

# From Black Holes to Information Erasure: Uniting Bekenstein's Bound and Landauer's Principle

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## Abstract

This research aims to integrate Bekenstein's bound and Landauer's principle, providing a unified framework to understand the limits of information and energy in physical systems. By combining these principles, we explore the implications for black hole thermodynamics, astrophysics, astronomy, information theory, and the search for new laws of nature. The result includes an estimation of the number of bits stored in a black hole (less than  $1.4 \times 10^{30}$  bits/m<sup>3</sup>), enhancing our understanding of information storage in extreme gravitational environments. This integration offers valuable insights into the fundamental nature of information and energy, impacting scientific advancements in multiple disciplines.

## Keywords

Astrophysics and Astronomy, Bekenstein Bound, Black Hole  
Thermodynamics, Information and Energy Limits, Information Theory and  
Quantum Mechanics, Landauer's Principle

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## 1. Introduction

Understanding the fundamental limits and principles governing information and energy is a key area of research in physics. Two important concepts that have emerged in this field are the Bekenstein bound [1] and Landauer's principle [2]. The Bekenstein bound, proposed by Jacob Bekenstein, sets an upper limit on the amount of thermodynamic entropy or information that can be contained within a finite region of space with a finite amount of energy. It provides insights into the maximum amount of information needed to fully describe a physical system at the quantum level within a specific spatial region and its associated

energy. On the other hand, Landauer's principle states that erasing one bit of information incurs a minimum energy cost of  $k_b \cdot T \cdot \ln 2$ , where  $T$  represents the temperature of a thermal reservoir and  $k_b$  is Boltzmann's constant.

While the Bekenstein formula has been well received in the scientific community, the response to Landauer's principle has been more nuanced. However, it has gained widespread acceptance as a fundamental physical law, with researchers demonstrating its derivation from the second law of thermodynamics and the change in entropy associated with acquiring information, including quantum and classical feedback systems [3] [4].

In [5], the generalization of Landauer's principle leading to an increase in entropy without consuming energy is a notable development. This insight provides a deeper understanding of the relationship between information processing and entropy, as it highlights that erasing information can have implications beyond energy consumption. By expressing this increase in entropy in terms of other conserved quantities, such as angular momentum, researchers have expanded our understanding of the underlying principles governing information and thermodynamics. This finding adds complexity to the concept of information erasure and its broader implications in physical systems.

A significant breakthrough in 2012 involved the first-ever measurement of the minuscule heat generated during the processing of a single bit of data [6]. Subsequent experiments confirmed Landauer's principle and quantified the energy dissipated during bit transitions [7] [8]. The performance of Landauer erasure at cryogenic temperatures using quantum molecular magnets further extended the application of the principle into the quantum realm [9]. These advancements highlighted the minimal thermodynamic costs of erasure and high-speed operation [9] [10].

Criticism of Landauer's principle has surfaced in recent years, with concerns regarding circular reasoning and flawed assumptions. However, proponents maintain its validity, noting its emergence from the second law of thermodynamics and the associated entropy change in information processing [11]-[16]. Moreover, studies have explored the connection between logical and thermodynamic reversibility, revealing nuanced implications for computation [17] [18]. In 2016, researchers from the University of Perugia claimed to have observed a violation of Landauer's principle [19]. However, Laszlo Kish [20] argued that their results are invalid due to their failure to account for the dominant source of energy dissipation—the charging energy of the capacitance of the input electrode.

In conclusion, the integration of the Bekenstein bound and Landauer's principle represents a significant advancement in our understanding of the fundamental limits and principles governing information and energy. By bridging information theory, thermodynamics, and quantum mechanics, this integration opens new avenues for discovery and practical applications. This chapter serves as an introduction to the integration of these concepts, setting the stage for further exploration and research in this exciting and promising field.

## 2. Integration of Concepts

In the 1980s, the Bekenstein bound was introduced as a groundbreaking formula to calculate the upper limit of information [1]:

$$Y \leq (2 \cdot \pi \cdot R \cdot E_m) / (\hbar \cdot c \cdot \ln 2) \quad (1)$$

Equation (1) quantifies the information content in bits  $Y$ , specifically referring to the number of quantum states within a given object sphere. The variables in the equation have the following meanings:  $R$  [ $M^1$ ] represents the radius of the object sphere,  $E_m$  [ $M^1 \cdot L^2 \cdot T^{-2}$ ] denotes the total mass-energy content,  $\hbar$  [ $M^1 \cdot L^2 \cdot T^{-1}$ ] denotes the reduced Planck constant, and  $c$  [ $M^1 \cdot T^{-1}$ ] symbolizes the speed of light. The Bekenstein bound formula combines these variables, using base SI (international system of units) quantities [21].

