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Option Pricing Model Driven by G-Lévy Process under the G-Expectation Framework

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Abstract

In this paper, we first present an option pricing model of stochastic differential equations driven by the G-Lévy process under the G-expectation framework, and prove the generalized Black-Scholes equations. Then, we present the algorithm for the time-homogeneous Poisson process versus the non-time- homogeneous Poisson process. Finally, we provide an explicit solution of generalized Black-Scholes equations and simulate it numerically with Matlab software.

Keywords

Generalized Black-Scholes Equations, G-Lévy Process, Matlab

1. Introduction

With the increasing complexity and diversification of financial markets, more and more financial problems cannot be solved directly by analytical formulas, but need to resort to numerical algorithms. In recent years, stochastic differential equations have been widely studied in financial engineering and have been applied to option pricing problems [1]. In 1973, Black and Scholes proposed the Black-Scholes formula, which became the most representative option pricing model. Based on this model, scholars have carried out a large number of option pricing studies and formed a wealth of results. With wide application, it is found that the market does not obey the assumptions of the Black-Scholes model. In 1976, Merton [2] started with stock prices and established the jump-diffusion behavior model of stock prices and the risk neutrality theorem, in which the jump-diffusion process represents the discontinuous and continuous fluctuations of stock prices. In 1976, Cox and Ross [3] proposed to modify the normal elastic volatility model to correct the reality that the model assumed too harsh conditions.

G-expectation space plays an important role in solving uncertain problems in the stock market, and Peng [4] [5] proposed G-Brownian motion, G-Itô formula and G-central limit theorem for us in the G-expectation framework. Yang and Zhao [6] introduced the simulation of G-Brownian and G-normal distributions under G-expectation. Although the G-Brownian movement solves many financial problems, some financial models that rely on Lévy processes remain unsolved. Therefore, Hu and Peng [7] studied the G-Lévy process which is a generalization of G-Brownian motion. Next, Chai [8] studied the option pricing problem for stochastic differential equations under the G-framework. And the simulation of G-Brownian motion under G-framework see [9]. In 2018, Quafoudi [10] worked out the exact solution of the fractional Black-Scholes European option pricing equation.

Because the Black-Scholes model is widely used in the financial market, and the interest rate and volatility in the equation are generally not constant in reality. This paper studies the stochastic interest rate and stochastic volatility based on this, which will be enlightening for further research of the option pricing model. In this paper, we study the stock price of St:

$$\mathrm{d}S_t = a(t)S_t\mathrm{d}t + b(t)S_t\mathrm{d}W_t + c(t)S_t\mathrm{d}L_t, \quad t\in[0,T], \tag{1}$$

where $a(t)$ is the stochastic interest rate, $b(t)$ is the stochastic volatility and $c(t)$ is the jump range of asset price, W_t is a G-Brownian motion and L_t is a G-Lévy process.

In this paper, we mainly study the Black-Scholes model driven by the G-Lévy process, and then give proof of this model by using the G-Itô formula and the properties of G-expectation. The main innovation of this paper is to change the original constant interest rate and volatility to stochastic interest rate and stochastic volatility, which will be closer to the actual financial market and can obtain more accurate and reasonable option pricing. At the same time, we use the new algorithm to give the corresponding numerical simulation analysis, and finally, get a more effective conclusion.

The outline of the paper is organized as follows. In Section 2, we introduce some fundamental definitions and theorems, such as G-expectation and G-Lévy processes. In Section 3, we present the proof of the generalized Black-Scholes equations and the Integro-PDE. Finally, in Section 4, we present a numerical simulation example of the option pricing model, and the simulation effect validates the conclusion well.

2. Preliminaries

In this section, we will introduce some concepts and notations that are the focus of this paper. We will use the notations as follows: Let $|x| = \langle x,x\rangle^{\frac{1}{2}}$ be the Euclidean norm in $\mathbb{R}^q$ and the scalar product of x, y is $\langle x,y\rangle$. The transpose of A which is a vector or matrix is denoted by A^{T}.

Definition 1 [7] (**Sublinear expectation**) $\hat{\mathbb{E}}:\mathbb{H}\to\mathbb{R}$ is a function on linear space $\mathbb{H}$. For any $X, Y\in\mathbb{H}$, we call the triple $(\Omega,\mathbb{H},\hat{\mathbb{E}})$ a sublinear expectation space if satisfy the following conditions:

- $\hat{\mathbb{E}}[X] \ge \hat{\mathbb{E}}[Y]$ for $X \ge Y$.
- $\hat{\mathbb{E}}[c] = c$ with $c \in \mathbb{R}$.
- $\hat{\mathbb{E}}[X+Y] \le \hat{\mathbb{E}}[X] + \mathbb{E}[Y]$.
- $\hat{\mathbb{E}}[\lambda X] = \lambda \hat{\mathbb{E}}[X]$ for $\lambda \ge 0$.

Definition 2 [5] (**G-Brownian motion**) For a d-dimensional stochastic process $(B_t)_{t\ge 0}$ in sublinear expectation space $(\Omega, \mathcal{H}, \hat{\mathbb{E}})$, we call it G-Brownian motion if it satisfies the following conditions:

- $B_0(\omega) = 0$.
- $\hat{\mathbb{E}}\left[|B_t|^3\right] t^{-1} = 0$.
- For any $t, s \ge 0$, $B_{t+s} - B_t$ and B_s are identically distributed, for any $n \in N$ and $0 \le t_1 \le t_2 \le \cdots \le t_n \le t$, $B_{t+s} - B_t$ is independent of $(B_{t_1}, B_{t_2}, \cdots, B_{t_n})$ $\lambda \ge 0$.

Definition 3 [7] (**Lévy process**) Let $(X_t)_{t\ge 0}$ be a d-dimensional stochastic process defined on the sublinear expectation space $(\Omega, \mathcal{H}, \hat{\mathbb{E}})$. We call X-Lévy process if it satisfies the following conditions:

- $X_0 = 0$.
- Independent increments: For each $t, s > 0$, $n \in N$ and $0 \le t_1 \le t_2 \le \cdots \le t_n \le t$, the increment $X_{t+s} - X_t$ is independent of $(X_{t_1}, X_{t_2}, \cdots, X_{t_n})$.
- Stationary increments: For all $t, s \ge 0$, the distribution of the increments $X_{t+s} - X_t$ is stable and does not depend on *t*.

Definition 4 [7] (**G-Lévy process**) $X = (X_t)_{t\ge 0}$ is a d-dimensional stochastic process defined on the sublinear expectation space $(\Omega, \mathcal{H}, \hat{\mathbb{E}})$. If satisfies the following properties, then X is said to be G-Lévy process.

- X satisfies the conditions in Definition 2.
- For each $t \ge 0$, exists $X_t = X_t^c + X_t^d$.
- $(X_t^c, X_t^d)_{t\ge 0}$ is a Lévy process which satifies the following conditions:

$$\lim_{t\downarrow 0} \hat{\mathbb{E}}\left[\left|X_t^c\right|^3\right] t^{-1} = 0 \hat{\mathbb{E}}\left[\left|X_t^d\right|\right] < Ct, t \ge 0,$$

where C is a constant that depends on *X*.

Lemma 1 [7] (**Lévy-Khintchine expression**) Let X a G-Lévy process in $\mathbb{R}^q$ and $h \in C_b^3(\mathbb{R}^q)$, we can obtain the following form:

$$G_X[h(\cdot)] := \lim_{t\downarrow 0} \mathbb{E}[h(X_t)] t^{-1}. \tag{2}$$

If Equation (2) is true, we have the following Lévy-Khintchine expression:

$$G_X[h(\cdot)] = \sup_{(\lambda,a,b)\in\mathbb{U}} \left\{ \langle Dh(0), a \rangle + \frac{1}{2} tr\left[D^2 h(0) bb^{\mathrm{T}}\right] + \int_{\mathcal{E}} h(e) \lambda(\mathrm{d}e) \right\},$$

where $h(0) = 0$, $\mathcal{E} = \mathbb{R}^q \setminus \{0\}$, $\mathbb{U} \subset \mathcal{V} \times \mathbb{R}^q \times \mathbb{Q}$, $\mathcal{V}$ is a set of all Borel measures of $\mathcal{E}$ and $\mathbb{Q}$ is a set of all positive definite symmetric matrix.

Lemma 2 [7] (**Integro-PDE**) Under the premise of Lemma 1, let X a G-Lévy process and function $u = u(x, s)$, we can obtain the following Integro-PDE:

$$\frac{\partial u}{\partial s} - \sup_{(\lambda,a,b)\in\mathbb{U}} \left\{ \langle Du, a \rangle + \frac{1}{2} tr\left[D^2 u bb^{\mathrm{T}}\right] + \int_{\mathcal{E}} \left(u(x + c(e), s) - u(x, s)\right) \lambda(\mathrm{d}e) \right\} = 0,$$

where $D^2 u$ is the Hessian matrix of u and $a \in \mathbb{R}^q, b \in \mathbb{R}^{q\times q}$.

Lemma 3 [11] (**G-Itô formula**) The k-th component of X_t is X_t^i for $1 \le i \le q$, and it meets the following form:

$$X_t^i = X_0^i + \int_0^t a_s^i \mathrm{d}s + \sum_{j=1}^{q} \int_0^t b_s^{i,j} \mathrm{d}W_s^j + \int_0^t \int_{\mathcal{E}} c(e,s) L(\mathrm{d}e, \mathrm{d}s),$$

where $\mathcal{E} \in \mathbb{R}^q \setminus \{0\}$, G-Brownian motion is represented by W_s and G-Lévy process is represented by $L(\mathrm{d}e, \mathrm{d}s)$. For $h \in C_b^2(\mathbb{R}^q)$, we can obtain

$$\begin{aligned} h(X_t) = {} & h(X_0) + \sum_{i=1}^{q} \int_0^t a_s^i \frac{\partial h(X_s)}{\partial x_i} \mathrm{d}s + \frac{1}{2} \sum_{i,k=1}^{q} \sum_{j=1}^{q} \int_0^t b_s^{i,j} b_s^{k,j} \frac{\partial^2 h(X_s)}{\partial x_i \partial x_k} \mathrm{d}\langle W \rangle_s \\ & + \sum_{i=1}^{q} \sum_{j=1}^{q} \int_0^t b_s^{i,j} \frac{\partial h(X_s)}{\partial x_i} \mathrm{d}W_s^j + \int_0^t \int_{\mathcal{E}} \left[h(X_{s-} + c(e,s)) - h(X_{s-}) \right] L(\mathrm{d}e, \mathrm{d}s). \end{aligned}$$

3. Generalized Black-Scholes Equations

In this section, our main aim is to use the G-Itô formula to prove integral partial differential equation. In the following, we will first give the theorem of the generalized Black-Scholes equations, and then prove this equation.

Theorem 1 (**Generalized Black-Scholes equations**) Based on Equation (1), we can derive the following integral-partial differential equation (Integro-PDE) under G-Lévy process. In particular, we assume that $u = u(S_t, t)$ is the option price and S_t is the stock price.

$$\begin{aligned} & \frac{\partial u}{\partial t} + \sup_{(\lambda,a,b,c)\in\mathbb{U}} \left\{ a(t) S_t \frac{\partial u}{\partial S} + \frac{b(t)^2 S^2}{2} \frac{\partial^2 u}{\partial S^2} + \ln(1+c(t)) \lambda(\mathcal{E}) S_t \frac{\partial u}{\partial S} \right. \\ & \left. + \ln^2(1+c(t)) \lambda(\mathcal{E}) \left(\frac{b^2(t) S_t^2}{2} \frac{\partial^2 u}{\partial S^2} + \frac{b^2(t) S}{2} \frac{\partial u}{\partial S} \right) \right\} - a(t) u = 0, \end{aligned}$$

where $a(t), b(t), c(t)$ are three functions of t, $\mathbb{U} \subset \mathcal{V} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}$, $\mathcal{V}$ is a set of all Borel measures of $\mathcal{E}$ and $\lambda(\mathcal{E}) = \int_{\mathcal{E}} \lambda(\mathrm{d}e)$.

Proof. On time interval $[0,T]$, we first define a uniform time partition. And $0 = t_0 < t_1 < \cdots < t_n < \cdots < t_N = T$, $\Delta t = t_{n+1} - t_n$ for $0 \le n \le N$. Let the function $u(S,t)$ be sufficiently smooth, $\Delta \langle W \rangle_n = \langle W \rangle_{t_{n+1}} - \langle W \rangle_{t_n}$ and $\Delta W_n = W_{t_{n+1}} - W_{t_n}$. We can deduce the explicit solution of Equation (1) by using the G-Itô formula:

$$\begin{aligned} S_{t_{n+1}} = S_{t_n} \exp \left\{ \int_{t_n}^{t_{n+1}} \left(a(t)\mathrm{d}t - \frac{1}{2} b^2(t) \mathrm{d}\langle W \rangle_t + b(t) \mathrm{d}W_t \right. \right. \\ \left. \left. + \int_{\mathcal{E}} \ln(1+c(t)) L(\mathrm{d}e, \mathrm{d}s) \right) \right\}. \end{aligned} \tag{3}$$

In the G-expectation space, we can obtain

$$\mathrm{d}W_t \cdot \mathrm{d}W_t = \mathrm{d}\langle W \rangle_t, \mathrm{d}L_t \cdot \mathrm{d}L_t = \lambda(\mathcal{E})\mathrm{d}t + (\lambda(\mathcal{E})\mathrm{d}t)^2, \mathrm{d}L_t \cdot \mathrm{d}t = 0, \mathrm{d}W_t \cdot \mathrm{d}L_t = 0.$$

Then, the well-known option pricing formula is as follows:

$$u(S_{t_n}, t_n) = \frac{1}{r} \mathbb{E}\left[u(S_{t_{n+1}}, t_{n+1}) - u(S_n, t_n) \mid S_{t_n} \right] + \frac{1}{r} u(S_{t_n},, t_n). \tag{4}$$

Next, we derive the generalized Black-Scholes model driven by G-Lévy process under the G-expectation framework. By using Taylor formula for $u\left(S_{t_{n+1}}, t_{n+1}\right)$, we can obtain the following equation:

$$\begin{aligned} u\left(S_{t_{n+1}}, t_{n+1}\right)-u\left(S_{t_n}, t_n\right)= & \frac{\partial u\left(S_{t_n}, t_n\right)}{\partial t} \Delta t+\frac{\partial u\left(S_{t_n}, t_n\right)}{\partial S}\left(S_{t_{n+1}}-S_a\right) \\ & +\frac{1}{2} \frac{\partial^2 u\left(S_{t_n}, t_n\right)}{\partial S^2}\left(S_{t_{n+1}}-S_{t_n}\right)^2+O(\Delta t)^{\frac{3}{2}} . \end{aligned} \tag{5}$$

Substituting Equation (3) into Equation (5), we have

$$\begin{aligned} u\left(S_{t_{n+1}}, t_{n+1}\right)-u\left(S_{t_n}, t_n\right)= & \frac{\partial u\left(S_{t_n}, t_n\right)}{\partial t} \Delta t+\frac{\partial u\left(S_{t_n}, t_n\right)}{\partial S}\left(S_{t_n} \exp \left\{X^n\right\}-S_a\right) \\ & +\frac{1}{2} \frac{\partial^2 u\left(S_{t_n}, t_n\right)}{\partial S^2}\left(S_{t_n} \exp \left\{X^n\right\}-S_{t_n}\right)^2+O(\Delta t)^{\frac{3}{2}}, \end{aligned}$$

where $X^n=a \Delta t-\frac{1}{2} b^2 \Delta\langle W\rangle_n+b \Delta W_n+\int_{t_n}^{t_{n+1}} \int_{\mathcal{E}} \ln (1+c(t)) L(\mathrm{d} e, \mathrm{d} s)$. Let $\lambda(\mathcal{E})=\int_{\mathcal{E}} \lambda(\mathrm{d} e)$, it induces from Taylor expansion for $\exp \left\{X^n\right\}$ that

$$\begin{aligned} & u\left(S_{t_{n+1}}, t_{n+1}\right)-u\left(S_{t_n}, t_n\right) \\ & =\left[\frac{\partial u}{\partial t}+a\left(t_n\right) S_{t_n} \frac{\partial u}{\partial S}\right] \Delta t-S_{t_n} \frac{b^2\left(t_n\right)}{2} \frac{\partial u}{\partial S} \Delta\langle W\rangle_n+S_{t_n} \ln \left(1+c\left(t_n\right)\right) \frac{\partial u}{\partial S} L_{\mathcal{E}} \\ & +S_{t_n} b\left(t_n\right) \frac{\partial u}{\partial S} \Delta W_n+\left[S_{t_n} \frac{\partial u}{\partial S}+S_{t_n}^2 \frac{\partial^2 u}{\partial S^2}\right] \frac{1}{2}\left(X^n\right)^2+O(\Delta t)^{\frac{3}{2}} \\ & =\left[\frac{\partial u}{\partial t}+a\left(t_n\right) S_{t_n} \frac{\partial u}{\partial S}\right] \Delta t-S_{t_n} \frac{b^2\left(t_n\right)}{2} \frac{\partial u}{\partial S} \Delta\langle W\rangle_n+S_{t_n} \ln \left(1+c\left(t_n\right)\right) \frac{\partial u}{\partial S} L_{\mathcal{E}} \\ & +S_{t_n} b\left(t_n\right) \frac{\partial u}{\partial S} \Delta W_n+\left[S_{t_n} \frac{\partial u}{\partial S}+S_{t_n}^2 \frac{\partial^2 u}{\partial S^2}\right] \frac{1}{2}\left(\frac{b\left(t_n\right)^4}{4}\left(\Delta\langle W\rangle_n\right)^2+b\left(t_n\right)^2\left(\Delta W_n\right)^2\right. \\ & \left.-b^3\left(t_n\right) \Delta W_n \Delta\langle W\rangle_n+\ln ^2\left(1+c\left(t_n\right)\right) \lambda(\mathcal{E}) \Delta t+\ln \left(1+c\left(t_n\right)\right) \lambda(\mathcal{E})(\Delta t)^2\right)+O(\Delta t)^{\frac{3}{2}}, \end{aligned}$$

