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Interaction of ^{28}Si with Emulsion Nuclei at 4.5 AGeV/c in View of Thermo-Statistical Approach

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Abstract

In this work, we study some changes of nuclear matter in the interactions of ^{28}Si with emulsion nuclei at 4.5 AGeV/c. From the experimental quantities investigated using Tsallis' statistics, we deduced the temperature, entropy density and non-equilibrium factor of the nuclear medium. These obtained parameters were used to reveal variations in the nuclear matter at the stated interaction energy. The results that came up from this study were compared with their corresponding results obtained from other heavy ion collision experiments at wide energy range.

Keywords

Silicon, Nuclear Emulsion, Rapidity, Heavy Ions Collision

1. Introduction

Over several past years, numerous number of nuclear reaction experiments at wide range of energy ranging from few GeV up to few TeV values have been offering a plenty of data, including nucleon-nucleon, nucleon-nucleus as well as nucleus-nucleus collisions. The increase in interaction energy at these experiments has been making the nuclear matter to go through different phases as it is exposed to higher pressure, energy density and temperature. The data that came out from high energy nuclear reactions have made the possibility to study particle production, the properties of which have led to valuable information about the phases through which nuclear matter passes during production of these particles. Different degrees of excitation of the interacting nuclei have been studied and some theoretical models have created pictures of what could be going on in the nuclear matter at different values of collision energy through measuring a number of the outcoming experimental physical quantities.

It is to be stated here that the physics of nuclear reactions is a rapid dynamical process and the phase transitions in a dynamical system is still an open field of research. Dynamics of phase transitions in both of "small" and ultra-relativistic particle collisions, have been widening the understanding of the symmetry problem and consequently the description of the equation of state of the nuclear system and revealing more mysteries of the quantum chromo-dynamics QCD.

In this study, we are working on the interactions of ^{28}Si with emulsion nuclei at 4.5 AGeV/c in view of thermo-statistical approach to get some information of the variations that take place in the nuclear medium by studying how the pseudo-rapidity of the created particles, entropy density, non-equilibrium factor and the temperature of the system change as the centrality of the collision changes.

We are going to follow Tsallis' statistics [1] [2] [3] [4] to estimate the possible macroscopic variables found in the Tsallis' distribution function that is appropriate for the pre-equilibrium thermodynamic states. The bulk of matter created in high-energy collisions can be quantitatively described in terms of hydrodynamic and thermo-statistical models [5] [6], which are governed mainly by the chemical freeze-out temperature. Tsallis' distribution describes the thermal fluctuation in the hadron or heavy ion systems at high energies. Tsallis' form is used to describe the pseudo-rapidity distribution of produced charged hadrons [7]:

$$\frac{\mathrm{d}N}{\mathrm{d}\eta} = N_o\left[1+(q-1)\frac{\sqrt{m^2+p_t^2\cosh^2(\eta)}}{T}\right]^{\frac{1}{1-q}}\frac{\cosh(\eta)}{\sqrt{m^2p_t^{-2}+\cosh^2(\eta)}} \quad (1)$$

The pseudo-rapidity density, $\frac{\mathrm{d}N}{\mathrm{d}\eta}$ is the number of produced pions, $\mathrm{d}N$ per pseudo-rapidity interval, $\mathrm{d}\eta$, T is the average temperature in a few sources which can describe local equilibrium states, q is the entropy index which represents the degree of non-equilibrium among different states, p_t is the average transverse momentum and m is the rest mass of the produced hadron. Values of $q \gg 1$ would indicate a system approaching equilibrium state, while values greater than 1 indicate a system far from equilibrium created in the early stage of the reaction.

For better fitting to our data, two parameters were added to Equation (1) to become:

$$\frac{\mathrm{d}N}{\mathrm{d}\eta} = o + N_o\left[1+(q-1)\frac{\sqrt{m^2+p_t^2\cosh^2(\eta-\eta_o)}}{T}\right]^{\frac{1}{1-q}}\frac{\cosh(\eta-\eta_o)}{\sqrt{m^2p_t^{-2}+\cosh^2(\eta-\eta_o)}} \quad (2)$$

where, o is the offset parameter in y-axis and η_o is a reference pseudo-rapidity. It denotes the fireball rapidity in its reference frame. In simple words, η_o is the rapidity of the center of mass system of collision, the peak position of the particle rapidity distribution (the mean value of rapidity of the particles).