Landauer's principle, introduced in [2], emphasizes the minimum energy cost incurred when erasing one bit of information. This relationship is defined by the equation

$$E_I = k_b \cdot T \cdot \ln 2 \quad (2)$$

In this context  $E_I$  represents the minimum possible energy required to erase one bit [ $M^1 \cdot L^2 \cdot T^{-2}$ ],  $k_b$  is the Boltzmann constant [ $M \cdot L^2 \cdot T^{-2} \cdot K^{-1}$ ], and  $T$  is the temperature of the heat sink [K].

By combining Equations (1) and (2) and considering [22], we can express Equation (1) in the following manner:

$$V_{bh} \cdot Q_{bh} \leq (2 \cdot \pi \cdot R \cdot E_{ml}) / (\hbar \cdot c \cdot \ln 2) \quad (3)$$

where

$$E_{ml} = V_{bh} \cdot d_{bh} \cdot c^2 + V_{bh} \cdot Q_{bh} \cdot k_b \cdot T \cdot \ln 2 \quad (4)$$

In this context,  $E_{ml}$  refers to the energy of the black hole, comprising mass-energy equivalence and information-energy equivalence.  $V_{bh}$  denotes the volume of the black hole, and  $d_{bh}$  signifies the specific density. Finally,  $Q_{bh}$  represents the amount of information (in bits/m<sup>3</sup>) contained within the black hole.

Further manipulation of Equations (3) and (4) yields the following expressions:

$$Q_{bh} \leq (2 \cdot \pi \cdot R \cdot d_{bh} \cdot c^2) / (\hbar \cdot c \cdot \ln 2 - 2 \cdot \pi \cdot R \cdot k_b \cdot T \cdot \ln 2) \quad (5)$$

To determine the value of  $Q_{bh}$ , we refer to numerical data provided in the referenced scientific publications, with the understanding that these calculations involve inherent approximations and are subject to refinement as ongoing research progresses.

The Event Horizon Telescope, as described in [23], conducted measurements that provided significant insights into a black hole. The study reported a mass of approximately  $6.5 \pm 0.7 \times 10^9$  solar masses for the black hole. Additionally, the estimated diameter of its event horizon was roughly 40 billion kilometers, which is approximately 2.5 times smaller than the observed shadow at the center of the image. By comparing the black hole's mass to that of the Sun (approximately  $2 \times 10^{30}$  kg), the calculated mass of the black hole ( $m_{bh}$ ) is approximately  $13 \pm 1.4 \times$

$10^{39}$  kilograms.

The mass density of the black hole ( $d_{bh}$ ) is estimated in [24] as  $1.9 \times 10^{-35 \pm 0.40}$   $\text{g}\cdot\text{cm}^{-3}$ , which is equivalent to  $1.9 \times 10^{-32 \pm 0.40}$   $\text{kg}\cdot\text{m}^{-3}$ . Using this information, we can calculate the volume of the black hole ( $V_{bh}$ ) as follows:

$$V_{bh} = m_{bh} / d_{bh} = 13 \pm 1.4 \times 10^{39} / 1.9 \times 10^{-32 \pm 0.4} \approx 6.84 \times 10^{71} \text{ m}^3 \quad (6)$$

Assuming a spherical shape for the black hole, we can proceed to calculate its radius (R) using the outcomes from Equation (6):

$$R = (3 \cdot V / (4 \cdot \pi))^{1/3} = (3 \times 6.84 \times 10^{71} / (4 \times 3.14))^{1/3} \approx 5.5 \times 10^{23} \text{ m} \quad (7)$$

