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Hyperchaotic Impulsive Synchronization and Digital Secure Communication

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Abstract

Based on chaos shift keying approach, impulsive signals from Hyperchaotic Chen system and Hyperchaotic Lü system are alternately emitted according to the transmission of binary signals "0" and "1". In the receiver, these two hyperchaotic systems are adopted as response systems at the same time. The digital signals are recovered via comparing the discrete signals of the two error systems. Numerical simulations show the effectiveness of the method.

Keywords

Impulsive Synchronization, Chaos Shift Keying, Switch Modulation, Secure Communication

1. Introduction

In 1990, Pecora and Carroll presented the conception of "chaotic synchronization" and introduced a method to synchronize two identical chaotic systems with different initial conditions [1] [2]. Since chaos control and synchronization have great potential applications in many areas such as information science, medicine, biology and Engineering, they have received a great deal of attention. Numerous researches have been done theoretically and experimentally [3] [4] [5] [6]. In recent years, many secure communication schemes based on chaotic synchronization have been proposed. For example, Kocarev *et al.* applied chaotic masking to secure communication in 1992 [7]. In 1993, Dedieu *et al.* proposed secure communication approach based on chaos shift keying [8] and Halle *et al.* presented secure communication scheme through modulation of chaos [9]. A lot of efforts have been devoted to chaotic secure communication [10] [11] [12] [13] [14], and some methods without synchronization were also proposed, such as in reference [15]. In 1999, Sushchik *et al.* applied impulsive synchronization to communication [16]. Because of transmitting signals in discrete times, impulsive synchronization demands less energy. Besides, it has faster synchronization speed and it's easy to implement via single channel. Thus it is more practical. Recently, Sun *et al.* used continuous chaotic signals switch modulation in their secure communication scheme [12]. In this paper, a improved method based on their work is presented, discrete drive signal is adopted instead of continuous signal. Impulsive signals from Hyperchaotic Chen system and Hyperchaotic Lü system are alternately sent according to the transmission of binary signals "0" and "1". In the receiver, these two hyperchaotic systems are adopted as response system at the same time. The digital signals are recovered via comparing the discrete signals of the two error systems. Because the information in the channel is discrete and the variables of Hyperchaotic Chen system and Hyperchaotic Lü system have similar amplitudes, original digital signals can't be judged from frequency and amplitude. Phase space reconstruction, neural networks and Return maps are not available here. Numerical simulations show the effectiveness and robustness of the method.

In Chapter 2, the analysis process and rough method are presented. In Chapter 3, the specific scheme and simulation results are given.

2. Impulsive Synchronization of Hyperchaotic Systems

2.1. Theory of Impulsive Synchronization

Suppose a n-dimensional chaotic system as

$$\dot{\boldsymbol{X}} = F\left(t, \boldsymbol{X}\right), \tag{1}$$

choose system (1) as drive system, response system is as follows

$$\begin{cases} \dot{\boldsymbol{Y}} = F\left(t, \boldsymbol{Y}\right)\left(t \neq t_i\right) \\ \Delta \boldsymbol{Y} = \boldsymbol{Y}\left(t_i^+\right) - \boldsymbol{Y}\left(t_i^-\right) = \boldsymbol{Y}\left(t_i^+\right) - \boldsymbol{Y}\left(t_i\right) = \boldsymbol{BE}\left(t = t_i, i = 1, 2, 3, \cdots\right) \\ \boldsymbol{Y}\left(t_0^+\right) = \boldsymbol{Y}\left(0\right) \end{cases}. \tag{2}$$

$\boldsymbol{B}$ is a matrix which stands for a linear combination of $\boldsymbol{Y} - \boldsymbol{X}$, let $\boldsymbol{B} = diag\left(b_1, b_2, \cdots, b_n\right)$; The error vector is $\boldsymbol{E} = \boldsymbol{Y} - \boldsymbol{X}$; t_i is the discrete time at which the impulse is transmitted. According to system (1) and system (2), we can get the error system

$$\begin{cases} \dot{\boldsymbol{E}} = F\left(t, \boldsymbol{Y}\right) - F\left(t, \boldsymbol{X}\right)\left(t \neq t_i\right) \\ \Delta \boldsymbol{E} = \boldsymbol{BE}\left(t = t_i\right) \end{cases}. \tag{3}$$

Suppose the impulsive interval η is invariable, $\eta = t_{i+1} - t_i$, if we have $\lim_{t \to \infty} \left\| \boldsymbol{E}\left(t\right) \right\| = 0$ under some conditions, system (1) and system (2) can be synchronized.

