the next 5 days is shown in **Table 7**.

It can be seen from **Table 6** that the mixed bifractional Brownian model simulates 1st qu., median, mean, and 3rd qu. of the stock price are closer to the real value of the stock than the classic B-S model. It can be obtained from **Table 7** that the simulation effect of the mixed bifractional Brownian model is better in the next 5 days.

The stocks of "Lingang B shares" from March 30, 1999, to April 7, 1999, and principles [14] [15]. Taking the "Lingang B shares" stock as the object, using the rescaled range (R/S) analysis method, the estimated value of the parameter H is calculated to be 0.7754.

Supposing the stock price is $S_0, S_1, \cdots, S_n$, and the rate of return is $\frac{S_1-S_0}{S_0}, \frac{S_2-S_1}{S_1}, \cdots, \frac{S_n-S_{n-1}}{S_{n-1}}$. Calculating the variance of the logarithmic return, and the parameter sigma is 0.02800305.

Calculating the average of the logarithmic return, and the parameter μ is 0.007753336.

Substituting the specific parameter values into Equation (2.1), and assuming that $S=1$. Taking the closing price of "Lingang B shares" on March 30, 1999, at 0.084 as the initial price S_0, and getting S_{0+dt}, S_{0+2dt}, $\cdots$, until $S_{0+ndt}=S_T$. The simulated value of the mixed bifractional Brownian model, the classic B-S model and the true value of the stock are shown in **Figure 5**. The specific statistical analysis of the simulation results is shown in **Table 8**, and the comparison between the model simulation value and the true value in the next 3 days is shown in **Table 9**.

