Journal of Applied Mathematics and Physics, 2022, 10, 3684-3692
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Fourth Rank Energy-Momentum Tensor

Vu B. Ho

Mulgrave, Australia
Email: vubho@bigpond.net.au

How to cite this paper: Ho, V.B. (2022) Fourth Rank Energy-Momentum Tensor. *Journal of Applied Mathematics and Physics*, **10**, 3684-3692.
https://doi.org/10.4236/jamp.2022.1012246

Received: October 15, 2022
Accepted: December 25, 2022
Published: December 28, 2022

Abstract

In this work, we introduce the new concept of fourth rank energy-momentum tensor. We first show that a fourth rank electromagnetic energy-momentum tensor can be constructed from the second rank electromagnetic energy-momentum tensor. We then generalise to construct a fourth rank stress energy-momentum tensor and apply it to Dirac field of quantum particles. Furthermore, since the established fourth rank energy-momentum tensors have mathematical properties of the Riemann curvature tensor, thus it is reasonable to suggest that quantum fields should also possess geometric structures of a Riemannian manifold.

Keywords

Fourth Rank Energy-Momentum Tensor, Riemannian Manifold, Riemann Curvature Tensor, Electromagnetic Field, Dirac Field

1. Introduction

The objective of this work is to introduce and construct a fourth rank energy-momentum tensor with the aim of providing the new tensor with the required mathematical structures of the Riemann curvature tensor. The compatibility would then allow to establish field equations in terms of the Riemann curvature tensor and a fourth rank energy-momentum tensor, in a manner analogous to Einstein field equations which associate the second rank Ricci curvature tensor to the second rank energy-momentum tensor in general relativity [1]. Since the concept of a fourth rank energy-momentum tensor had emerged from our previous work on the geometric formulation of classical physics, therefore for completeness we give a brief outline of the formulation.

We have shown in our work on geometric formulation of physics that classical physics can be formulated as field equations written in the general form [2]

$$\nabla_\mu \mathrm{M} = \mathrm{J} \tag{1}$$

where M is a mathematical object, J is a physical current, and ∇_β is a covariant derivative. For Newton mechanics in Euclidean space, we can set $\mathrm{M}=E$ with $E=(m/2)\sum_{\mu=1}^{3}\left(\mathrm{d}x^\mu/\mathrm{d}t\right)^2+V$ and $\mathrm{J}=0$. For Maxwell electromagnetic field, $\mathrm{M}=F^{\alpha\beta}$, where the electromagnetic tensor $F^{\alpha\beta}$ expressed in terms of the four-vector potential $A^\mu\equiv(V,\mathbf{A})$ as $F^{\alpha\beta}=\partial A^\beta/\partial x^\alpha-\partial A^\alpha/\partial x^\beta$ with the four-current $\mathrm{J}=j^\beta\equiv(\rho_e,\mathbf{j}_e)$. On the other hand, from Einstein field equations of general relativity

$$R_{\alpha\beta}=k\left(T_{\alpha\beta}-\frac{1}{2}g_{\alpha\beta}T\right) \tag{2}$$

we can obtain general relativistic field equations for the Ricci curvature tensor as

$$\nabla_\beta R^{\alpha\beta}=j^\alpha \tag{3}$$

where $j^\alpha=-(1/2k)g^{\alpha\beta}\nabla_\beta T$. Then, also using Einstein field equations in Equation (2), general relativistic field equations for the Riemann curvature tensor in the form of the general equation in Equation (1) can also be obtained as

$$\nabla_\mu R^\mu_{\beta\nu\lambda}=J_{\beta\nu\lambda} \tag{4}$$

where the third rank current $J_{\beta\nu\lambda}$ is defined in terms of the second rank energy-momentum tensor as $J_{\beta\nu\lambda}=\nabla_\nu R_{\beta\lambda}-\nabla_\lambda R_{\beta\nu}=k\nabla_\nu\left(T_{\beta\lambda}-g_{\beta\lambda}T/2\right)-k\nabla_\lambda\left(T_{\beta\nu}-g_{\beta\nu}T/2\right)$. Furthermore, since the second rank Ricci curvature tensor is associated with a second rank energy-momentum tensor $T_{\alpha\beta}$, therefore we also suggested that there should be a fourth rank energy-momentum tensor $T_{\alpha\beta\mu\nu}$ correlated with the Riemann curvature tensor in the way that they should possess comparable intrinsic mathematical structures. In fact, in the following we first show that such a fourth rank electromagnetic energy-momentum tensor can be constructed from the second rank electromagnetic energy-momentum tensor, and then we generalise to construct a fourth rank stress energy-momentum tensor and apply it, as an illustration, to Dirac field of quantum particles.

2. Fourth Rank Electromagnetic Energy-Momentum Tensor

In classical electrodynamics, as the electromagnetic field tensor $F_{\mu\nu}$ is expressed in terms of the four-vector potential A_μ in the form $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$, therefore the electromagnetic field tensor $F_{\mu\nu}$ is antisymmetric with respect to the covariant indices μ and ν. The electromagnetic energy-momentum tensor is then given by

$$T_{\alpha\mu}=k\left(-F_{\alpha\beta}F^\beta_\mu+\frac{1}{4}\eta_{\alpha\mu}F_{\lambda\sigma}F^{\lambda\sigma}\right) \tag{5}$$

where $\eta_{\mu\nu}=\mathrm{diag}(1,-1,-1,-1)$ is the Minkowski metric tensor and k is a dimensional constant [3]. It is observed that from the energy-momentum tensor $T_{\alpha\mu}$ in Equation (5), by renaming the contravariant index β of F^β_μ to F^ν_μ, a fourth rank electromagnetic energy-momentum tensor can be specified to take the following form

$$T^{\nu}_{\alpha\beta\mu} = k\left(-F_{\alpha\beta}F^{\nu}_{\mu} + \frac{1}{4}\eta_{\alpha\beta}\eta^{\nu}_{\mu}F_{\lambda\sigma}F^{\lambda\sigma}\right) \tag{6}$$

From the tensor $T^{\nu}_{\alpha\beta\mu}$, then a fourth rank covariant electromagnetic energy-momentum tensor $T_{\alpha\beta\mu\nu}$ is obtained by lowering the contravariant index ν

$$T_{\alpha\beta\mu\nu} = k\left(-F_{\alpha\beta}F_{\mu\nu} + \frac{1}{4}\eta_{\alpha\beta}\eta_{\mu\nu}F_{\lambda\sigma}F^{\lambda\sigma}\right) \tag{7}$$

It is seen that the fourth rank electromagnetic energy-momentum tensor in Equation (7) is essentially a tensor product of two antisymmetric tensors of rank two. The fourth rank tensor $T_{\alpha\beta\mu\nu}$ also possesses some tensorial properties of the Riemann curvature tensor, such as it is antisymmetric with respect to the two indices α and β, and to the two indices μ and ν. Furthermore, if the fourth rank tensor $T_{\alpha\beta\mu\nu}$ is also required to satisfy conservation laws, like the Bianchi identities for the Riemann curvature tensor, then we may form general relativistic field equations determined by the electromagnetic field as

$$R_{\alpha\beta\mu\nu} = \kappa T_{\alpha\beta\mu\nu} \tag{8}$$

In the next section, using the mathematical structures that have been established for the electromagnetic field, we will construct a fourth rank stress energy-momentum tensor.

