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Yang-Mills Mass Gap Problem: A Possible Solution

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Abstract

As known, the *spontaneous symmetry breaking* (SSB) and the Brout-Englert-Higgs Mechanism (BEH-M) solved the Yang-Mills *Mass Gap* Problem. However, various mathematicians, even prestigious ones, consider the basic assumptions of the *gauge theories* to be wrong, as well as *in conflict with the experimental evidence* and in clear disagreement with the facts, *distorting the physical reality itself*. Likewise, these theories are *mathematically inconsistent*, adopting a mathematical structure somewhat complicated and arbitrary, which does not satisfy the strong demands for coherence. The weakest point of the gauge theories, in our opinion, consists in imposing that all the particles must be free of an intrinsic mass. On the contrary, even for the particle considered universally massless, *i.e.* the photon, our calculations show a dynamic-mass, a push-momentum ($\boldsymbol{p}$) of 1.325×10^{-22} [g·cm/s]. With this work we try to provide a possible solution to the Yang-Mills *Mass Gap* Problem, but without taking into account the SSB, nor using the BEH-M. We try to provide a *mathematical explanation* for this phenomenon, considering that in the spectrum of the Yang-Mills theory, there is a *mass gap*, that is, the difference between the energy of the vacuum state and the first excited state is different from zero. In other words, the lightest of the particles predicted by the theory must have a strictly positive mass to explain the short range of strong nuclear forces. It is clear, indeed, that if we replaced this value with the null value of the photon inserted in the equations of the Perturbation Theory, the Quantum Fields Theory and the Yang-Mills theories, all *divergences*, that is all zeroes and infinities, would suddenly disappear. Consequently, the limits imposed by the SSB disappear so that there is no longer any need to deny the mass to the Nuclear Forces bosons, including the Yang-Mills ***b*** *quantum*.

Keywords

Electromagnetic (EM), Quantum Fields Theory (QFT), Spontaneous Symmetry Breaking (*SSB*), Gluon (G), Photon (P)

1. Introduction

As it is known, in agreement with *Quantum Fields Theory*, a Dirac field (Ψ), with a mass *m* and not interacting, is described by Dirac's Lagrangian, $Ł_D$:

$$Ł_D = i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - m\bar{\Psi}\Psi \tag{1}$$

where γ^μ is the Dirac matrix, which satisfies Clifford's algebra: $\{\gamma^\mu, \gamma^\nu\} = 2g\gamma^{\mu\nu}I$; *i* is the imaginary unit, $\bar{\Psi}$ is the anti-particle. The magnitude ($Ł_D$) is, by construction, invariant under the action of the Poincaré group. At the same time, it presents a further symmetry, not associated with space-time transformations.

1.1. Gauge Transformations and *Gauge Theories*

The $Ł_D$ form, indeed, remains unchanged if we rotate the phase of the field (Ψ) of a real angle (θ), that is if we perform the *gauge transformation*:

$$\Psi(x) \to e^{ie\theta}\Psi(x) \tag{2}$$

Well, the real quantity that appears is the so-called gauge coupling parameter; it depends on the particular field Ψ on which the transformation acts; it coincides, except for the sign, with its electric charge. The value of the continuous parameter θ univocally identifies each particular *gauge transformation*, as shown in Equation (2). Now, the *gauge connection* can be established.

The set of these transformations forms a *Lie group*, namely the symmetry group *U*(1), or group of *Unitary* transformations (*U*) of a *complex variable* (1).

From a geometric point of view, it concerns transformations analogous to the continuous rotations of the circle. However, while the circle is drawn in a bi-dimensional plane at real dimensions, the transformations of group *U*(1) concern the rotations in the 2-dimensional *complex plane*: the latter is formed by two real dimensions, one of which is multiplied by the imaginary number *i*.

The group *U*(1) can also be represented in terms of continuous transformations of the *phase angle* (θ for example) of a sinusoidal wave.

As known, Maxwell's equations do not change, that is they are *invariant*, so Weyl believed that it was possible to extend this invariance to the gravitational field too, as well as to General Relativity, thus trying to unify electromagnetism and gravity. So, working on the theory of continuous symmetry groups (or *Lie's groups*) and bearing in mind the Noether theorem [1], Weyl was convinced that the Conservation Laws are related to *local transformations of symmetry*, which gave the generic name of *Eichinvarianz* or *gauge invariance* (*eich*=gauge) under the change of measurement scale: or *gauge*, precisely; a term unfortunately rather obscure. However, *each* means *phase*, or even *scale* or *caliber*, that is, a *measure of length*. In fact, Weyl sought *invariance* by dilatations, *i.e.* with real *iθ*, instead he found invariance in the case of real θ. Weyl conjectured that a *gauge symmetry* might also be a *local* symmetry of General Relativity. Then, in 1918 Weyl formulated a *gauge theory* to be applied to General Relativity [2]. In this framework, Weyl formulates its gauge theory of Electro-Magnetic Interactions. Weyl postulates that the invariance for *local* coordinate transformations also ex-tends to the calibration of physical lengths:

$$\mathrm{d}x \rightarrow e^{\lambda(x)}\mathrm{d}x \tag{3}$$

with λ (wave length) real function of the coordinates. Equation (3) shows that the scale factor, $\lambda(x)$, (as saying *gauge* factor) is determined by the coefficients of a differential form, $A\mu(x)$:

$$\lambda(x) = \int^{x} A\mu(y)\,\mathrm{d}y^{\mu} \tag{4}$$

The integral must be performed on a *path* that, starting from the origin, arrives at point x. The path is arbitrary and the same result is or isn't obtained for all paths, depending on whether the condition of integrability occurs:

$$F_{\mu\upsilon}(x) = \frac{\partial A\mu}{\partial\upsilon} - \frac{\partial A\upsilon}{\partial\mu} = 0 \tag{5}$$

When this does not happen, there are new forces determined by $F\mu v$ and the corresponding equation of motion takes the form:

$$\frac{\delta}{\delta x^{\nu}} \cdot F^{\mu\upsilon}(x) = eJ^{\mu}(x) \tag{6}$$

Along with Maiani, the latter is an equation of the type of Einstein's equation, a geometric entity equaled by a dynamic entity, which determines the forces exerted on the matter that carries the quality associated with the current J^{μ} [3]. Weyl identifies J^{μ} with the electromagnetic current, the constant e with the elementary electric charge, and $F^{\mu\nu}$ with the Maxwell tensor, which sums up the electric and magnetic fields: $A\mu(x)$.

As known, Einstein immediately replied that the laws of Physics are *not invariant* under *gauge transformations* and that the Weyl's *gauge theory* was in conflict with Relativity Theory.

Nevertheless, later Fock and London modified *gauge transformations* by replacing the scale factor with a complex quantity and turned the scale transformation into a change of phase, which is a $U(1)$ gauge symmetry. This explained the electromagnetic field effect on the wave function of a charged quantum mechanical particle. In this way, both Fock, 1926 [4], in relation to the Schrödinger equation for the electron (generalized the Klein-Gordon equation), and London, 1927 [5], in formulating the superconductivity theory, observed that the minimal substitution of classical electro-magnetism

$$\boldsymbol{H}_{p} \rightarrow \boldsymbol{H}_{p} - qA^{0} \tag{7}$$

$$\boldsymbol{p} \rightarrow \boldsymbol{p} - q\boldsymbol{A} \tag{8}$$

takes, in Schrödinger equation, to the substitution:

$$i\frac{\partial}{\partial t} \rightarrow i\frac{\partial}{\partial t} + e\varphi(\boldsymbol{x}, t) \tag{9}$$

$$-i\frac{\partial}{\partial \boldsymbol{x}} = -i\frac{\partial}{\partial \boldsymbol{x}} + e\boldsymbol{A}(\boldsymbol{x}, t) \tag{10}$$

which allows us to make the Schrödinger equation invariant for substitutions:

$$\Psi(x) \to e^{ie\lambda(x)}\Psi(x) \tag{11}$$

$$\boldsymbol{A}_\mu(x) \to \boldsymbol{A}_\mu(x) + \frac{\delta}{\delta x^\mu}\lambda(x) \tag{12}$$

on the *wave function* of the electron, $\Psi(x)$, and on the electromagnetic (EM) field, $\boldsymbol{A}_\mu(x)$.

So, Weyl accepted the crucial introduction of the imaginary unit (*i*) in the exponent and proposed that the invariance for transformations (11) and (12) should be the principle from which to derive the laws of electrodynamics, principle which he gave the name of *gauge principle*.

Hence, taking inspiration from Fock's works on the *electron's Wave Function equation* of Schrödinger, or the London's works on *superconductivity*, in 1929 Weyl published another work in which he attributed great importance to the *gauge theories* [6].

Weyl's 1929 article, indeed, marks the beginning of modern *gauge theories* [3].

1.2. The Mass *Breaks* the Symmetry

Thus, with his 1929 article Weyl associates the transformation on potentials with a *gauge tranformation* of the *wave function* (WF). This article, however, also fully preserves the same parameters and mathematical procedures previously contested by Einstein, as the *assumption* that in an invariant *gauge theory*, all the particles should have zero mass like the photon [3].

To be honest, the downside of the *Gauge Symmetry Theories*, in our opinion, lies in the fact, really paradoxical from a *logical* point of view, that the introduction of a simple mass parameter, necessary to describe the *intrinsic mass of a particle*, is in contradiction with the existence of this *gauge symmetry*: it is said, that is, that *the mass breaks the symmetry*.

In line with the *Standard Model* of the elementary particles, the problem can be solved by *assuming* (as a *dogma*) that all particles have a null intrinsic mass and postulating the existence of a *complex scalar field* permeating the space. The re-introduction of the mass parameter causes the gauge symmetry to be no more explicit, but that is spontaneously broken: *Spontaneous Symmetry Breaking* (*SSB*) [7] [8] [9]. It is in this case a *symmetry hidden from the mass.*

To this purpose, in 1956 Nambu attended a seminar by Schrieffer on *Superconductivity*. This theory, developed by Bardeen, Cooper and Schrieffer (*BSC theory*), explained how certain crystalline materials, when cooled below a critical temperature, lose all electrical resistance, becoming *Superconductors*. In fact, although charges of the same sign repel each other, the electrons of a *superconductor* suffer a weak mutual attraction. This happens because a free electron, passing close to a positive ion of the crystal lattice, moves it slightly from its position distorting the lattice. The electron continues on its way, but the lattice continues to vibrate, and this vibration produces a slight excess of positive charge, which attracts a second electron. The result is pairs of electrons with opposite spin and momentum (called *Cooper pairs*). These *pairs of electrons* behave like bosons, so they can accumulate in large numbers and *condense* into a single state, traveling in the lattice without any resistance. For Nambu this theory did not seem to respect the *gauge invariance* of the electro-magnetic (EM) field since it did not respect the conservation of the electric charge.

Thus, in keeping with Nambu, the *superconductivity BSC theory* was an example of *SSB* applied to the gauge field of electromagnetism. Namely, the *SSB* is due to tiny fluctuations in the surrounding environment, which are part of the *background noise*. That's the *SSB* regards the minimum energy state of a system, called *vacuum* state. But, the possibility of coordinated motions of *Cooper pairs*, mediated by lattice vibrations, creates a lower energy vacuum state. Consequently, the *gauge symmetry* of electromagnetism $U(1)_Q$ is *broken* by the presence of another field, which *quanta* are represented by *Cooper pairs*. The laws describing the dynamics of electrons in the material remain *invariant* with respect to the *local* gauge symmetry $U(1)$, but the vacuum state is no longer so.

So, Nambu realized that, since the Cooper pairs exist in a lower energy state, it is necessary to supply energy to dismember them: the free electrons thus created would have an additional energy, equal to half of that necessary to separate a pair. This additional energy would appear as a *mass*. Thus, Nambu wondered, it is just necessary to *break the symmetry* to have massive particles.

Hence, in 1961 Nambu and Jona-Lasinio published an article describing this *SSB mechanism*, but they had to postulate a background field that creates a *false void* [7]. *Breaking symmetry* in a quantum field requires a *background* with which to interact. This implies that the empty space is not really empty, but contains energy in the form of an all-pervasive quantum field [10] [11]. The false vacuum provides the *background* necessary to break the symmetry in a theory of Strong Interactions, containing hypothetical massless nucleons: the *symmetry breaking* (*SB*) had *lit* the masses [12].

