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Iwueze's Distribution and Its Application

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Abstract

Increased usage of single parameter life-time distributions for reference in development of other life-time distributions and data modeling has attracted the interest of researchers. Because performance ratings differ from one distribution to another and there are increased need for distributions that delivers improved fits, new distributions with a better performance rating capable of providing improved fits have evolved in the Literature. One of such distribution is the Iwueze's distribution. Iwueze's distribution is proposed as a new distribution with Gamma and Exponential baseline distributions. Iwueze's distribution theoretical density, distribution functions and statistical features such as its moments, factors of variation, skewness, kurtorsis, reliability functions, stochastic ordering, absolute deviations from average, absolute deviations from mid-point, Bonferroni and Lorenz curves, Bonferroni and Gini indexes, entropy and the stress and strength reliability have been developed. Iwueze's distribution curve is not bell-shaped, but rather skewed positively and leptokurtic. The risk measurement function is a monotone non-decreasing function, while the average residual measurement life-time function is a monotone non-increasing function. The parameter of the Iwueze's distribution was estimated using the likelihood estimation approach. When used for a real-life data modeling, the new proposed Iwueze's distribution delivers improved and superior fits better than the Akshya, Shambhu, Devya, Amarendra, Aradhana, Sujatha, Akash, Rama, Shanker, Suja, Lindley, Ishita, Prakaamy, Pranav, Exponential, Ram Awadh and Odoma distributions. Iwueze's distribution is definitely tractable and offers a better distribution than a number of well-known distributions for modeling life-time data, with greater superiority of fit performance ratings.

Keywords

Life-Time Distributions, Baseline Distributions, Akash Distribution, Sujatha Distribution, Method of Maximum Likelihood Estimation

1. Introduction

Life-time distributions are statistical distributions whose density functions can be utilized to describe the behavioral structure of life-time data in reliability and life data analysis, model the life and failure time to the occurrence of an event of interest [1]. Single parameter life-time distributions, in which the probability density functions consist of only one scale parameter of reference, are most popular because of the simplicity and flexibility of the structure of its density functions and mathematical operations, easy interpretations of its physical properties, and consequently increased usage as a reference in further development of other life-time distributions in the existing literature.

Researchers in the fields of biological sciences, demography, economics, engineering, insurance, and medical sciences have developed and used single parameter life-time distributions to model the varying behavioral structure of univariate life-time data [1] [2]. The Lindley [3], Shanker [4], Akash [5], Rama [6], Suja [7], Sujatha [8], Amarendra [9], Devya [10], Shambhu [11], Aradhana [12], Akshaya [14], Pranav [15], Ishita [16], Ram Awadh [17], Prakaamy [18] and Odoma [19] distributions are examples. The aforementioned distributions are module compositions of gamma and exponential distributions. Both gamma and exponential distributions have disadvantages. First, the gamma distribution's survivorship or existence measurement function cannot be described in closed form [4] [5]. The aforementioned distributions' survivorship or existence measurement functions, on the other hand, can be summed in closed form. Second, the exponential distribution has a constant risk measurement function that makes it unsuitable for modeling data with monotonically non-decreasing risk measurement rates [17] [18] [20] [21] [22].

It's worth noting that each of the aforementioned distributions can be used to solve problems involving data with monotonically non-decreasing risk measurement rates. When dealing with data with monotonically non-decreasing risk measurement rates, however, evidence of goodness or superiority of fit of distributions fitted to given data is always sought. Because performance ratings differ from one distribution to the next, an alternate distribution is explored when such evidence is lacking. In some cases, the available distributions may not adequately fit the data, which is why researchers are constantly working to modify or introduce new distributions that can be used to model some of the most commonly encountered complex data. As a result, the Iwueze's distribution, a new single parameter distribution, is proposed. Materials and Methods, mathematical and statistical features of the distribution, parameter estimation and applicability, Results, Discussion, and Conclusion are all discussed.

2. Materials and Methods

The theoretical density function (also known as probability density function; denoted p.d.f.) and distribution function (also known as cumulative distribution function; denoted c.d.f.) of the new single parameter Iwueze's life-time distribution, simply known as the Iwueze distribution, as well as the mixture combinations cum ratios of corresponding standard or baseline distributions are given as

$$f(v,\theta)=\begin{cases}\dfrac{\theta^5}{\theta^4+2\theta^3+6\theta^2+12\theta+24}\left[1+v+v^2\right]^2\mathrm{e}^{-\theta v};\theta,v>0\\ 0;\ \text{otherwise}\end{cases} \tag{2.1}$$

and

$$G(v,\theta)=1-\left[1+\frac{\left[\begin{array}{l}\theta^4v^4+2\theta^3(\theta+2)v^3+3\theta^2\left[\theta^2+2(\theta+2)\right]v^2\\ +2\theta\left[\theta^3+3\left[\theta^2+2[\theta+2]\right]\right]v\end{array}\right]}{\theta^4+2\theta^3+6\theta^2+12\theta+24}\right]\mathrm{e}^{-\theta v}. \tag{2.2}$$

where $V=v$ is the random variable and θ is the scale parameter of the distribution.

Remark 1: Iwueze's theoretical density function, Equation (2.1), is a five-module composition of an exponential (θ) and gamma $(2,\theta)$, $(3,\theta)$, $(4,\theta)$ and $(5,\theta)$ baseline distributions such that

$$f(v,\theta)=\sum_{i=1}^{5}p_i g_i(v,\theta) \tag{2.3}$$

where p_i are the *i*th mixing ratios or weights given specifically as;

$$p_1=\frac{\theta^4}{\theta^4+2\theta^3+6\theta^2+12\theta+24} \tag{2.4}$$

$$p_2=\frac{2\theta^3}{\theta^4+2\theta^3+6\theta^2+12\theta+24} \tag{2.5}$$

$$p_3=\frac{6\theta^2}{\theta^4+2\theta^3+6\theta^2+12\theta+24} \tag{2.6}$$

$$p_4=\frac{12\theta}{\theta^4+2\theta^3+6\theta^2+12\theta+24} \tag{2.7}$$

$$p_5=1-\sum_{i=1}^{4}p_i=\frac{24}{\theta^4+2\theta^3+6\theta^2+12\theta+24} \tag{2.8}$$

and $g_i(v,\theta)$ is the *i*th p.d.f. of the standard or baseline distributions also given specifically as;

$$g_i(v,\theta)=\begin{cases}\dfrac{\theta^i v^{i-1}\mathrm{e}^{-\theta v}}{\Gamma(i)};v>0\text{ and for fixed }i=1,2,3,4,5\\ 0,\quad\text{otherwise}\end{cases} \tag{2.9}.$$

Henceforth in the paper, $f(v,\theta)$ and $G(v,\theta)$ will be simply written as $f(v)$ and $G(v)$. We shall say that a random variable V follows Iwueze's distribution, simply written $V\sim IW(\theta)$, if its behavioral structure could be attributed to Equation (2.1). The graphical plots of the theoretical density and distribution function (for some selected but different real points of θ) of an Iwueze's distribution are shown in **Figure 1** and **Figure 2**. The curves shown in

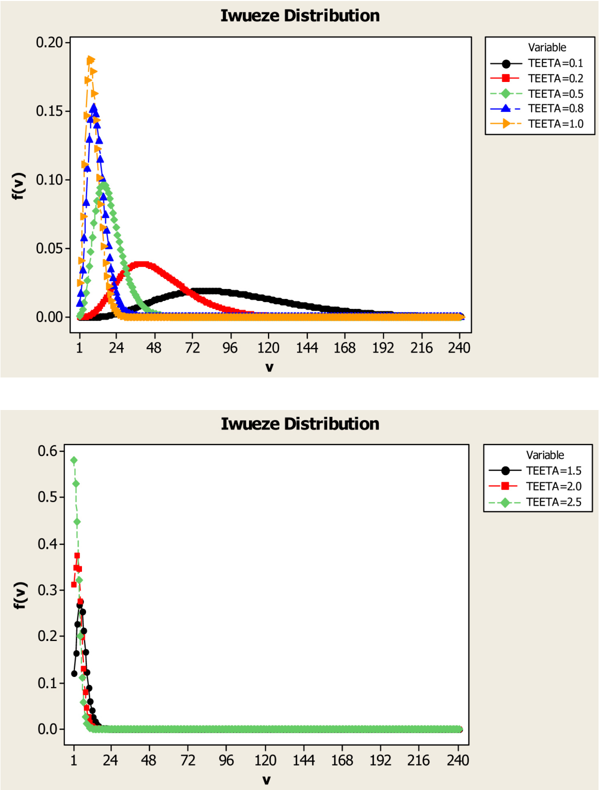


Figure 1. The graphical plots of the theoretical density function (for some selected but different real points of θ) of an Iwueze's distribution.