According to Fermi model [8], the heavy-ion collisions look like two flying disks towards each other with high speed that approaches that of light, surrounded by a cloud of pions, this cloud extends to a region of distance of the order $\hbar c/m_\pi$, and it is assumed to have a spherical radius R given as:

$$R = \frac{\hbar c}{m_\pi} \cong 1.4 \text{ fm} \tag{3}$$

where $\hbar c$ is a conversion constant and m_π is the pion mass. At high energy, the radius suffers Lorentz' contraction by a factor taken as the ratio between the beam energy in the colliding frame and the rest mass energy of the proton, m_p. The interaction area s is estimated as [9] [10]:

$$s = \pi R^2 \frac{N_p m_p}{E_{beam}} \tag{4}$$

N_p, is the number of nucleons that participate in the overlap region of the collisions. It is evaluated according to the measured outgoing total charge of non-interacting projectile particles, Q. The quantity E_{beam} is the projectile energy per nucleon (4.6 AGeV).

The initial entropy density is given by:

$$\sigma_s = \frac{3}{2} \frac{\frac{\mathrm{d}N}{\mathrm{d}\eta}}{s} \tag{5}$$

where s, denotes the interaction area, according to which, the values of entropy density are varied.

2. Experiment

A stack plates of Br-2 nuclear emulsions exposed to a 4.5 AGeV/c ^{28}Si beam at Dubna Synchro-Phastron of intensity of irradiation ≈ 10^4 particles/cm^2 and beam diameter ≈ 1 cm, was scanned by our group members under high power microscopes. Particle multiplicity, grain density and the emission angles of the emitted particles were accurately measured. According to the emulsion techniques [11], the tracks of the coming out secondary particles from the interactions, were classified into black (slow), grey (intermediate) and shower (fast), according to their relative velocity β and specific ionization I^*. The quantity $\beta = v/c$, where v and c are the particle velocity and speed of light. The quantity $I^* = I/I_o$, where I is the measured grain density of the secondary particle track (the number of developed grains in the photographic emulsion per 100 μm) and I_o is grain density of the minimum ionizing particle track. These particles are emitted at space angles $\theta \leq 3^\circ$ and exposed to further multiple scattering measurements for momentum determination in order to separate the produced pions from the singly charged projectile fragments. These particles having $I^* < 1.4$ would have $\beta > 0.7$. In this study, we are interested in such single charged fast particles being mostly produced charged pions.

For comparison, we used some data of other heavy ion collision experiments [12] [13] [14] [15] that have wide spectrum of collision energy.

3. Results and Analysis

The rapidity distribution of the produced particles in a nuclear reaction refers to the size of the reaction and the origin of the produced particles. Figure 1 shows schematic features of such parameter at different classes of interaction energy [16].

In Figure 1(a), around AGS (Alternating Gradient Synchrotron) region of energy, the colliding nuclei stop each other forming a hot baryon rich volume in the middle, while Figure 1(b), in the RHIC (Relativistic Heavy Ion Collider) and LHC (Large Hadron Collider) regions, the Lorenz-contracted nuclei penetrate through each other forming a very hot baryon-free volume in the middle. η_P and η_T in Figure 1 are the projectile and target particle rapidity, respectively.

The pseudo-rapidity of the produced charged hadrons are calculated using the measured space angle θ_s of the fast charged hadrons according to the formula:

$$\eta = -\ln \tan \frac{\theta_s}{2} \tag{6}$$

Figure 2 presents the pseudo rapidity - distributions for four degrees of centrality, classified according to the Q - number (the outgoing total charge of non-interacting projectile particles), as follows:

Central: $Q = (0 - 1)$

Semi-central: $Q = (2 - 5)$

Semi-peripheral: $Q = (6 - 10)$

Peripheral: $Q = (11 - 14)$

In Figure 2, the rapidity distributions appear as semi-Gaussian function, as expected at that relative low interaction energy. This Gaussian distribution, actually, contains three mixed Gaussians inside it; one from the fireball and the other two from the fragmentation of the projectile and target sources, according to the three source model [17]. This is because, at the center of mass interaction

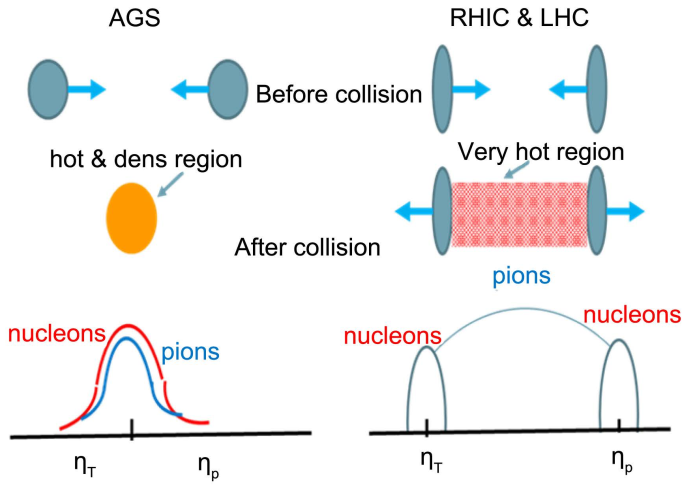

Figure 1. A schematic diagram to the origin of the rapidity of the produced particles in heavy ion collisions in the AGS region (Figure 1(a)) and in the RHIC & LHC energy region (Figure 1(b)).