Utilizing the results from Equations (6) and (7), we can derive a numerical formulation for  $Q_{bh}$ :

$$Q_{bh} \leq 1.4 \times 10^{30} \text{ (bits/m}^3\text{)} \quad (8)$$

In this context,  $c = 299,792,458 \text{ m}\cdot\text{c}^{-1}$ ,  $c^2 = 8.988 \times 10^{16} \text{ m}^2\cdot\text{c}^{-2}$ ,  $\hbar = 6.62607015 \times 10^{-34} \text{ kg}\cdot\text{m}^2\cdot\text{c}^{-1}$ ,  $k_b = 1.380649 \times 10^{-23} \text{ kg}\cdot\text{m}^2\cdot\text{s}^{-2}\cdot\text{K}^{-1}$ ,  $\pi = 3.14$ ,  $\ln 2 = 0.69$ . It is a well-known fact that the temperature of a black hole decreases as its mass increases. Stellar black holes have extremely low temperatures, approaching absolute zero at around zero Kelvin or  $-273.15$  degrees Celsius. For the purpose of analysis, we assume a temperature value of approximately  $T = 1 \times 10^{-2} \text{ }^\circ\text{K}$ .

The value obtained in Equation 8 holds significant implications for advancing scientific research across various fields, including black hole thermodynamics, astrophysics, astronomy, information theory, the Bekenstein limit, and the exploration of new laws of nature. It is important to note that the precise value of  $Q_{bh}$  remains uncertain in the existing scientific literature, raising questions about its magnitude and the extent of documented findings.

Several studies have addressed this question since the late 1970s. Davies [25] utilized the Bekenstein-Hawking formula for black hole entropy to calculate the information content of the universe, resulting in an estimation of  $10^{120}$  bits. Wheeler [26] estimated the present universe's information capacity at approximately  $8 \times 10^{88}$  bits based on entropy considerations at a temperature of  $2.735 \text{ }^\circ\text{K}$ . Similarly, Lloyd [27] estimated the total information capacity of the universe to be around  $10^{90}$  bits.

Applying Landauer's principle, Gough [28] developed an information equation of state and found that  $10^{87}$  bits of intrinsic universe information content could account for all the Dark Energy. These estimates were made during the cosmic time of star formation when high-temperature baryons represented the majority of the universe's bit information content, with an average bit energy value of  $120 \text{ eV}$ .

Considering the mass-energy-information equivalence principle, Vopson [29] [30] estimated that approximately  $5.2 \times 10^{93}$  bits could account for all the missing Dark Matter in the observable universe. However, this estimation may be overestimated as it assumed that all information bits were stored at a temperature of  $2.735 \text{ }^\circ\text{K}$ , disregarding the higher temperatures of baryonic matter contained within stars, intergalactic gas, and dust.

Another approach employed by Vopson [31] utilized Shannon's information theory to estimate the number of bits contained in matter particles of the observable universe. This independent approach, which does not rely on the dynamics of universe expansion or internal dynamics, yielded a total information content of baryonic matter in the observable universe as  $6.036 \times 10^{80}$  bits.

Considering these findings, the value presented in Equation 8 must be regarded as substantial and represents a pioneering estimation. Further investigation is necessary to determine the precise number of bits contained within one cubic meter of a black hole, contributing to an enhanced understanding of these enigmatic cosmic entities and unraveling the complexities of information storage within them.

### 3. Discussion

While the individual significance of the Bekenstein bound and Landauer's principle in The integration of the Bekenstein bound and Landauer's principle represents a significant advancement in our understanding of the fundamental limits and principles governing information and energy. By combining these two principles, researchers can explore new avenues for comprehending the intricate relationship between information, energy, and the underlying laws of physics.

The motivation behind integrating the Bekenstein bound and Landauer's principle lies in the quest to establish a unified framework that encompasses the limitations on information storage and the energetic costs associated with information processing. This integrated approach offers the potential for a deeper understanding of the fundamental connections between thermodynamics, information theory, and quantum mechanics. By understanding their significance and exploring their integration, we can gain insights that go beyond their individual contributions, leading to a more comprehensive understanding of the fundamental limits and principles governing information and energy.

The integration of the Bekenstein bound and Landauer's principle enables us to achieve a more comprehensive understanding of the fundamental limits and principles governing information and energy. This integrated framework allows us to explore the interplay between information storage, processing, and energy dissipation in physical systems. It provides a deeper understanding of the limitations and fundamental principles governing information and energy.

The significance of integrating the Bekenstein bound and Landauer's principle extends beyond theoretical considerations and holds practical implications for various fields of study. In the realm of quantum computing, where information is stored and processed at the quantum level, understanding the fundamental limits of information storage and the energetic costs associated with information manipulation is crucial for the development of efficient and scalable quantum computing systems.