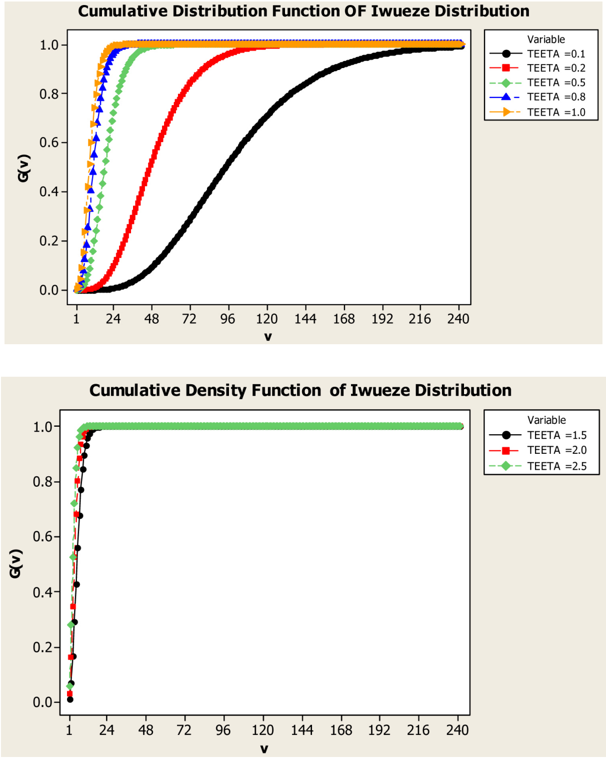


Figure 2. The graphical plots of the theoretical distribution function (for some selected but different real points of θ) of an Iwueze's distribution.

Figure 1 are not bell-shaped, skewed positively and an increase in the value of θ causes a considerable increase in the peak of the curve.

2.1. Generating Moments of Iwueze's Distribution

The moment generating function (denoted $M_V(t)$) of a continuous random variable V with Iwueze's distribution structure can be calculated as follows:

$$\begin{aligned} M_V(t) &= \frac{\theta^5}{\theta^4+2\theta^3+6\theta^2+12\theta+24}\int_0^\infty \left[1+v+v^2\right]^2 \mathrm{e}^{-(\theta-t)v}\mathrm{d}v \\ &= \frac{\theta^5}{\theta^4+2\theta^3+6\theta^2+12\theta+24}\Big[(\theta-t)^{-1}+2(\theta-t)^{-2}+6(\theta-t)^{-3} \\ &\quad +12(\theta-t)^{-4}+24(\theta-t)^{-5}\Big] \\ &= \frac{1}{\theta^4+2\theta^3+6\theta^2+12\theta+24}\left[\theta^4\left(1-\frac{t}{\theta}\right)^{-1}+2\theta^3\left(1-\frac{t}{\theta}\right)^{-2}\right. \\ &\quad \left. +6\theta^2\left(1-\frac{t}{\theta}\right)^{-3}+12\theta\left(1-\frac{t}{\theta}\right)^{-4}+24\left(1-\frac{t}{\theta}\right)^{-5}\right] \end{aligned} \tag{2.10}$$

Noting the famous binomial series expansion [23] that

$$(1-\partial)^{-\ell} = \sum_{\pi=0}^{\infty}\binom{\ell+\pi-1}{\pi}\partial^\pi; \ell>0 \text{ and } |\partial|<1, \tag{2.11}$$

Equation (2.10) reduces to

$$M_V(t) = \frac{\sum_{\pi=0}^{\infty}\begin{bmatrix}\theta^4+2(\pi+1)\theta^3+3(\pi+1)(\pi+2)\theta^2 \\ +2(\pi+1)(\pi+2)(\pi+3)\theta+(\pi+1)(\pi+2)(\pi+3)(\pi+4)\end{bmatrix}\left(\frac{t}{\theta}\right)^\pi}{\theta^4+2\theta^3+6\theta^2+12\theta+24} \tag{2.12}$$

From Equation (2.12), the ωth generalized derivation of the moments about zero or origin) is

$$\mu_\omega^{/} = \frac{\omega!\begin{bmatrix}\theta^4+2(\omega+1)\theta^3+3(\omega+1)(\omega+2)\theta^2 \\ +2(\omega+1)(\omega+2)(\omega+3)\theta+(\omega+1)(\omega+2)(\omega+3)(\omega+4)\end{bmatrix}}{\theta^\omega\left[\theta^4+2\theta^3+6\theta^2+12\theta+24\right]}; \omega=1,2,\cdots \tag{2.13}$$

As a result, precise instances about ωth specific moments about the origin for $\omega=1,2,3,4$ of Iwueze's distribution are defined as follows:

$$\mu_1^{/} = \frac{\theta^4+4\theta^3+18\theta^2+48\theta+120}{\theta\left[\theta^4+2\theta^3+6\theta^2+12\theta+24\right]} = \frac{\theta^4+2\left[2\theta^3+3\left[3\theta^2+4[2\theta+5]\right]\right]}{\theta\left[\theta^4+2\theta^3+6\theta^2+12\theta+24\right]} \tag{2.14}$$

$$\mu_2^{/} = \frac{2\theta^4+12\theta^3+72\theta^2+240\theta+720}{\theta^2\left[\theta^4+2\theta^3+6\theta^2+12\theta+24\right]} = \frac{2\theta^4+3\left[4\theta^3+4\left[6\theta^2+10[2\theta+6]\right]\right]}{\theta^2\left[\theta^4+2\theta^3+6\theta^2+12\theta+24\right]} \tag{2.15}$$

$$\mu_3^{/} = \frac{6\theta^4+4\left[12\theta^3+5\left[18\theta^2+36[2\theta+7]\right]\right]}{\theta^3\left[\theta^4+2\theta^3+6\theta^2+12\theta+24\right]} \tag{2.16}$$

$$\mu_4^{/} = \frac{24\theta^4 + 5\left[48\theta^3 + 6\left[72\theta^2 + 168\left[2\theta + 8\right]\right]\right]}{\theta^4\left[\theta^4 + 2\theta^3 + 6\theta^2 + 12\theta + 24\right]} \quad (2.17)$$

Its essential moments about the mean, on the other hand, are

$$\mu_2 = \sigma^2 = \frac{\theta^8 + 8\theta^7 + 56\theta^6 + 240\theta^5 + 876\theta^4 + 1344\theta^3 + 2304\theta^2 + 2880\theta + 2880}{\theta^2\left[\theta^4 + 2\theta^3 + 6\theta^2 + 12\theta + 24\right]^2} \quad (2.18)$$

$$\mu_3 = \frac{2\begin{bmatrix}\theta^{12} + 12\theta^{11} + 114\theta^{10} + 664\theta^9 + 3132\theta^8 + 7776\theta^7 + 170640\theta^6 \\ + 302400\theta^5 + 492480\theta^4 + 777600\theta^3 + 1036800\theta^2 + 1036800\theta + 69120\end{bmatrix}}{\theta^3\left[\theta^4 + 2\theta^3 + 6\theta^2 + 12\theta + 24\right]^3} \quad (2.19)$$

$$\mu_4 = \frac{3\begin{bmatrix}3\theta^{16} + 48\theta^{15} + 560\theta^{16} + 41920\theta^{13} + 249200\theta^{12} + 965760\theta^{11} \\ + 3198720\theta^{10} + 9000960\theta^9 + 22577760\theta^8 + 49628160\theta^7 \\ + 97873920\theta^6 + 166579200\theta^5 + 242887680\theta^4 + 287539200\theta^3 \\ + 298598400\theta^2 + 232243200\theta + 11612160\end{bmatrix}}{\theta^4\left[\theta^4 + 2\theta^3 + 6\theta^2 + 12\theta + 24\right]^4} \quad (2.20)$$

The factors for evaluating the Iwueze's distribution variation (VF), dispersion (FD), skewness ($\sqrt{\beta_1}$) and kurtorsis (β_2) are as follows:

$$VF = \frac{\sqrt{\theta^8 + 8\theta^7 + 56\theta^6 + 240\theta^5 + 876\theta^4 + 1344\theta^3 + 2304\theta^2 + 2880\theta + 2880}}{\theta^4 + 4\theta^3 + 18\theta^2 + 48\theta + 120} \quad (2.21)$$

$$FD = \frac{\theta^8 + 8\theta^7 + 56\theta^6 + 240\theta^5 + 876\theta^4 + 1344\theta^3 + 2304\theta^2 + 2880\theta + 2880}{\theta\left[\theta^4 + 2\theta^3 + 6\theta^2 + 12\theta + 24\right]\left[\theta^4 + 4\theta^3 + 18\theta^2 + 48\theta + 120\right]} \quad (2.22)$$

$$\sqrt{\beta_1} = \frac{2\begin{bmatrix}\theta^{12} + 12\theta^{11} + 114\theta^{10} + 664\theta^9 + 3132\theta^8 + 7776\theta^7 + 170640\theta^6 \\ + 302400\theta^5 + 492480\theta^4 + 777600\theta^3 + 1036800\theta^2 + 1036800\theta + 69120\end{bmatrix}}{\begin{bmatrix}\theta^8 + 8\theta^7 + 56\theta^6 + 240\theta^5 + 876\theta^4 \\ + 1344\theta^3 + 2304\theta^2 + 2880\theta + 2880\end{bmatrix}^{3/2}} \quad (2.23)$$

$$\beta_2 = \frac{3\begin{bmatrix}3\theta^{16} + 48\theta^{15} + 560\theta^{16} + 41920\theta^{13} + 249200\theta^{12} + 965760\theta^{11} \\ +3198720\theta^{10} + 9000960\theta^9 + 22577760\theta^8 + 49628160\theta^7 \\ +97873920\theta^6 + 166579200\theta^5 + 242887680\theta^4 + 287539200\theta^3 \\ +298598400\theta^2 + 232243200\theta + 11612160\end{bmatrix}}{\left[\theta^8 + 8\theta^7 + 56\theta^6 + 240\theta^5 + 876\theta^4 + 1344\theta^3 + 2304\theta^2 + 2880\theta + 2880\right]^2} \quad (2.24)$$

Figure 3 depicts a graphical depiction of the values of β_2 at various positions of θ. **Figure 3** shows that β_2 for all varying points of θ, the values of β_2 are greater than three, indicating that the Iwueze's distribution is leptokurtic in nature. However, the values of β_2 for $0 < \theta < 1.179$ drop, while the values of β_2 remain constant for $1.179 \le \theta < 1.459$ and increase exponentially for $\theta \ge 1.459$. As a result, **Table 1** shows varying values of θ and associated dispersion structures of Iwueze's and some single parameter distributions.

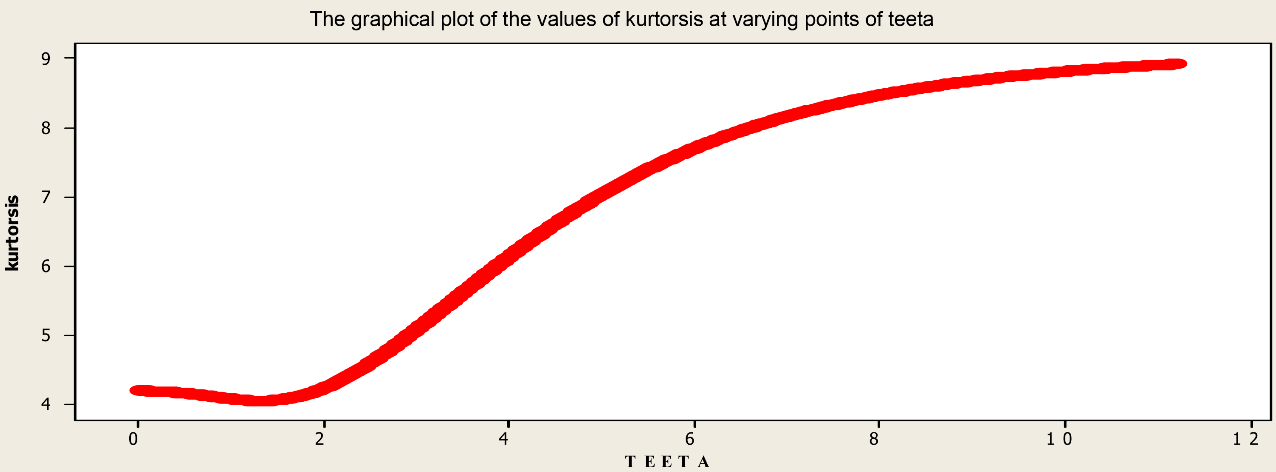


Figure 3. Graphical depiction of the values of β_2 at various positions of θ.

Table 1. Varying values of θ and associated dispersion structures (over (denoted A*), equi (denoted B*) and under (denoted C*) dispersions) of Iwueze's and few number of single parameter distributions.

Distribution Names	A* $\mu < \sigma^2$	B* $\mu = \sigma^2$	C* $\mu > \sigma^2$
Iwueze	$\theta < 1.370727266$	$\theta = 1.370727266$	$\theta > 1.370727266$
Sujatha	$\theta < 1.364271174$	$\theta = 1.364271174$	$\theta > 1.364271174$
Akash	$\theta < 1.515400063$	$\theta = 1.515400063$	$\theta > 1.515400063$
Lindley	$\theta < 1.170086487$	$\theta = 1.170086487$	$\theta > 1.170086487$
Exponential	$\theta < 1$	$\theta = 1$	$\theta > 1$

2.2. Some Statistical Qualities of Iwueze's Distribution

Some statistical properties of Iwueze's distribution are derived, including the survivorship or existence measurement function, risk measurement function, and average residual measurement life-time function, stochastic ordering of random variate, absolute deviations from average and mid points, bonferroni curve, lorenz curve, bonferroni and lorenz indices, entropy, and stress-strength reliability.

2.3. Reliability Indices: Survivorship or Existence Measurement Function, Risk Measurement Function and Average Residual Measurement Life-Time Function

The survivorship or existence measurement function (known as survival rate function [20] [21] [22]) is specified as $\left(s(v,\theta)\right)$, risk measurement function (known as hazard or failure rate function [8] [20] [21] [22] [24]) is specified as $\left(h(v,\theta)\right)$ and average residual measurement life-time function (also known as mean residual life function [8] [21] [24]) is specified as $\left(a_r(v,\theta)\right)$ for a random variable $V, V > 0$ with theoretical density function $f(v)$ and distribution function $G(v)$ are specified as

$$s(v,\theta)=1-G(v) \tag{2.25}$$

$$\begin{aligned}h(v,\theta)&=\lim_{\Delta v\to 0}\left(\frac{P(\text{failure in the interval}(v,v+\Delta v))/\Delta v}{P(\text{surviving up till time } v)}\right)\\&=\frac{1}{s(v,\theta)}\lim_{\Delta v\to 0}\left(\frac{G(v+\Delta v)-G(v)}{\Delta v}\right)=\frac{G'(v)}{s(v)}=\frac{f(v,\theta)}{s(v,\theta)}\end{aligned} \tag{2.26}$$

and

$$a_r(v,\theta)=\frac{1}{s(v,\theta)}\int_x^\infty\left[1-G(t)\right]\mathrm{d}t \tag{2.27}$$

$s(v,\theta)$, $h(v,\theta)$ and $a_r(v,\theta)$ will be simply written as $s(v),h(v)$ and $a_r(v)$ respectively. Therefore, the mathematical expressions for evaluating the Iwueze's distribution $s(v),h(v)$ and $a_r(v)$ are as follows:

$$s(v)=\left[1+\frac{\left[\begin{array}{l}\theta^4v^4+2\theta^3(\theta+2)v^3+3\theta^2\left[\theta^2+2(\theta+2)\right]v^2\\+2\theta\left[\theta^3+3\left[\theta^2+2\left[\theta+2\right]\right]\right]v\end{array}\right]}{\theta^4+2\theta^3+6\theta^2+12\theta+24}\right]\mathrm{e}^{-\theta v} \tag{2.28}$$

$$h(v)=\frac{\theta\left[\theta^2+\theta^2v+\theta^2v^2\right]^2}{\left[\begin{array}{l}\theta^4x^4+2\theta^3(\theta+2)x^3+3\theta^2\left[\theta^2+2(\theta+2)\right]x^2\\+2\theta\left[\theta^3+3\left[\theta^2+2\left[\theta+2\right]\right]\right]x+\left(\theta^4+2\theta^3+6\theta^2+12\theta+24\right)\end{array}\right]} \tag{2.29}$$

$$a_r(v)=\frac{\left[\begin{array}{l}\theta^4v^4+2\theta^3(\theta+4)v^3+3\theta^2\left[\theta^2+4(\theta+3)\right]v^2\\+2\theta\left[\theta^3+6\left[\theta^2+2\theta+8\right]\right]v+\left(\theta^4+4\theta^3+18\theta^2+48\theta+120\right)\end{array}\right]}{\theta\left[\begin{array}{l}\theta^4x^4+2\theta^3(\theta+2)x^3+3\theta^2\left[\theta^2+2(\theta+2)\right]x^2\\+2\theta\left[\theta^3+3\left[\theta^2+2\left[\theta+2\right]\right]\right]x+\left(\theta^4+2\theta^3+6\theta^2+12\theta+24\right)\end{array}\right]} \tag{2.30}$$

Observe that $s(0)=1$, $h(0)=\dfrac{\theta^5}{\theta^4+2\theta^3+6\theta^2+12\theta+24}=f(0)$ and $a_r(0)=\dfrac{\theta^4+4\theta^3+18\theta^2+48\theta+120}{\theta\left(\theta^4+2\theta^3+6\theta^2+12\theta+24\right)}=\mu_1'$. **Figure 4** and **Figure 5** show graphs of $h(v)$ and $a_r(v)$ of Iwueze's distribution for some selected but different real points or values of θ respectively. The graphical plot of $h(v)$ clearly shows that $h(v)$ is monotone non-decreasing function in v and θ, while $a_r(v)$ is monotone non-increasing function in v and θ.

2.4. Stochastic Ordering of Iwueze's Random Variables

The comparative behaviour of continuous variables from Iwueze's distribution can be appropriately examined and\or determined using the stochastic ordering of any two selected random variables, say $T,V>0$. Then, $T<V$ in

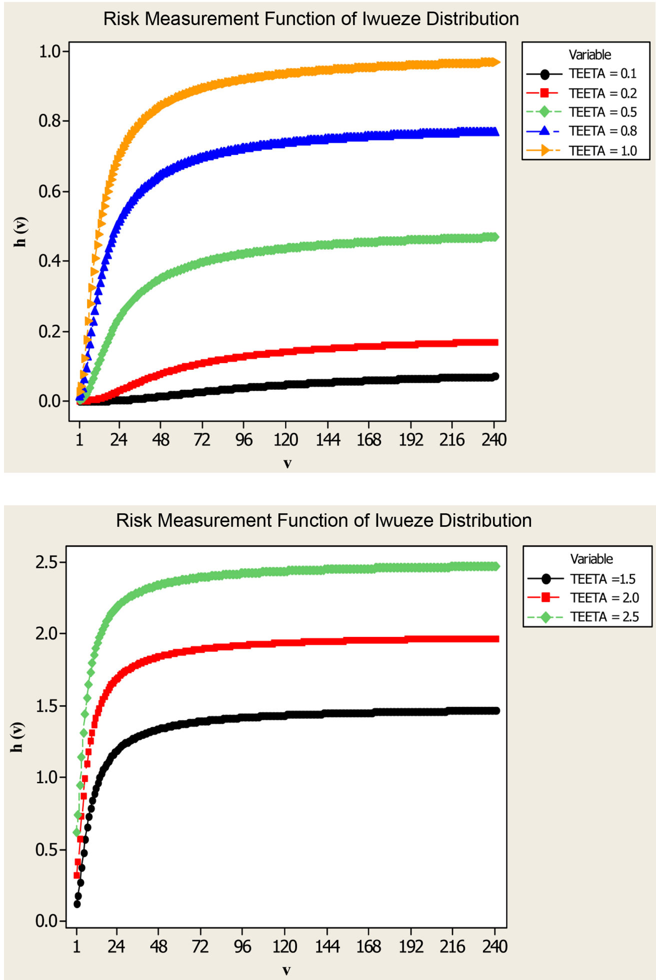


Figure 4. The graphical plots of the risk measurement function $(h(v))$ of Iwueze's distribution for some selected but different real points of θ.

1) Stochastic ordering; that is

$$T \leq_{stor} V \quad \text{if} \quad G_T(\varsigma) \geq G_V(\varsigma) \quad \text{for all} \quad \varsigma \tag{2.31}$$

2) Risk Measurement ordering; that is

$$T \leq_{Ror} V \quad \text{if} \quad h_T(\varsigma) \geq h_V(\varsigma) \quad \text{for all} \quad \varsigma \tag{2.32}$$

3) Average residual Measurement life ordering; that is

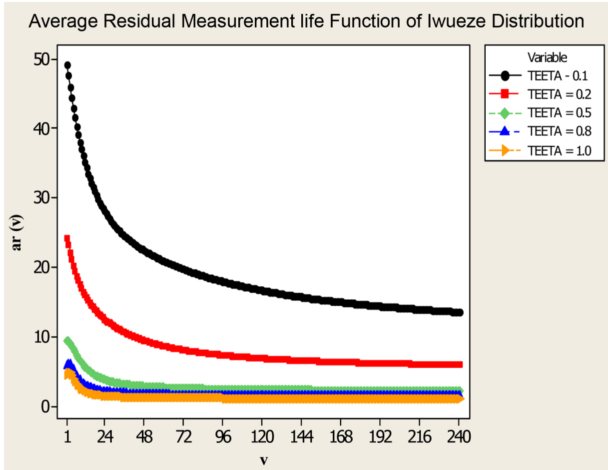
$$T \leq_{arl} V \quad \text{if} \quad ar_T(\varsigma) \geq ar_V(\varsigma) \quad \text{for all} \quad \varsigma \tag{2.33}$$

4) Likelihood Ratio ordering; that is

$$T \leq_{lor} V \quad \text{if} \quad \frac{f_T(\varsigma)}{f_V(\varsigma)} \quad \text{for all} \quad \varsigma \tag{2.34}$$

These results had been established in the literature for stochastic ordering of distributions [9] [25];

$$T \leq_{lor} V \Rightarrow T \leq_{Ror} V \Rightarrow \begin{cases} T \leq_{stor} V \\ T \leq_{arl} V \end{cases} \tag{2.35}$$



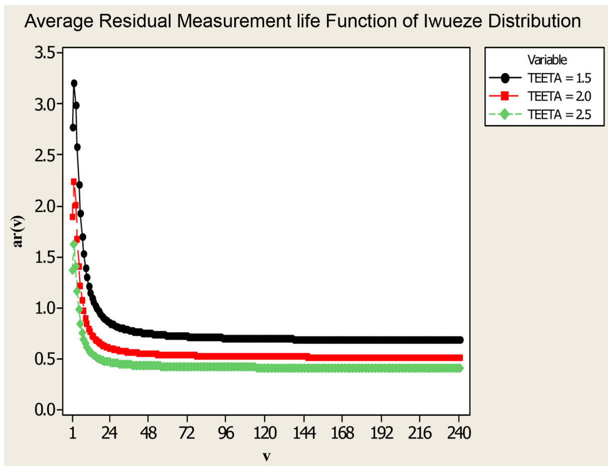


Figure 5. The graphical plots of average residual measurement life-time function $a_r(v)$ of Iwueze's distribution for some selected but different real points of θ.