3. Fourth Rank Stress Energy-Momentum Tensor

In continuum mechanics, the Cauchy stress tensor σ_{ij} that associates the traction vector T_i with the normal vector n_j across a surface is defined according to the equation [4] [5] [6]

$$T_i = \sum_{j=1}^{3}\sigma_{ij}n_j \tag{9}$$

In the Minkowski spacetime, the Cauchy stress tensor σ_{ij} has been generalised to the stress energy-momentum tensor $T_{\alpha\beta}$ that represents the energy density, the flux of energy, and the flux of momentum. The electromagnetic energy-momentum tensor given in Equation (5) demonstrates that the concept of the stress energy-momentum tensor in continuum mechanics can also be applied to a physical field. We may in fact speculate that the established form given in Equation (5) for the electromagnetic field tensor can be considered as a general form of the stress energy-momentum tensor, hence if we apply the same procedure then we may be able to generalise the second rank stress energy-momentum tensor $T_{\alpha\beta}$ to a fourth rank stress energy-momentum tensor. With this consideration, following the tensorial structure given in Equation (5), we first assume that the Cauchy stress energy-momentum tensor $T_{\alpha\beta}$ can also be expressed in terms of a second rank antisymmetric tensor $A_{\alpha\beta}$ in the composite form

$$T_{\alpha\mu} = k\left(-A_{\alpha\beta}A^{\beta}_{\mu} + \frac{1}{4}\eta_{\alpha\mu}A_{\lambda\sigma}A^{\lambda\sigma}\right) \tag{10}$$

Then to regain the symmetric form for the Cauchy stress energy-momentum tensor, we let the second rank antisymmetric tensor $A_{\alpha\beta}$ be written in the matrix form

$$A_{\alpha\beta} = \begin{pmatrix} 0 & a_x & a_y & a_z \\ -a_x & 0 & -b_z & b_y \\ -a_y & b_z & 0 & -b_x \\ -a_z & -b_y & b_x & 0 \end{pmatrix} \tag{11}$$

A convincing reason for suggesting this form for the construction of the stress energy-momentum tensor $T_{\alpha\mu}$ is that the spatial part of the tensor $A_{\alpha\beta}$ can be used to characterize an oriented area, which is an essential element for the establishment of the Cauchy stress tensor. It is also noticed that because the components of the Cauchy stress tensor have the physical dimension N/m^2, thus the components of the tensor $A_{\alpha\beta}$ should have the physical dimension $\sqrt{N}/m$. This type of physical dimension may be related to wave motion because it is known that the quantity $\sqrt{N}$ is related to the speed of the transverse wave motion on a string under tension. Consequently, the wave dynamics of a quantum particle may be the result of coupling of two vibrations, similar to the coupling of the electric and magnetic field in electromagnetism. Then, quantum particles may exist as localised waves at the quantum level and their wave character can only be revealed by a particular experimental arrangement [7]. The stress energy-momentum tensor in Equation (10) can be expressed in terms of the components of the antisymmetric tensor $A_{\alpha\beta}$ in Equation (11) as follows

$$T_{\alpha\mu} = k \begin{pmatrix} \frac{1}{2}\left(a^2+b^2\right) & -a_y b_z + a_z b_y & a_x b_z - a_z b_x & -a_x b_y + a_y b_x \\ -a_y b_z + a_z b_y & \frac{1}{2}\left(-a_x^2 + a_y^2 + a_z^2 - b_x^2 + b_y^2 + b_z^2\right) & -a_x a_y - b_x b_y & -a_x a_z - b_x b_z \\ a_x b_z - a_z b_x & -a_x a_y - b_x b_y & \frac{1}{2}\left(a_x^2 - a_y^2 + a_z^2 + b_x^2 - b_y^2 + b_z^2\right) & -a_y a_z - b_y b_z \\ -a_x b_y + a_y b_x & -a_x a_z - b_x b_z & -a_y a_z - b_y b_z & \frac{1}{2}\left(a_x^2 + a_y^2 - a_z^2 + b_x^2 + b_y^2 - b_z^2\right) \end{pmatrix} \tag{12}$$

Now, similar to the establishment of the fourth rank electromagnetic energy-momentum tensor given in Section 2, a fourth rank stress energy-momentum tensor can be derived to take the form similar to Equation (6)

$$T_{\alpha\beta\mu}^{\nu} = k\left(-A_{\alpha\beta} A_{\mu}^{\nu} + \frac{1}{4}\eta_{\alpha\beta}\eta_{\mu}^{\nu} A_{\lambda\sigma} A^{\lambda\sigma}\right) \tag{13}$$

Then, a fourth rank covariant stress energy-momentum tensor $T_{\alpha\beta\mu\nu}$ can be obtained as

$$T_{\alpha\beta\mu\nu} = k\left(-A_{\alpha\beta} A_{\mu\nu} + \frac{1}{4}\eta_{\alpha\beta}\eta_{\mu\nu} A_{\lambda\sigma} A^{\lambda\sigma}\right) \tag{14}$$

Since the fourth rank covariant stress energy-momentum tensor in Equation (14) also has mathematical properties similar to that of the Riemann curvature tensor, therefore, in addition to required conservation laws, we may establish a system of general relativistic field equations in the form given in Equation (8) as $R_{\alpha\beta\mu\nu} = \kappa T_{\alpha\beta\mu\nu}$. As an illustration, in the next section, we discuss the possibility of applying the established fourth rank stress energy-momentum tensor to Dirac field of quantum particles.

4. Dirac Field Fourth Rank Energy-Momentum Tensor

We now show how to establish a fourth rank energy-momentum tensor for Dirac field of quantum particles. The prospect that Dirac and Maxwell fields behave physically in the same manner can be predicted from the fact that, as we have shown in our previous work, both Maxwell field equations of electromagnetism and Dirac field equations for quantum particles can be formulated from a general system of linear first order partial differential equations [8]. As a consequence, the field equations of the two physical fields have many common features that specify characteristics that are not typical in classical physics. The similarity between the Maxwell and Dirac field can be extended by constructing a fourth rank energy-momentum tensor for Dirac field which has similar structure to that of Maxwell field. For reference, we give a brief summary of our previous formulation. The system of linear first order partial differential equations that we need to use in this work is given as

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}^{r} \frac{\partial \psi_i}{\partial x_j} = k_1 \sum_{l=1}^{n} b_l^r \psi_l + k_2 c^r, \quad r = 1, 2, \cdots, n \tag{15}$$

The system of equations in Equation (15) can be rewritten in a matrix form as

$$\left(\sum_{i=1}^{n} A_i \frac{\partial}{\partial x_i} \right) \psi = k_1 \sigma \psi + k_2 J \tag{16}$$

where $\psi = (\psi_1, \psi_2, \cdots, \psi_n)^{\mathrm{T}}$, $\partial\psi/\partial x_i = (\partial\psi_1/\partial x_i, \partial\psi_2/\partial x_i, \cdots, \partial\psi_n/\partial x_i)^{\mathrm{T}}$, A_i, σ and J are matrices representing the quantities a_{ij}^k, b_l^r and c^r, and k_1 and k_2 are undetermined constants. By applying the operator $\sum_{i=1}^{n} A_i \partial/\partial x_i$, assuming further that the coefficients a_{ij}^k and b_l^r are constants and $A_i \sigma = \sigma A_i$, then we obtain

$$\begin{aligned} &\left(\sum_{i=1}^{n} A_i^2 \frac{\partial^2}{\partial x_i^2} + \sum_{i=1}^{n}\sum_{j>i}^{n} \left(A_i A_j + A_j A_i \right) \frac{\partial^2}{\partial x_i \partial x_j} \right) \psi \\ &= k_1^2 \sigma^2 \psi + k_1 k_2 \sigma J + k_2 \sum_{i=1}^{n} A_i \frac{\partial J}{\partial x_i} \end{aligned} \tag{17}$$

In the case of Maxwell and Dirac field, the matrices A_i must be determined to take a form so that Equation (17) can be reduced to a wave equation

$$\left(\sum_{i=1}^{n} A_i^2 \frac{\partial^2}{\partial x_i^2} \right) \psi = k_1^2 \sigma^2 \psi + k_1 k_2 \sigma J + k_2 \sum_{i=1}^{n} A_i \frac{\partial J}{\partial x_i} \tag{18}$$