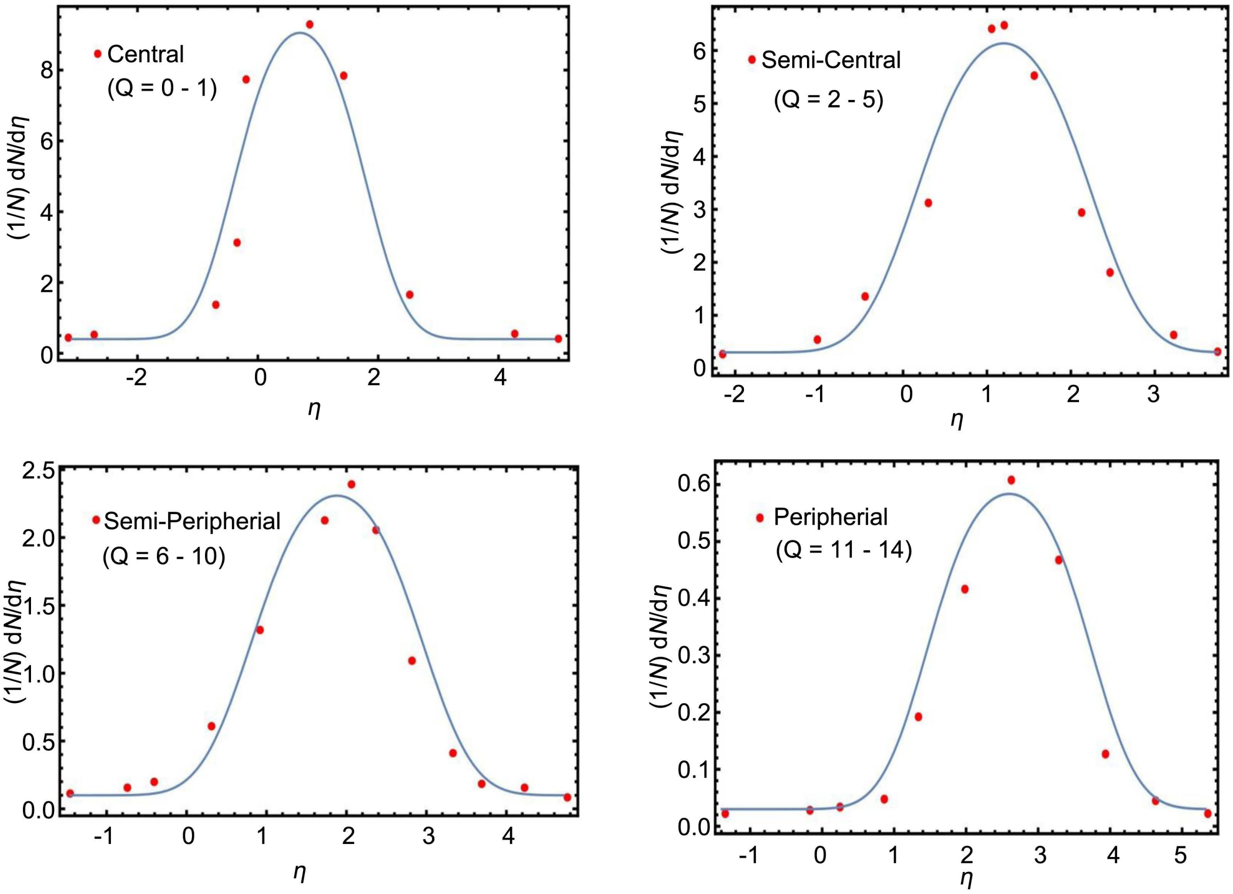

Figure 2. Pseudo-rapidity distribution of the shower particles emitted in 4.5A GeV/c ^{28}Si interactions with emulsion for central, semi-central, semi-peripheral and peripheral.

energy values $\sqrt{s_{NN}}$ ≤ 20 GeV, these three distributions reduce to the two fragmentation sources due to the very low effect of the gluonic source. These two sources overlap in the rapidity space and appear as a single Gaussian (thermal) source. The three stated distributions from their emitting sources get dispersed from each other as we get out of the AGS energy region. This dispersion spreads more and more, further apart, as the collision energy passes through the RHIC and enters the LHC values [18]. Some parameters from the Gaussian fit to the data of **Figure 2** have been extracted and presented in **Table 1** to help in quantitative description.