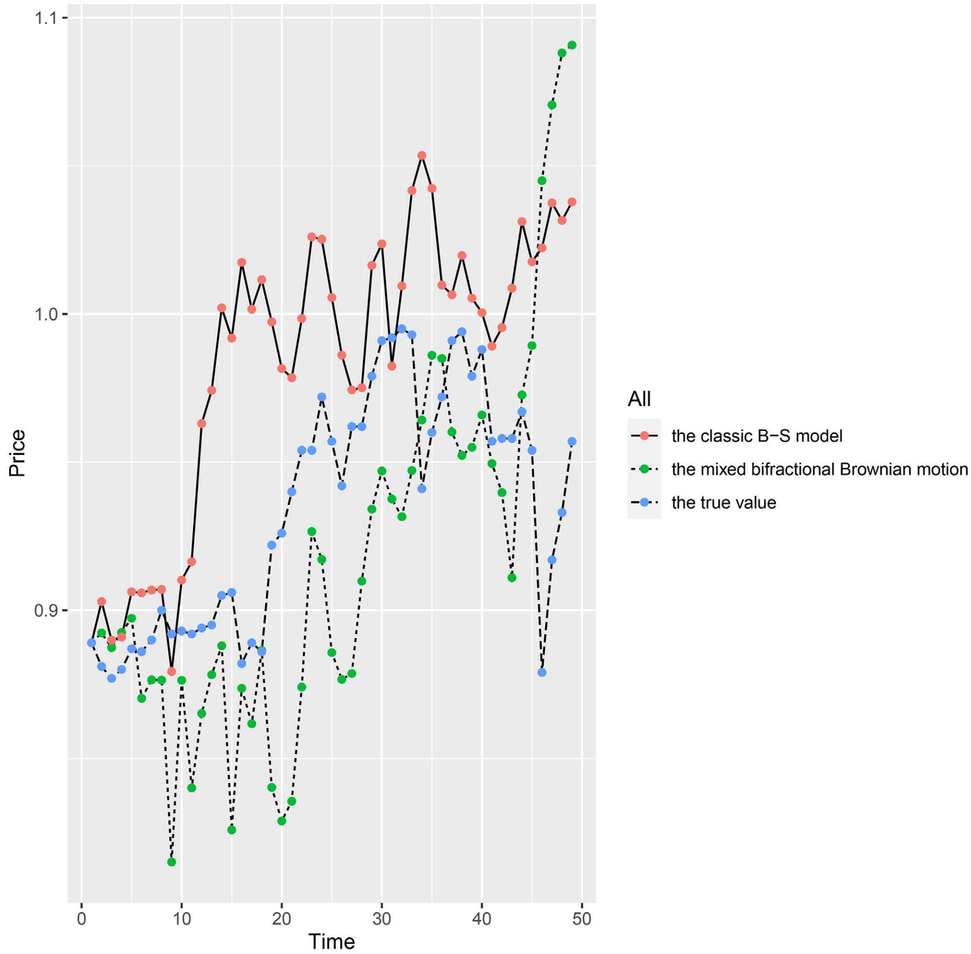

Figure 4. The comparison of mixed bifractional Brownian model, classic B-S model and true value.

Table 6. Statistics of simulation and real values.

	Mixed bifractional Brownian model simulation value	Classic B-S model simulation value	True value
1st qu.	0.8764	0.9742	0.8920
median	0.8973	1.0005	0.9410
mean	0.9181	0.9836	0.9341
3rd qu.	0.9523	1.0177	0.9620
variance	0.004227694	0.002446244	0.001605285

Table 7. Comparison of model simulation value and true value in the next 5 days.

	stock price simulation of mixed bifractional Brownian model	stock price simulation of classic B-S model	Yitai B shares
January 5, 2022	0.892334392848952	0.902934518732361	0.881
January 6, 2022	0.887369902098056	0.889889236033983	0.877
January 10, 2022	0.897293567111286	0.906221737423565	0.887
January 11, 2022	0.870261172117313	0.905866480583088	0.886
January 12, 2022	0.876528731385001	0.90681048346985	0.89

