

Modeling of Residential Photovoltaic (PV) System Connected to Low Voltage (LV) Network: Application to Public Distribution Network of Burkina Faso

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Abstract

Residential photovoltaic (PV) systems connected to the grid are used for self-consumption. Any surplus production is fed into the grid and contributes to improving the voltage. Several techniques are developed to model their connection. However, studies on methods of injecting energy production into the Low Voltage (LV) network are nowadays a problem. This paper proposes a mathematical model to determine the current to be injected and calculate each node's voltage. The current equation is a recurrence relation with an initial condition. This initial condition is for the case of a single PV system connected to the LV grid. The equation can also be written in matrix form. Similarly, the voltage solution is a recurrence relation. It also has an initial condition for the first node. Both mathematical formulae with the proposed initial conditions are consistent and can be used for the determination of the current and voltage of the different nodes in the grid.

Keywords

Residential PV System, Low Voltage Grid, Node, Mathematical Model, Electric Current

1. Introduction

Solar energy captured through photovoltaic (PV) panels represents a viable energy alternative for electricity generation. The latter is a renewable source, both clean, unlimited and with a very low level of accident risk according to B. Bouke-

zata *et al.*, 2014 [1]. In Burkina Faso, current studies seek to propose the optimal option of injecting this renewable energy into the LV network to improve the voltage at the user's premises. To this end, several methods have been developed.

Guingané, 2018 in his thesis [2] proposes the case of grid-connected PV systems without storage as a solution to enhance local power generation. According to the latter, a generalized integration study of these PV systems on the power grid allows determining with precision the influence of PV production on the power grids of Burkina-Faso. Zongo *et al.*, 2020 in their article [3] carried out theoretical studies that helped to identify the critical lengths of low voltage lines. This allowed them to determine for any LV network the maximum length that should not be exceeded at the risk of not providing a regulated voltage to consumers. Similarly, Zongo *et al.*, 2022 in their paper [4], proved that the peak hours of national electricity consumption are observed between 19:00 and 23:00 Local Time (LT). By analogy M. Du *et al.*, 2017 [5] proposed a voltage drop calculation software by a distribution network node to alert the network operator. Together, these mechanisms prove that power generation from a grid-connected PV system can contribute to substantially improving the voltage quality. Considering that during the high-demand period there is no sunshine in Burkina Faso, it is necessary to use PV systems with storage to optimize the critical lengths of LV lines. In his Ph.D. thesis G.O. Cimuca, 2016 [6] asks whether these PV systems with energy storage, the object of modeling, can increase the efficiency of the existing system of transmission and distribution of electrical energy. Also, with the advent of new technologies, several methods of interconnection between solar PV systems and the power grid have been successfully developed. This is the case for both old and new configurations of grid-connected PV systems. Nowadays, research is ongoing to define the best technique to inject the output of renewable sources into the grid to efficiently improve the voltage of the users. It can be with or without storage. Thus, we will carry out work to study the influence of renewable sources connected to the grid on voltage. The objective of this study is to model a public distribution network with residential PV systems and to analyze the voltage variability.

In the following, we first present the single-phase model of grid-connected residential PV systems, then establish and solve the mathematical equations. Then, we analyze and discuss the results obtained, and finally, we conclude with some perspectives.

2. Single-Phase Model of Grid-Connected Residential PV Systems

The work on critical lengths has made it possible to realize the voltage drops that we have on the public distribution network in Burkina Faso according to Zongo *et al.*, 2022 [4]. However, users of electric current who are connected to isolated networks, as well as those connected to large networks, use the same types of electrical appliances because the requirements of voltage quality are generally the

same, G.A. Koucoï, 2017 [7]. To this end, let us ask the question of whether voltage dips can be mitigated by the use of residential grid-connected PV systems. The problem is that there is no sunshine during the periods of high power demand (7 pm to 11 pm) while voltage drops are higher according to Zongo et al., 2022 [4].