From **Figure 2** and **Table 1**, one may notice that, as we move from peripheral to central collisions:

1) The peak value of the rapidity gets higher. This could be referred to the increase in the number of participants from both the interacting nuclei. This causes larger amount of deposited energy in the interaction region and hence greater number of created particle multiplicity, according to the principle of the "participant scaling" [18].

2) The peak position shifts towards the lower values. This might be attributed to the increase in the number of collisions that increases the values of transverse

Table 1. Peak height, position and width of the Gaussian fit.

Interaction type	Peak height	Peak position	Peak width
Peripheral	0.65	2.6	1.27
Semi-peripheral	2.5	1.8	1.42
Semi-central	6.2	1.2	1.46
Central	9.0	0.86	1.58

momentum of the interacting and created particles leading to wider scattering angles of the coming out particles.

3) The width of the distributions gets fatter by small amount, as would be expected from the widening of the gap between the projectile and target participants particle sources, in the rapidity space. Assuming that at high energy, the two interacting nuclei can be viewed as two groups of individual nucleons. Then, according to the "Thermalized Cylinder Model" [19] which uses the simple picture of the one dimensional "string" model [20] and the "fireball" model [21]; the "string" model assumes that in relativistic nucleon-nucleon collisions, a string is formed consisting of two endpoints acting as energy reservoirs and the interior with constant energy per length. The string breaks into a number of substrings along the direction of the incident beam. The distribution length of substrings will define the width of the pseudo-rapidity distribution. The "fireball" model assumes that the incident nucleon penetrates through the target nucleon, then a fire streak is formed along the direction of the incident beam. The length of the fire streak expresses the width of pseudo-rapidity distribution. As the energy content in the collision gets larger; greater number of strings or fire-streaks is formed along the incident direction. The "Thermalized Cylinder Model" assumes that these strings or fire streaks mix in the transverse direction; thus leading to increases in the width of the distribution as one moves from the peripheral to central collisions. However, our Si-Emulsion data show small increases in the width as is expected in this relative low interaction energy, in which expansion of the interaction volume does not increase drastically, relative to that expected to happen at the RHIC and the LHC energies.

The values of kinetic freeze-out temperature T, the average transverse momentum $\langle p_t \rangle$ and the non-equilibrium factor q, extracted as fitting parameters from the Tsallis' distribution (Equation (2)) for each interaction type are presented in **Table 2**. The entropy density was calculated from Equation (5).

There are some facts that manifest themselves in **Table 2**, as the centrality of collision increases. These facts may be summarized as follows:

1) The temperature of the emitting particle system rises due the increasing average values of transverse momentum with a consequence of larger amount of deposited energy in the interaction region, thus, higher temperature.

2) The entropy density increases to probably indicate a slow gradual hadron dissociation, forming quark pairs, getting into the QGP phase. So, we have a sit-

uation of growing number of created particles and increased amount of content in the interaction volume.

3) The non-equilibrium factor, q, looks to be decreasing with very small values as the centrality increases tending to approach equilibrium. This behavior may be understood in view of the fact that, in peripheral collision, particles are produced in the most forward direction (high rapidity intervals, having small transverse momenta and low entropy density) at low temperature and little bit far from equilibrium. As centrality increases, particles spread out with small rapidity to attain the final state with larger temperature and closer to pre-equilibrium. In other words, deeper centrality of interaction means longer time for the particles to stay in the system, with increased quark-gluon interactions, be produced in larger cone angle (smaller rapidity), leading to higher temperature and larger entropy and become closer to equilibrium. However, all of the q values, of the 4.6 GeV data, are pretty close to unity, indicating a good equilibrium state for the whole interacting system at our interaction energy.

In **Figure 3**, we present the values of q of our data together with other values that came out from other experiments at higher energies [12] [13] [14] [15].

Table 2. The kinetic freeze-out temperature T, average transverse momentum $<p_t>$, the non-equilibrium factor q, for each interaction type, and the entropy density.