For Dirac field, the matrices A_i are required to satisfy the conditions $A_i A_j + A_j A_i = 0$ and $A_i^2 = \pm 1$. For Maxwell field, the conditions required for the matrices A_i can be determined from the classical form of Maxwell field equations and Gauss's law $\sum_{i=1}^{n}\partial\psi_i/\partial x_i=\rho$. From Equation (16), Maxwell field equations of electromagnetism can be written in the form

$$\left(A_0\frac{\partial}{\partial t}+A_1\frac{\partial}{\partial x_1}+A_2\frac{\partial}{\partial x_2}+A_3\frac{\partial}{\partial x_3}\right)\psi=k_1\psi+k_2J \tag{19}$$

where the matrices A_i are given as

$$A_0=\begin{pmatrix}-1&0&0&0&0&0\\0&-1&0&0&0&0\\0&0&-1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{pmatrix},\quad A_1=\begin{pmatrix}0&0&0&0&0&0\\0&0&0&0&0&-1\\0&0&0&0&1&0\\0&0&0&0&0&0\\0&0&-1&0&0&0\\0&1&0&0&0&0\end{pmatrix},$$
$$A_2=\begin{pmatrix}0&0&0&0&0&1\\0&0&0&0&0&0\\0&0&0&-1&0&0\\0&0&1&0&0&0\\0&0&0&0&0&0\\-1&0&0&0&0&0\end{pmatrix},\quad A_3=\begin{pmatrix}0&0&0&0&-1&0\\0&0&0&1&0&0\\0&0&0&0&0&0\\0&-1&0&0&0&0\\1&0&0&0&0&0\\0&0&0&0&0&0\end{pmatrix} \tag{20}$$

On the other hand, the Dirac field equations for a free particle can be written in the form [9]

$$\gamma^{\mu}\partial_{\mu}\psi=-im\psi \tag{21}$$

where the matrices $A_i=\gamma_i$ are given as

$$\gamma_0=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix},\quad \gamma_1=\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix},$$
$$\gamma_2=\begin{pmatrix}0&0&0&-i\\0&0&i&0\\0&i&0&0\\-i&0&0&0\end{pmatrix},\quad \gamma_3=\begin{pmatrix}0&0&1&0\\0&0&0&-1\\-1&0&0&0\\0&1&0&0\end{pmatrix} \tag{22}$$

It is seen from the above formulation for Dirac field that Dirac field equations require only four components therefore the second rank antisymmetric tensor $A_{\alpha\beta}$ for Dirac field is reduced to a form written in terms of the field components as

$$A_{\mu\nu}=\begin{pmatrix}0&\psi_1&\psi_2&0\\-\psi_1&0&0&\psi_4\\-\psi_2&0&0&-\psi_3\\0&-\psi_4&\psi_3&0\end{pmatrix} \tag{23}$$

With the form of the second rank antisymmetric tensor $A_{\alpha\beta}$ given in Equation (23), the stress energy-momentum tensor for the Dirac field can be expressed in terms of the components of the antisymmetric tensor $A_{\alpha\beta}$ as

$$T_{\alpha\mu} = k\left(-A_{\alpha\beta}A_{\mu}^{\beta} + \frac{1}{4}\eta_{\alpha\mu}A_{\lambda\sigma}A^{\lambda\sigma}\right)$$
$$= k\begin{pmatrix} \frac{1}{2}\left(\psi_1^2+\psi_2^2\right) & 0 & 0 & -\psi_1\psi_4+\psi_2\psi_3 \\ 0 & \frac{1}{2}\left(-\psi_1^2+\psi_2^2-\psi_3^2+\psi_4^2\right) & -\psi_1\psi_2-\psi_3\psi_4 & 0 \\ 0 & -\psi_1\psi_2-\psi_3\psi_4 & \frac{1}{2}\left(\psi_1^2-\psi_2^2+\psi_3^2-\psi_4^2\right) & 0 \\ -\psi_1\psi_4+\psi_2\psi_3 & 0 & 0 & \frac{1}{2}\left(\psi_1^2+\psi_2^2+\psi_3^2+\psi_4^2\right) \end{pmatrix} \quad (24)$$

