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Event-Triggered Finite-Time H_∞ Filtering for Discrete-Time Nonlinear Stochastic Systems

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Abstract

This paper addresses the problem of event-triggered finite-time H_∞ filter design for a class of discrete-time nonlinear stochastic systems with exogenous disturbances. The stochastic Lyapunov-Krasoviskii functional method is adopted to design a filter such that the filtering error system is stochastic finite-time stable (SFTS) and preserves a prescribed performance level according to the pre-defined event-triggered criteria. Based on stochastic differential equations theory, some sufficient conditions for the existence of H_∞ filter are obtained for the suggested system by employing linear matrix inequality technique. Finally, the desired H_∞ filter gain matrices can be expressed in an explicit form.

Keywords

Event-Triggered Scheme, Discrete-Time Nonlinear Stochastic Systems, Stochastic Finite-Time Stable, Linear Matrix Inequalities (LMIS)

1. Introduction

During the past few decades, there has been a rapidly growing interest in nonlinear stochastic systems. Based on the fundamental stochastic stability theory [1] and Lyapunov-Krasovskii functional [2], some results can be found in the literature [3]-[15]. Specifically, problems of stochastic stabilization and destabilization were studied for nonlinear differential equations by noise and impulsive stochastic nonlinear systems respectively in [4] [6] [10] [14]. References [5] [7] [11] [12] [15] investigated state-feedback and output feedback stabilization problems for stochastic nonlinear systems and stochastic delay nonlinear systems. Fault detection filter and full-order H_∞ filter were provided for nonlinear stochastic systems and nonlinear switched stochastic systems in terms of second-order nonlinear Hamilton-Jacobi inequalities and T-S fuzzy framework respectively in references [3] [8]. Dissipativity and tracking control problems were presented for nonlinear stochastic dynamical systems in references [9] [13].

On the other hand, increasing effort has been paid to the study of event-triggered control (ETC) of nonlinear stochastic systems due to their significance in science and engineering applications. Many important results have been presented for event-triggered control of nonlinear stochastic systems in references [16]-[24]. Dynamic event-triggered control, dynamic self-triggered control and event-triggered stability were investigated for a class of nonlinear stochastic systems by introducing an additional internal dynamic variable in [16] [17]. Based on event-triggered predictive control (ETPC) scheme, a novel discrete-time feedback law was designed for the stabilization of continuous-time stochastic systems with output delay in [24]. The input-to-state practically exponential mean-square stability of stochastic nonlinear delay systems with exogenous disturbances was provided and a framework of event-triggered stabilization was received for the stochastic systems without applying the well-known Lyapunov theorem respectively in [18] and [21]. Periodic event-generators and continuous event-enerators were studied in both static and dynamic cases in [19]. Reference [20] addressed the dynamic event-based fault detection problem of nonlinear stochastic systems influenced by random nonlinearity, data transmission delays and packet dropout. Based on fuzzy technique, the problem of event-triggered optimized control for uncertain nonlinear Itô-type stochastic systems with time-delay was addressed in [22]. The modified unscented Kalman filter was proposed for stochastic nonlinear system with Markov packet dropout in [23].

Although the problem of event-triggered control for nonlinear stochastic systems has been investigated, there has little literature on filtering problem for discrete-time nonlinear stochastic systems. With above inspirations, we aim to propose an event-triggered finite-time filtering scheme for discrete-time nonlinear stochastic systems with exogenous disturbance. We present the definition of SFTS into a class of discrete-time nonlinear stochastic systems. By employing the event-triggered strategy, we construct a detection filter such that the resulting filter error augmented system is SFTS. Sufficient conditions for SFTS of the filter error system is established by constructing the Lyapunov-Krasovskii functional candidate combined with LMIs. The desired event-triggered finite-time filter can be constructed by solving a set of LMIs.

This paper is organized as the following. First, some preliminaries and the problem formulation are introduced in Section 2. In Section 3, in terms of event-triggered technique, a sufficient condition for SFTS of the filter error system is established and a method for designing the corresponding filter is presented. Finally, some conclusions are drawn in Section 4.