Interaction type	T [MeV]	$<p_t>$ [MeV/c]	q	N_o	η_o	o	$<\sigma_s>$ [fm^{-2}]
Peripheral	101	156	1.034	5.5	2.6	0.03	0.07
Semi-peripheral	105	172	1.030	22	1.9	0.10	0.18
Semi-central	115	187	1.028	52	1.2	0.30	0.37
Central	134	192	1.025	60	0.7	0.40	0.43

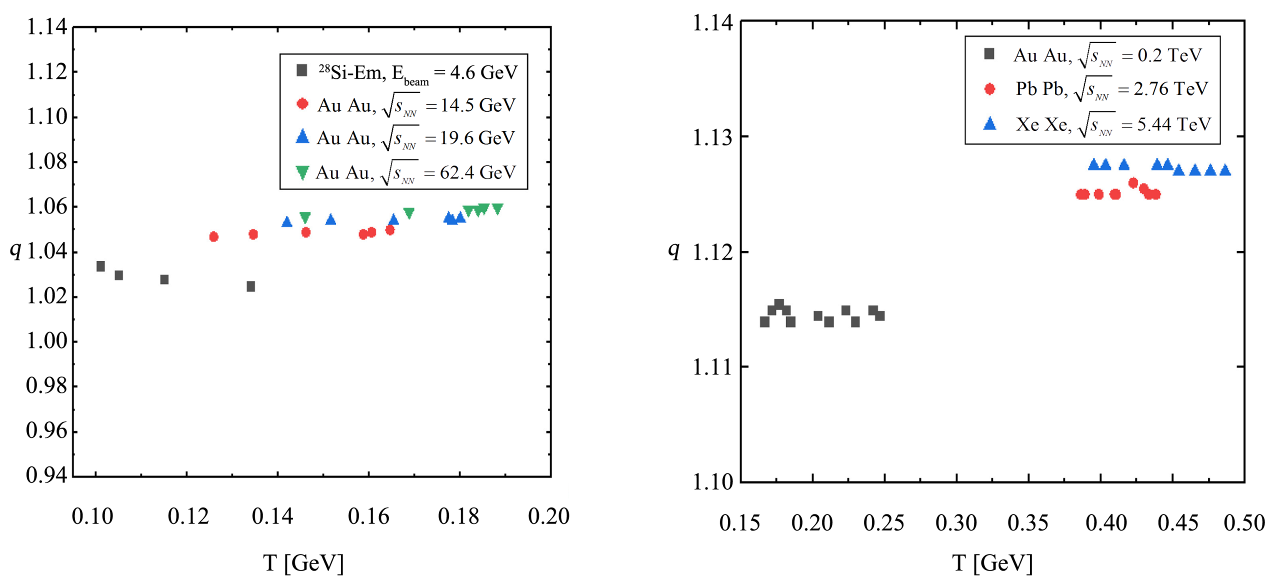

Figure 3. The change of the non-equilibrium parameter q with the temperature T of the produced particles in heavy ion collisions for AGeV (**Figure 3(a)**) and TeV group of energy (**Figure 3(b)**).

In **Figure 3(a)**, we notice that q increases by very small rate with the temperature in the stated range of interaction energy, keeping fairly close to equilibrium state of the interacting system. In **Figure 3(b)**, we notice that q has different levels of saturations that get higher as the interaction energy increases, meaning that such saturation levels get further apart from the equilibrium level for higher interaction energy. This might be interpreted by recalling the fact that at such high energy density of the system, there is no way the matter could be tightly packed as collection of hadrons but, instead, it is a gas of quarks and gluons. The interaction between these quarks and gluons is very weak and equilibrium state of matter is far to be reached, as could be imposed by the interplay between the two crucial characters of the QCD; the color confine and the asymptotic freedom [18].

4. Conclusion

The pseudo-rapidity distribution of the created particles followed one Gaussian function for the Si-Emulsion interactions, as is expected at energy. As the centrality of collision increased, we noticed higher peak values (obeying the "participant scaling" principle) and a shift of peak positions to lower rapidity with a small growth of the widths (in agreement with the "fireball", "fire-streak" model basics). These effects were expected to be small at such relative low interaction energy, compared to those at RHIC and LHC energies. The values of kinetic freeze-out temperature, of the emitting particle system were found to increase with centrality, as a consequence of larger average values of transverse momentum, that added larger multiplicity of created particles as well as rise energy content in the interaction region. The non-equilibrium parameter showed small bits of decreases with centrality approaching equilibrium in our data. In the range of few tens of interaction energy, the systems keep, somehow, close to the border of equilibrium with small bits of getting further from it. In the TeV energy, this parameter has shown different levels of saturations that got away from equilibrium value and from each other for higher interaction energy.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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