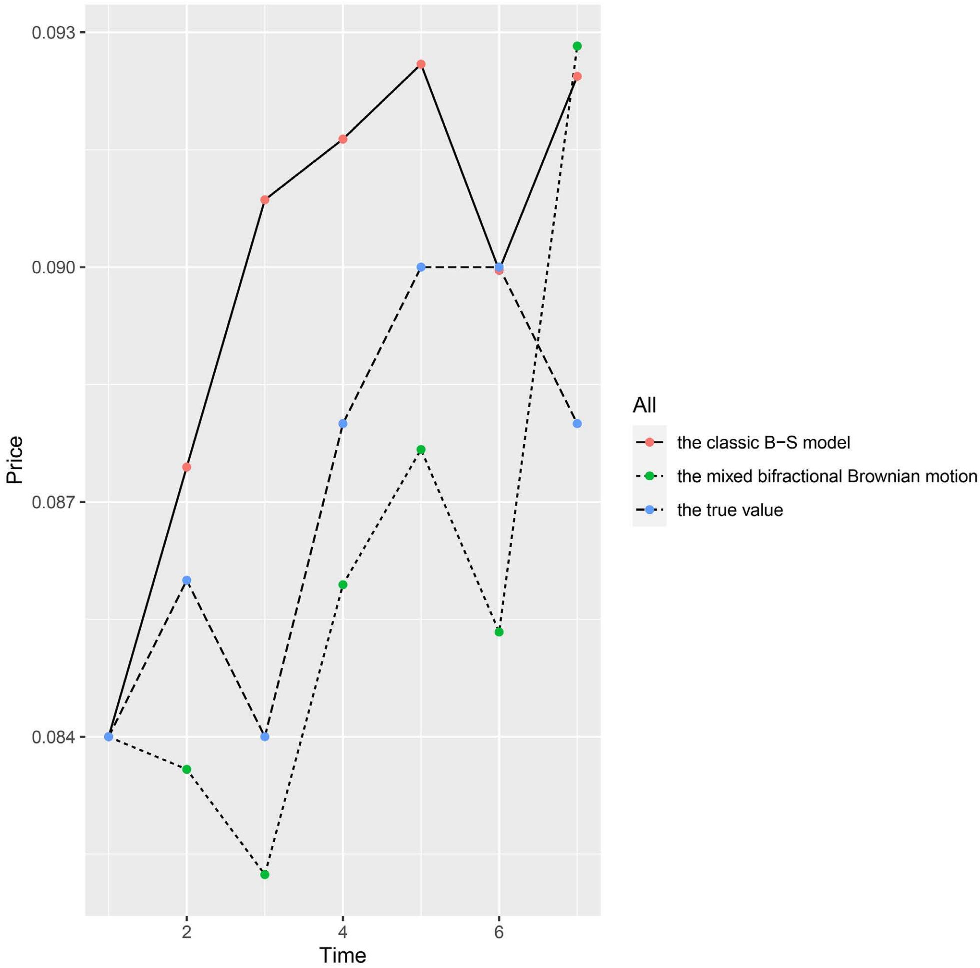

Figure 5. The comparison of mixed bifractional Brownian model, classic B-S model and true value.

Table 8. Statistics of simulation and true values.

	Mixed bifractional Brownian model simulation value	Classic B-S model simulation value	True value
1st qu.	0.08379	0.08870	0.08500
median	0.08534	0.09086	0.08800
mean	0.08594	0.08985	0.08714
3rd qu.	0.08681	0.09204	0.08900
variance	0.00001229301	0.000009739342	0.00000647619

Table 9. Comparison of model simulation value and true value in the next 3 days.

	stock price simulation of mixed bifractional Brownian model	stock price simulation of classic B-S model	Lingang B shares
April 1, 1999	0.0822392263855708	0.0908593435257147	0.084
April 2, 1999	0.0859432584986611	0.0916349232332731	0.088
April 5, 1999	0.0876700893229475	0.0925926061191512	0.09

It can be seen from **Table 8** that the mixed bifractional Brownian model simulates 1st qu., median, mean, and 3rd qu. of the stock price are closer to the real value of the stock than the classic B-S model. It can be obtained from **Table 9** that the simulation effect of the mixed bifractional Brownian model is better in the next 3 days.

The stocks of "Yitai B shares" from October 26, 1999, to November 1, 1999, and principles [14] [15]. Taking the "Yitai B shares" stock as the object, using the rescaled range (R/S) analysis method, the estimated value of the parameter H is calculated to be 0.7195.

Supposing the stock price is $S_0, S_1, \cdots, S_n$, and the rate of return is $\frac{S_1 - S_0}{S_0}, \frac{S_2 - S_1}{S_1}, \cdots, \frac{S_n - S_{n-1}}{S_{n-1}}$. Calculating the variance of the logarithmic return, and the parameter sigma is 0.0413144.

Calculating the average of the logarithmic return, and the parameter μ is 0.01327746.

Substituting the specific parameter values into Equation (2.1), and assuming that $S = 1$. Taking the closing price of "Yitai B shares" on October 26, 1999, at 0.11 as the initial price S_0, and getting S_{0+dt}, S_{0+2dt}, $\cdots$, until $S_{0+ndt} = S_T$. The simulated value of the mixed bifractional Brownian model, the classic B-S model and the true value of the stock are shown in **Figure 6**. The specific statistical analysis of the simulation results is shown in **Table 10**, and the comparison between the model simulation value and the true value in the next 3 days is shown in **Table 11**.

It can be seen from **Table 10** that the mixed bifractional Brownian model simulates 1st qu., median, mean, and 3rd qu. of the stock price are closer to the true value of the stock than the classic B-S model. It can be obtained from **Table 11** that the simulation effect of the mixed bifractional Brownian model is better in the next 3 days.

Table 10. Statistics of simulation and true values.