Theorem 1.0

Let $T \sim IW(\theta_1)$ and $V \sim IW(\theta_2)$. If $\theta_1 > \theta_2$ then $T \leq_{lor} V$ and by implication $T \leq_{Ror} V$, $T \leq_{arl} V$ and $T \leq_{stor} V$.

Proof: Let $T \sim IW(\theta_1)$ and $V \sim IW(\theta_2)$. We obtain that

$$\frac{f_T(\varsigma)}{f_V(\varsigma)} = \frac{\theta_1^5\left(\theta_2^4 + 2\theta_2^3 + 6\theta_2^2 + 12\theta_2 + 24\right)}{\theta_2^5\left(\theta_1^4 + 2\theta_1^3 + 6\theta_1^2 + 12\theta_1 + 24\right)} e^{-(\theta_1-\theta_2)\varsigma}; \varsigma > 0$$

and

$$\log_e \frac{f_T(\varsigma)}{f_V(\varsigma)} = \log_e \left(\frac{\theta_1^5\left(\theta_2^4 + 2\theta_2^3 + 6\theta_2^2 + 12\theta_2 + 24\right)}{\theta_2^5\left(\theta_1^4 + 2\theta_1^3 + 6\theta_1^2 + 12\theta_1 + 24\right)}\right) - (\theta_1 - \theta_2)\varsigma$$

Hence,

$$\frac{d}{d\varsigma} \log_e \frac{f_T(\varsigma)}{f_V(\varsigma)} = -(\theta_1 - \theta_2)$$

By implication, $\frac{d}{d\varsigma} \log_e \frac{f_T(\varsigma)}{f_V(\varsigma)} < 0$ if $\theta_1 > \theta_2$. This result implies that

$T \le_{lor} V \Rightarrow T \le_{Ror} V \Rightarrow \begin{cases} T \le_{stor} V \\ T \le_{arl} V \end{cases}$. The result indicated clearly that Iwueze distribution is ordered in the likelihood ratio and consequently has risk measurement, average residual measurement life and stochastic orderings.

2.5. Absolute Deviations from Average and Mid-Points

The absolute deviations about the average and mid-points respectively can be used to assess the attitude of variation inherent in a group of observations. The absolute deviations about the average point (denoted $\psi_1(v)$) and that about the mid-point (denoted $\psi_2(v)$) is defined by

$$\psi_1(v) = E\left(\left|V-\mu^*\right|\right) = \int_0^\infty \left|v-\mu^*\right| f(v)\,\mathrm{d}v \tag{2.36}$$

$$\psi_2(v) = E\left(\left|V-M^*\right|\right) = \int_0^\infty \left|v-M^*\right| f(v)\,\mathrm{d}v \tag{2.37}$$

The absolute deviation about the average point can be calculated using

$$\begin{aligned} \psi_1(v) &= \int_0^\infty \left|v-\mu^*\right| f(v)\,\mathrm{d}v \\ &= \int_0^{\mu^*} \left|\mu^*-v\right| f(v)\,\mathrm{d}v + \int_{\mu^*}^\infty \left|v-\mu^*\right| f(v)\,\mathrm{d}v \qquad (2.38) \\ &= 2\mu^* G\left(\mu^*\right) - 2\int_0^{\mu^*} v f(v)\,\mathrm{d}v \end{aligned}$$

While, the absolute deviation about the mid-point is calculated using

$$\begin{aligned} \psi_2(v) &= \int_0^\infty \left|v-M^*\right| f(v)\,\mathrm{d}v \\ &= \int_0^{m^*} \left|M^*-v\right| f(v)\,\mathrm{d}v + \int_{m^*}^\infty \left|v-M^*\right| f(v)\,\mathrm{d}v \\ &= \mu^* - 2\int_0^{m^*} v f(v)\,\mathrm{d}v \qquad (2.39) \\ &= -\mu^* + 2\int_{m^*}^\infty v f(v)\,\mathrm{d}v \qquad (2.40) \end{aligned}$$

where $\mu^* = E(V)$ is the average of the random variable *V*, M^* is the mid-point of the random variable V and $G\left(\mu^*\right) = \int_0^{\mu^*} f(v)\,\mathrm{d}v$. By using p.d.f. (2.1), expressions (2.41) through (2.44) were obtained as follows:

$$\int_0^{\mu^{**}} v f(v)\,\mathrm{d}v = \mu - \frac{\begin{bmatrix} \theta^5\mu^{*5} + \left[2\theta+5\right]\theta^4\mu^{*4} + \left[3\theta^2+8\theta+20\right]\theta^3\mu^{*3} \\ +\left[2\theta^3+9\theta^2+24\theta+60\right]\theta^2\mu^{*2} \\ +\left[\theta^4+4\theta^3+18\theta^2+48\theta+120\right]\theta\mu^* \\ +\left[\theta^4+4\theta^3+18\theta^2+48\theta+120\right] \end{bmatrix} \mathrm{e}^{-\theta\mu^*}}{\theta\left[\theta^4+2\theta^3+6\theta^2+12\theta+24\right]} \tag{2.41}$$

$$\int_0^{m^*} vf(v)\mathrm{d}v = \mu^* - \frac{\begin{bmatrix} \theta^5 m^{*5} + [2\theta+5]\theta^4 m^{*4} + [3\theta^2+8\theta+20]\theta^3 m^{*3} \\ +[2\theta^3+9\theta^2+24\theta+60]\theta^2 m^{*2} \\ +[\theta^4+4\theta^3+18\theta^2+48\theta+120]\theta m^* \\ +[\theta^4+4\theta^3+18\theta^2+48\theta+120] \end{bmatrix} \mathrm{e}^{-\theta m^*}}{\theta[\theta^4+2\theta^3+6\theta^2+12\theta+24]} \quad (2.42)$$

$$\int_{m^*}^{\infty} vf(v)\mathrm{d}v = \frac{\begin{bmatrix} \theta^5 m^{*5} + [2\theta+5]\theta^4 m^{*4} + [3\theta^2+8\theta+20]\theta^3 m^{*3} \\ +[2\theta^3+9\theta^2+24\theta+60]\theta^2 m^{*2} \\ +[\theta^4+4\theta^3+18\theta^2+48\theta+120]\theta m^* \\ +[\theta^4+4\theta^3+18\theta^2+48\theta+120] \end{bmatrix} \mathrm{e}^{-\theta m^*}}{\theta[\theta^4+2\theta^3+6\theta^2+12\theta+24]} \quad (2.43)$$

$$\int_0^{\mu^{**}} f(v)\mathrm{d}v = 1 - \frac{\begin{bmatrix} \theta^4 \mu^{*4} + [2\theta+4]\theta^3 \mu^{*3} + [3\theta^2+6\theta+12]\theta^2 \mu^{*2} \\ +[2\theta^3+6\theta^2+12\theta+24]\theta\mu^* \\ +[\theta^4+2\theta^3+6\theta^2+12\theta+24] \end{bmatrix} \mathrm{e}^{-\theta\mu^*}}{[\theta^4+2\theta^3+6\theta^2+12\theta+24]} \quad (2.44)$$