2.2. Implement of Impulsive Synchronization

Hyperchaotic Chen system [17] [18] is described as

$$\begin{cases} \dot{x} = a(y-x)+w \\ \dot{y} = dx - xz + cy \\ \dot{z} = xy - bz \\ \dot{w} = yz + rw \end{cases}, \tag{4}$$

in this paper choose $a=35$, $b=3$, $c=12$, $d=7$, $r=0.5$ so that system (4) exhibits a hyperchaotic behavior [17] [18], **Figure 1** shows the projections of hyperchaotic Chen system's attractor.

Hyperchaotic Lü system [19] is described as

$$\begin{cases} \dot{x} = \hat{a}(y-x)+w \\ \dot{y} = -xz + \hat{c}y \\ \dot{z} = xy - \hat{b}z \\ \dot{w} = xz + \hat{d}w \end{cases}, \tag{5}$$

in this paper choose $\hat{a}=36$, $\hat{b}=3$, $\hat{c}=20$, $\hat{d}=1$ so that system (5) exhibits a hyperchaotic behavior [19], **Figure 2** shows the projections of hyperchaotic Lü system's attractor.

From **Figure 1** and **Figure 2**, we can see that the relevant variables of Hyperchaotic Chen system and Hyperchaotic Lü system have similar amplitudes, so signals from the two systems can't be identified by amplitude.

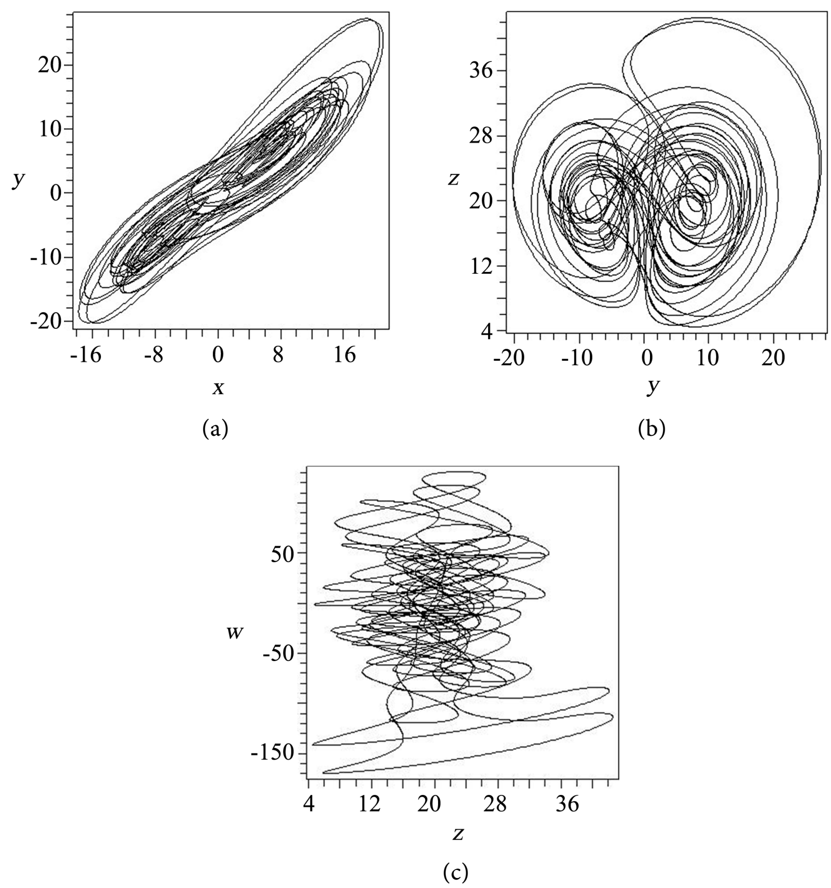

Figure 1. The projections of hyperchaotic Chen system's attractor.

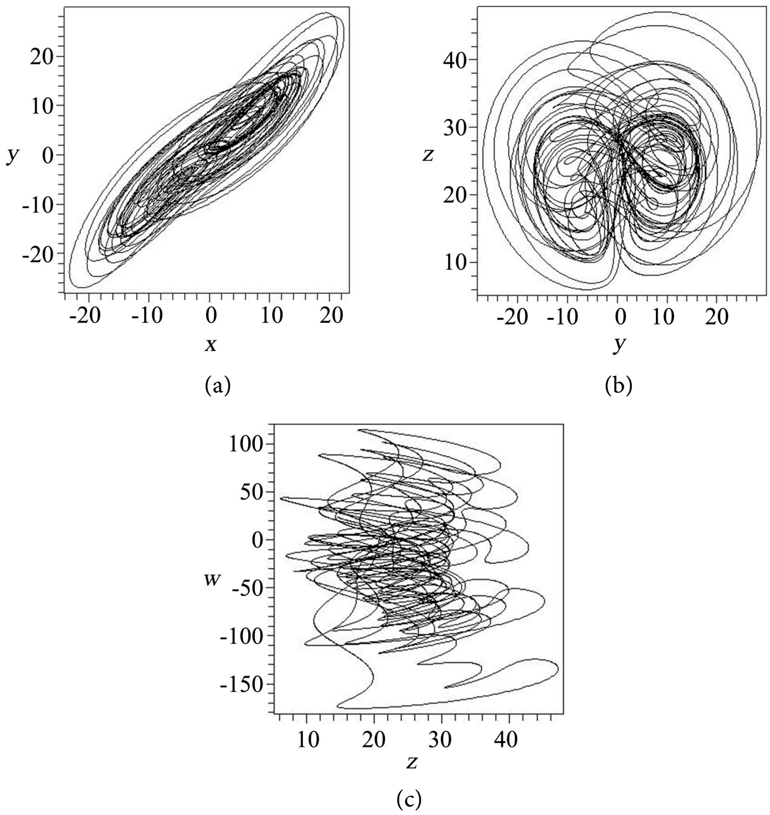

Figure 2. The projections of hyperchaotic Lü system's attractor.

Mohammad *et al.* proposed sufficient confidents for the impulsive synchronization of hyperchaotic Chen system [20]. According to reference [20], we can conclude that as far as system (1) and system (2) are concerned, if $\boldsymbol{B} = diag(b_1, b_2, \cdots, b_n) \approx -I_n$ (*i.e.* $b_1, b_2, \cdots, b_n$ are very near −1) and the invariable impulsive interval η is very small, system (1) and system (2) can be synchronized.

As to communication, transmitting signals via one channel is best. Impulsive signals are only transmitted in discrete times, so it's likely to synchronize drive system and response system via one channel. Choose system (1) as drive system, the response system is described as follows

$$\begin{cases} \dot{\boldsymbol{Y}} = F(t, \boldsymbol{Y}) \left(t \neq t_i + \eta/n, t_i + 2\eta/n, \cdots, t_i + (n-1)\eta/n, t_{i+1}; i = 0,1,2,\cdots \right) \\ \Delta \boldsymbol{Y} = \boldsymbol{Y}(t_i^+) - \boldsymbol{Y}(t_i) = \boldsymbol{BE} \left(t = t_i + \eta/n, t_i + 2\eta/n, \cdots, t_i + (n-1)\eta/n, t_{i+1}; i = 0,1,2,\cdots \right) \\ \boldsymbol{Y}(t_0^+) = \boldsymbol{Y}(0) \end{cases}, \quad (6)$$

where

$$\boldsymbol{B} = \begin{cases} diag(b_1, 0, \cdots, 0, 0) \left(t = t_i + \eta/n; i = 0,1,2,\cdots \right) \\ diag(0, b_2, \cdots, 0, 0) \left(t = t_i + 2\eta/n; i = 0,1,2,\cdots \right) \\ \vdots \\ diag(0, 0, \cdots, b_{n-1}, 0) \left(t = t_i + (n-1)\eta/n; i = 0,1,2,\cdots \right) \\ diag(0, 0, \cdots, 0, b_n) \left(t = t_{i+1}; i = 0,1,2,\cdots \right) \end{cases}. \quad (7)$$

All information of the variables of the drive system are transmitted during η, that is to say, during every η/n, the response system will receive one impulsive signal and adjust its relevant variable. By this means, system (1) and system (6) can be synchronized via single channel.

In PC synchronization scheme, hyperchaotic Chen system described as system (4) can be self-synchronized if y, w are chosen as drive signals. Hyperchaotic Lü system can be self-synchronized in the same way. So we can consider only transmitting the information of y, w instead of all the information of drive system.

Suppose hyperchaotic Chen system described as system (1) is drive system, the response system is as follows.

$$\begin{cases} \dot{\boldsymbol{Y}} = F\left(t, \boldsymbol{Y}\right)\left(t \neq t_i + \eta/2, t_{i+1}; i = 0,1,2,\cdots\right) \\ \Delta \boldsymbol{Y} = \boldsymbol{Y}\left(t_i^+\right) - \boldsymbol{Y}\left(t_i\right) = \boldsymbol{BE}\left(t = t_i + \eta/2, t_{i+1}; i = 0,1,2,\cdots\right), \\ \boldsymbol{Y}\left(t_0^+\right) = \boldsymbol{Y}\left(0\right) \end{cases} \tag{8}$$

where $\boldsymbol{X} = \left[x_1, x_2, x_3, x_4\right]^{\mathrm{T}}$, $\boldsymbol{Y} = \left[y_1, y_2, y_3, y_4\right]^{\mathrm{T}}$, $\boldsymbol{E} = \left[e_1, e_2, e_3, e_4\right]^{\mathrm{T}} = \left[y_1 - x_1, y_2 - x_2, y_3 - x_3, y_4 - x_4\right]^{\mathrm{T}}$,

$$\boldsymbol{B} = \begin{cases} diag\left(0, -1.01, 0, 0\right)\left(t = t_i + \eta/2; i = 0,1,2,\cdots\right) \\ diag\left(0, 0, 0, -1.01\right)\left(t = t_{i+1}; i = 0,1,2,\cdots\right) \end{cases}. \tag{9}$$

It means x_2 signal is transmitted to adjust y_2 at $t_i + \eta/2$ time, and x_4 signal is transmitted to adjust y_4 at t_{i+1} time. In this numerical simulation, let $\eta = 0.02(\text{sec.})$. A time step of size 0.001 (second) is employed and fourth-order Runge-Kutta method is used to solve Equation (1) and Equation (8). The initial states of the drive system (1) and the response system (8) are taken as $\boldsymbol{X}(0) = (-10, 5, 8, 15)$ and $\boldsymbol{Y}(0) = (8, -12, -20, 30)$. Hence the error system has the initial state $\boldsymbol{E}(0) = (18, -17, -28, 15)$. **Figure 3** shows the history of $e_1(t)$, $e_2(t)$, $e_3(t)$, $e_4(t)$ in the error system. From **Figure 3**, we can see that $e_1(t)$, $e_2(t)$, $e_3(t)$, $e_4(t)$ are steady near zero at last, *i.e.*, system (1) and system (8) can be synchronized when $\eta = 0.02(\text{sec.})$ and $\boldsymbol{B}$ is described as Equation (9).

In the same way, suppose hyperchaotic Lü system described as system (1) is drive system; the response system is described as system (8). Let $\boldsymbol{X}(0) = (-20, 15, 5, -8)$, $\boldsymbol{Y}(0) = (13, -10, -5, 3)$, other conditions are the same as above, the two systems can be synchronized too. The errors $e_1(t)$, $e_2(t)$, $e_3(t)$, $e_4(t)$ are shown in **Figure 4**.

3. The Digital Secure Communication Scheme and Numerical Simulations

Suppose $m(t)$ as the useful signal, $s(t)$ is the signal sent into the channel, $s'(t)$ is the signal received by the receiver, which has been influenced by noise. Hyperchaotic Chen system described as Equation (4) sends signal $d_1(t)$, which is composed by y, w signals alternatively. Hyperchaotic Lü system described as Equation (5) sends signals $d_2(t)$, which is composed by y, w signals alternatively.

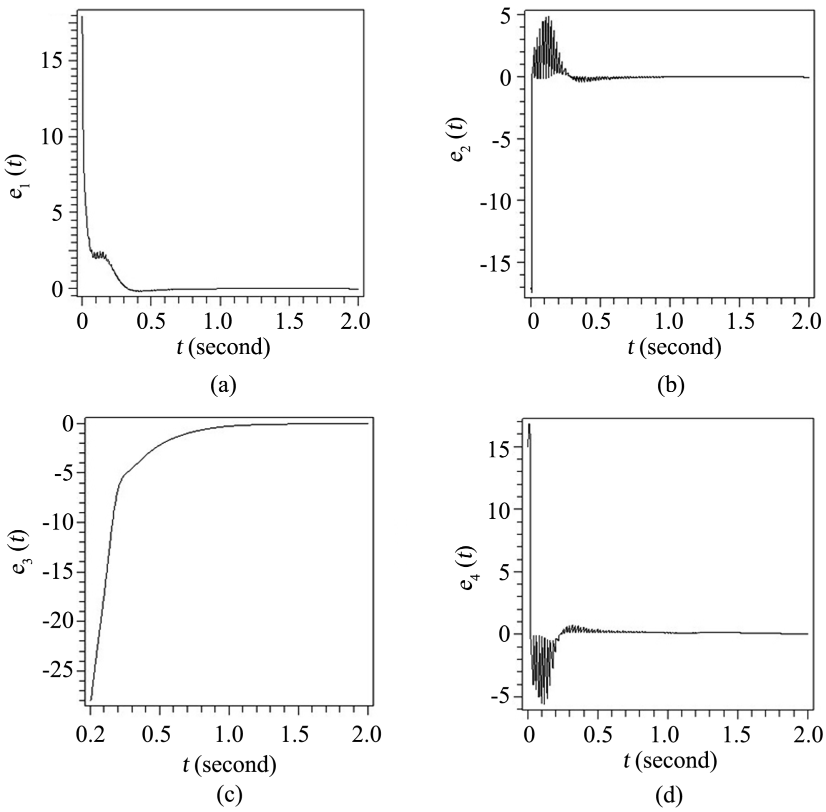

Figure 3. The error of the impulsive self-synchronization of hyperchaotic Chen system.