The objective of this section is to present the equivalent scheme of grid-connected residential PV systems, **Figure 1**.

Therefore, we have chosen nodes and connected residential PV systems with storage and performed tests to verify their impacts on the voltage plan As a reminder, these types of tests were carried out among users of the Societ  National d'Electricit  du Burkina (SONABEL) at substation N 55 in the Sarfalao district of the city of Bobo-Dioulasso, Burkina Faso according to Zongo et al., 2022[4]. The data collected was the subject of a published article [4].

To facilitate the study, we have reducepublishedam to a single-phase, **Figure 1**. To change to a three-phase, we simply multiply the measured voltages and currents by the root of three.

The meaning of the different annotations can be consulted after the bibliographic references.

We notice in **Figure 1** that from the Medium Voltage (MV) line, we pass through a transformer that lowers the High Voltage category A to Low Voltage (HTA/BT) to adapt it to the consumers' needs. According to Zongo et al. [4], most of the voltage drops resulting from the study are higher than the maximum regulatory value of 8% of the nominal values. Thus we make this mathematical study to determine the impact of residential PV systems on critical voltages and lengths.

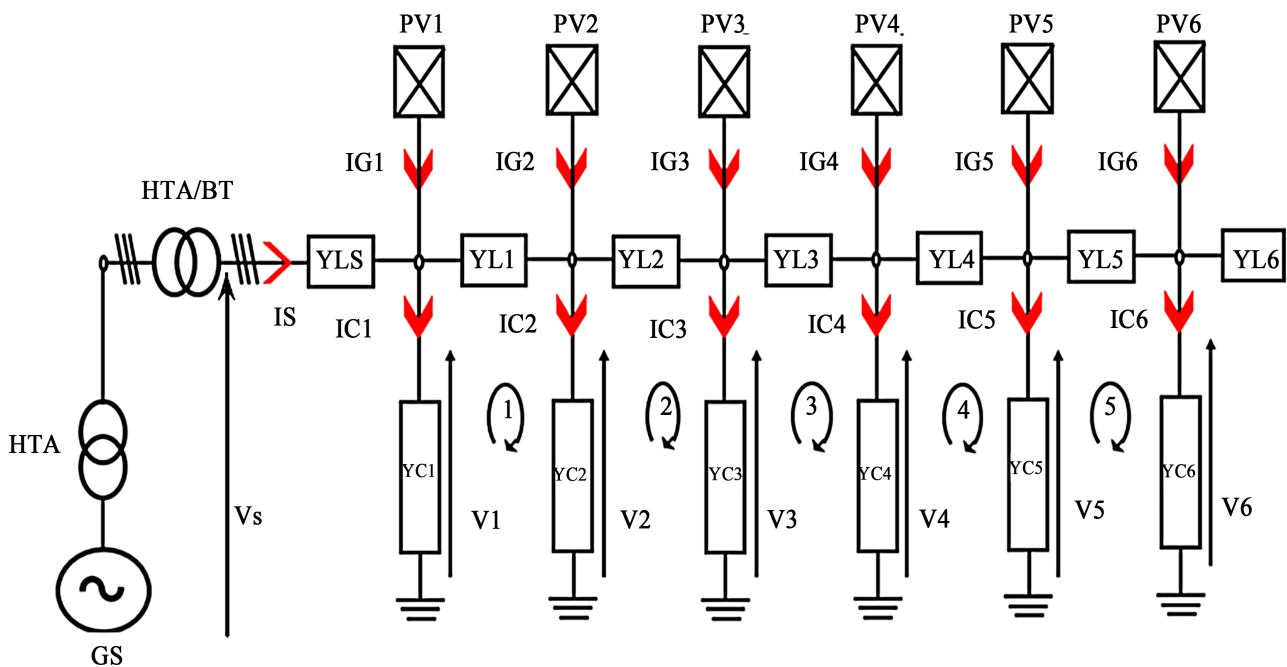


Figure 1. Model equivalent diagram of grid-connected residential PV systems.

3. