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Initial and Stopping Condition in Possibility Principal Factor Rotation

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Abstract

Uemura [1] discovered the mapping formula for Type 1 Vague events and presented an alternative problem as an example of its application. Since it is well known that the alternative problem leads to sequential Bayesian inference, the flow of subsequent research was to make the mapping formula multidimensional, to introduce the concept of time, and to derive a Markov (decision) process. Furthermore, we formulated stochastic differential equations to derive them [2]. This paper refers to type 2 vague events based on a second-order mapping equation. This quadratic mapping formula gives a certain rotation named as possibility principal factor rotation by transforming a non-mapping function by a relation between two mapping functions. In addition, the derivation of the Type 2 Complex Markov process and the initial and stopping conditions in this rotation are mentioned.

Keywords

Extension Principle, Vague Event, Type 2 Possibility Different Equation, Possibility Principal Factor Analysis, Initial and Stopping Condition

1. Introduction

Uemura [1] discovered the mapping formula. It is often called Vague in order to distinguish it from Fuzzy of Zadeh [3]. Therefore, our study named Vague Sets and Theory [2]. Zadeh's Fuzzy deals with vertical ambiguity, while our Vague deals with horizontal ambiguity. Also, The Zadeh's modeling is conceptually very close to the interval modeling of subjective Bayesian theory, and the rotation based on our quadratic mapping formula is very related to factor analysis or independent component analysis [4]. First, Uemura [1] defined the mapping of complex events. Next, Hori *et al.* showed that this definition was the formula. In this flow of formalization research, the proposed decision-making method can be applied to the alternative problem in order to apply the formula of the mapping function of complex events to the utility function theory and expand it to the decision-making method based on the complex utility function, and to make a decision by asking myself in the case of no-data problem. Also, since it is well known that alternative questions result in sequential Bayesian inference, it expanded to multi-dimensionalization for states of nature by introduction of the concept of time, next to derivation of complex Markov (decision) processes, at last to complex stochastic differential equations. Therefore, the solution of alternative question based on the sequential Bayesian interference, especially Uemura [1] deals with the no-data problems, so it can be said that it shows the flow of solution of sequential Bayesian inference in the no-data problems. Finally, this article refers to type 2 vague events based on the quadratic mapping formula. This quadratic mapping formula gives a certain rotation to a non-mapping function by transforming it with a relationship between the 2 mapping functions. Wherein it refers to the derivations of the type 2 complex Markov process and the initial and stop conditions for its rotation.

2. Mapping Formula in the Vague Events

Uemura [1] defined the formula for mapping a utility function $f(x)$ with a membership function $g_1(x)$ by Equation (1), making full use of Zadeh's extension principle for mapping. This principle is Fourier Transform under the ergodic condition in the state of nature or linear condition in membership functions.

However, wherein the alternative problem is shown as an application example. Later it is shown to be a theorem [2].

$$SUP_{y=f(x)} g_1(x) = g_1\left(f^{-1}(y)\right) \tag{1}$$

(Proof)

When stochastic differential equation representing the flow in Sequential Bayesian inference is formulated as with Equation (2), the complex Markov process is found by the Equation (3). This pole of complex Markov process is the mapping formula in the complex event of Equation (1).

$$\frac{dF}{dt} = b\left(t, f_t^{-1}(y_t)\right) + \sigma\left(t, f_t^{-1}(y_t)\right) \cdot W_t \tag{2}$$

where let b be the average term of state equation in the normal events, σ be the variance term, and W be the error term.

$$F_t = L^{-1}\left(t, g_1\left(f^{-1}(y_t)\right)\right) \tag{3}$$

where L is a transition rate matrix of Markov process in a normal event.

3. Type 2 Complex Markov Process

The quadratic mapping formula is formulated by Hori [4] like Equation (2).

$$SUP_{\substack{y=f(x) \\ Z=g_1\left(f^{-1}(y)\right)}} g_2(Z) = g_2(g_1^{-1}\left(f^{-1}(y)\right)) \tag{4}$$

where the Equation (2) is the quadratic mapping formula that maps Equation (1) again with $g_2(x)$ A notable of the quadratic mapping formula is that it inverts 180-degree, when mapping functions are equivalent like Equation (3). This shows that it is a kind of principal factor analysis. Wherein 180-degree rotation requires 2 rotations every 90 degree. However, note that the quadratic mapping formula fillips 180 degrees in the one rotation.

$$\text{if } g_1(\cdot) = g_2(\cdot) \text{ then } x = f^{-1}(y) \tag{5}$$

Then, supposing that the transition rate matrix is L, the Markov process D_t is formulated as the following equation [5].

$$D_t = L(t, x_t) \tag{6}$$

Lastly, the type 1 complex Markov process which introduces concept of complex events is Equation (5), and the type 2 complex Markov process is derived by Equation (6).

$$F_t = L^{-1}\left(t, g_1(x_t)\right) \tag{7}$$

$$\begin{aligned} F_{F_t} &= \underset{y_t = L^{-1}\left(t, g_1\left(f(x_t)\right)\right)}{SUP} L^{-1}\left(t, g_2(x_t)\right) \\ &= L^{-1}\left(t, L^{-1}\left(t, g_2\left(g_1^{-1}\left(f^{-1}(y_t)\right)\right)\right)\right) \end{aligned} \tag{8}$$

where the Markov process of Equation (8) is derived from the following Simultaneous stochastic differential equations. Note that Equation (9) represents the change in the x-axis direction, and in the y-axis direction.

$$\begin{cases} \dfrac{dZ}{dt} = m_1\left(t, g_{1t}\left(f_t^{-1}(Z_{t1})\right)\right) + \sigma_1\left(t, g_{1t}\left(f_t^{-1}(Z_t)\right)\right) \cdot W_{1t} & (9) \\ \dfrac{dx}{dt} = m_2\left(t, g_{2t}\left(f_t(x_t)\right)\right) + \sigma_2\left(t, g_{2t}\left(f_t(x_t)\right)\right) \cdot W_{2t} & (10) \end{cases}$$

where it is $Z_t = f_t(x_t)$, so Equation (10) is equivalent to Equation (11).

$$\frac{dZ}{dt} = m_2\left(t, g_{2t}(Z_t)\right) + \sigma_2\left(t, g_{2t}(Z_t)\right) \cdot W_{2t} \tag{11}$$

4. Possibility Principal Factor Rotation

Type 2 Fuzzy Events simultaneously encompass a two-dimensional necessity variable error model that considers longitudinal and transverse possibility errors. The 180-degree orthogonal rotation is the case of Equation (5), where possibility theory [6] is applied to these possibility variable error models. Note that since both longitudinal and transverse fuzzy variables are considered, possibility theory can be applied. In this paper, particular attention to the measure of the size relationship of the fuzzy set is paid. Here, the possibility measure (POS) and the necessity measure (NES) are defined as followed [6]. In addition, M and N are assumed to be Orthogonal Fuzzy Events with orthogonal degrees of attribution.

$$POS(M \geq N) \triangleq \underset{U \geq V}{SUP} \min\left(\mu_M(U), \mu_N(V)\right) \tag{12}$$

$$POS(M > N) \triangleq \underset{U}{SUP}\ \underset{V \ge U}{inf} \min\left(\mu_M(U), \mu_N(V)\right) \tag{13}$$

$$NES(M \ge N) \triangleq \underset{U}{inf}\ \underset{V \le U}{SUP} \max\left(1-\mu_M(U), \mu_N(V)\right) \tag{14}$$

$$NES(M > N) \triangleq 1-\underset{U \ge V}{SUP} \min\left(\mu_M(U), \mu_N(V)\right) \tag{15}$$

The possibility principal factor rotation matrix for type 2 fuzzy is as follows:

$$\begin{bmatrix} x_{t+1} \\ Z_{t+1} \end{bmatrix} = \begin{bmatrix} POS(M \ge N) & NES(M > N) \\ NES(M > N) & POS(M \ge N) \end{bmatrix} \begin{bmatrix} x_t \\ Z_t \end{bmatrix} \tag{16}$$

In particular, note that in (16), when the possibility measure is 1, it is the identity matrix, and when the necessity measure is 1, it is the inversion matrix. Therefore, the possibility main factor rotation matrix in Equation (16) indicates that the sum of the weights of Equation (9) and (10) in the simultaneous fuzzy stochastic differential equation is 1 [7].