Substituting Equations (2.41) and (2.44) in Equation (2.38), and Equation (2.42) in Equation (2.39) or Equation (2.43) in Equation (2.40), we obtain the expressions for calculating the absolute deviations about the average and mid-points respectively of an Iwueze's distribution as:

$$\psi_1(v) = \frac{2\begin{bmatrix} [\mu^{*2}+\mu^*+1]^2\theta^4 + 4[2\mu^{*3}+3\mu^{*2}+3\mu^*+1]\theta^3 \\ +18[2\mu^{*2}+2\mu^*+1]\theta^2 + 48[2\mu^*+1]\theta + 120 \end{bmatrix} \mathrm{e}^{-\theta\mu^*}}{\theta[\theta^4+2\theta^3+6\theta^2+12\theta+24]} \quad (2.45)$$

$$\psi_2(v) = \frac{2\begin{bmatrix} m^*[m^{*2}+m^*+1]^2\theta^5 + [5m^{*4}+8m^{*3}+9m^{*2}+4m^*+1]\theta^4 \\ +2[10m^{*3}+12m^{*2}+9m^*+2]\theta^3 + 6[10m^{*2}+8m^*+3]\theta^2 \\ +24[5m^*+2]\theta+120 \end{bmatrix} \mathrm{e}^{-\theta m^*}}{\theta[\theta^4+2\theta^3+6\theta^2+12\theta+24]} - \mu^* \quad (2.46)$$

2.6. Bonferroni Curve, Lorenz Curve, Bonferroni Index and Gini Index

The Bonferroni curve, Lorenz curve, Bonferroni index and Gini index [26] have a wide range of applications in economics, medicine, insurance, demography, and reliability. The Bonferroni curve is defined as follows [26]:

$$B(c) = \frac{1}{c\mu^*}\int_0^q vf(v)\mathrm{d}v = \frac{1}{c\mu^*}\left[\int_0^\infty vf(v)\mathrm{d}v - \int_q^\infty vf(v)\mathrm{d}v\right] = \frac{1}{c\mu^*}\left[\mu^* - \int_q^\infty vf(v)\mathrm{d}v\right] \quad (2.47)$$

or alternatively, Equation (2.47) is written equivalently as

$$B(c)=\frac{1}{c\mu^{*}}\int_{0}^{p}G^{-1}(v)\,\mathrm{d}v \tag{2.48}$$

while, the Lorenz curve [26] is defined as

$$L(c)=\frac{1}{\mu^{*}}\int_{0}^{q}vf(v)\,\mathrm{d}v=\frac{1}{\mu^{*}}\left[\int_{0}^{\infty}vf(v)\,\mathrm{d}v-\int_{q}^{\infty}vf(v)\,\mathrm{d}v\right]=\frac{1}{\mu^{*}}\left[\mu^{*}-\int_{q}^{\infty}vf(v)\,\mathrm{d}v\right] \tag{2.49}$$

or alternatively, Equation (2.49) is written equivalently as

$$L(c)=\frac{1}{\mu^{*}}\int_{0}^{p}G^{-1}(v)\,\mathrm{d}v \tag{2.50}$$

where $\mu^{*}=E(V)$ and $q=G^{-1}(c)$. However, the Bonferroni index [26] is defined as

$$B=1-\int_{0}^{1}B(c)\,\mathrm{d}c \tag{2.51}$$

while the Gini index [26] is

$$G=1-2\int_{0}^{1}L(c)\,\mathrm{d}c \tag{2.52}$$

But using Equation (2.1), we obtain

$$\int_{q}^{\infty}vf(v)\,\mathrm{d}v=\frac{\begin{bmatrix}\theta^{5}q^{5}+[2\theta+5]\theta^{4}q^{4}+[3\theta^{2}+8\theta+20]\theta^{3}q^{3}\\+[2\theta^{3}+9\theta^{2}+24\theta+60]\theta^{2}q^{2}\\+[\theta^{4}+4\theta^{3}+18\theta^{2}+48\theta+120]\theta q\\+[\theta^{4}+4\theta^{3}+18\theta^{2}+48\theta+120]\end{bmatrix}\mathrm{e}^{-\theta q}}{\theta[\theta^{4}+2\theta^{3}+6\theta^{2}+12\theta+24]} \tag{2.53}$$

On substituting Equation (2.53) in Equations (2.47) and (2.49), the expressions for calculating the Bonferroni and Lorenz curves respectively of an Iwueze's distribution are obtained as:

$$B(c)=\frac{1}{c}\left[1-\frac{\begin{bmatrix}q[q^{2}+q+1]^{2}\theta^{5}+[5q^{4}+8q^{3}+9q^{2}+4q+1]\theta^{4}\\+2[10q^{3}+12q^{2}+9q+2]\theta^{3}+6[10q^{2}+8q+3]\theta^{2}\\+24[5q+2]\theta+120\end{bmatrix}\mathrm{e}^{-\theta q}}{[\theta^{4}+4\theta^{3}+18\theta^{2}+48\theta+120]}\right] \tag{2.54}$$

$$L(c)=1-\frac{\begin{bmatrix}q[q^{2}+q+1]^{2}\theta^{5}+[5q^{4}+8q^{3}+9q^{2}+4q+1]\theta^{4}\\+2[10q^{3}+12q^{2}+9q+2]\theta^{3}+6[10q^{2}+8q+3]\theta^{2}\\+24[5q+2]\theta+120\end{bmatrix}\mathrm{e}^{-\theta q}}{[\theta^{4}+4\theta^{3}+18\theta^{2}+48\theta+120]} \tag{2.55}$$

In addition, on substituting Equations (2.54) and (2.55) in Equations (2.51) and (2.52) respectively, the expression for Bonferroni index of an Iwueze's distribution is obtained as

$$B = 1 - \frac{\begin{bmatrix} q\left[q^2+q+1\right]^2\theta^5+\left[5q^4+8q^3+9q^2+4q+1\right]\theta^4 \\ +2\left[10q^3+12q^2+9q+2\right]\theta^3+6\left[10q^2+8q+3\right]\theta^2 \\ +24\left[5q+2\right]\theta+120 \end{bmatrix}\mathrm{e}^{-\theta q}}{\left[\theta^4+4\theta^3+18\theta^2+48\theta+120\right]} \tag{2.56}$$

while, the expression for the Gini index of an Iwueze's distribution is:

$$G = \frac{2\begin{bmatrix} q\left[q^2+q+1\right]^2\theta^5+\left[5q^4+8q^3+9q^2+4q+1\right]\theta^4 \\ +2\left[10q^3+12q^2+9q+2\right]\theta^3+6\left[10q^2+8q+3\right]\theta^2 \\ +24\left[5q+2\right]\theta+120 \end{bmatrix}\mathrm{e}^{-\theta q}}{\left[\theta^4+4\theta^3+18\theta^2+48\theta+120\right]} - 1. \tag{2.57}$$

2.7. Entropy

Entropy is the most well-known statistical feature and/or quality that accounts for the uncertainty in a continuous random variable's probability distribution. The most prominent type of entropy, known as the Renyi entropy [27], is defined for V with. $f(v)$ as follows:

$$T_{RE}(\Im) = \frac{1}{1-\Im}\log\left[\int_0^\infty f^{\Im}(v)\mathrm{d}v\right]; \Im > 0 \text{ and } \Im \neq 1 \tag{2.58}$$

By substituting Equation (2.1) into Equation (2.58), we obtain the expression necessary for obtaining the entropy of Iwueze distribution as

$$\begin{aligned} T_{RE}(\Im) &= \frac{1}{1-\Im}\log\left[\int_0^\infty \frac{\theta^{5\Im}}{\left[\theta^4+2\theta^3+6\theta^2+12\theta+24\right]^{\Im}}\left[1+v+v^2\right]^{2\Im}\mathrm{e}^{-\Im\theta v}\mathrm{d}v\right] \\ &= \frac{1}{1-\Im}\log\left[\frac{\theta^{5\Im}}{\left[\theta^4+2\theta^3+6\theta^2+12\theta+24\right]^{\Im}}\int_0^\infty\sum_{i=0}^{\infty}\binom{2\Im}{i}v^i\left[1+v\right]^i\mathrm{e}^{-\Im\theta v}\mathrm{d}v\right] \\ &= \frac{1}{1-\Im}\log\left[\frac{\theta^{5\Im}}{\left[\theta^4+2\theta^3+6\theta^2+12\theta+24\right]^{\Im}}\sum_{i=0}^{\infty}\binom{2\Im}{i}\sum_{k=0}^{\infty}\binom{i}{k}\int_0^\infty v^{i+k}\mathrm{e}^{-\Im\theta v}\mathrm{d}v\right] \end{aligned} \tag{2.59}$$