Nevertheless, Goldstone thinks that the *SB* generates a new massless particle [8]. Even Nambu and Jona-Lasinio, in effect, had run into the same problem as Goldstone. Besides giving mass to the nucleons, the theory foresaw the existence of massless particles, formed by nucleons and antinucleons, though they tried to argue that these particles could acquire a small mass, so as to identify them with the pions. As follows, the new massless particles are the well-known *Nambu-Goldstone bosons*.

Even these massless bosons were subject to the same *objections* that weighed on the massless particles of the *Quantum Fields Theory* (QFT): any new massless particle predicted by the theory should have been ubiquitous, like the photon. However, these additional particles had never been observed.

In sum, *SSB* promised a solution to the problem of massless particles in Yang-Mills field theories, but at the price of introducing once again new massless particles, never observed.

One problem was solved and a new one was created [12].

1.3. Brout-Englert-Higgs Mechanism

It was conjectures more or less at the same time, and independently by Englert and Brout [13], by Higgs [14], Guralnik, Hagen and Kibble [15], that particles would tend to interact, to mate with this *complex scalar field*, now known as Higgs field (HF), acquiring an energy at rest which is not null, which for almost all respects is analogous to a value of mass at rest, then describable as a parameter mass. As we all know, the mechanism just described is the Brout-Englert-Higgs *Mechanism*(BEH-*M*). The BEH-*M* requires the intervention of a permeating particle the HF, the so-called Higgs Boson (HB) [14] [16]. And 'interesting to emphasize, in this respect, that the coupling between the various particles (among bosons only those bearers of *weak charge* [17]) and HF (steeped in *weak charge*) complies with the *gauge symmetry* and explains the presence of non-null rest masses.

1.4. Incompatibility between Physics and *Gauge Theories*

1.4.1. *Gauge Theories* in Conflict with Relativity Theory

In short, 3 years after Einstein's introduction of his General Relativity, Weyl suggested a generalization in which the notion of *length* itself became *dependent* from the *path* [2].

Along with Weyl's theory, for example, the way a clock measures time does not depend solely on its current position, but also on the previously positions. Likewise, the emission frequencies of a hydrogen atom will depend both on its current and past positions. As to say, the behavior of the atom will depend on its history, despite contradicting experimental evidence [18].

Nevertheless, Weyl's idea contained a fatal mistake, which Einstein clearly saw from the beginning.

In effect, Einstein explained that the laws of physics are not invariant under *gauge transformations* and the elegant electromagnetic field theory had to be abandoned. Namely, Einstein had shown that the mathematical formalism introduced by Weyl was excessively *incoherent* and incongruous, as well as blatantly clashing with the experimental evidence. In fact, when Einstein got to know of the *gauge theory*, he informed Weyl that he had a fundamental *physical* objection. The spectral frequencies, for example, are not at all influenced by the *history of an atom*, as predicted by Weyl's theory. And even more fundamental, Weyl's theory is in conflict with the necessarily exact identity between particles of the same kind. There is, in particular, a direct relationship between the rhythms of clocks and masses of particles. A particle with rest mass m has a natural frequency $mc^2 \cdot h^{-1}$, where h is the Planck constant and c is the speed of light. In this way, in Weyl's geometry, not only the rhythms of the clocks but also the *mass* of a particle would depend on its history. In this way two protons with different histories would almost certainly have different masses, according to Weyl's theory. This would violate another quantum principle, namely, all particles of the same type must be *exactly* identical. Indeed, Einstein's objection to Weyl's original gauge idea was based on the fact that the mass of a particle, and so its natural frequency, is directly measurable, so that it cannot be used as a *gauge field* in the required sense. This matter gets muddy in some modern uses of the *gauge* idea [19].

As Penrose points out, Noether's theorem shows various limitations in the case of Gravitational Theory: when gravity is included, there must be the *gauge invariance* appropriate to gravity, *i.e.* the *invariance* with respect to the coordinates, using the mathematical formalism of *tensors* [19].

In Weyl's theory, indeed, *null cones* retain the fundamental role they play in Einstein theory (they define the boundary velocities for massive particles and give us the local *Lorentz group* that must act in the vicinity of each point), so that a Lorentzian metric (eg, + − − −) g is still locally required in order to define these cones. There are, however, some additional structures to this structure of *null cone* (that is to say the *conformal* structure), and precisely a gauge connection, that Weyl introduced so that its *curvature* was Maxwell's tensor F (*i.e.* F_{ab}). This curvature measures the discrepancy of the clocks' rhythms.

Einstein had shown that the mathematical formalism introduced by Weyl was excessively *incoherent* and incongruous, as well as blatantly clashing with the experimental evidence. In fact, initially Weyl attributed the *gauge invariance* to the space itself. But, as Einstein soon pointed out, this implied that the measure of the length of a ruler, or the hour marked by a clock, depended on their recent history. So, a clock, moved from one point to another of a room, would no longer mark the correct time [12].

In short, the Mathematics supported by Weyl belied and contradicted the basic principles of the Theory of Relativity! It was really unacceptable for Einstein.

Pauli also was in complete discordance with the Weyl's *gauge theory*. To this purpose, he immediately published two articles. In the first Pauli pointed out a sign error ("a little oversight" [20]) in one of Weyl's formulas. In the 2nd article, however, there is a pitiless and dry criticism [21]. In this respect, the Mathematics used by Pauli refers to the *tensor calculation* developed by Gregorio Ricci Curbastro and his pupil Tullio Levi Civita. It is the same mathematical formalism suggested to Einstein by Marcel Grossmann. Well, modern textbooks use a more general and abstract context, that of the theory of *differential varieties*, in which some passages and formulations are more direct. On the contrary, the calculations elaborated by Pauli are rarely found in the most modern manuals [22].

1.4.2. *Gauge Theories* in Conflct with Causality Principle

At this regard, it is interesting to highlight that the physical meaning of this *gauge invariance* formulated by Weyl lies in the possibility of assigning the *phase* to the fields in an arbitrary way, without changing the observable quantities. In effect, this way of thinking contradicts the Causality Principle, since it requires to assign the phase of the fields simultaneously at all space-time points.

It looks more physical to require the possibility of assigning the phase in an arbitrary way at each space-time point [23]. Furthermore, as concerns the *Quan-*

tum Fields Theory (QFT), with reference to a Dirac (Ψ) field of non-interacting mass *m*, as illustrated by Equation (1), it is not clear whether the value of the continuous parameter θ uniquely identifies each particular gauge transformation represented by the aforementioned Equation (1). Namely, if the angle θ had a physical meaning, the *globality* of the gauge transformations represented there (that is the fact that θ is constant in space-time) would entail a violation of the Causality Principle: in this case, indeed, it would be necessary that the information of a phase change at one point spreads instantaneously throughout the space [24].

Therefore, it is necessary to understand whether it is possible to promote the examined symmetry to *local gauge invariance*, which means to have the opportunity to choose point by point which phase to assign to the field. To do this, let us consider the generalization of Equation (2) to the case where θ is a *smooth* function of the variable *x*:

$$\Psi(x) \to e^{ie\theta(x)}\Psi(x) \tag{13}$$

And interesting to emphasize that the second term of Equation (2) is *locally* gauge invariant and does not need any modification. The first term of Equation (2), on the other hand, requires particular attention, because under the *transformation* illustrated by Equation (3), it is transformed in a non-trivial way:

$$i\bar{\Psi}\gamma^{\mu}\partial_{\mu}\Psi \to i\bar{\Psi}e^{-ie\theta}\gamma^{\mu}\partial_{\mu}\left[e^{ie\theta}\Psi\right] = i\bar{\Psi}\gamma^{\mu}\partial_{\mu}\Psi - e\bar{\Psi}\gamma^{\mu}\Psi\partial_{\mu}\theta \tag{14}$$

The extra term that appears in Equation (14), and that *breaks* the gauge invariance of the Dirac lagrangian (Equation (1)), is generated by the presence of a derivation operator (∂).

Moreover, as concerns the *isospin symmetry*, the basic rule of isospin symmetry can be summarized in the

$$\text{Substitution: } \begin{pmatrix} p \\ n \end{pmatrix} \to U \begin{pmatrix} p \\ n \end{pmatrix} \tag{15}$$

where *p* is the proton, *n* the neutron and *U* is any *complex matrix* 2 × 2.

According to Maiani, you need to keep in mind that the substitution shown in Equation (15) is similar to Equation (13), but with two substantial differences: 1) The product of the matrices does not have the commutative property, which the product of the gauge factors in Equation (13) have [3]. 2) The transformations illustrated in Equation (15) are *global transformations*, unlike Equation (13), which shows *local* transformations, where the phase attributed to Ψ(*x*) is different from one point to another, but without significant variations of the examined physical system [3]. The *transformation* shown by Equation (13), then, is *invariant*, which implies exclusively massless particles!

In Equation (15), instead, the *p* states refer to a definition of proton and neutron which must be shared, at a given instant of time, by all observers of the Universe and which is transformed by the *U* matrix in all points of the Universe, *simultaneously*.

1.4.3. *Gauge Theories* in Conflict with Baryon Number Conservation Law

As known, the most general renormalizable Lagrangian invariant under the *Standard Model gauge group* (containing only color singlet Higgs fields) is automatically invariant under *global* Abelian symmetries. These may be identified with the baryon (*B*) and lepton (*L*) symmetries, which are accidental symmetries and as a result it is not possible to violate *B* and *L* at tree-level or at any order of *Perturbation Theory*. However, in many cases the perturbative expansion does not describe all the dynamics of the theory.

As we all know, the Conservation of the Electric Charge finds its theoretical basis in the *gauge invariance* of Maxwell eqations. On the contrary, as Maiani reminds us, the Conservation of the Baryon Number is not associated with any *gauge invariance* and has always appeared as an artificial rule, however it applies with great precision [25]. Yet the Baryon Number Law is always preserved!

Actually, this leaves us perplexed because the *gauge invariance* does not coincide with one of the fundamental laws of Physics: the Law of Conservation of the Baryon Number.

It is even possible to consider that maybe something "artificial" lies in the "rules", or dogmas, which are the basis of *gauge theories*, after all, in keeping with Einstein and Pauli, that Mathematics is not up to standards. In fact, as regards the *Perturbation Theory*, unfortunately the approximate calculation methods available (the *Perturbative Calculus*) are not completely reliable [26].

Hence, let's go into this last topic.

2. Methods

2.1. *Divergences* in Perturbative Calculus

Notoriously, an approximation method is useful for finding the changes in the discrete energies and the associated *wave functions* of a physical system resulting from a small disturbance, or *perturbation*, provided the energies and the wave functions of the undisturbed system are known. So, in this method, usually referred to as the *Rayleigh-Schrödinger Perturbation Theory*, the changes in the energies and the wave functions are expressed as an infinite power series in the perturbation parameter. The approximation, then, consists in neglecting terms in the *infinite series* after the first few terms.

Approximating the series to the first *n* terms in the series, gives the *nth order approximation* [27].

Successively, in the 30s of the last century, scientists began to notice that in the equations of *Perturbative Development* of the Quantum Electro-Dynamics (QED) *divergences* emerged, which were considered un-eliminable: these equations, indeed, resulted in zeroes and/or infinities!

As it is known, the *Perturbative Development* is a mathematical technique for finding an approximate solution to a problem that cannot be solved exactly, by starting from the exact solution of a related problem; used in modelling physical interactions between particles, etc.

In effect, at that time there was a widespread belief that the *infinities* coming from the equations inherent to the *Perturbative Calculating* in the QED were absolutely ineliminable.

Namely, QED is a *Quantum Theory* of the Electromagnetic Field, *i.e.* a *Quantum Fields Theory* (QFT).

The first mathematical formulation of a QFT describing the interaction between the electro-magnetic radiation and matter (*i.e.* between photons and electrons) is Dirac's [28]. Heisemberg and Pauli also formulated one of the first QFT, and even they could not solve the relative *field equations*, which tended to infinite values [29]. Thus, it was not possible to write a solution of the equations in the form of a single mathematically compact expression, applicable in all circumstances. They had to resort to an alternative solution method: the so-called *Perturbative Development* (or *Perturbation Theory*). Adopting this technique, the equation is rewritten as a potentially infinite sum of a series of terms: $x^0 + x^1 + x^2 + x^3 + \cdots$ that is, the series begins with an expression of 'order zero', or x°, or *unperturbed*, corresponding to the total absence of interactions, so the equation is perfectly solvable. The other terms of the series, on the other hand, are *perturbative*: they represent corrections to the 1˚ order, as in the case of x^1, or corrections to the 2˚ order (x^2), or to the 3˚(x^3) and so on. The subsequent terms of the *Perturbative Development* make ever smaller corrections to the zero order result, progressively bringing the calculation closer to the exact solution. Hence, the accuracy of the final result depends on the number of perturbative terms included in the calculation. However, instead of ever smaller corrections, Heisemberg and Pauli found that some terms of the *Perturbative Development* 'exploded', tending to *infinite* values. These terms, applied to QED, were identified with the so-called electron *eigenenergy*, due to the *eigeninteraction* of the electron with the *quanta* (ie photons) of the Electromagnetic Field (EMF) generated by itself.

In other words, the common interaction electron-photon generates *infinities*. Why?

Because the photon is considered massless. Consequently, the most elementary Algebra teaches that multiplying a value by zero we get 0, and dividing by zero we get ∞.

This also occurs with the radius of the electron, considered null, *i.e.* equal to zero.

Briefly, the QED describes all phenomena relating to electrically charged particles interacting through EM Interaction. As known, m*athematically* the QED presents the structure of an *Abelian* gauge theory, with the symmetry group $U(1)$ where, *physically*, it means that charged particles interact with each other by the exchange of null-mass particles: the photons. In effect, the *gauge field*, which mediates the interaction between the charged spin – 1/2 fields, is the EMF. At this regard, the spinorial QED, or QED Lagrangian (L_{QED}), for a spin – 1/2 field interacting with the EMF, is represented as follows:

$$L_{QED} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi} \left(1/2\, i\partial - M + e A \right) \psi \quad (16)$$

where ψ and its antiparticle ($\bar{\psi}$) are the fields that represent charged particles (Dirac spinors: e.g. electron–positron field); i is the imaginary unity; M indicates the mass of the electron or positron; e is the coupling constant, equal to the electric charge of the bispinor field; A is the covariant four-potential of the EMF generated by the electron itself; $F_{\mu\nu}$ is the EMF *tensor*, which represents the evolution of the free field, that is in the absence of additional potentials.

Hence, Equation (16) describes the interactions between a quantized material spinorial field (*i.e.* the electronic field) and a non-massive vector field that describes the electromagnetic radiation (EMR), *i.e.* the EMF managed by the photons, considered massless [30].

Thus, Oppenheimer in 1930 demonstrated that at the origin of the infinities there was the term expressing the interaction between the electronic current and the EMF produced by the electron [31]. Namely, the *eigeninteraction* of the electron, considering at the 2nd order the processes in which the electron emits and resets a photon, causes an infinite shift (with quadratic *divergence*) of the hidrogen spectral lines. Of course, this occurs because in the equations a point value for the radius of the electron (a) is introduced, thus $a \to 0$ (which is as to give the value $a = 0$).

In this way the calculation results in an *infinite shift*: for $a \to 0$ diverges as $1/a^2$, where a is the electron radius, considered size point, therefore equal to 0. Namely, the EM energy of an electron(E_{em}), thought as a charge sphere, $E_{em} = e^2/4\pi a = \infty$ (e is the electron's charge), is *divergent* in the limit $a \to 0$.

To this purpose, as Oppenheimer remind us, the paper develops a method for the systematic integration of the relativistic wave equations for the coupling of electrons and protons with each other and with the EMF. It is shown that, when the velocity of light is made *infinite*, these equations reduce to the Schrödinger equation in configuration space for the *many body* problem [32].

Likely, there is something wrong: the speed of light will never be able to reach infinity values!

As known, the literature of the time is full of results similar to those found by Oppenheimer At the 7th Solvay Congress (Paris, 1933), the *polarization of the vacuum* was explored, among other things. In this typical quantum phenomenon the vacuum continuously generates pairs of particles, such as electron-positron. What happens is that positrons, surrounding the electron, create an asymmetry in the electron charge distribution. So, according to Barrow, a virtual cloud of positron reduces the charge of the electron [10] and the calculation of this effect highlights a *new infinity*, which is added to the infinity generated by the *electron eigeninteraction*. In this respect, Dirac proposes to mutually subtract these 2 *infinities*. The method proposed by Dirac, indeed, consists of a procedure of *subtraction of the infinities*, similar to the one used to subtract the *infinities* emerged from the calculations related to the *vacuum polarization* [33].

After the Solvay Congress, Pauli instructs Weisskopf to recalculate the electron *eigenenergy* (cause of the 1st infinitive highlighted by Oppenheimer), taking into account the production of electron-positron pairs (generated by quantum vacuum fluctuations) and correlated to the *polarization of the vacuum*: another cause of *infinitives*. However, the result was depressing: an infinite was always obtained. In fact the divergence always existed, even if it was only logarithmic:

$$E = \frac{3e^2 mc^2}{hc} \log\left[\frac{h}{mca} + \sqrt{1 + \left(\frac{\hbar}{mca}\right)^2}\right] \tag{17}$$

where E is the electron *eigenenergy*, m its mass and a its ray, considered as a point (thus = 0). In Equation (17) the *null* value of a appears twice in the denominators: we shouldn't marvel at the *infinities*!

Of course, this occurs because in the equations a point value for the radius of the electron (a) is introduced, thus $a \to 0$ (which is as to give the value $a = 0$). Consequently, the calculation results in an infinite shift: for $a \to 0$ diverges as $1/a^2$.

Besides in Equation (17), twice, the mass of the electron and the speed of light, both multiplied by zero (the point electron), reset to zero, they cancel each other out! But it is not possible, it is clear that there is an error, which certainly does not lie in the values of m or c; thus it must be in the value given to a, that is to the radius of the electron, considered equal to a point, that is equal to zero. At the same time, as Equation (17) shows, the energy of the electron tends to ∞.

Conversely, the electron *rest energy* is *only* equal to 0.511 MeV/c^2! Moreover, being massive particles, the electrons can in no way occupy a void or point volume of space, that is, equal to 0. No! That's no good. So, staring around 1934 the rumor began to spread that something was definitely wrong in the QED, since on one hand the Dirac equation could not explain the experimental data, while, on the other, the QFT even produced *infinite* results. At this regard, Oppenheimer and Furry are convinced that the difficulties and *divergences* in the calculations concerning the *eigenenergy* of electrons are based on the illegitimate application of the methods of Quantum Mechanics (*QM*) to the electro-magnetic field (EMF) [34]. In fact, according to Oppenheimer, it is further shown that it is impossible on the present theory to eliminate the interaction of a charge with its own field, and that *the theory leads to false predictions* when it is applied to compute the energy levels and the frequency of the absorption and emission lines of an atom [31].

Perhaps, unconsciously, Oppenheimer had been prophetic.

Briefly, the common interaction electron-photon generates *infinities*. Why? It's obvious! For the fact that the photon is considered massless and the radius of the electron is considered null, *i.e.* equal to zero, so that multiplying a value by zero we get 0, and dividing by zero we get ∞.

We believe this represents a fundamental crossroads.

Hence, let's try to analyze these zero values more carefully.

2.2. On the Zero Mass Photon

At this regard, we deepen the nature of such a radiation. As we all know, the electro-magnetic radiation (EMR) carries a large number of *light quanta*, or photons (P_s), second after second. The energetic values of each photon(P), without considering its oscillating frequency, corresponds to the *Planck's constant* (h), which is just an energetic value, corresponding to 6.626×10^{-27} [erg·sec].

In the impacts between the atoms and the EMR, according to Barrow, the value of h is large enough to take a rather strong "*stroke*" to push the electrons to the immediately higher permissible level. Well, this non-null value of h is important for the *stability of matter* [35]. As known, h identifies with Planck "grain", with the quantum of light, that is with P. And yet, a massless P is capable of inferring such a *stroke*, besides giving "stability to matter!" [35]. Unless the P is not so massless.

The P goes with the speed of light: this value(c) is known too, it is 299,792.458 (±0.4) Km/sec. Let's now consider the equation related to the Principle of Equivalence Mass-Energy (MEEP) [36]:

$$E = mc^2 \tag{18}$$

That is:

$$m = \frac{E}{c^2} \tag{19}$$

with such words Einstein commented upon his MEEP: "The value of the considered mass refers to the value of an *inertial mass*" [37]. Hence, let's apply Equation (18) to the P, keeping in mind that one of the three parameters is well known, that is c, the speed of the P in the vacuum. The 2nd parameter is the *energy* (E) of the P which, as described by Planck [38], is expressed by the formula:

$$E = h \cdot \nu \tag{20}$$

where ν is its oscillation frequency. However, here things get more complicated, since Equation (20) expresses the energetic value of a single light quantum(or P) in motion, that is at the highest speed, oscillating a number of times per second, depending on the EMR band to which the quantum of light is associated. Conversely, Equation (18) and Equation (19) represent the value of an *inertial mass*, just because it is involved the MEEP, it will express an *inertial energy,* as to say the *minimal energy*, or *Zero Point Energy* (ZPE) [39] [40] of the particle we are considering.

Thus, to a very small energy, as in the case of the P, corresponds a very small mass, however $\neq 0$ [41].

In short, the value of the *density of mass energy* carried out by h, by the Planck's *grain*, although infinitesimal (and without considering its number of oscillations per second) will always be $\neq 0$!

So, in the case of a P at the *inertial state,* that is when it interacts with another particle, so it stops running, at least for that infinitesimal instant of time it will probably oscillate much less. Obviously, we will never be able to know with *accuracy* how much an interacting P can oscillate, that is what could be the number of oscillations [c/s] in that instant. Let's indicate this unknown value with 10^n [c/s], which is an *uncertainty factor*. The P stops running when hitting another particle, as it happens during a *measurement* [42], so it will not oscillate as when it was running, though it never stops running completely. It is the Heisenberg Uncertainty Principle(HUP) to deny it, since in this case we would know simultaneously the *position* and the *momentum* of the particle [43] [44]. Thus, also in the inertial state, the oscillating frequency(ν) of the P can never be 0, but always ≥ 1/s, that is ≥one oscillation (10°) per second (if not even 1/2 oscillation per s., or a fraction of its). Therefore, if we want to consider the Energy of the P in its *inertial state*, indicated with E_0, we should have:

$$E_0 = h \cdot \nu = h \cdot 10^n \left[\text{c/s}\right] \tag{21}$$

$$E_0 = 6.626 \times 10^{-27} \left[\text{erg} \cdot \text{s}\right] \cdot 10^n \left[\text{c/s}\right] \tag{22}$$

$$\text{that is:} \quad E_0 = 6.626 \times 10^{-27+n} \left[\text{erg}\right] \tag{23}$$

Hence, this should be the Energy value of a P at an inertial state. As to say its *minimal energy value*, or ZPE; as we can see this value is not easy to determine, rather, it is *undetermined*, as stated by the *QM*. As the *erg* value is expressed in [g·cm/s²·cm], that is in [g·cm²/s²], we have:

$$E_0 = 6.626 \times 10^{-27+n} \left[\text{g} \cdot \text{cm}^2/\text{s}^2\right] \tag{24}$$

So we can have information, with a certain approximation, about a 2nd parameter of Equation (19), referred to the P. Thus, we can easily have the 3rd parameter, the *equivalent rest-mass* or *equivalent inertial mass* (m_0) of the P [45]:

$$m_0 = \frac{E_0}{c^2} = \frac{6.626 \times 10^{-27+n} \left[\text{g} \cdot \frac{\text{cm}^2}{\text{s}^2}\right]}{\left(2.9979 \times 10^{10}\right)^2 \left[\text{cm/s}\right]^2} \tag{25}$$

Let us calculate this value following the *cgs* system:

$$m_0 = \frac{6.626 \times 10^{-27+n}}{2.9979^2} \times 10^{-20} \frac{\text{g} \cdot \frac{\text{cm}^2}{\text{s}^2}}{\frac{\text{cm}^2}{\text{s}^2}} \tag{26}$$

and we have:

$$m_0 = \frac{6.626}{2.9979^2} \times 10^{-27-20+n} \left[\text{g} \cdot \frac{\text{cm}^2}{\text{s}^2}\right] \cdot \frac{\text{s}^2}{\text{cm}^2} \tag{27}$$

$$m_0 = \frac{6.626}{2.9979^2} \times 10^{-47+n} \left[\text{g}\right] \tag{28}$$

$$m_0 = 7.732 \times 10^{-48+n} \left[\text{g}\right] \tag{29}$$

Well, what we get is that the *inertial mass-energy* of the P corresponds to

10^{-48+n} grams. Therefore, if the value of n was 10^0, that is one oscillation per second, m_o would be 10^{-48} [g]. Whereas if n was 10^3 oscillation per second, we would have $m_0 = 10^{-45}$ [g].

In all cases, of course, it is an extremely small value, but it is ≠ 0, according to Relativity and *QM*.

Moreover, one of characteristics of the P is to travel most of the time, so it also gets a *momentum*.

2.3. The *Momentum* of Photon

In agreement with Fermi, the photon (P) too, as other particles, is a corpuscle, a *light's quantum* and has a its own *momentum* (***p***), through which transfers all its energy to the hit particle [46].

In Newtonian Mechanics ***p*** is thus represented:

$$\boldsymbol{p} = m \cdot \boldsymbol{v} \tag{30}$$

where m is the mass and v the velocity of the involved particle [47].

In Quantum Mechanics (*QM*), in its turn, ***p*** is described by the *de Broglie formula*:

$$\boldsymbol{p} = \frac{h}{\lambda} \tag{31}$$

where λ is the wavelength of the considered P (or other *quantum object*).

de Broglie, indeed, suggested to give particles the same property as waves. He gave each particle a its own wave length depending only on the *momentum* (***p***) of the particle itself [48]. Thus, along with de Broglie, any *quantum object* (*i.e.* any particle) with a *momentum* (***p***) seems to be something periodic, oscillating as a wave, with an universal relation between the wave length of the particle, indicated by λ, and modulus ***p*** of its *momentum* [49].

Thus, in line with *de Broglie formula*, let us now to analyze the ***p*** value of photons with different wave length (λ). The mean wave length of a photon in the optical band corresponds to ≈5 × 10^−5^ [cm] and its ***p*** is:

$$\boldsymbol{p} = \frac{h}{\lambda} = \frac{6.626\times10^{-27}\,[\text{erg}\cdot\text{s}]}{5\times10^{-5}\,[\text{cm}]} \tag{32}$$

$$\boldsymbol{p} = \frac{6.626\times10^{-27}\left[\text{g}\cdot\dfrac{\text{cm}^2}{\text{s}}\right]}{5\times10^{-5}\,[\text{cm}]} \tag{33}$$

$$\boldsymbol{p} = 1.325\times10^{-22}\left[\text{g}\cdot\frac{\text{cm}}{\text{s}}\right] \tag{34}$$

As Equation (34) shows, the momentum (***p***) of a visible photon carries out a *dynamic-mass*, a *pushing momentum* bigger than the *rest mass* of 100 protons. No surprise! At this regard, Zeilinger asks himself: what is the deep meaning of a relationship like $E = mc^2$? What is *hidden* behind these symbols? For many physicists the equation $E = mc^2$ is to say that energy and mass are the same thing, two faces of the same medal; there is therefore *equivalence between mass and* *energy*: **energy is just another form of mass**, and vice versa, **mass is another form of energy**" [50].

To this purpose, we find very interesting to emphasize what Eddington said in 1919: the simplest interpretation of the deflection of the light beam is the one that considers it as an effect of **the weight of light** [51]. At the dinner of that meeting, Eddington read out some verses he had composed; we will quote the last quatrain: *We will compare the measures taken, one thing at least is certain,* ***light has weight****. One thing is certain and the rest debate. Light rays, when near the Sun, do not go straigh*t [51].

In other words, Lord Eddington clearly points out the *mechanical effect* exerted by light, in complete agreement with our conviction that light carries with it also a mass (the *dynamic-mass* of P) [52] [53].

2.4. Removal of Massless Photons from *Perturbative Equations*

According to Einstein's MEEP, as shown in the Equation (19), to an "energetic" particle, carrying energy, forces etc., should correspond a *mass equivalent* to the energy carried, divided c^2. Moreover, since there is no zero energy for the *zero point energy* (ZPE), as Chandrasekhar reminds us [39], there should not be any particle carrying energy, with a zero mass.

In short, it may be incongruous to say that a particle with energy does not have an *equivalent mass* [45], it does not "conceal", at least, a mass. It is Einstein's equation to show that this particle has a mass, otherwise the equation would be null, the result would be zero.

In other words, there should not be real particles, having any energy, with a zero mass. If there are, they should "subtend" a tiny mass, a *Zero Point Mass* [40].

Therefore, to a very small energy, as in the case of P, corresponds with a very small mass, however $\neq 0$.

In sum, we think that the base concept of the *gauge theories*: 'the mass *breaks* the symmetry' is not applicable to the Planck constant. No! Planck's constant is a real value, ineradicable: represented by an *intrinsic* value, 6.626×10^{-27} [erg·sec], it expresses the value of the density of energy-(*equivalent mass*) of the *Planck Quantum.* On the contrary, reduce this value to zero, as *gauge theories* dictate, with consequent and inevitable *divergences* and *infinities* emerging from the equations of the *Perturbation Calculus*, would totally cancel the very existence of *Planck Quantum* and, consequently, also the energy of light. In this way, we would have a world everywhere *dark* and totally devoid of power! No, it is not possible.

In this context, in our view, especially a way could provide a solution: Correct the *divergenes*, without Renormalization, but by removing the massless photons (P_s) from equations of *Perturbative Calculus.*

As previously reported in paragraph **3.2**, from our simple calculations, taken from *Planck's formula* E = $h\nu$ (shown with the equation 20) and Einstein' MEEP, emerges that the photon(P) is not completely massless since, even in its *mini-*

mum energy state, or ZPE, or *inertial mass* (m_0), it shows a mass value which is not null, but corresponding to: $m_0 = 7.372 \times 10^{-48+n}$ [g], as shown in Equation (29), where n indicates the oscillation number per second of the involved P. Well, this is certainly a very small value, of no value in our macroscopic world and without the slightest meaning in our daily life. Nevertheless, although it is infinitesimal, it is still ≠ 0, so it can assume, in our view, a its value, a its role, both in the sub-atomic world and in the mathematical formalism.

In addition, if we take into account the value of the energy charge of the P, we must calculate its *momentum* ($\boldsymbol{p}$), obtainable from the *de Broglie*'s *formula* ($\boldsymbol{p} = h/\lambda$), as illustrated by Equation (31). Hence, considering the mean wave length (λ) of a P of the optical band, we have: $\boldsymbol{p} = 1.325 \times 10^{-22}$ [g·cm/s], as shown in Equation (34). It is really surprising! We have that a common optic P carries a mass-energy value over 5 orders of magnitude greater than the rest-mass of an electron: other than massless P!

Thus, it is obvious that we are going to replace this last value of the P with the massless P inserted in all equations of the *Perturbation Theory* and of the *Quantum Fields Theory*(QFT), including the Yang-Mills equation. Well, wat do we expect?

It is clear: the disappearance of *divergences* and *infinities*.

Likewise, even the calculation of the *electron eigenenergy* will not give null results any more. No! With a P value no more massless, the zeros disappear. They appeared whenever one tried to multiply the electron mass-energy with the *quanta* of his field, *i.e.* with P_s!

In brief, with this value of P other than zero, all *divergences* emerging from the equations of Perturbation Calculus, Quantum Electro-Dynamics(QED) and QFT disappear.

Concluding, in our opinion, the *removal of the infinites* emerging from the perturbative QED and the other QFT, can be obtained with 2 modes: 1) Replacing in the equations of such theories the zero value of a P massless, with the real *energy-mass value* of P, as represented by Equation (29) or Equation (34).

2) Replacing in the equations of the QFT the point value attributed to the radius of the electron, therefore → 0, with the real value of its radius.

2.5. Removal of Point-Like Electron

To be honest, it is natural and logical to think that, being massive particles. the electrons can in no way occupy a void or a point volume of space, that is, equal to 0.

Besides, considering the value of the minimum distance two particles can come close, no infinites should emerge from *perturbation calculations* of QED and other QFT. To this purpose, indeed, and in full compliance with Feynman, maybe the idea that two points may be infinitely close is incorrect, it is false the assumption that geometry will continue to be invariably unchanged. But if instead of including all the possible points of interaction until a 0 distance, the calculation is *cut off* when the distance between the points is very small, there exist defined values of the mass of the electron and of the its charge, such that the calculated mass coincides with the value of the mass of the electron measured experimentally, and the calculated charge coincides with the experimental value of the electric charge of the electron [54].

Moreover, as you can read from literature, "as regards the *problem of infinities*, just think about the energy of the electric field of a charged sphere, which radius (r) tends to zero: $r \to 0$; *i.e.* the energy $\to \infty$, diverges, such as $1/r$. For the theory of Special Relativity, part of the mass of the sphere comes from the (divergent!) energy contained in the surrounding electromagnetic field (EMF). However, one might think that no electrical charge is actually point size and that the problem is simply due to a *mathematical abstraction*" [55].

Thus, let's try to calculate *mathematically* and *physically* the actual value of the electron ray. To this purpose, we consider the value of the electron energy-mass density in its state of *minimal energy*, or *inertial mass* (m_o), which in the *cgs* metric system corresponds to 9.109383×10^{-28} [g]. From Planck formula $E = h\nu$, as Equation (34) shows, where ν is the frequency, thus $\nu = E/h$, we get the value of ν. Hence:

$$\nu = \frac{E}{h} = \frac{mc^2}{h} = \frac{9.109383 \times 10^{-28}\,[\mathrm{g}] \times \left(2.9979^2 \times 10^{20}\left[\frac{\mathrm{cm}^2}{\mathrm{s}^2}\right]\right)}{6.626 \times 10^{-27}\,[\mathrm{erg} \cdot \mathrm{s}]}$$
$$= \frac{81.8697 \times 10^{-8}\left[\mathrm{g} \cdot \frac{\mathrm{cm}^2}{\mathrm{s}^2}\right]}{6.626 \times 10^{-27}\left[\mathrm{g} \cdot \frac{\mathrm{cm}^2}{\mathrm{s}^2}\right]} = 1.23558 \times 10^{20} \tag{35}$$

So, the electron frequency, in its *minimal energy state*, or ZPE, corresponds to $\approx 10^{20}$ oscillations per second, or Hertz, or cycles per second (c/s). Let us now consider the formula of the electromagnetic (EM) waves, *i.e.*: $\lambda \cdot \nu = c$, of which we now know 2 parameters, *i.e.* c and ν. Let's calculate the 3rd parameter, *i.e.* λ, which refers to wavelength of the electron in its *minimal energy state:*

$$\lambda = \frac{c}{\nu} = \frac{2.9979 \times 10^{10}\left[\frac{\mathrm{cm}}{\mathrm{s}}\right]}{1.23558 \times 10^{20}\left[\frac{\mathrm{cm}}{\mathrm{s}}\right]} = 2.4263 \times 10^{-10}\,[\mathrm{cm}] \tag{36}$$

This is the value that, in our judgment, should be inserted in the equations of the Perturbation Calculus and QFT (QED included) to represent the radius of the electron (a), replacing the null value which has been considered so far.

Thus, it is clear that no longer dividing by a zero value, *infinities* and *divergences* will disappear.

Furthermore, it is obvious that, as in all material particles, the more the electron is accelerated, the more its wavelength will be restricted, but without never reaching zero or close to zero values!

Consequently, if we replace this value with the null value of the electron ray inserted in the equations of the Perturbation Theory, of the QFT and the Yang-Mills theories, all *divergences*, that is all zeroes and infinities, would suddenly disappear.

2.6. Isospin Symmetry

As known, the isotopic spin symmetry (or *isospin symmetry*), as shown by Equation (15), had been introduced by Heisenberg in relation to the surprising similarity of the masses of the proton and neutron (called "Nucleons" by Heisenberg [56]) and consisted in supposing that the nuclear forces were symmetrical for the proton and neutron substitution with arbitrary linear superpositions of these two states. Obviously, this symmetry is not respected by the EM Interaction (EMI), which distinguishes the proton (positive electric charge) from the neutron (zero electric charge). In analogy with what happens for particle spin (hence the name of symmetry), symmetry implied that the nuclei occurred in *multiplets* of isotopic spin I, with $2I + 1$ states and electrical charges one unit apart, according to the rule:

$$Q = I_3 + \frac{1}{2}B, \quad I_3 = -I, -I+1, \cdots, +I, \tag{37}$$

where Q is the electric charge in units of the proton charge, B is the Barionic Number and I_3 is the third component of the isotopic spin, analogous to the magnetic quantum number of the *angularmomentum* [3]. The surprise was that even hadrons respected isospin symmetry and presented themselves in *multiplets*, each characterized by an I value of the isotopic spin and by electric charges given by a formula analogous to Equation (37):

$$Q = I_3 + \frac{1}{2}(B+S), \quad I_3 = -I, -I+1, \cdots, +I, \tag{38}$$

where S is a new quantum number introduced by Gell-Mann to characterize the *strange particles* ($S = 0$ for nucleons and pions, $S = +1$ for K^+, K^o, $S = -1$ for hyperone Λ, etc.).