Also, as in the case of the general stress energy-momentum tensor, we obtain fourth rank stress energy-momentum tensors for Dirac field of quantum particles which take the form given in Equations (13) and (14). It is also noted that these forms of energy-momentum tensors may be related to the spin of quantum particles because, as shown in the case of electromagnetism, that the Poynting vector can be used to describe a circulating flow of internal energy, which in turns indicates the existence of an intrinsic angular momentum [10]. In fact, these intrinsic dynamic properties of Dirac quantum particles can be realised from Dirac field equations given in Equation (21) when they are rewritten as a system of real equations [11]

$$-\frac{\partial\psi_1}{\partial t} = \frac{\partial\psi_4}{\partial x} + \frac{\partial\psi_3}{\partial z}, \quad -\frac{\partial\psi_2}{\partial t} = \frac{\partial\psi_3}{\partial x} - \frac{\partial\psi_4}{\partial z} \quad (25)$$

$$-\frac{\partial\psi_3}{\partial t} = \frac{\partial\psi_2}{\partial x} + \frac{\partial\psi_1}{\partial z}, \quad -\frac{\partial\psi_4}{\partial t} = \frac{\partial\psi_1}{\partial x} - \frac{\partial\psi_2}{\partial z} \quad (26)$$

$$\frac{\partial\psi_4}{\partial y} = m\psi_1, \quad \frac{\partial\psi_3}{\partial y} = -m\psi_2, \quad \frac{\partial\psi_2}{\partial y} = -m\psi_3, \quad \frac{\partial\psi_1}{\partial y} = m\psi_4 \quad (27)$$

From the system of real equations in Equations (25)-(27) we can derive the system of equations for all components ψ_i as

$$\frac{\partial^2\psi_i}{\partial^2 y} - m^2\psi_i = 0 \quad (28)$$

$$\frac{\partial^2\psi_i}{\partial t^2} - \frac{\partial^2\psi_i}{\partial x^2} - \frac{\partial^2\psi_i}{\partial z^2} = 0 \quad (29)$$

Solutions to Equation (28) are

$$\psi_i = c_{1i}\left(x,z\right)\mathrm{e}^{my} + c_{2i}\left(x,z\right)\mathrm{e}^{-my} \quad (30)$$

where c_{1i} and c_{2i} are undetermined functions. These solutions give a distribution of physical matter along the *y*-axis. On the other hand, solutions to Equation (29) describe the dynamics, for instance, of a vibrating membrane. In polar coordinates $x = r\cos\theta$, $z = r\sin\theta$, the two-dimensional wave equation in Equation (29) becomes [12]

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi}{\partial r^2} - \frac{1}{r}\frac{\partial\psi}{\partial r} - \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} = 0 \quad (31)$$

General solutions to Equation (31) can be found as

$$\psi(r,\theta,t)=\sum_{m=1}^{\infty}J_0\left(\sqrt{\lambda_{0m}}r\right)\left(C_{0m}\cos\sqrt{\lambda_{0m}}ct+D_{0m}\sin\sqrt{\lambda_{0m}}ct\right)$$
$$+\sum_{m,n=1}^{\infty}J_n\left(\sqrt{\lambda_{nm}}r\right)\left(A_{nm}\cos n\theta+B_{nm}\sin n\theta\right) \tag{32}$$
$$\times\left(\left(C_{nm}\cos\sqrt{\lambda_{nm}}ct+D_{nm}\sin\sqrt{\lambda_{nm}}ct\right)\right)$$

where $J_n\left(\sqrt{\lambda_{nm}}r\right)$ is the Bessel function, and the quantities A_{nm}, B_{nm}, C_{nm} and D_{nm} are determined from the initial and boundary conditions. Furthermore, real-valued Dirac field equations can be used to describe a standing wave in a fluid due to the motion of two waves in opposite directions. At its steady state in which the system is time-independent, the system of equations in Equations (25)-(27) reduces to the following system of equations

$$\frac{\partial\psi_2}{\partial x}+\frac{\partial\psi_1}{\partial z}=0,\quad \frac{\partial\psi_1}{\partial x}-\frac{\partial\psi_2}{\partial z}=0 \tag{33}$$

$$\frac{\partial\psi_4}{\partial x}+\frac{\partial\psi_3}{\partial z}=0,\quad \frac{\partial\psi_3}{\partial x}-\frac{\partial\psi_4}{\partial z}=0 \tag{34}$$