But on solving $\int_0^\infty v^{i+k}\mathrm{e}^{-\Im\theta v}\mathrm{d}v$ and noting the well known gamma relational form $\int_0^\infty h^{\eta}\mathrm{e}^{-\theta h}\mathrm{d}h = \frac{\Gamma(\eta+1)}{\theta^{\eta+1}} = \frac{\eta!}{\theta^{\eta+1}}$; $\eta > 0$ and η is an interger, Equation (2.59) reduces to

$$T_{RE}(\Im)=\frac{1}{1-\Im}\log\left[\frac{\theta^{5\Im}}{\left[\theta^4+2\theta^3+6\theta^2+12\theta+24\right]^{\Im}}\sum_{i=0}^{\infty}\binom{2\Im}{i}\sum_{k=0}^{\infty}\binom{i}{k}\frac{(i+k)!}{(\Im\theta)^{I+k+1}}\right]$$
$$=\frac{1}{1-\Im}\log\left[\sum_{i=0}^{\infty}\binom{2\Im}{i}\sum_{k=0}^{\infty}\binom{i}{k}\frac{(i+k)!\theta^{5\Im-(i+k+1)}}{\left[\theta^4+2\theta^3+6\theta^2+12\theta+24\right]^{\Im}\Im^{I+k+1}}\right] \quad (2.60)$$

2.8. Stress and Strength Reliability

The stress and strength reliability of an independent strength; say V and stress, say B, of random variables from an Iwueze's distribution with parameters θ_1 and θ_2 is evaluated and calculated using [28]:

$$R_{SS}=P(V<B)=\int_0^{\infty}P(V<B/B=b)\,\mathrm{d}b=\int_0^{\infty}f(v,\theta_1)G(v,\theta_2)\,\mathrm{d}v$$
$$=\int_0^{\infty}\frac{\theta_1^5\left[1+v+v^2\right]^2\mathrm{e}^{-\theta_1 v}}{\left[\theta_1^4+2\theta_1^3+6\theta_1^2+12\theta_1+24\right]}\left[1-\left[\frac{\begin{bmatrix}\left[\theta_2^4+2\theta_2^3+6\theta_2^2+12\theta_2+24\right]+\theta_2^4v^4+2\theta_2^3(\theta_2+2)v^3\\+3\theta_2^2\left[\theta_2^2+2(\theta_2+2)\right]v^2+2\theta_2\left[\theta_2^3+3\left[\theta_2^2+2[\theta_2+2]\right]\right]v\end{bmatrix}\mathrm{e}^{-\theta_2 v}}{\theta_2^4+2\theta_2^3+6\theta_2^2+12\theta_2+24}\right]\right]\mathrm{d}v \quad (2.61)$$
$$=1-\frac{\theta_1^5\begin{bmatrix}\theta_2^4t_1(\theta_1,\theta_2)+2\theta_2^3t_2(\theta_1,\theta_2)+6\theta_2^2t_3(\theta_1,\theta_2)\\+12\theta_2t_4(\theta_1,\theta_2)+24t_5(\theta_1,\theta_2)\end{bmatrix}}{\left[\theta_1^4+2\theta_1^3+6\theta_1^2+12\theta_1+24\right]\left[\theta_2^4+2\theta_2^3+6\theta_2^2+12\theta_2+24\right]\left[\theta_1+\theta_2\right]^9}$$

where

$$t_1(\theta_1,\theta_2)=\left[[\theta_1+\theta_2]^8+4[\theta_1+\theta_2]^7+20[\theta_1+\theta_2]^6+96[\theta_1+\theta_2]^5+456[\theta_1+\theta_2]^4\right.$$
$$\left.+1920[\theta_1+\theta_2]^3+7200[\theta_1+\theta_2]^2+20160[\theta_1+\theta_2]+40320\right] \quad (2.62)$$

$$t_2(\theta_1,\theta_2)=[\theta_1+\theta_2]\left[[\theta_1+\theta_2]^7+5[\theta_1+\theta_2]^6+24[\theta_1+\theta_2]^5+114[\theta_1+\theta_2]^4\right.$$
$$\left.+480[\theta_1+\theta_2]^3+1800[\theta_1+\theta_2]^2+5040[\theta_1+\theta_2]+10080\right] \quad (2.63)$$

$$t_3(\theta_1,\theta_2)=[\theta_1+\theta_2]^2\left[[\theta_1+\theta_2]^6+4[\theta_1+\theta_2]^5+18[\theta_1+\theta_2]^4+72[\theta_1+\theta_2]^3\right.$$
$$\left.+264[\theta_1+\theta_2]^2+720[\theta_1+\theta_2]+1440\right] \quad (2.64)$$

$$t_4(\theta_1,\theta_2)=[\theta_1+\theta_2]^3\left[[\theta_1+\theta_2]^5+4[\theta_1+\theta_2]^4+14[\theta_1+\theta_2]^3+48[\theta_1+\theta_2]^2\right.$$
$$\left.+120[\theta_1+\theta_2]+240\right] \quad (2.65)$$

$$t_5(\theta_1,\theta_2)=[\theta_1+\theta_2]^4\left[[\theta_1+\theta_2]^4+2[\theta_1+\theta_2]^3+6[\theta_1+\theta_2]^2+12[\theta_1+\theta_2]+24\right] \quad (2.66)$$

2.9. Iwueze's Distribution Parameter Estimation Technique

The method of likelihood estimation is a methodology for estimating parameters

θ from a random sample of size s ($V_1, V_2, V_3, \cdots, V_s$) that follows an Iwueze's distribution with a theoretical density function in Equation (2.1) and the likelihood function $L^*(\theta)$ is given as

$$L^*(\theta) = L^* = \left(\frac{\theta^5}{\theta^4 + 2\theta^3 + 6\theta^2 + 12\theta + 24}\right)^s \prod_{i=1}^{s}\left[1 + v + v^2\right]^2 e^{-\theta\sum_{i=1}^{s} v_i} \quad (2.67)$$

The natural logarithm of $L(\theta)$ is calculated as follows:

$$L^{*1} = \ln L^*(\theta)$$

$$= 5s\ln\theta - s\ln\left(\theta^4 + 2\theta^3 + 6\theta^2 + 12\theta + 24\right) + \sum_{i=1}^{s}\ln\left[1 + v + v^2\right]^2 - \theta\sum_{i=1}^{s} v_i \quad (2.68)$$

$$= 5s\ln\theta - s\ln\left(\theta^4 + 2\theta^3 + 6\theta^2 + 12\theta + 24\right) + \sum_{i=1}^{s}\ln\left[1 + v + v^2\right]^2 - s\theta\bar{v}$$

Hence,

$$\frac{dL^{*1}}{d\theta} = \frac{5s}{\theta} - \frac{s\left(4\theta^3 + 6\theta^2 + 12\theta + 12\right)}{\theta^4 + 2\theta^3 + 6\theta^2 + 12\theta + 24} - s\bar{v}, \quad (2.69)$$

By equating Equation (2.69) to zero, the estimate for $\hat{\theta}$ of θ that maximizes Equation (2.68) was calculated using

$$\frac{5s}{\theta} - \frac{s\left(4\theta^3 + 6\theta^2 + 12\theta + 12\right)}{\left(\theta^4 + 2\theta^3 + 6\theta^2 + 12\theta + 24\right)} - s\bar{v} = 0, \quad (2.70)$$

Equation (2.70) reduces to

$$\bar{v}\theta^5 + (2\bar{v} - 1)\theta^4 + 2(3\bar{v} - 2)\theta^3 + 6(2\bar{v} - 3)\theta^2 + 24(\bar{v} - 2)\theta - 120 = 0 \quad (2.71)$$

Where the sample average is $\bar{v}$. The Quintic Equation (2.71) can be solved analytically, numerically and using Mathematical Software such as R, Mathematica etc. Solving the Quintic Equation (2.71) yields five values for $\hat{\theta}$ in which only one satisfies $\hat{\theta} > 0$.