The Equation (38) is known as Gell-Man and Nishijima's *formula*.

2.7. Yang-Mills' Isospin Symmetry Theory

Hence, taking inspiration from the Isospin Symmetry introduced by Heisenberg [56], Yang and Mills propose to formulate an Isospin Symmetry Theory that does not suggest, to put it to Einstein, any *spooky action-at-a-distance* [3]. We learn from A.A.: "We wish to explore the possibility of requiring all interactions to be invariant under *independent* rotations of the isotopic spin at all space-time points, so that the relative orientation of the isotopic spin at two space-time points becomes a physically meaningless quantity: the EM field(EMF) being neglected. We define *isotopic gauge* as an arbitrary way of choosing the orientation of the isotopic spin axes at all space-time points, in analogy with the electro-magnetic gauge, which represents an arbitrary way of choosing the complex phase factor of a charged field at all space-time points. We then propose that all physical processes (not involving the EMF) be invariant under an isotopic gauge transformation, $\Psi \to \Psi'$, $\Psi' = S^{-1}\Psi$, where S represents a space-time dependent isotopic spin rotation. Let Ψ be a two-component wave function describing a field with isotopic spin 1/2. Under an isotopic gauge transformation it transforms by:

$$\Psi = S\Psi' \tag{39}$$

where S is a 2×2 unitary matrix with determinant unity" [57].

In short, the synthesis of the construction of the Abelian London [5] and Weyl *gauge theory* [6] is extended to a not-Abelian gauge theory. To do this, Yang and Mills replace the one-dimensional unitary symmetry group $U(1)$, to be considered as the set of rotations on the plane, with a *compact Lie group*, expression of a set of rigid movements in a multi-dimensional space. Nevertheless, while $U(1)$ is Abelian, or commutative (a series of rotations add up), the *compact Lie group* is not Abelian, giving rise to a much more complicated *gauge theory*. Hence, Yang and Mills suggest that even the Nuclear Interactions can be described by a gauge theory: a *false step*, in our view.

The main problem with this model, indeed, is that the *gauge simmetry* prohibits the presence of mass terms for the vector bosons mediating the interaction. However an interaction mediated by a null mass particles has to produce long-range effects which, on the contrary, are completely absent in the phenomenology of Nuclear Interactions [24].

2.8. Yang-Mills Equation

Yang and Mills introduce the reader to the *new field* they highlighted and called ***b** field.* Bold-face letters and type denote three-component vectors in isotopic space, not in space-time. We get then to the equations describing the ***b** field*: "To write down the field equations for the ***b*** field we clearly only want to use isotopic gauge invariant quantities. In analogy with the electromagnetic case we therefore write down the following Lagrangian density: $-\frac{1}{4} f_{\mu\nu} \cdot f_{\mu\nu}$. We shall use the following *total Lagrangian* density:

$$L = -\frac{1}{4} f_{\mu\nu} \cdot f_{\mu\nu} - \bar{\Psi}\gamma_\mu \left(\partial_\mu - ie\tau \cdot \boldsymbol{b}_\mu\right)\Psi - m\bar{\Psi}\Psi \tag{40}$$

One obtains from this the following equations of motion" [57]:

$$\frac{\partial \boldsymbol{f}_{\mu\nu}}{\partial x_\nu} + 2\varepsilon\left(\boldsymbol{b}_\nu \times \boldsymbol{f}_{\mu\nu}\right) + \boldsymbol{J}_\mu = 0 \tag{41}$$

$$\gamma_\mu \left(\partial_\mu - i\varepsilon\boldsymbol{\tau} \cdot \boldsymbol{b}_\mu\right)\Psi - m\Psi = 0 \tag{42}$$

and where:

$$\boldsymbol{J}_\mu = i\varepsilon\bar{\Psi}\gamma_\mu\boldsymbol{\tau}\Psi \tag{43}$$

As we all know, the (41) is the famous Yang-Mills equation: it represents the motion equation of the ***b** field*, or *Yang-Mills field,* that is the *nuclear* and *in-*

tra-nuclonic strong field, which today we can call *gluon field* (or *color field*). Moreover, concerning Equation (41), it can be useful remember that $\boldsymbol{f}_{\mu\nu}$ describes the intensity of Yang-Mills field; $\partial/\partial x_\nu$ specifies that this equation depends on the way the intensity of the field changes with space and time. In effect, being the derivatives of the spatial coordinates in the denominator, we have that as the distance increases, the intensity of the strength of the field decreases proportionately. The parameter ε represents the *charge*; $\boldsymbol{J}_\mu$ is "the spin −1/2 field; $\boldsymbol{b}_\nu$ is the Yang-Mills ***b* quantum**, *i.e.* the potential of the ***b*** *field*, which can be identified with the *quanta* going through the *Yang-Mills field.*

According to Maiani, in Yang-Mills theory, as in electrodynamics and in General Relativity, the symmetry (invariance under *local* not-Abelian transformations) determines the interaction of vector fields with matter (the nucleons). The intensity of the interaction is fixed by a constant, *g*, completely analogous to the electric charge (*e*) that appears in Equation (2). Unlike electrodynamics, however, the vector fields ($\boldsymbol{b}_\nu \times \boldsymbol{f}_{\mu\nu}$) are themselves sensitive to not-Abelian transformations and therefore interact with each other in a way also completely determined by the symmetry and the interaction constant *g* [3]. In fact, $\boldsymbol{b}_\nu \times \boldsymbol{f}_{\mu\nu}$ provides important difference compared to Maxwell's equations, since it emphasizes the dependence of the Yang-Mills *field* from itself [58]. To this regard, indeed, Yang and Mills specify the isotopic spin (J_μ): "We define:

$$J_\mu = \boldsymbol{J}_\mu + 2\varepsilon \boldsymbol{b}_\nu \times \boldsymbol{f}_{\mu\nu} \tag{44}$$

Equation (44) shows that the isotopic spin arises both from the spin-1/2 field (J_μ) and from the $\boldsymbol{b}_\mu$ field itself" [57]. Nevertheless, doubts began to arise.

3. Results

3.1. Yang-Mills *Mass Gap* Problem

3.1.1. Glashow Model

Since Glashow couldn't find a mathematically congruent solution, in contrast to the *gauge theories* and QFT, in order to solve the Yang-Mills *mass gap* problem, he forced the issue and introduced *ad hoc* massive WI bosons. In line with the Glashow theory, indeed, the Yang-Mills Lagrangian is reduced, in the limit of null coupling constant ($g = 0$), to a Maxwell Lagrangian for each guage field:

$$L^{(g=0)} = -\frac{1}{4} W^i_{\mu\nu} \cdot W^i_{\mu\nu} - \frac{1}{4} B^i_{\mu\nu} \cdot B^i_{\mu\nu} \tag{45}$$

This point Glashow, to avoid the presence of zero mass bosons, adds a mass term (M) [59]:

$$L = \frac{1}{2}\left[M^2 W_\mu \cdot W^\mu + M_0^2 B_\mu B^\mu{}_? + 2M_{03}^2 W_\mu^3 B^\mu \right] \tag{46}$$

In the case of charged fields ($i = 1, 2$), it is possible to define:

$$W_\mu = \frac{W_\mu^1 + iW_\mu^2}{\sqrt{2}}; \quad W_\mu^\dagger = \frac{W_\mu^1 - iW_\mu^2}{\sqrt{2}} \tag{47}$$

and we get:

$$
\begin{aligned}
L = W_{\mu\nu}^{\dagger} W^{\mu\nu} + M^2 W_{\mu}^{?} \cdot W^{\mu} - \frac{1}{4}\left[W_{\mu\nu}^{3} \left(W^3\right)^{\mu\nu} + B_{\mu\nu} B^{\mu\nu} \right] \\
+ \frac{1}{2}\left[M^2 W_{\mu}^{3} \left(W^3\right)^{\mu} + M_0^2 B_{\mu} B^{\mu}{}_{?} + M_{03} W_{\mu}^{3} B^{\mu} \right]
\end{aligned} \tag{48}
$$

Maiani describes Equation (48): the 1st line of (48) defines two integer (1) spin bosons, with electric charge ± 1 and mass *M*. As for the neutral fields, *i.e.* 2nd and 3rd line of (48), the physical fields (with defined mass) are identified by the self-vectors of the *mass matrix* (*M*) which, in the bases (W^3, *B*) are written as:

$$
M = \begin{pmatrix} \dfrac{M^2}{M_{03}^2} & \dfrac{M_{03}^2}{M_0^2} \end{pmatrix} \tag{49}
$$

This matrix is not completely arbitrary because it must have a null auto-value, corresponding to the zero mass of the photon. We must therefore impose:

$$
\det M = 0 \Rightarrow \left(M_{03}^2\right)^2 = M^2 M_0^2 \tag{50}
$$

We write the self-vectors of the matrix (*M*) illustrated by Equation (51) as:

$$
\begin{aligned}
Z_{\mu} &= \cos\theta W_{\mu}^{3} - \sin\theta B_{\mu} \\
A_{\mu} &= \sin\theta W_{\mu}^{3} + \cos\theta B_{\mu}
\end{aligned} \tag{51}
$$

where A_μ is the electromagnetic field and Z_μ is a new electrically neutral vector field. The self not-zero value of M is simply given by its *trace* [60]:

$$
\begin{aligned}
M_Z^2 &= M^2 + M_0^2 \\
&= \left(\cos\theta, -\sin\theta\right) M \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix} \\
&= \cos^2\theta M^2 - 2\cos\theta\sin\theta M M_0 + \sin^2\theta M_0^2
\end{aligned} \tag{52}
$$

From which we get:

$$
\frac{M_0^2}{M^2} = \tan^2\theta \tag{53}
$$

$$
\text{hence:} \quad M_Z^2 = \frac{M^2}{\cos^2\theta} \tag{54}
$$

In short, Glashow knows that the bosons of a *Nuclear Force* cannot be massless, otherwise their range of action would extend to infinity! Consequently, in order to try to unify the Electromagnetic Interaction (EMI) with the Weak Interaction (WI), as suggested by Schwinger, Glashow has to solve a very complicated problem.

On the one hand, indeed, the *Quantum Fields Theory* (QFT) equations, related to the *gauge theories*, categorically impose that all the particles are massless. On the other hand, on the contrary, there emerges an absurd complication. In order not to collapse the whole theoretical construction of the *gauge invariance*, and with it the QFT, in obvious opposition with the Yukawa Principle, the bosons of a *Nuclear Force* are also considered to be massless, thus, like the photon (P), they should exercise their action for unlimited distances. Of course, Glashow cannot accept these absurd concepts, in complete conflict with physical reality and experimental events, so he *manually introduces massive bosons*, as shown by Equation (48). This equation, however, violates the *symmetry*, a fundamental presupposition for *gauge theories*, and furthermore it cannot be *renormalized*.

3.1.2. Weinberg Theory

Weinberg activity, in turn, was based on the same principles adopted by Glashow, however at the beginning he tried to assign a mass to the bosons of the other *nuclear force*: the Strong Interaction (SI). Weinberg, in effect, had spent a couple of years studying the effects of the *Spontaneous Symmetry Breaking* (*SSB*) in SI described by a $SU(2) \otimes SU(1)$ gauge theory. As Nambu and Jona-Lasinio had discovered a few years earlier [7], the result of the *symmetry breaking* was that protons and neutrons acquire a mass. Weinberg was convinced that the Nambu-Goldstone bosons so created could be identified, to a certain approximation, with the pions [12]. Weinberg thought it promising to use the ideas of *symmetry breaking* in a Yang-Mills theory to describe SI. At the beginning, as he tried to assimilate the particles with and without mass, which appeared in his theory, with particles of strong interaction, his efforts seemed in vain" [58].

As Baggott reports, Weinberg had tried to apply the Higgs Mechanism to the SI, and now he realized that the mathematical structures he had tried to use for SI were precisely what was needed to solve the problems of WI and its heavy bosons [12]. As known, the mathematical difficulty encountered by both Glashow and Weinberg and Salam, in including hadrons in their unified theories, emerges when one tries to extend Cabibbo theory to a unified Yang-Mills theory. At this regard, indeed, it is necessary to bear in mind that Cabibbo observed that WI may not respect the scheme:

$$Q = \begin{pmatrix} u \\ d \end{pmatrix}, (s) \tag{55}$$

where the *strange* Quark (*s* or *s*Q) would have isospin 0 and would not be coupled to W^-, that is the boson carrying the WI that governs the Neutron Decay, or *negative beta decay* (βd^-) [61].