where $L_{\mathcal{E}}=\int_{t_n}^{t_{n+1}} \int_{\mathcal{E}} L(d e, d s)$. By inserting the above result into Equation (4), we can obtain the following rule:

$$\begin{aligned} u=\frac{1}{r} \mathbb{E}\Bigg[& \left(\frac{\partial u}{\partial t}+a\left(t_n\right) S_{t_n} \frac{\partial u}{\partial S}\right) \Delta t-S_a \frac{b^2\left(t_n\right)}{2} \frac{\partial u}{\partial S}\left(\Delta\langle W\rangle_n\right)+S_{t_n} b\left(t_n\right) \frac{\partial u}{\partial S}\left(\Delta W_n\right) \\ & +S_{t_n} \ln \left(1+c\left(t_n\right)\right) \frac{\partial u}{\partial S} L_{\mathcal{E}}+\left[S_{t_n} \frac{\partial u}{\partial S}+S_{t_n}^2 \frac{\partial^2 u}{\partial S^2}\right] \frac{1}{2}\left(\frac{b^4\left(t_n\right)}{4}\left(\Delta\langle W\rangle_n\right)^2\right. \\ & +b^2\left(t_n\right)\left(\Delta W_n\right)^2-b^3\left(t_n\right) \Delta W_n \Delta\langle W\rangle_n \\ & \left.\left.+\ln ^2\left(1+c\left(t_n\right)\right) \lambda(\mathcal{E}) \Delta t+O(\Delta t)^{\frac{3}{2}}\right) \right\rvert\, S_{t_n}\Bigg]+\frac{1}{r} u . \end{aligned}$$

On the base of $u=u\left(S_{t_n}, t_n\right)$ and $\Delta W_n \sim N\left(0 ;\left[\underline{\sigma}^2 \Delta t, \bar{\sigma}^2 \Delta t\right]\right)$, combined with the G-expectation property, we can derive the following equation:

$$\left(1-\frac{1}{r}\right)u=\frac{\Delta t}{r}\left(\frac{\partial u}{\partial t}+\sup_{(\lambda,a,b,c)\in\mathbb{U}}\left\{a\left(t_n\right)S_{t_n}\frac{\partial u}{\partial S}+\left(\frac{b^2\left(t_n\right)S_{t_n}^2}{2}\frac{\partial^2 u}{\partial S^2}\right)^+\bar{\sigma}^2\right.\right.$$

$$-\left(\frac{b^2\left(t_n\right)S_{t_n}^2}{2}\frac{\partial^2 u}{\partial S^2}\right)^-\underline{\sigma}^2+\ln\left(1+c\left(t_n\right)\right)S_{t_n}\frac{\partial u}{\partial S}\lambda(\mathcal{E})$$

$$\left.\left.+\ln^2\left(1+c\left(t_n\right)\right)\left(\frac{b^2\left(t_n\right)S_{t_n}^2}{2}\frac{\partial^2 u}{\partial S^2}+\frac{b^2\left(t_n\right)S_{t_n}}{2}\frac{\partial u}{\partial S}\right)\lambda(\mathcal{E})\right\}+O(\Delta t)^{\frac{1}{2}}\right),$$

where $r=1+a\left(t_n\right)\Delta t$ and $a\left(t_n\right)$ is risk-free rate. In the end, we have the following integro-partial differential equation:

$$\frac{\partial u}{\partial t}+\sup_{(\lambda,a,b,c)\in\mathbb{U}}\left\{a(t)S_t\frac{\partial u}{\partial S}+\frac{b^2(t)S_t^2}{2}\frac{\partial^2 u}{\partial S^2}+\ln\left(1+c(t)\right)\lambda(\mathcal{E})S_t\frac{\partial u}{\partial S}\right.$$

$$\left.+\ln^2\left(1+c(t)\right)\lambda(\mathcal{E})\left(\frac{b^2(t)S_t^2}{2}\frac{\partial^2 u}{\partial S^2}+\frac{b^2(t)S_t}{2}\frac{\partial u}{\partial S}\right)\right\}-au=0.$$

The proof is completed.

4. Numerical Experiment

In this section, we first give the simulation algorithm of time homogeneous and non-time-homogeneous Poisson processes. Then, we study the stock price S_t driven by G-Lévy process under the G-expection frame. Finally, we give an explicit solution of a numerical example of option pricing model and simulate it numerically by Matlab software. And the equation for constant terms of interest rates and volatility see [12].

Algorithm 1 (**Time-homogeneous Poisson process**)

The simulation of the time-homogeneous Poisson process is based on the time interval of each increment of $N(t)$, that is, the time interval of each jump is independent and follows the exponential distribution of the parameter λ.

Our algorithm is to generate a random number x between 0 and 1, and then take $Y=-\lambda In(X)$, where Y is the exponential distribution of parameter λ. For the simulation process of n jumps, this method is used to generate $nY(i)$, then the time of the N jumps is $S_N=\sum_{i=0}^{N}Y(i)$.

Algorithm 2 (**Non-time-homogeneous Poisson process**)

For non-time-homogeneous Poisson process, the function $m(t)=\int_0^t\lambda(s)\mathrm{d}s$ is computed by $\lambda(t)$ first. When $\lambda=1$ and $M(u)$ is time-homogeneous Poisson process, for $N(t)=M\left(m(t)\right)$, then $N(t)$ is the Poisson process for which the intensity function is $\lambda(t)$.

Our algorithm first simulates a time-homogeneous Poisson process with parameter 1, and then for each jump time, we can get each jump time of the non-time-homogeneous Poisson process by substituting the inverse function of $m(t)$.

Next, we will present relevant examples of option pricing based on the generalized Black-Scholes equations.

Example 1 Consider the stock price S_t has the following form:

$$\frac{\mathrm{d}S_t}{S_t} = a(t)\mathrm{d}t + b(t)\mathrm{d}W_t + c(t)\mathrm{d}L_t, \quad t \in [0,T], \tag{6}$$

where the initial value $S_0 = 0$, the interest rate $a(t)$ and volatility $b(t)$ are positive, W_t is a G-brownian motion and L_t is a G-Lévy process. Next, we give the explicit solution of Equation (6) on $t \in [0.1]$

$$S_t = S_0 \exp\left\{\int_0^t a(t)\mathrm{d}t - \frac{1}{2}(b(t))^2 \mathrm{d}\langle W\rangle_t + \int_0^t b(t)\mathrm{d}W_t + \int_0^t \int_{\varepsilon}\left[\ln(1+c(t))\right] L(\mathrm{d}e, \mathrm{d}t)\right\}.$$

In this example, we first use three different functions $a_1(t) = 0.2t$, $b_1(t) = 0.3t$, $c_1(t) = 0.1t$, $a_2(t) = 0.3t$, $b_2(t) = 0.1t$, $c_2(t) = 0.3t$ and $a_3(t) = 0.1t$, $b_3(t) = 0.2t$, $c_3(t) = 0.2t$ to simulate the stock price S_t. And the simulation of S_t is given in Figure 1 with three different functions.

From Figure 1, we can see that the stochastic volatility $b(t)$ and the jump intensity $c(t)$ have a more significant impact on the stock price S_t, so this paper mainly explores variation of stochastic volatility $b(t)$ and jump intensity $c(t)$.

Firstly, let coefficients $a(t) = 0.1t, c(t) = 0.1t$, we plot the stock price S_t with the time t under the different coefficients $b(t) = 0.1t$, $b(t) = 0.2t$, $b(t) = 0.3t$ in Figure 2. Figure 2 reflects that the stock price S_t will decrease with the increase of the stochastic volatility $b(t)$.