Notation: Throughout this paper, the notations used are quite standard. We use R^n to denote the n-dimensional Euclidean space. $R > 0$ denotes a symmetric positive definite matrix. The symbol * in a matrix denotes a term that is defined by symmetry of the matrix. I and 0 denote the identity and zero matrices with appropriate dimensions. $\lambda_{\max}(R)$ and $\lambda_{\min}(R)$ denote the maximum and the minimum of the eigenvalues of a real symmetric matrix *R*. The superscript T denotes the transpose for vectors or matrices. $\Xi(P)$ is the mathematical expectation of *P*. Matrices, if not explicitly stated, are with compatible dimensions.

2. Problem Formulation and Preliminaries

We shall consider the following discrete-time nonlinear stochastic system:

$$\begin{cases} x(k+1) = Ax(k) + f(k,x(k)) + D_1 v(k) + g(k,x(k))\varpi(k) \\ y(k) = Cx(k) + D_2 v(k) \\ z(k) = Lx(k), x(0) = x_0 \in R^n \end{cases} \tag{1}$$

where $x(k) \in R^n$, $y(k) \in R^m$, $v(k) \in R^p$, $z(k) \in R^q$, are state vector, measurement output, external disturbance, and controlled output respectively, $\varpi(k)$ is a one-dimensional zero-mean process which satisfies

$$\Xi[\omega(k)] = 0, \Xi[\omega(i)\omega(j)] = 0, i \neq j, \Xi[\omega^2(k)] = \delta \tag{2}$$

where Ξ is the expected value. Here $\delta > 0$ is a known scalar. The matrices $A, C, D_1 D_2, L$ are constant matrices with appropriate dimensions.

Assumption 1 The nonlinear functions $f(k,x(k))$ and $g(k,x(k))$ satisfy the following quadratic inequalities:

$$\left|f(k,x(k)) - f(k,\tilde{x}(k))\right|^2 \le \varepsilon_1^2 \left|x(k) - \tilde{x}(k)\right|^2$$

$$\left|g(k,x(k)) - g(k,\tilde{x}(k))\right|^2 \le \varepsilon_2^2 \left|x(k) - \tilde{x}(k)\right|^2$$

for all $x(k), \tilde{x}(k) \in R^n$, where $\varepsilon_1, \varepsilon_2 > 0$ are constants related to the function $f(k,x(k))$, $g(k,x(k))$.

Assume that $\{t_k\}_{k \in N}$ denotes the triggered instants and there is no time-delay in sampler and actuator, $t_0 < t_1 < t_2 < \cdots < t_k < t_{k+1}$, $t_k \le k < t_{k+1}$. $x(t_k)$ is the current sampled system state, t_{k+1} is the next sampled instant, which can be determined by the event-trigger, and $x(t_0) = x_0$ is chosen as the initial sampled state.

In this paper, the event-triggering schemes are described by

$$t_{k+1} = \inf\left\{k \ge t_k \mid e_{y(k)}^{\mathrm{T}} Q e_{y(k)} - \eta y^{\mathrm{T}}(k) Q y(k) > 0\right\}, \tag{3}$$

where $e_{y(k)} = y(k) - y(t_k)$, η is a constant and $Q = \Pi^{\mathrm{T}}\Pi$ is a symmetric and positive definite matrix with appropriate dimension to be determined.

We now consider the following filter:

$$\begin{cases} \hat{x}(k+1) = A_f \hat{x}(k) + B_f y(t_k) \\ \hat{z}(k) = L_f \hat{x}(k) \end{cases} \tag{4}$$

where $\hat{x}(k) \in R^n$ is the filter state, and matrices A_f, B_f, L_f are filter parameters with compatible dimensions to be determined.

Define $\bar{x}(k) = \begin{bmatrix} x^{\mathrm{T}}(k) & \hat{x}^{\mathrm{T}}(k) \end{bmatrix}$, $\bar{z}(k) = z(k) - \hat{z}(k)$. Then the filtering error system is

$$\begin{cases} \bar{x}(k+1) = \bar{A}\bar{x}(k) + \bar{F}(k, x(k)) + \bar{D}\bar{v}(k) - \bar{B}_f K\bar{e}_{y(k)} + \bar{G}(k, x(k))\varpi(k) \\ \bar{z}(k) = \bar{L}\bar{x}(k) \end{cases} \tag{5}$$

where

$$\bar{A} = \begin{bmatrix} A & 0 \\ B_f C & A_f \end{bmatrix},\ \bar{F}(k, x(k)) = \begin{bmatrix} f(k, x(k)) \\ 0 \end{bmatrix},\ \bar{G}(k, x(k)) = \begin{bmatrix} g(k, x(k)) \\ 0 \end{bmatrix},$$

$$\bar{D} = \begin{bmatrix} D_1 & 0 \\ B_f D_2 & 0 \end{bmatrix},\ \bar{B}_f = \begin{bmatrix} 0 \\ B_f \end{bmatrix},\ \bar{L} = \begin{bmatrix} L & -L_f \end{bmatrix},\ \bar{e}_{y(k)} = \begin{bmatrix} e_{y(k)} \\ 0 \end{bmatrix},$$

$$\bar{v}(k) = \begin{bmatrix} v(k) \\ 0 \end{bmatrix},\ K = \begin{bmatrix} I & 0 \end{bmatrix}.$$

Before providing the main results, we summarize several needed definitions and lemmas from the literature.

Definition 2.1 The filtering error system (5) with event-triggered scheme (3) and $v(k) = 0$ is said to be stochastic finite-time stable (SFTS) with respect to (c_1, c_2, P, N), where $P > 0, 0 < c_1 < c_2$, if the following relation holds:

$$\Xi\left[x^{\mathrm{T}}(0)Px(0)\right] < c_1 \Rightarrow \Xi\left[x^{\mathrm{T}}(k)Px(k)\right] < c_2 \quad \text{for all} \quad k \in 1, 2, \cdots, N.$$

Definition 2.2 For $\gamma > 0$, suppose the event-triggered residual system in (5) is stochastic finite-time stable (SFTS) with respect to (c_1, c_2, P, N), then system (5) is said to have a weighted H_∞ attenuation level γ for all nonzero $v(k) \in l_2[0, \infty)$, if the following inequality holds:

$$\Xi\left\{\sum\nolimits_{k=k_0}^{\infty} \bar{z}^{\mathrm{T}}(k)\bar{z}(k)\right\} < \gamma^2 \sum\nolimits_{k=k_0}^{\infty} \bar{v}^{\mathrm{T}}(k)\bar{v}(k) \tag{6}$$

Lemma 2.1 ([25]). Let $\Omega \in R^{n \times n}$ be a symmetric matrix, and let $x \in R^n$, then the following inequality holds

$$\lambda_{\min}(\Omega)x^{\mathrm{T}}x \le x^{\mathrm{T}}\Omega x \le \lambda_{\max}(\Omega)x^{\mathrm{T}}x. \tag{7}$$

Lemma 2.2 (Schur complement [26] [27]) Given a symmetric matrix $\phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$, the following three conditions are equivalent to each other:

1) $\phi < 0$;
2) $\phi_{11} < 0$, and $\phi_{22} - \phi_{12}^{\mathrm{T}}\phi_{11}^{-1}\phi_{12} < 0$;
3) $\phi_{22} < 0$, and $\phi_{11} - \phi_{12}\phi_{22}^{-1}\phi_{12}^{\mathrm{T}} < 0$.

3. Main Results

In this section, we focus on stochastic finite-time stable (SFTS) with respect to (c_1, c_2, P, N) of the event-triggered residual system in (5), and propose sufficient conditions of system performance analysis.

Theorem 3.1 For given constants $\gamma, \eta, \delta > 0$ and $\mu > 1$, suppose that there exist symmetric positive definite matrices $R = P^{\frac{1}{2}}\Omega P^{\frac{1}{2}}$ such that the following LMIs hold

$$\Theta < 0 \tag{8}$$

where

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} & \Theta_{15} & \Theta_{16} \\ * & \Theta_{22} & \Theta_{23} & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & -\eta^{\frac{1}{2}} I & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\Omega^{-1} \end{bmatrix}$$

$$\frac{\mu^{N-k_0}\lambda_{\max}(\Omega)c_1}{\lambda_{\min}(\Omega)} \le c_2 ,$$

$$\Theta_{11} = \bar{A}^{\mathrm{T}} R\bar{A} + \bar{A}^{\mathrm{T}} R\Psi_1 + \Psi_1^{\mathrm{T}} R\bar{A} + \Psi_1^{\mathrm{T}} R\Psi_1 + \delta\Psi_2^{\mathrm{T}} R\Psi_2 - \mu R ,$$

$$\Theta_{12} = -\bar{A}^{\mathrm{T}} R\bar{B}_f K - \Psi_1^{\mathrm{T}} R\bar{B}_f K , \quad \Theta_{13} = \bar{A}^{\mathrm{T}} R\bar{D} + \Psi_1^{\mathrm{T}} R\bar{D} , \quad \Psi_i = \begin{bmatrix} \varepsilon_i & 0 \\ 0 & 0 \end{bmatrix}, i = 1,2 ,$$

$$\Theta_{14} = \begin{bmatrix} \bar{\Pi}\bar{C} & 0 & \bar{\Pi}\bar{D}_2 \end{bmatrix}^{\mathrm{T}} , \quad \Theta_{15} = \begin{bmatrix} \bar{L} & 0 & 0 \end{bmatrix}^{\mathrm{T}} , \quad \Theta_{15} = \begin{bmatrix} 0 & 0 & P^{\frac{1}{2}}\bar{D} \end{bmatrix}^{\mathrm{T}} ,$$

$$\Theta_{22} = K^{\mathrm{T}} \bar{B}_f^{\mathrm{T}} R\bar{B}_f K , \quad \Theta_{23} = -K^{\mathrm{T}} \bar{B}_f^{\mathrm{T}} R\bar{D} , \quad \bar{Q} = \begin{bmatrix} Q & 0 \\ 0 & 0 \end{bmatrix} = \bar{\Pi}^{\mathrm{T}}\bar{\Pi} ,$$

$$\bar{Q}_1 = \bar{C}^{\mathrm{T}}\bar{\Pi}^{\mathrm{T}}\bar{\Pi}\bar{C} , \quad \bar{Q}_2 = \bar{C}^{\mathrm{T}}\bar{\Pi}^{\mathrm{T}}\bar{\Pi}\bar{D}_2 , \quad \bar{Q}_3 = \bar{D}_2^{\mathrm{T}}\bar{\Pi}^{\mathrm{T}}\bar{\Pi}\bar{D}_2 ,$$

$$\bar{\Pi} = \begin{bmatrix} \Pi & 0 \end{bmatrix} , \quad \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix} , \quad \bar{D}_2 = \begin{bmatrix} D_2 & 0 \end{bmatrix} .$$

Proof: Consider the following Lyapunov function candidate for system (5):

$$V(\bar{x}(k)) = \bar{x}^{\mathrm{T}}(k) R\bar{x}(k) \tag{9}$$

Then, based on assumption 1, (8) and Schur complement, it follows that

$$\begin{aligned} \Gamma(k) &\triangleq \Xi\left[V(\bar{x}(k+1)) - \mu V(\bar{x}(k))\right] \\ &= \left[\bar{A}\bar{x}(k) + \bar{F}(k,x(k)) - \bar{B}_f K\bar{e}_{y(k)} + \bar{G}(k,x(k))\varpi(k)\right]^{\mathrm{T}} R\left[\bar{A}\bar{x}(k)\right. \\ &\quad \left. + \bar{F}(k,x(k)) - \bar{B}_f K\bar{e}_{y(k)} + \bar{G}(k,x(k))\varpi(k)\right] - \mu\bar{x}^{\mathrm{T}}(k) R\bar{x}(k) \\ &= \begin{bmatrix} \bar{x}^{\mathrm{T}}(k) & \bar{e}_{y(k)}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ * & \Gamma_{22} \end{bmatrix} \begin{bmatrix} \bar{x}(k) \\ \bar{e}_{y(k)} \end{bmatrix} < 0 \end{aligned} \tag{10}$$

where $\Gamma_{11} = \Theta_{11}$, $\Gamma_{12} = \Theta_{12}$, $\Gamma_{22} = \Theta_{22}$.

Then for $\forall k \in [t_k, t_{k+1})$, we have

$$\Xi\left[V(\bar{x}(k))\right] \le \Xi\left[\mu V(\bar{x}(k-1))\right] . \tag{11}$$

Proceeding in an iterative fashion, we obtain the following inequality:

$$\Xi\left[V(\bar{x}(k))\right] < \mu^{k-k_0}\Xi\left[V(\bar{x}(k_0))\right] = \mu^{k-k_0}\Xi\left[V(\bar{x}(0))\right] \le \mu^{N-k_0}\lambda_{\max}(\Omega)c_1 \tag{12}$$

On the other hand, it can be derived from (9) and lemma 2.1 that

$$\Xi\left[V(\bar{x}(k))\right] \ge \lambda_{\min}(\Omega)\Xi\left[\bar{x}^{\mathrm{T}}(k) P\bar{x}(k)\right] . \tag{13}$$

Thus we have that

$$\Xi\left[\bar{x}^{\mathrm{T}}(k)P\bar{x}(k)\right]\leq\frac{\mu^{N-k_0}\lambda_{\max}(\Omega)c_1}{\lambda_{\min}(\Omega)}\leq c_2. \tag{14}$$

According to Definition 2.1, the filter error systems (5) with $v(k)=0$ is SFTS.

Next, we prove the event-based residual system in (5) satisfies H_∞ performance value.

In view of event condition (3) (8), together with Lemma 2.2, the following inequality can be deduced:

$$\begin{aligned}\mathcal{T}(k)\triangleq{}&\Xi\Big[V(\bar{x}(k+1))-\mu V(\bar{x}(k))+\eta y^{\mathrm{T}}(k)Qy(k)-e_{y(k)}^{\mathrm{T}}Qe_{y(k)}\\&+\bar{z}^{\mathrm{T}}(k)\bar{z}(k)-\gamma^2\bar{v}^{\mathrm{T}}(k)\bar{v}(k)\Big]\\={}&\left[\bar{A}\bar{x}(k)+\bar{F}(k,x(k))+\bar{D}\bar{v}(k)-\bar{B}_fK\bar{e}_{y(k)}+\bar{G}(k,x(k))\varpi(k)\right]^{\mathrm{T}}\\&\times R\left[\bar{A}\bar{x}(k)+\bar{F}(k,x(k))+\bar{D}\bar{v}(k)-\bar{B}_fK\bar{e}_{y(k)}+\bar{G}(k,x(k))\varpi(k)\right]\\&-\mu\bar{x}^{\mathrm{T}}(k)R\bar{x}(k)+\eta\left[Cx(k)+D_2v(k)\right]^{\mathrm{T}}R\left[Cx(k)+D_2v(k)\right]\\&-e_{y(k)}^{\mathrm{T}}Qe_{y(k)}+\bar{z}^{\mathrm{T}}(k)\bar{z}(k)-\gamma^2\bar{v}^{\mathrm{T}}(k)\bar{v}(k)\\={}&\left[\bar{x}^{\mathrm{T}}(k)\quad\bar{e}_{y(k)}^{\mathrm{T}}\quad\bar{v}^{\mathrm{T}}(k)\right]\begin{bmatrix}T_{11}&T_{12}&T_{13}\\ *&T_{22}&T_{23}\\ *&*&T_{33}\end{bmatrix}\begin{bmatrix}\bar{x}(k)\\ \bar{e}_{y(k)}\\ \bar{v}(k)\end{bmatrix}<0\end{aligned} \tag{15}$$

where

$$T_{11}=\bar{A}^{\mathrm{T}}R\bar{A}+\bar{A}^{\mathrm{T}}R\Psi_1+\Psi_1^{\mathrm{T}}R\bar{A}+\Psi_1^{\mathrm{T}}R\Psi_1+\delta\Psi_2^{\mathrm{T}}R\Psi_2-\mu R+\eta\bar{Q}_1+\bar{L}^{\mathrm{T}}\bar{L},$$

$$T_{12}=-\bar{A}^{\mathrm{T}}R\bar{B}_fK-\Psi_1^{\mathrm{T}}R\bar{B}_fK,\quad T_{13}=\bar{A}^{\mathrm{T}}R\bar{D}+\Psi_1^{\mathrm{T}}R\bar{D}+\eta\bar{Q}_2,$$

$$T_{22}=K^{\mathrm{T}}\bar{B}_f^{\mathrm{T}}R\bar{B}_fK-\bar{Q},\quad T_{23}=\Theta_{23},\quad T_{33}=\bar{D}^{\mathrm{T}}R\bar{D}+\eta\bar{Q}_3-\gamma^2I,$$

$$\bar{Q}=\begin{bmatrix}Q&0\\0&0\end{bmatrix},\ \bar{Q}_1=\begin{bmatrix}C^{\mathrm{T}}QC&0\\0&0\end{bmatrix},\ \bar{Q}_2=\begin{bmatrix}C^{\mathrm{T}}QD_2&0\\0&0\end{bmatrix},\ \bar{Q}_3=\begin{bmatrix}D_2^{\mathrm{T}}QD_2&0\\0&0\end{bmatrix}.$$

Then we can conclude that (6) holds. It can be concluded that the event-triggered residual system in (5) possesses a prescribed H_∞ performance index proposed in Definition 2.2. Thus the proof is completed.

The following theorem will set forth our filter design method for the system (1).

Theorem 3.2 For given constants $\gamma,\eta,\delta>0$ and $\mu>1$, the filtering error system (5) with the event-triggering strategy (3) is SFTS with respect to (c_1,c_2,P,N) and the error signal satisfies (6), if there exist positive definite matrix $R=\begin{bmatrix}R_{11}&R_{12}\\R_{12}^T&R_{22}\end{bmatrix}$ and matrices G_1,G_2,G_3 with appropriate dimensions satisfying:

$$\tilde{\Theta}<0, \tag{16}$$

where

$$\tilde{\Theta}=\begin{bmatrix}\tilde{\Theta}_{11} & \tilde{\Theta}_{12} & \tilde{\Theta}_{13} & \tilde{\Theta}_{14} & \tilde{\Theta}_{15} & \tilde{\Theta}_{16} \\ * & \tilde{\Theta}_{22} & \tilde{\Theta}_{23} & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & -\eta^{\frac{1}{2}} I & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -P^{\frac{1}{2}} R^{-1} P^{\frac{1}{2}}\end{bmatrix}, \quad (17)$$

$$\tilde{\Theta}_{11}=\tilde{A}^{\mathrm{T}} R \tilde{A}+\tilde{A}^{\mathrm{T}} R \Psi_1+\Psi_1^{\mathrm{T}} R \tilde{A}+\Psi_1^{\mathrm{T}} R \Psi_1+\delta \Psi_2^{\mathrm{T}} R \Psi_2-\mu R,$$

$$\tilde{\Theta}_{12}=-\tilde{A}^{\mathrm{T}} R \tilde{B}_f K-\Psi_1^{\mathrm{T}} R \tilde{B}_f K, \quad \tilde{\Theta}_{13}=\tilde{A}^{\mathrm{T}} R \bar{D}+\Psi_1^{\mathrm{T}} R \bar{D}, \quad \tilde{\Theta}_{14}=\Theta_{14},$$

$$\tilde{\Theta}_{15}=\begin{bmatrix}\tilde{L} & 0 & 0\end{bmatrix}, \quad \tilde{\Theta}_{16}=\Theta_{16}, \quad \tilde{\Theta}_{22}=\tilde{B}_f^{\mathrm{T}} R \tilde{B}_f K,$$

$$\tilde{\Theta}_{23}=-K^{\mathrm{T}} \tilde{B}_f^{\mathrm{T}} R \bar{D} \tilde{A}=\begin{bmatrix}A & 0 \\ R_{22}^{-1} G_2 C & R_{12}^{-1} G_1\end{bmatrix},$$

$$\tilde{B}_f=\begin{bmatrix}0 \\ R_{22}^{-1} G_2\end{bmatrix}, \quad \tilde{L}=\begin{bmatrix}L & -G_3\end{bmatrix}.$$

Moreover, the suitable filter parameters A_f, B_f, L_f in system (4) can be given by

$$A_f=R_{12}^{-1} G_1, B_f=R_{22}^{-1} G_2, L_f=G_3. \quad (18)$$

Proof By Theorem 3.1, let $A_f=R_{12}^{-1} G_1, B_f=R_{22}^{-1} G_2, L_f=G_3$, then the condition (8) is equivalent to (16).

4. Conclusion

In this paper, we have introduced the concept of SFTS into a class of discrete-time nonlinear stochastic systems with exogenous disturbances. We have addressed the event-triggered finite-time filter designing problem. A sufficient condition is provided to guarantee the SFTS of the filter error system. For the presented event-triggering schemes, the criteria for the event-based filter residual systems with a prescribed performance level γ were established by adopting Lyapunov-Krasovski function method. Sufficient conditions for H_∞ performance analysis and corresponding filter designing technique have been provided in a given finite-time interval in terms of LMIs technique, respectively.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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