2.10. Application and Test for Superiority (Goodness) of Fit

The likelihood estimate for θ (abbreviated MLE $\hat{\theta}$) was employed to fit Iwueze's, Lindley [3], Shanker [4], Akash [5], Rama [6], Suja [7], Sujatha [8], Amarendra [9], Devya [10], Shambhu [11], Aradhana [12], Akshya [14], Pranav [15], Ishita [16], Ram Awadh [17], Prakaamy [18], Odoma [19] and Exponential [20] [21] [22] distributions to the real-life data set on relief times (in minutes) of twenty (20) patients receiving analgesic treatment. Iwueze's and the aforementioned distributions constitute the competing distributions for modeling the real-life data. The real life-time data used in the study is shown in **Table 2**.

Table 2. Relief times (in minutes) of twenty (20) patients receiving an analgesic.

1.1	1.4	1.3	1.7	1.9	1.8	1.6	2.2	1.7	2.7
4.1	1.8	1.5	1.2	1.4	3	1.7	2.3	1.6	2

See [1].

Then, the superiority (goodness) of fit for the competing distributions fitted to the real-life data set was compared and judged using the values of the superiority of fit indices. The probability (denoted p), $-2L^{*1}$, Akaike Information Criterion (denoted AIC) [29] [30] [31] [32], AICC (Akaike Information Criterion Corrected [30]), BIC (Bayesian Information Criterion) [30] [31] [32] [33] and Kolmogorov-Smirnov (denoted $K^{*}-S^{*}$) [31] values were evaluated and utilized as the superiority of fit indices for comparison's in order to judge the superiority (or goodness) of the fit of the competing distributions. The computational expressions for evaluating AIC, AICC, BIC and $K^{*}-S^{*}$ statistics are:

$$\text{AIC} = -2L^{*1} + 2m \tag{2.72}$$

$$\text{AICC} = \text{AIC} + \frac{2m(m+1)}{z-m-1} \tag{2.73}$$

$$\text{BIC} = -2L^{\wedge 1} + mLn(z) \tag{2.74}$$

$$D = \sup_{v} \left| G_z(v) - G_0(v) \right| \tag{2.75}$$

where the totality of estimated parameters equals one is m, the totality of all observations is z and $G_Z(v)$ is the theoretical distribution function. The decision will then be made to accept the distribution with the lower Probability (p) and $K^{*}-S^{*}$ values, as well as the least and/or lowest value for $-2L^{\wedge 1}$, AIC, AICC and BIC among all considered distributions, as the distribution that provides superior fit to the life-time data.

3. Results

With gamma and exponential distributions as the baseline distributions, the theoretical density of an Iwueze's distribution has been formulated. Its distribution function has also been derived. The curve pattern for Iwueze's distribution is depicted in **Figure 1** while, that for the distribution function is shown in Figure2for some selected but different real points of θ. **Figure 3** depicts the curve pattern for the values of the kurtorsis, denoted β_2, at various positions of θ. **Table 1** illustrates varying values of θ and associated dispersion structures of Iwueze's and few number of single parameter distributions. **Table 3** shows the values of the likelihood estimates (denoted $\hat{\theta}$) and superiority of fit indices (particularly, $-2L^{\wedge 1}$, AIC, AICC, BIC, and p values) for Iwueze's and some well known single distributions fitted to the real life-time data. **Table 3** clearly illustrates that among all competing distributions, Iwueze's has lower p and $K^{*}-S^{*}$ values, as well as the least and/or lowest value for, AIC, AICC, and BIC. As a result, Iwueze's distribution delivers improved and superior fits to the real life-time data.

4. Discussion

Iwueze's distribution has been proposed as a new single parameter distribution for defining the behavioral structure of univariate life-time data, with monotonically non-decreasing Risk measurement and non-increasing average residual

Table 3. Likelihood Estimates of θ (denoted MLE $\hat{\theta}$) and superiority of fit indices.

Distribution Names	MLE $\hat{\theta}$	$-2L^{*1}$	AIC	AICC	BIC	$K^* - S^*$	p-values
IWUEZE	**1.801254**	**51.89**	**53.89**	**54.11**	**54.88**	**0.28**	**0.0050**
AKSHYA	1.441686	53.01	55.01	55.23	56.01	0.46	0.0000
SHAMBHU	2.21539	53.89	55.89	56.11	56.89	0.50	0.0000
DEVYA	1.841946	54.50	56.50	56.73	57.50	0.55	0.0000
AMARENDRA	1.480769	55.63	57.63	57.85	58.63	0.47	0.0003
ARADHANA	1.12319	56.37	58.37	58.59	59.37	0.44	0.0008
SUJATHA	1.136745	57.49	59.49	59.71	60.49	0.44	0.0007
AKASH	1.15692	59.52	61.52	61.74	62.52	0.44	0.0007
RAMA	1.52133	59.70	61.7	61.92	62.70	0.47	0.0003
SHANKER	0.8039	59.78	61.78	62.00	62.78	0.44	0.0008
SUJA	1.895379	60.40	62.40	62.62	63.40	0.49	0.0001
LINDLEY	0.81612	60.49	62.49	62.71	63.49	0.45	0.0012
ISHITA	1.094847	60.16	62.16	62.38	63.16	0.33	0.0049
PRAKAAMY	2.2735	61.43	63.43	63.65	64.43	0.52	0.0290
PRANAV	1.401401	62.38	64.38	64.60	65.38	0.49	0.0049
EXPONENTIAL	0.52632	65.67	67.67	67.89	68.67	0.47	0.0002
RAM AWADH	2.04587	68.52	70.52	70.74	71.52	0.51	0.5240
ODOMA	2.1863030	72.01	74.01	74.23	75.01	0.19	0.0510

See [13].

measurement functions. Moments, skewness, kurtosis, and dispersion, reliability functions (survivorship or existence measurement, risk measurement, and average residual measurement life-time functions), stochastic ordering, absolute deviations from average, and mid points have all been stated analytically. The Iwueze distribution's Bonferroni curve, Lorenz curve, Bonferroni Index, Gini index, Entropy, and stress-strength reliability have all been fully derived. The Iwueze' distribution is positively skewed, not symmetric, as shown by the curve pattern in Figure 1, and increasing the value of θ causes a significant increase in the curve's peak. Similarly, the curve pattern in Figure 3 indicates that values of β_2 at various positions of θ are greater than three. As a result, the Iwueze distribution curve in Figure 3 is leptokurtic in nature. Using the maximum likelihood estimate methodology, the parameter estimation of Iwueze's distribution was investigated and presented. A real life-time data set was used to demonstrate the applicability of Iwueze's distribution and to determine its superiority of fit over various rival distributions (see Table 3 for the listed competing distributions). Results in Table 3 show that Iwueze's distribution has the lowest $K^* - S^*$

and p values, as well as the lowest AIC, AICC, and BIC values, among all competing distributions. The results show that Iwueze's distribution fits the real-life data better. As a result, it is preferred for representing the behavioral structure of the real life-time data set over all other considered distributions.

5. Conclusion

Iwueze's distribution is a new single life-time distribution that can be used to describe the behavioral structure of life-time data. Iwueze's distribution is a better model for describing the behavioral structure of life-time data, because it has lower $-2L^{*1}$, AIC, AICC, and BIC values than all selectedrival or competing distributions. As a result, Iwueze's distribution is a crucial distribution for modeling life-time data, because it delivers improved and superior fits to life-time data.

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Conflicts of Interest

None.

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