Let functions $a(t) = 0.1t, b(t) = 0.1t$, we plot the stock price S_t with the time t under the jump intensity functions $c(t) = t$, $c(t) = 5t$, $c(t) = 10t$ in Figure 3.

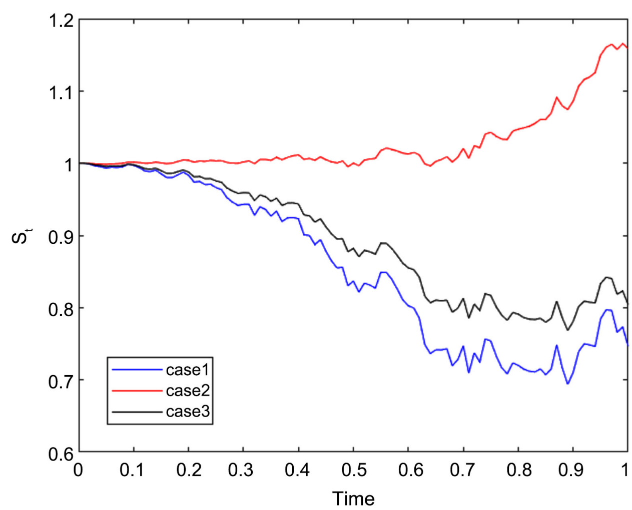

Figure 1. The simulation of stock price S_t with three different coefficients.

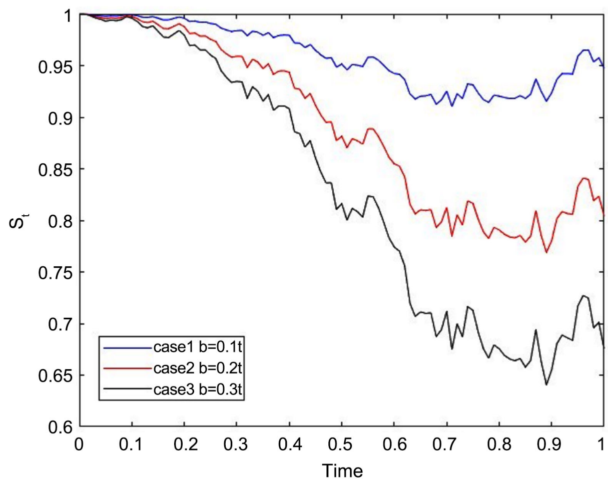

Figure 2. Stock price S_t with $b(t)=0.1t, b(t)=0.2t, b(t)=0.3t$.

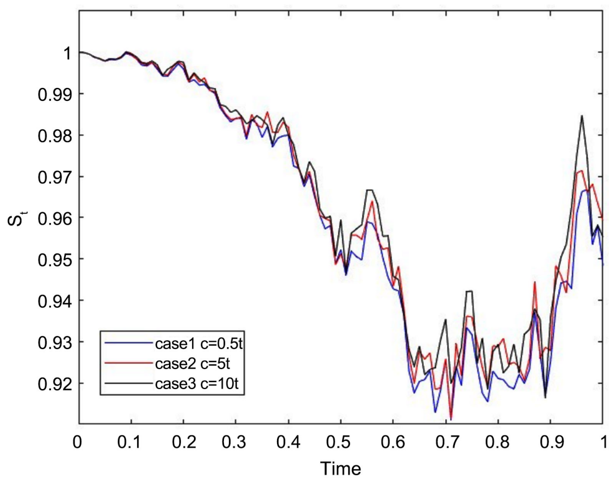

Figure 3. Stock price S_t with $c(t)=t, c(t)=5t, c(t)=10t$.

When the jump intensity $c(t)$ increase or decrease, the stock price S_t only has slight changes by comparing **Figures 1-3**. And, we can draw the conclusion that the stochastic volatility $b(t)$ have a great influence on stock price S_t than the jump intensity $c(t)$.

5. Conclusion

In this paper, we first derive the generalized Black-Scholes equations, and then prove the integral-partial differential equation driven by the G-Lévy process under the G-expectation framework. Finally, in the fourth part, we study the influence of coefficients on the stock price S_t. Through the research, we find that coefficients $a(t), b(t)$ have a great influence on the stock price S_t. This has heuristic implications for solving the uncertainty problem in option pricing. In the future, we will further investigate numerical schemes for stochastic differential equations under G-expectation, including Euler schemes and so forth.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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