However the βd^- of the *strange particles* (Λ), as

$$\Lambda \to p + e^- + \bar{\upsilon}_e \tag{56}$$

(where $\bar{\upsilon}_e$ is the electronic anti-neutrino, p the proton and e^- the electron) corresponds to the transition *uds* → *uud*, or *s*Q → *u*Q,(where *u*Q is an up Quark) which could not occur in this scheme because the *s*Q would have isospin 0 and would not be coupled to W^-. To this purpose Cabibbo deduced that the down Quark (*d*Q), with defined isospin, is a superposition of the quarks *d* and *s* with a *mixing angle*, since then known as *Cabibbo angle*. In this case, the weak isospin scheme is:

$$Q = \begin{pmatrix} u \\ \cos\theta d + \sin\theta s \end{pmatrix}, (-s\theta d + \cos\theta s) \tag{57}$$

Comparing the decays of the baryons having *strangeness* with the neutron βd^{-}, the value we obtain

$$\text{is:}\quad \sin\theta = 0.225 \tag{58}$$

In keeping with Maiani, the classification in Equation (57) is not yet satisfactory to extend the Cabibbo theory to a unified Yang-Mills theory. If we do this, the neutral boson, Zº, would produce processes with change of *strangeness*, of the type:

$$\mathrm{K}^{\circ} \to \mu^{+}\mu^{-} \tag{59}$$

which are observed to proceed with much lower probabilities than the processes mediated by the W particle, for example βd^{-} shown in Equation (56). If we did so, the neutral boson, Z˚, would produce processes with strangeness change, of the type K˚ → $\mu^{+}\mu^{-}$ (equation 59), which are much less frequent than the processes mediated by boson W [3].

In closing, this was the reason that had prevented Glashow and, subsequently, Weinberg and Salam, from including hadrons in their unifying theories.

Namely, Weinberg knew well that, if the masses of the $W^{\pm}$ and Z˚ particles were added by hand, as in Glashow Electro-Weak Theory SU(2) ⊗ SU(1), the result was a not renormalizable theory. Thus Weinberg wondered if breaking the symmetry with the Brout-Englert-Higgs Mechanism (BEH-*M*), besides giving mass to the particles and eliminating the unwanted Nambu-Goldstone bosons, a renormalizable theory could have resulted. The still remained the problem of neutral currents, that is the interactions due to the Z˚ particle, of which there was no experimental proof.

Weinberg decided to avoid the problem by restricting his theory to leptons. Weinberg no longer trusted either hadrons (the particles subject to SI) nor the strange particles, which had become the main terrain of exploration on the Weak Interaction (WI).

3.1.3. Weinberg-Salam Model

Three years after Glashow formulated his theory, as known it was invented *ad hoc* the BEH-*M*, a cumbersome *mechanism*, curiously asymmetrical. According to Randall, it lavishes mass only on WI-sensitive particles thus, among the bosons vectors of the *fundamental forces*, only the particles carrying the WI acquire mass, while the photon and the gluon remain massless [17]. This is how Brout-Englert-Higgs's message is readily collected. In fact, in 1967 both Weinberg and Salam, independently, get to the same solution.

The scheme is the one outlined by Glashow in 1961, starting from the Yang-Mills theory of 1954.

In the Weinberg and Salam scheme the symmetry group is the same as Glashow, but the *Action* is perfectly symmetrical [62]. We have, that is, that in the *Weinberg-Salam model* some additional *scalar fields* have been introduced, whose condensed *breaks the symmetry* providing, at the same time, to the masses required. In this *model*, as it is known, the starting point is the theory based on the symmetry $SU(2)_L \otimes U(1)_Y$ in its perfectly symmetrical version, *i.e.* without *ad hoc* mass terms for vector fields and for the electron field. The symmetry $SU(2)_L \otimes U(1)_Y$ indicates the symmetry group of weak isospin that unifies the EMI and the WI. The subscript Y distinguishes this copy of $U(1)$ from electromagnetism's, indicated with $U(1)_Q$. To be precise, the interaction $SU(2)_L$ represents the weak isospin, while $U(1)_Y$ is the weak hyper-charge.

Along with Maiani, the Lagrangian follows from the classification below $SU(2)_L \otimes U(1)_Y$ of lepton fields:

$$l = \begin{pmatrix} (\nu_e)L \\ e_L \end{pmatrix}_{Y=-1} ;\ (e_R)_{Y=2} \tag{60}$$

The corresponding Yang-Mills Lagrangian of the Electro-Weak Interaction (L_{eW}) is therefore:

$$L_{eW} = \bar{l} i y^{\mu} D_{\mu} l + \bar{e}_R i y^{\mu} D_{\mu} e_R - \frac{1}{2}\left[\boldsymbol{W}_{\mu\nu} \boldsymbol{W}^{\mu\nu} + B_{\mu\nu} B^{\mu} \right] \tag{61}$$

Covariant derivatives and field tensors are given by:

$$D_{\mu} l = \left[\partial_{\mu} + ig \boldsymbol{W}_{\mu} \cdot \frac{\tau}{2} + ig' \left(-\frac{1}{2} \right) B_{\mu} \right] l$$

$$D_{\mu} e_R = \left[\partial_{\mu} + ig' (-1) B_{\mu} \right] e_R$$

$$\boldsymbol{W}_{\mu\nu} = \partial_{\mu} \boldsymbol{W}_{\mu} - \partial_{\mu} \boldsymbol{W}_{\nu} \tag{62}$$

The Yang-Mills Lagrangian describes fermions and vector fields, all massless [60]. The novelty proposed by Weinberg and Salam was to introduce, in this Lagrangian, a *scalar field*, in turn capable of inducing the *symmetry breaking*, but leaving the gauge symmetry of electromagnetism unchanged, as shown in the diagram:

$$SU(2)_L \otimes U(1)_Y \leftrightarrow U(1)_Q \tag{63}$$

To this purpose, in keeping with Maiani, on the *scalar field* we have little information and different possibilities. The choice of Weinberg and Salam allows the *spontaneous breaking mechanism* to also generate the mass of the electron and, subsequently, of the quarks (in the extension to the other nuclear particles) in order to take us to a completely realistic theory. The choice in question consists in introducing a *doublet* of $SU(2)_L$, with $Y = +1$ [60]:

$$\phi = \begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix}_{Y=+1} \tag{64}$$

$$D_{\mu} \phi = \left[\partial_{\mu} + ig \boldsymbol{W}_{\mu} \cdot \frac{\boldsymbol{\tau}}{2} + ig' \left(+\frac{1}{2} \right) B_{\mu} \right] \phi \tag{65}$$

where ϕ represents the *Higgs doublet*, which is equivalent to 4 real fields:

$$\phi = \begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix} = \begin{bmatrix} \phi^{+} \equiv (\phi_1 + i\phi_2)/\sqrt{2} \\ \phi^{0} \equiv (\phi_3 + i\phi_4)/\sqrt{2} \end{bmatrix} \tag{66}$$

At this point Weinberg and Salam add to the Yang-Mills electro-weak Lagrangian (L_{eW}) the BEH-*M* in order to give a mass to the gauge bosons (well, not really to all of them) and to the fermions:

$$L_{eW} = \left| \left(i\partial_{\mu} - g\frac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{W}_{\mu} - g'\frac{Y}{2}B_{\mu} \right)\phi \right|^2 - V(\phi) \quad (67)$$

The added field, with 4 components, must be a *multiplet* of $SU(2)_L \otimes U(1)_Y$ in order to preserve the *gauge invariance* of the L_{eW}(68): *minimal* choice. The potential chosen for L_{eW}(67) is the usual ($\mu^2 < 0$, $\lambda > 0$):

$$V(\phi) = \mu^2\phi^+\phi + \lambda\left(\phi^+\phi\right)^2 \quad (68)$$

We choose the *vacuum point* (in the 3-dimensional space: $|\phi|^2 = v^2$):

$$\phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \rightarrow \phi_0' e^{i\alpha(x)IQ_0}\phi_0 \equiv \phi_0 \quad (69)$$

If $Q_0 = 0$. This void breaks symmetry $SU(2)_L \otimes U(1)_Y$ but it preserves the invariance for $U(1)_{em}$ (if Q_0, a charge of the *Higgs boson*, is 0). This guarantees the presence of a neutral boson, without mass (the photon), and of other 3 gauge bosons with mass: the particles W^+, W^- and Z˚ [63].

Weinberg adds "The spontaneous breakdown of $SU(2) \otimes U(1)$ to the $U(1)$ of ordinary electromagnetic gauge invariance would give masses to three of the four vector gauge bosons: the charged bosons $W^{\pm}$, and a neutral boson that I called the Z˚. The fourth boson would automatically remain massless, and could be identified as the photon. Knowing the strength of the ordinary charged current weak interactions like beta decay which are mediated by $W^{\pm}$, the mass of the $W^{\pm}$ was determined as about 40 GeV/sinθ, where θ is the γ-Z˚ mixing angle. To go further, one had to make some hypothesis about the mechanism for the breakdown of $SU(2) \otimes U(1)$. The only kind of field in a renormalizable $SU(2) \otimes U(1)$ theory whose vacuum expectation values could give the electron a mass is a spin zero $SU(2)$ doublet (φ^+, φ^0), so for simplicity I assumed that these were the only scalar fields in the theory. The mass of the Z˚ was then determined as about 80 GeV/sin2θ" [64].

According to Weinberg, one of the essential elements of the Standard Model (*SM*) is the *Symmetry* between 2 of the 3 Forces included in the *SM*: the Electro-Magnetic Interaction (EMI) and the Weak Nuclear Interaction (WI). This *symmetry* unites the two Forces in a single *electro-weak* (EW) structure. One of the consequences of the EW Symmetry is that, if no other ingredients are added to the theory, all elementary particles, including electrons and quarks, are massless, and this is patently false. Therefore it is necessary to add something new to the theory: some new type of field or matter. Somehow the EW Symmetry, an exact property of the fundamental equations underlying particle physics, had to be *broken*: in other words it is not directly applicable to the particles and forces we observe in reality [65]. Weinberg specifies: "as early as 1960-61 with Nambu and Goldstone it was known that a symmetry breaking of this kind or *Spontaneous Symmetry Breaking* (*SSB*) is possible in several theories: this implied the existence of new massless particles (the Nambu-Goldstone bosons) which instead, it was known, do not exist. It was the independent studies of Brout and Englert, Higgs, Guralnik, Hagen and Kibble, all of 1964, that showed that in some theories these Nambu-Goldstone bosons, without mass, disappear, giving instead mass to the force mediating particles (gauge bosons): this is what happens in the EW theory proposed by Salam and myself in 1967-68. What kind of matter or field breaks the EW Symmetry?" [65].

In other words, as Weinberg reminds us, there were two possibilities: 1) The existence of fields, never observed, which pervade the vacuum and which (as the Earth's magnetic field distinguishes the north from the other directions) distinguish EMI from WI, giving mass to the mediating particles of WI and to other particles, but leaving photons (the mediators of EMI) massless. These fields are called *scalars* because, unlike the EM field, they do not identify any direction in ordinary space. *Scalar fields* of this type were introduced in the illustrative examples of *symmetry breaking* (SB) used by Goldstone and then in 1964 by the various A.A. just mentioned. Salam and I used this *SSB* to elaborate the EW theory, assuming that the *breaking* was due to *scalar fields* of the type described, pervasive of all space (a *SSB* of this kind had already been hypothesized by Glashow, Salam and Ward, but not as exact property of the equations of the theory, for which they were not induced to introduce scalar fields). One of the consequences of the theories in which symmetries are broken by scalar fields (including the models considered by Goldstone or in the cited 1964 articles, as well as the EW Salam theory) is that, although some of these fields serve only for giving mass to the mediating force particles, other fields appeared in nature as physical particles observable in accelerators and particle colliders. Three of these *scalar fields* were used to give mass to the W^+, W^- and Z° particles, ie the *heavy photons* that in our theory carry WI. A 4th *scalar field* remained outside, which showed as a physical particle, that is, a *concentration* of energy and momentum of the field itself: the *Higgs particle* [65].

Weinberg concludes: "But there was always a 2nd possibility: 2) There could be no new omnipervasive scalar field, nor any Higgs particle. The symmetry could be broken by *strong forces*, called *Technicolor Forces*, acting on new type particles, **never seen so far because too heavy** [65]."