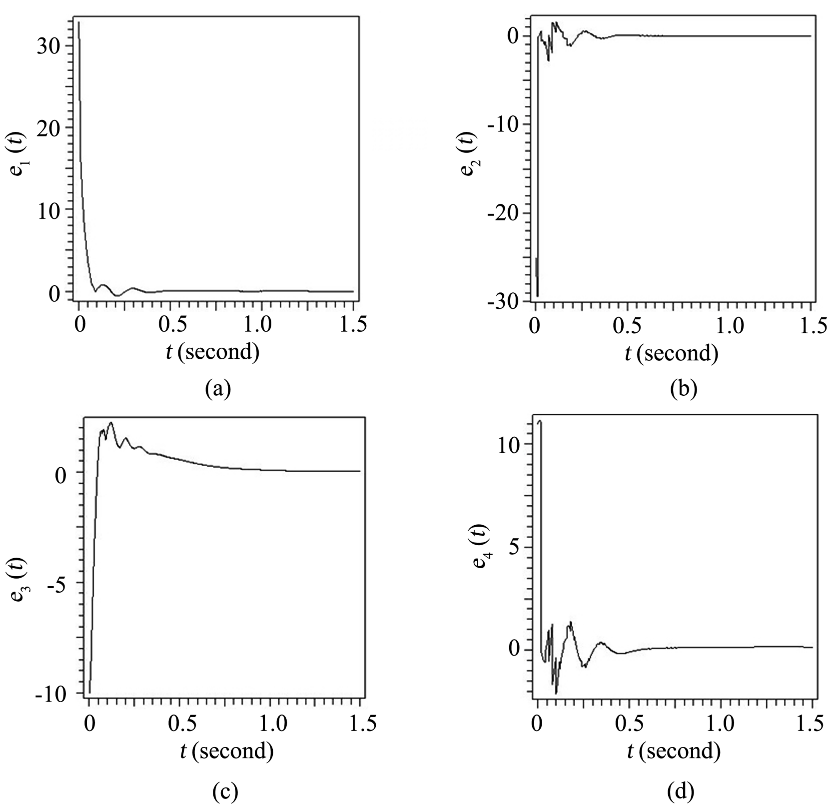

Figure 4. The error of the impulsive self-synchronization of hyperchaotic Lü system.

When $m(t)$ is "1", $s(t)=d_1(t)$; When $m(t)$ is "0", $s(t)=d_2(t)$. Choose hyperchaotic Chen system as the response system I and hyperchaotic Lü system as the response II. Two series of discrete response signals are obtained as $r_1(t), r_2(t)$, which are composed by y, w signals of the relevant response system alternatively. Suppose T stands for the short period of time to decode a digital signal. The former half of T is used for synchronization. The latter half of T is used to collect and sum the discrete error signals together to obtain the summation error E_1 and E_2 for every digital signal. When $E_1 > E_2$, we can conclude the discrete drive signals during the previous T come from hyperchaotic Lü system, "0" is recovered. On the contrary, when $E_1 < E_2$, "1" is recovered. The theory of this scheme is shown in Figure 5.

To avoid the drive signals sudden change, the values of the variables of the current drive system are transferred to the next drive system before every switch. Accurate synchronization is not necessary to analysis the summation error, so we can choose $T=1$ second. Suppose there are N useful signals, transmitting starts at t_0, then $t_0+K-1 \sim t_0+K$ is used to transmit the Kth signal. Choose the latter 0.5 second of every T to collect the summation error E_1 and E_2, we have

$$E_{1K}=\sum\left|r_1(t)-s'(t)\right|\left(t=t_0+K-0.5+\eta/2, t_0+K-0.5+\eta, t_0+K-0.5+3\eta/2, \cdots, t_0+K\right), \quad (10)$$

$$E_{2K}=\sum\left|r_2(t)-s'(t)\right|\left(t=t_0+K-0.5+\eta/2, t_0+K-0.5+\eta, t_0+K-0.5+3\eta/2, \cdots, t_0+K\right). \quad (11)$$

Let $\eta=0.02(\text{sec.})$, *i.e.* during every 0.01 second one impulsive signal is transmitted, 50 error values compose the summation error. If $E_{1K} > E_{2K}$, the Kth useful signal is "0"; If $E_{1K} < E_{2K}$, the Kth useful signal is "1".

Suppose $m(t)=\{1,1,0,1,0,0,0,1,1,0\}$, transmitting starts at $t_0=0$. The initial states of the drive system are taken as $(-1,-2,1,2)$. The initial states of the response system I and response system II are taken as $(1,2,-1,-2)$ and $(2,-1,-2,1)$. Suppose the noise in the channel is random between 0 and δ, in order to decrease its influence, adjust the impulsive signals sent into the two response system to $s''(t)=s'(t)-\delta/2$. Figure 6 shows the simulation results when $\delta=1$.