Dirac field equations for steady states with the field (ψ_1,ψ_2) and (ψ_3,ψ_4) satisfy the Cauchy-Riemann equations in the (x,z)-plane, therefore it is possible to consider a Dirac quantum particle as a physical system which exists in a fluid state as defined in the classical fluid dynamics. Finally, it should also be mentioned that with the real-valued tensor for Dirac field in Equation (24), we can apply Einstein field equations $R_{\alpha\beta}=k\left(T_{\alpha\beta}-g_{\alpha\beta}T/2\right)$ to describe geometric structures of Dirac quantum particles, and their most intrinsic geometric structures can be described by using the Riemann field equations $R_{\alpha\beta\mu\nu}=\kappa T_{\alpha\beta\mu\nu}$.

5. Conclusion

We have shown that it is possible to construct a fourth rank energy-momentum tensor which may be considered as a generalisation of the second rank electromagnetic energy-momentum tensor and the second rank Cauchy stress energy-momentum tensor. Despite the fact that the generalisation is rather speculative, it is possible to test experimentally for the case of the fourth rank electromagnetic energy-momentum tensor. On the other hand, the Cauchy fourth rank stress energy-momentum tensor may be used to construct geometric structures of a quantum particle, such as Dirac field, by applying Einstein and Riemann field equations.

Acknowledgements

We would like to thank the reviewers for their constructive comments, and we would also like to thank the administration of JAMP, in particular Nancy Ho, for their editorial advice during the preparation of this work.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Einstein, A. (1952) The Principle of Relativity. Dover Publications, New York.

[2] Ho, V.B. (2022) On the Field Equations of General Relativity. *Journal of Applied Mathematics and Physics*, **10**, 49-55. https://doi.org/10.4236/jamp.2022.101005

[3] Landau, L.D. and Lifshitz, E.M. (1987) The Classical Theory of Fields. Pergamon Press, Sydney.

[4] Landau, L.D. and Lifshitz, E.M. (1989) Fluid Mechanics. Pergamon Press, Sydney.

[5] Batchelor, G.K. (1988) An Introduction to Fluid Dynamics. Cambridge University Press, Melbourne.

[6] Irgens, F. (2008) Continuum Mechanics. Springer-Verlag Berlin Heidelberg.

[7] Roychoudhuri, C. (2021) Cosmic Ether, Possessing Electric-Tension and Magnetic-Resistance, Is the Unified Field for Physics. *Journal of Modern Physics*, **12**, 671-699. https://doi.org/10.4236/jmp.2021.125044

[8] Ho, V.B. (2020) Classification of Relativity. *Journal of Modern Physics*, **11**, 535-564. https://doi.org/10.4236/jmp.2020.114036

[9] Dirac, P.A.M. (1928) The Quantum Theory of the Electron. *Proceedings of the Royal Society A*: *Mathematical*, *Physical and Engineering Sciences*, **117**, 610-624. https://doi.org/10.1098/rspa.1928.0023

[10] Feynman, R.P., Leighton, R.B. and Sands, M. (1964) The Feynman Lectures on Physics: Mainly Electromagnetism and Matter. Addison-Wesley Inc., London.

[11] Ho, V.B. (2018) Fluid State of Dirac Quantum Particles. *Journal of Modern Physics*, **9**, 2402-2419. https://doi.org/10.4236/jmp.2018.914154

[12] Kreyszig, E. (1975) Introduction to Differential Geometry and Riemannian Geometry. University of Toronto Press, Toronto.