In the field of thermodynamics, the combination of the Bekenstein bound and Landauer's principle contributes to a deeper understanding of the fundamental

connections between information and entropy. It sheds light on the relationship between the microscopic properties of physical systems and their macroscopic thermodynamic behavior. This understanding can have implications for the design and optimization of energy-efficient thermal energy storage systems, where the management of information and energy plays a crucial role.

Equations (5), (8) introduce a thought-provoking concept that raises the possibility of explaining “dark matter” from a fresh perspective rooted in information physics. This notion is based on the premise that information represents the fifth form of physical matter. Previous studies [25]-[31] have explored the information content of the Universe. If information possesses mass, its influence would solely manifest through gravitational interactions, rendering it invisible to detection via electromagnetic radiation [32]. This radical proposition suggests that information might account for the elusive dark matter present in the universe. Furthermore, ongoing efforts in the field of information physics, such as those undertaken by the Information Physics Institute (IPI) [32] and in [33], are anticipated to yield significant advancements, enhancing our comprehension of the universe and its fundamental laws.

By delving deeper into the integration of the Bekenstein bound and Landauer’s principle, we can unlock new insights, advance technological developments, and pave the way for further research in understanding the fundamental nature of information and energy. This chapter aims to explore the theoretical foundations, implications, and practical applications arising from the combination of these two concepts. Through this exploration, we strive to provide a comprehensive understanding of the importance and potential of integrating the Bekenstein bound and Landauer’s principle in the scientific community.

## 4. Conclusions

In conclusion, the integration of the Bekenstein bound and Landauer’s principle represents a significant advancement in our understanding of the fundamental limits and principles governing information and energy in physical systems. By combining these concepts, we have established a comprehensive framework that sheds light on the interplay between information storage, processing, and energy consumption. This integration holds immense importance for various scientific disciplines, including black hole thermodynamics, astrophysics, astronomy, information theory, and the exploration of new laws of nature.

The combination of the Bekenstein bound and Landauer’s principle has revealed three key points that highlight its significance:

**Unifying Limits:** The integration of the Bekenstein bound and Landauer’s principle has provided a unified perspective on the limitations of information and energy in physical systems. By considering the finite nature of information within a given region of space, as quantified by the Bekenstein bound, and the energetic costs associated with information manipulation, as described by Landauer’s principle, we have established a holistic understanding of the fundamen-

tal boundaries that govern these processes.

**Fundamental Connections:** This integration has unveiled the fundamental connections between thermodynamics, information theory, and quantum mechanics. By recognizing the interplay between information storage, processing, and energy dissipation, we have gained insights into the intricate relationship between microscopic properties and macroscopic thermodynamic behavior. This deeper understanding enhances our grasp of the underlying principles that govern the behavior of physical systems at various scales.

**Implications and Future Research:** The integration of the Bekenstein bound and Landauer's principle has profound implications for practical applications and further scientific research. In the field of quantum computing, understanding the fundamental limits of information storage and the energetic costs of information manipulation is crucial for the development of efficient and scalable quantum computing systems. Additionally, this integration has implications for information theory, communication systems, and thermodynamics, offering opportunities for optimizing energy utilization, designing energy-efficient thermal storage systems, and advancing our knowledge of the fundamental laws of nature.

The obtained result (Equation (8)) regarding the number of bits contained within one cubic meter of a black hole is a significant contribution to scientific research in black hole thermodynamics, astrophysics, and related disciplines. As no precise value for  $Q_{bh}$  has been provided in existing scientific literature, the presented estimation in Equation (8) marks an important step forward. However, further investigation is necessary to refine this estimation and determine the precise number of bits. This pursuit will not only deepen our understanding of black holes as enigmatic cosmic entities but also expand our knowledge of information storage in extreme gravitational environments.

In summary, the integration of the Bekenstein bound and Landauer's principle has unveiled fundamental insights into the limits of information and energy in physical systems. This comprehensive framework has broad implications for diverse fields, ranging from quantum computing to thermodynamics. By delving deeper into the integration of these concepts, we will continue to unlock new knowledge, drive technological advancements, and advance our understanding of the intricate relationship between information and energy in the natural world.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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