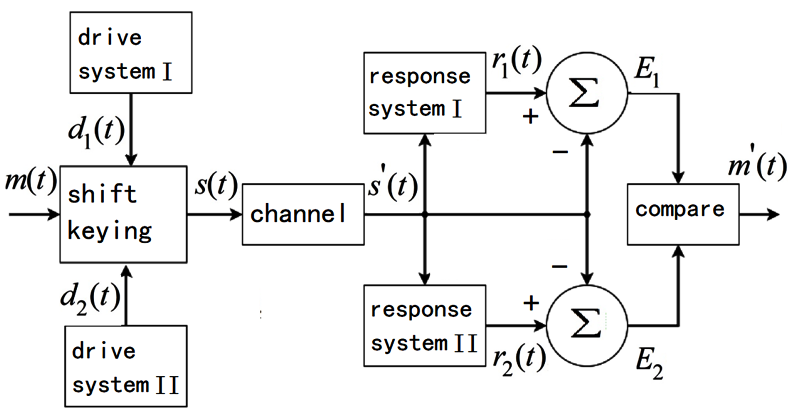

Figure 5. Pictorial diagram of the digital secure communication scheme.

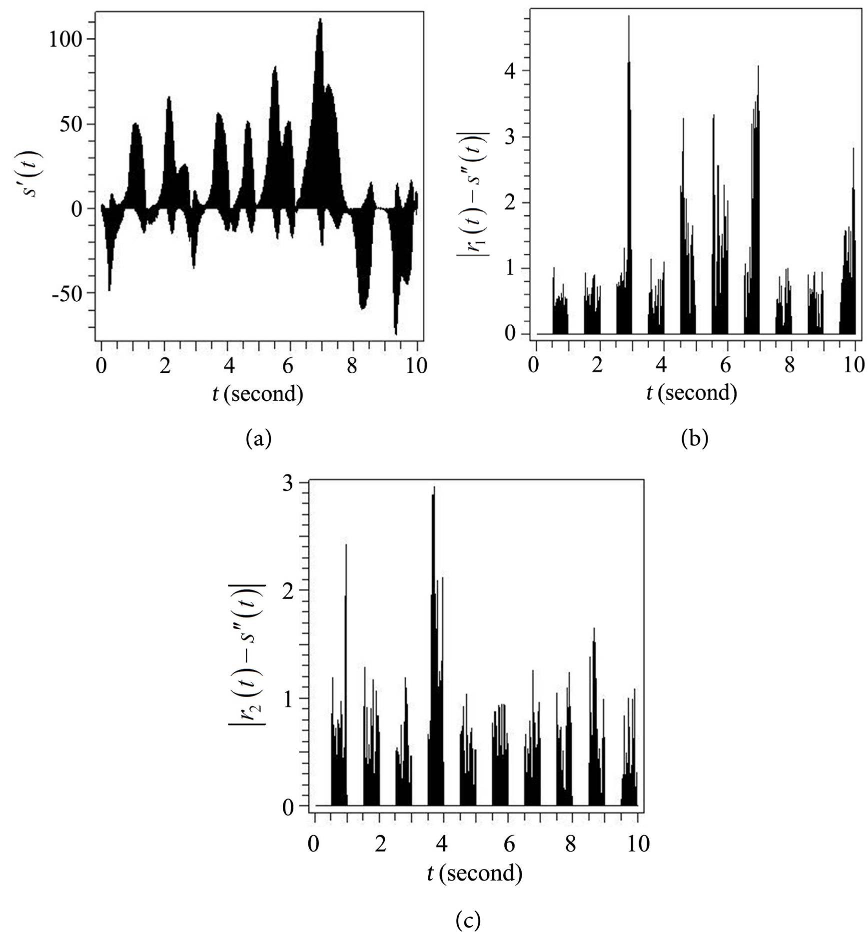

Figure 6. The simulation results of the digital secure communication scheme.