3.2. Yang-Mills *b Quantum*

It is very important to emphasize that Weinberg considers the possibility that the Yang-Mills ***b*** *quantum* could be a particle with its own mass.

In other words Weinberg is in perfect agreement with Yang and Mills, who were convinced, just for "physical reasons" [57], that the gauge particle of the ***b*** *field* (as to say the nuclear and intra-nucleonic *strong field*) and represented by the ***b*** *quantum*, could not be massless.

As concerns the possible value of the ***b*** *quantum* mass, we are not in discordance with Yang and Mills, who consider it at least greater than the mass of pions, which as Yang himself says corresponds to "134.97 MeV for the π°, and to 139.58 MeV for $\pi^{\pm}$" [66]. Successively, with the introduction of the concepts of gluon (G) and quarks(Q_s), the problem of the Q_s symmetry in baryons arose for spin and flavor.

This problem finds a natural solution if we assume that a Q of a given flavor has an additional quantum number (the so-called *color*) which takes three values. It is possible to satisfy the Pauli Exclusion Principle, if we assume that the baryons are, in the state completely anti-symmetric in the new quantum numbers, an invariant configuration for color transformations (*color singlet*) [3].

In 1965, indeed, Han and Nambu gave an elegant formulation of this hypothesis, introducing a $SU(3)$ symmetry that operates on color indexes and hypothesizing that the *color symmetry* were gauge symmetry, and gluons(G_s) were Yang-Mills fields associated with the color itself. At this regard, in line with Han and Nambu, it is shown that in a $U(3)$ scheme of triplets with integral charges, one is naturally led to three triplets located symmetrically under the constraint that the Nishijima-Gell-Mann relation remains intact [67].

Thus, even if Yang and Mills have not quantified the mass of the ***b** quantum*, they are still convinced that it has a mass at least higher than pions'. Otherwise, a ***b** quantum* with a mass similar to or lower than pions' would have already been highlighted easily [57].

Well, this is another crucial point which, in our view, further contributes to denying those who want to continue to consider the gluon (G) massless, *i.e.* the ***b** quantum* of Yang and Mills, and so all the other particles having charge, which as Yang points out "could not be massless" [68]!

At this point, one may wonder: what, then, is the mass of Yang-Mills *b quantum*, now identifiable with the G? As it is known, the Yang-Mills ***b** quantum*, following their reasoning, must have a mass-energy density certainly "higher than pions" [57]. In effect, being the vector of a Nuclear Interaction, thus acting exclusively within the very restricted nuclear space, the mass of the ***b** quantum* cannot be too lower than the values found for the Weak Nuclear Interaction (WI) bosons, corresponding to 80.4 and 91 GeV/c^2. But reflecting further, if it had similar values, the G, or Yang-Mills ***b** quantum*, would have already been detected.

The next step, of experimentally detected massive particles, corresponds to the values given, at CERN, to the Higgs Boson (HB): 125 - 126.5 GeV/c^2 [69]. A particle of that weight, at the same time, could very well be also the G, if it were not massless. In addition, from the literature we learn that many products and channels of decay are in common between $W^{\pm}$ and Z° bosons, G and HB.

However, following the experiments carried out at the Petra of Hamburg in 1979 [70] it was deduced that the mass of G is equal to zero, in agreement with the requirements of the *gauge theories*. Nevertheless, we believe that the zero mass attributed to G is patently incongruous and inconsistent: it is in full discordance with the simplest and most basic concepts of Arithmetic.

The massless G, moreover, would deny one of the basic principles of Special

Relativity, the MEEP: $E = mc^2$, as shown by Equation (18). In this respect, a massless G implies an energyless G! In effect, considering the gluon mass as zero, we would have: $E = 0 \cdot c^2$, and thus $E = 0$, as to say that the boson of a nuclear force, considered the most energetic boson, is *massless* and *energyless*, where the bosons of the another nuclear force, the WI, are highly massive! It is really against the reality of the facts: the MEEP categorically forbids that the G can be massless: in that case, *ex abrupto*, its energy (which is enormous) would instantly vanish. In other words, anyone who claims that G is massless affirms at the same time that the most famous equation in the world is not true, but is misleading, wrong.

On the contrary, Yang and Mills, and so many A.A., knew that the bosons of a Nuclear Force cannot, for any reason, be massless (in this case their range of action would extend to infinity).

In addition, it is in open and unacceptable contrast with the Yukawa Principle [71], according to which the mass (m) of the boson carrying a *fundamental force* must absolutely be inversely proportional to the range (R) of the force it conveys:

$$R = \frac{h}{2\pi mc} \tag{70}$$

where h is the Planck's constant and c is the speed of light in the vacuum.

We could also imagine a slightly larger mass, conveyed by the ***b*** *quantum*. In such an eventuality, we must wait for experiments to be performed at even higher energies.

The next step, among the particle surveys carried out, corresponds to that of the *top* Quark (*t*Q): ≈177 GeV/c^2. Even in this case, considering a massive ***b*** *quantum*, it cannot be completely excluded that those decay products are not referable also to G.

Moreover, it should be borne in mind that, probably, a fixed mass-energy value could not be assumed for the ***b*** *quantum*, since this value may differ depending on the energy retained by the particle at the time of detection, and non yet returned to the surrounding field. Therefore, it is likely that a possible massive Yang-Mills ***b*** *quantum* can carry an *energy-mass density* between 125 and 177 GeV/c^2 (maybe even a little further).

What would be its radius of action, *i.e.* its *range*?

3.3. Radius of Action of Yang-Mills *b Quantum*

Let's examine the *range*, that is the space that the Yang-Mills ***b*** *quantum* can pass through.

Of course, in line with the Yukawa Principle, the range varies in a ratio inversely proportional to the mass of the particle.

3.3.1. Radius of Action ≈ Higgs Boson Range

At this point we start from the alleged lower value, which should correspond to the *range* of decay products detected at the Large Hadron Collider (LHC) [69].

In this respect, we made some calculations to evaluate the possible radius of action of the HB. In truth, it was very simple, as we knew the mass. Thus, we applied the Yukawa Principle, as shown in Equation (70).

But yet, to this purpose, one wonders: where does the Higgs boson (HB) take all this mass-energy? From the field in which it is immersed. In line with Quantum Mechanics (*QM*), the higher the value of the mass of the particle, *i.e.* the more the energy (Δ_E) taken from the field, the sooner (Δ_t) the energy must be returned to the field itself. As known, this is an inviolable rule of *QM*, dictated by the Heisenberg Uncertainty Principle (HUP):

$$\Delta_E \cdot \Delta_t \geq h \tag{71}$$

where h is Planck's constant, equal to 6.626×10^{-27} [erg · sec]. Applying the HUP to HB, we have that the Δ_E of Equation (71) corresponds to the energy value of HB, *i.e.* 125.5 GeV/c^2.

What we do not know, in this case, is the value of Δ_t, *i.e.* of duration (t) of the HB' life, before it returns to the field all the energy (E) taken, so to speak, *borrowed*.

The duration of this energy loan, in favor of HB, is provided by Equation (71), from which we have:

$$t = \frac{h}{E} \tag{72}$$

ence, Equation (72) tells us that time(t) and energy are inversely proportional. That's why the higher the energy value borrowed, as saying subtracted from the field, the sooner this energy must be returned. To this point we take into account the Einstein's MEEP ($E = mc^2$) represented in Equation (18).

Hence, by replacing the value of E in Equation (18) with that of Equation (72), we obtain:

$$t = \frac{h}{mc^2} \tag{73}$$

Equation (73), as Fermi reminds us "it is the time in which the boson issued may remain in free space.

If then it is assumed that its speed is the maximum speed at which a particle can move, that is the speed of light (c), it is seen that the maximum distance (d) it can reach, before being recalled to weld the debt, is given, as order of magnitude, by the product of time (t) for the maximum rate at which the particle can move" [72], namely:

$$d = tc \tag{74}$$

So we put in Equation (74) the value of t expressed by Equation (73):

$$d = \frac{h}{mc^2} \cdot c \tag{75}$$

$$\textit{i.e.:}\ \ d = \frac{h}{mc} \tag{76}$$

It is interesting to detect that the *distance* (d) illustrated by the latter equation corresponds to the radius of action (R)

obtainable from the Yukawa potential, as illustrated by Equation (70).

Thus, one expressed by Equation (76) is the maximum distance the HB can take, ie the upper limit of its range. It comes more useful to express in grams [g] the mass HB, using the *cgs* system.

Since 1 GeV/c² = 1.782 × 10⁻²⁴ [g], it follows that the mass of HB (m_{HB}) will be:

$$m_{HB} = 125.5 \times \left(1.782 \times 10^{-24}\,[\mathrm{g}]\right) \tag{77}$$

$$\text{That is: } m_{HB} = 2.23641 \times 10^{-22}\,[\mathrm{g}] \tag{78}$$

So we replace this value to m of Equation (76):

$$d = \frac{6.626 \times 10^{-27}\,[\mathrm{erg \cdot s}]}{2.23641 \times 10^{-22}\,[\mathrm{g}] \times 2.99792 \times 10^{10}\,[\mathrm{cm/s}]} \tag{79}$$

Since 1 erg = g·cm/s² · cm, we can write:

$$d = \frac{6.626 \times 10^{-27}\,[\mathrm{g \cdot cm^2/s}]}{6.7045782 \times 10^{-12}\,[\mathrm{g \cdot cm/s}]} \tag{80}$$

$$d_{HB} = 9.8828 \times 10^{-16}\,[\mathrm{cm}] \tag{81}$$

And 'interesting to emphasize that the value expressed by Equation (81) represents the maximum limit of the HB *range*, *i.e.* the maximum distance (d) passable by HB, before it returns the energy to the field in which it is immersed. Our calculations reveal a range of HB really very small, slightly smaller than 10^{-15} [cm], but this value is justified by the considerable mass that the HB acquires [73]. Of course, this is certainly a very small value, which shows a very marked space limitation of this boson, but these are the rules imposed by *QM* through one of its most profound concepts: the HUP.

By closing, the range of HB will never exceed the distance expressed by Equation (74), otherwise the HUP would be violated [16].

This very narrow range of HB (particle with a considerable mass) is perfectly congruent and in full accordance with the Yukawa Principle, along with which the range of a *fundamental force* must be inversely proportional to the mass of its bosons.

Furthermore, we have confirmation of these concepts from the bosons of the WI, whose mass is notoriously lower than that of HB. Even in this case, in fact, knowing the mass, it is very easy to calculate the *range* of such bosons. Our calculations show that the Z° boson (mass = 91.1876 ± 0.0021 GeV/c²) would have a radius of action equal to 1.36×10^{-15} [cm], where the $W^\pm$ particles (mass = 80.385 ± 0.015 GeV/c²) extend their action up to a limit of 1.543×10^{-15} [cm] [74].

As it is easy to see, indeed, even with nuanced mass differences, the range changes. Thus the most massive particle, the Z°, has a barely narrower range [74].

Nevertheless, the Strong Interaction (SI) boson, *i.e.* the G, also operating exclusively at the intra-nuclear and intra-nucleonic levels, *i.e.* in the same spaces in which the WI operates, is considered massless! Likewise also the Yang-Mills ***b*** *quantum*, now identifiable with G, is considered massless, although Yang and Mills themselves, like so many authoritative Authors, were firmly convinced of the massiveness of this particle [57].

Then why did they accept this *compromise*?

Because the mathematical formalism of *gauge invariance* is used, *i.e.* a formalism in which the mass of particles tilts the equations: the mass *breaks the symmetry*. Subsequently, in order to deal with the problems, the *gauge theories* require that all the particles are massless.

Then, since 1964, with the invention of the BEH-*Mechanism* and the alleged Higgs Field, various particles can acquire mass by reacting with this field, but not all: only those sensitive to WI. Therefore the G, that is the ***b*** *quantum*, being sensitive to the SI, but not to the WI, remains massless!

3.3.2. Radius of Action between the Range of Higgs Boson and Top Quark

As reported, the next step, among the mass particle surveys greater than the HB, is the *top Quark* (*t*Q) ≈ 177.16 GeV/c². A very high value, which really left the researchers baffled, since they did not expect such high values attributable to a Q.

Even in this case, knowing the mass of the particle, it is very easy to obtain its action radius (d_{tQ}):

$$d_{tQ} = 7\times10^{-16}\ [\text{cm}] \quad (82)$$

This is the maximum distance that a particle with such a mass can travel, before having to return the energy loan to the field in which it is immersed. If a possible massive ***b*** *quantum* (or G) were to have values corresponding to those detected for the *t*Q, it would operate in spaces really very small, barely above the size of the quarks, considered around ≈ 10^{-16} [cm].