	Mixed bifractional Brownian model simulation value	Classic B-S model simulation value	True value
1st qu.	0.1162	0.1100	0.1140
median	0.1171	0.1108	0.1160
mean	0.1167	0.1111	0.1148
3rd qu.	0.1188	0.1109	0.1160
variance	0.00001763709	0.000004520498	0.0000092

Table 11. Comparison of model simulation value and true value in the next 3 days.

	stock price simulation of mixed bifractional Brownian model	stock price simulation of classic B-S model	Yitai B shares
October 27, 1999	0.117060782333182	0.110888726473413	0.118
October 29, 1999	0.11615517085485	0.110828080867588	0.114
November 1, 1999	0.121240963185306	0.108954831459311	0.116

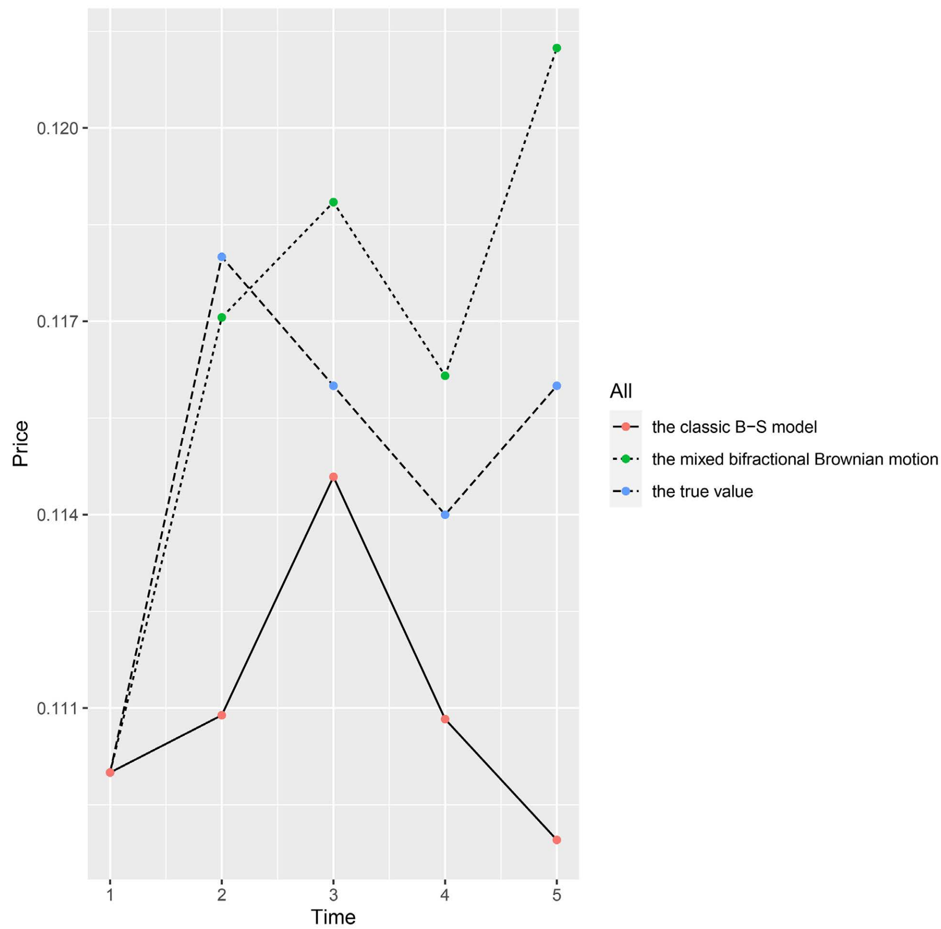

Figure 6. The comparison of mixed bifractional Brownian model, classic B-S model and true value.

5. Conclusions

From **Figures 1-6** and **Tables 1-11**, the results of using mixed bifractional Brownian motion to simulate stock prices are closer to the true price of stocks than the classic B-S model. Because stocks are an important component of options, so the more accurate the stock price simulation is, the more accurate the simulation value of the corresponding option value will be.

In summary, the mixed bifractional Brownian motion model can better simulate the trend of stock prices than the classic B-S model, so its corresponding option value will be more accurate.

Acknowledgements

The author thanks the anonymous referee for reading the paper carefully and giving several useful suggestions. This work was supported by the Master Talent Project of the President's Fund of Tarim University [Grant No. TDZKSS202258].

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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