Figure 6(a) shows the impulsive signals in the channel. **Figure 6(b)** and **Figure 6(c)** show the absolute values of the discrete signals of the two error systems in every latter 0.5 second of *T*. According to Equation (10) and Equation (11), we have

$$E_1 = \{19.21, 18.53, 54.38, 18.52, 49.85, 54.06, 63.23, 17.58, 19.65, 42.14\},$$

$$E_2 = \{28.05, 23.68, 16.73, 49.68, 18.32, 25.32, 21.07, 19.37, 28.46, 14.70\},$$

then obtain $m'(t) = \{1,1,0,1,0,0,0,1,1,0\}$.

There is a potential problem in the scheme. If the receiver's actions are driven by the signals from the emitter, an excessive strategy is required. The drive signal is y or w and it belongs to the latter half of T or not, which should be answered, or missing a signal will cause a fatal consequence.

According to **Figure 1** and **Figure 2**, the two drive systems all have $-30 < y < 30$, $-200 < w < 150$. Before sending signals, for the former half of *T*, Let $y' = y + 50$, $w' = w + 300$, then $20 < y' < 80$, $100 < w' < 450$; for the latter half of *T*, let $y' = y + 500$, $w' = w + 750$ then $470 < y' < 530$, $550 < w' < 900$. Sending y', w' instead of y, w, the actions of the receiver can be decided by the range of the signals, and y, w can be recovered from y', w'.

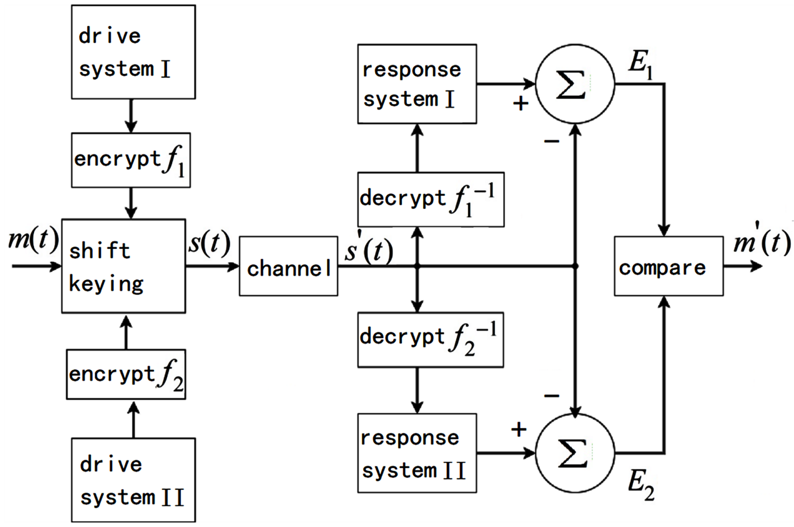

Figure 7. Pictorial diagram of the improved digital secure communication scheme.

Of course, if the actions of the receiver are driven by timer, the above strategy is not necessary, but it can be used in another way. If the amplitudes of the two drive systems are different, they can be adjusted by this means. Moreover, the drive signals can be encrypted by this strategy, the emitter can transmit all information of the variables of the drive system to the receiver, so almost all chaotic systems are available in the digital secure communication scheme and we could transmit more digital information in a very short time because the synchronization speed is much faster. The improved digital secure communication scheme is shown in **Figure 7** (f_1, f_2 are used to encrypt, f_1^{-1}, f_2^{-1} are used to decrypt).

4. Conclusions

The digital secure communication scheme based on impulsive synchronization has more advantages over the scheme in reference [12]: 1) Discrete drive signals demand less energy but provide more security; 2) By the improved communication scheme described in **Figure 7**, almost all chaotic systems can be chosen as drive system; 3) More digital information can be transmitted in a very short time in the improved scheme; 4) Comparing with the scheme in reference [12], summator takes the place of integrator, the complexity of the communication system is decreased; 5) Lots of simulations show that the digital information can be recovered accurately when $\delta < 2$, which is a little better than reference [12] (In reference [12], the value is 1.9).

Choose different combination of drive-response systems, the robustness against noise will be different. We believe the robustness can be increased via adopting more suitable systems.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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