5. Initial and Stopping Condition in Possibility Principal Factor Rotation

The initial condition and stopping condition for a normal Markov process are shown in [5]. Since we deal with horizontal ambiguity, we introduce the concept of quadratic possibility theory to the rotation according to a complex Markov process. The initial and stopping condition are shown in Equations ((17), (18) and (19), (20)), respectively. Where the rotation can start from (18) satisfying the initial condition (17). And the rotation can stop under (20) satisfying the stopping condition (19).

1) $\left(F_{10t}, F_{20t}\right) = \left(Z_{10t}, Z_{20t}\right)$ (17)

2) $POS\left(\left(F_{1t}, F_{2t}\right) \ge \left(Z_{1t}, Z_{2t}\right) \mid x_0\right) \le POS\left(\left(DF_{1t}x_0, DF_{2t}x_0\right) \ge \left(DZ_{1t}x_0, DZ_{2t}x_0\right)\right)$

3) $NES\left(\left(F_{1t}, F_{2t}\right) \ge \left(Z_{1t}, Z_{2t}\right) \mid x_0\right) \ge NES\left(\left(DF_{1t}x_0, DF_{2t}x_0\right) \ge \left(DZ_{1t}x_0, DZ_{2t}x_0\right)\right)$

where $\left(DF_{1t}x_0, DF_{2t}x_0\right) = \left(DX_{1t}, DZ_{2t}\right)$

※ $F_{10}(\cdot)_t = F_{20}(\cdot)_t$ (18)

1) $F_{ti0} = Z_{ti0} \quad (i = 1.2)$

2) $POS\left(F_{it} \ge Z_{it} \mid x_{i0}\right) \le POS\left(DF_{xi0} \ge DZ_{xi0}\right) \quad (i = 1, 2)$

3) $NES\left(F_{it} \ge Z_{it} \mid x_{i0}\right) \ge NES\left(DFx_{i0} \ge DZx_{i0}\right) \quad (i = 1, 2)$

where $DFx_{i0} = DZx_{i0}\ (i = 1, 2)$

where F_{i0t} and $DF_{i0t}(i = 1, 2)$ represents 2 complex events, and the quadratic possibility theory is applied. If the mapping function is equivalent, they invert 180-degree, and the initial condition and stopping condition is reversed. Note that the complex event becomes also one in a situation like this.

1) $\left(F_{1t0}, F_{2t0}\right) = \left(Z_{1t0}, Z_{2t0}\right)$ (19)

2) $POS\left(\left(F_{1t0}, F_{2t0}\right) \ge \left(Z_{1t}, Z_{2t}\right)\right) \le POS\left(\left(DF_{10}, DF_{20}\right) \ge \left(DZ_{10}, DZ_{20}\right)\right)$

3) $NES\left(\left(F_{1t}, F_{2t}\right) \ge \left(Z_{1t}, Z_{2t}\right)\right) \ge NES\left(\left(DF_{10}, DF_{20}\right) \ge \left(DZ_{10}, DZ_{20}\right)\right)$

where $\left(DF_{10}, DF_{20}\right) = \left(DZ_{10}, DZ_{20}\right)$.

$$※\ F_{10}(\cdot)_t = F_{20}(\cdot)_t$$
$$※\ F_{10}(\cdot)_t = F_{20}(\cdot)_t \tag{20}$$

1) $F_{ti0} = Z_{ti0} \quad (i = 1,2)$
2) $POS(F_{it} \ge Z_{it}) \le POS(DF_{i0} \ge DZ_{i0}) \quad (i = 1,2)$
3) $NES(F_{it} \ge Z_{it}) \ge NES(DF_{i0} \ge DZ_{i0}) \quad (i = 1,2)$

where $DF_{i0} = DZ_{i0}$.

6. Conclusion

In this article, we refer to the type 2 complex Markov process, possibility principal factor rotation that derive the initial condition and stopping condition from the quadratic possibility theory. The future subject is to obtain the initial and stopping condition in possibility oblique factor rotation. In conclusion, the quadratic mapping formula is regarded as multidimensional non-linear factor analysis and is closely connected with the artificial intelligence.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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