Moreover, comparing the distances (*d*) that can be traveled by particles of different mass, which are the particles with mass corresponding to that of the HB or of the *t*Q, we can note that *Yukava Principle* is perfectly respected. The heavier particle, in fact, shows a minor radius of action.

Briefly, we believe that the most likely and appropriate solution is to consider the range of the Yang-Mills ***b*** *quantum* (d_{bq}) corresponding to the intermediate value included among those calculated for the HB (Equation (81)) and *t*Q (Equation (83)),

$$\textit{i.e.}:\ d_{bq} = 8.4414(\pm1.4414)\times10^{-16}\ [\text{cm}] \quad (83)$$

3.3.3. Radius of Action > Top Quark Range

Yet, it is not reasonable to hypothesize that the ***b*** *quantum* can have a mass lower than that detected for the HB, otherwise it would have been detected at the LHC without much difficulty.

At most we can consider the possibility that the G, as to say the ***b*** *quantum*, may have a mass slightly higher than that

detected for the *t*Q, but not too much further, otherwise its *range* would be narrowed beyond acceptable limits. Indeed, it is difficult to imagine that the *range* of ***b** quantum* can be <(6.5 – 6) × 10^{-16} [cm], that is, always close to the size of the Q.

Thus, if the mass of G corresponds to ≈190.786 GeV/c², its radius of action (d_{bq}) is:

$$d_{bq} = 6.5 \times 10^{-16} \, [\text{cm}] \tag{84}$$

It is less likely that this *range* is even shorter, however, if the ***b** quantum* had a mass of ≈206.7 GeV/c², its range of action would drop to:

$$d_{bq} = 6 \times 10^{-16} \, [\text{cm}] \tag{85}$$

Consider that this range corresponds to the space occupied by ≈6 Q_s arranged in a row and contiguous, which is not possible, since between the Q_s, as among other fermions, there is always a space, *something* interposed between them, which separates them. We find it incongruous to go down to even lower *ranges*, since too narrow spaces would not be sufficient, in our opinion, so that the SI has the space and time to carry out its various tasks. This is because, according to *QM and* Yukawa Principle, as the boson mass increases, its range and lifetime will decrease in parallel [46] [71].

3.4. Lifetime of a Massive Yang-Mills *b Quantum*

Let's check it out if our theory and our calculations are still in agreement with Yang and Mills for the *lifetime* of the ***b** quantum* (as to say the G) in their "less than 10^{-20} sec" [57]. In this respect, we believe that the most likely range for a massive ***b** quantum* is between 125 and 177 GeV/c².

To this purpose, even considering for the ***b** quantum* a higher mass, roughly equal to ≈191 GeV/c², in *cgs* metric system it corresponds to ≈3.4 × 10^{-22} [g].

Therefore, we insert this value into Equation (73) and get the possible value of the lifetime (*t*) of the G:

$$t = \frac{h}{mc^2} = \frac{6.626 \times 10^{-27} \, [\text{erg} \cdot \text{s}]}{3.4 \times 10^{-22} \, [\text{g}] \times \left(2.99792 \times 10^{10} \, [\text{cm/s}]\right)^2} \tag{86}$$

That is:

$$t = \frac{6.626 \times 10^{-27} \left[\left(\text{g} \cdot \text{cm}^2/\text{s}^2\right) \cdot \text{s}\right]}{3.4 \times 10^{-22} \, [\text{g}] \times 8.9874 \times 10^{20} \left[\text{cm}^2/\text{s}^2\right]} \tag{87}$$

$$t = \frac{6.626 \times 10^{-27} \left[\text{g} \cdot \text{cm}^2/\text{s}\right]}{30.557 \times 10^{-2} \left[\text{g} \cdot \text{cm}^2/\text{s}^2\right]} \tag{88}$$

From which we get:

$$t = 2.168 \times 10^{-26} \text{ sec} \tag{89}$$

This value inherent the lifetime of a G (*i.e.* the Yang-Mills ***b** quantum*) corresponds exactly to the maximum term of the loan granted under the HUP, as it is shown by Equation (72).

Well, from these calculations emerges a data of particular scientific importance: the life of the G corresponds to an infinitely short time, about 3 *orders of magnitude* lower than the common decay times governed by the Strong Interaction(SI), and 3 *orders of magnitude* lower than the time it takes light to cross an atomic nucleus. Thus, based on its extremely short existence, even the most powerful particle accelerators it is really extremely difficult to detect the trace of a G! However, although in agreement with the forecasts of Yang e Mills, it is a really too short time. About the SI's bosons, indeed, Fermi states: "Its speed is the maximum speed at which a particle can move, that is the speed of light" [72].

A less massive G would have a slightly longer lifetime.

Therefore, it seems more reasonable to expect a massive ***b*** *quantum* between 125 and 177 GeV/c^2.

To this purpose, if the possible mass of the G coincides roughly with the HB's, *i.e.* ≈125.5 GeV/c^2; the corresponding in grams is 2.2364 × 10^{-22} [g]. Hence, using the method just used, we have:

$$t = \frac{6.626\times10^{-27}\left[\text{g}\cdot\text{cm}^2\cdot\text{s}\right]}{2.2364\times10^{-22}\left[\text{g}\right]\times8.9874\times10^{20}\left[\text{cm}^2/\text{s}^2\right]} = 3.2966\times10^{-26}\,\text{sec} \quad (90)$$

Hence, this last value expresses the lifetime of a particle as heavy as the HB. We then calculate the intermediate value between the values expressed in Equations (89) and (90), obtaining:

$$t = 2.7323\left(\pm0.5643\right)\times10^{-26}\ \text{sec} \quad (91)$$

In short, these should be the life times of the ***b*** *quantum* provided with a mass likely between the weight of the HB and that of a particle barely heavier than the *t*Q. The HUP does not allow the lifetime of the ***b*** *quantum* to go beyond these limits, since the G must immediately return the energy loan. Hence, even the distances (d_{bq}) this particle travel, as represented in the Equations (81)-(84), do not exceed the very limited space of 10^{-16} [cm]!

4. Discussion

Now, we need to make a consideration: very probably it is precisely the very short lifetime of the G (probably among the shortest found for a particle) to determine the **Q$_s$ Confinement**, the **G$_s$ Confinement** and the **Colors Confinement**!

And why? Because the very short lifetime of ***b*** *quantum* is associated in parallel the reduction beyond measure of the space granted and practicable by the Q$_s$ and G$_s$, since the HUP imposes that the energy debt must be repaid in the shortest possible time.

Really, we think that it is *physically* very complicated, if not impossible, to be able to detect a G, *i.e.* the Yang-Mills ***b*** *quantum*. And why? Because of the G very short lifetime which, from our calculations, is equal to 2.7323 (±0.5643) ×

10^{-26} seconds, as shown in Equation (91).

At this regard, according to Maiani, the Strong Interaction(SI), which as we know operate for distances ≤ 10^{-13} [cm], allow the formation of *Resonances (R)*: unstable states that disintegrate into final particles, due to the Interaction itself. The typical times of decay (τ), by the SI, correspond to [75]:

$$\tau \simeq \frac{R}{c} \simeq \frac{\sim 10^{-13}\,[\mathrm{cm}]}{3\times 10^{10}\left[\frac{\mathrm{cm}}{\mathrm{s}}\right]} \simeq 10^{-23}\ \mathrm{sec} \qquad (92)$$

Well, the typical decay times managed by SI are of the order it takes the light to cross the *resonance*, which has a linear dimension of order *R* [75], corresponding to the radius of the atomic nucleus (being a *nuclear force*).

Here, a reflection is obligatory: comparing the Equations (91) and (92), a difference of 3 orders of magnitude immediately stands out. Namely, the Yang-Mills ***b*** *quantum* lifetime is as many as 3 *orders of magnitude* shorter, compared to times that are already infinitely short themselves. In other words, the creation and disappearance of a G is resolved in less than one thousandth of the time taken by light to pass through a proton, a light nucleus! Or: the whole life-time (so to speak) of the Yang-Mills ***b*** *quantum* resolves in less than a thousandth of the time necessary for an operation managed by the SI. They are really times beyond reach, like saying: *inaccessible*.

At this point, one wonders: are we able to examine and study physical phenomena that occur in such short times? We really don't think so! That is, they are *virtual* phenomena. This could also be said for ***b*** *quantum*, *i.e.* G, a massive G, which can never be massless, is to be considered a *virtual particle*; obviously, this does not mean that G does not exist, but that it exists for such a short time that we cannot access it in time.

Thus, for this reason, we believe that, likely, we will never be able to study ***b*** *quantum* directly, nor will we ever have enough time to detect it.

We may say that the *b quantum*, the G is *Temporally Confined* by its very short lifetime!

Hence, a new parameter may be added: the *Temporal Confinement of Gluon* (and their *Colours* and *anti-Colours*).

This Gluon Temporal Confinement should be added to the *Spatial Confinement* of Q_s (and their *Colours* and *Anti-Colours*) due, instead, to the extremely narrow radius of action of a massive ***b*** *quantum*, likely equal to 8.4414 (±1.4414) × 10^{-16} [cm], as shown in Equation (83).

Instead, as regards the Yang-Mills *Mass Gap* Problem, as widely described in the paragraphs **2.4** and **3.1**, the solution we propose consists, first of all, in not considering anymore the photon(P) as a massless particle, but having its own *equivalent* rest *mass* equal to 7.372 × 10^{-48+n} grams, as shown in Equation (29), and characterized by a *momentum* of 1.325 × 10^{-22} [g·cm/s], in the case of optic P_s, as shown in Equation (34). This automatically results in the immediate *Removal of Infinities and Divergences* from the equations of the *Perturbative Cal-*

culations, of the *Gauge Theories* and of the QED and QFT.

This also involves the removal of the zeroes from the equations concerning the *electron eigenenergy*, ie the interaction of the electron with P_s. This is completed, as described in the paragraph **2.5**, if the obvious contradiction of the *point electron* is eliminated (another inappropriate and incongruous cause of the aforementioned *infinities* and *divergences*)!

5. Conclusions

These, we reiterate, are the 2 fundamental stages to try to solve the Yang-Mills *Mass Gap* Problem.

In fact, once it has been made the *Removal* of *Infinities* and *Divergences* in equations of the Perturbative Calculus, the *Symmetry Breaking* caused by massive particles fails!

In short, replacing in these equations also the null value of a P massless with its real mass-energy value, the limits imposed by the *spontaneous symmetry breaking* (*SSB*) vanish, so that there is no longer any need to deny the mass to the Nuclear Forces bosons, as the Yang-Mills ***b*** *quantum*, which corresponds to the boson of the Strong Nuclear Interaction (SI): the gluon (G).

In this way, moreover, that incomprehensible and unjustifiable asymmetry between the two nuclear forces is resolved.

Namely, as *gauge theories* dictate, they have bosons with antipodal masses: on one side the Weak Interaction (WI), carrying very heavy gauge bosons, between 80 and 91 GeV, and on the other hand the SI, conveyed by bosons considered massless, although it also operates in the very restricted space of a nucleus or a nucleon.

Well, it is absolutely unjustified: a clear contradiction, physically and mathematically unacceptable, that a massless particle shows a range of action $\leq 10^{-13}$ [cm]. No! It is not possible. In this case, as we all know, a massless particle should have an infinite range of action.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix: Mathematical Symbols

$Ł_D$ is the Dirac's Lagrangian

L_{eW} is the Lagrangian of the Electro-Weak Interaction

L_{QED} indicates the Lagrangian of the QED (Quantum Electro-Dynamics)

h is the *Planck's constant*

$\boldsymbol{p}$ is the *momentum* of any subatomic particle, or *quantum object* (QO)

i is the imaginary unit

θ indicates the *phase angle* of a sinusoidal wave

$\Psi(x)$, or simply Ψ, is the *wave function* of any particle

F_{ab} indicates the Maxwell's *tensor*

$U(1)$ is a *Lie group*, namely the symmetry group $U(1)$, or group of *Unitary* transformations (U) of a *complex variable*(1)

the symmetry $SU(2)_L \otimes U(1)_Y$ indicates the symmetry group of the *weak isospin*

βd^- represents the *negative neutron beta-decay*

γ^μ is the Dirac matrix, which satisfies Clifford's algebra: $\{\gamma^\mu, \gamma^\nu\} = 2g\gamma^{\mu\nu}I$

I is the isotopic spin or *isospin*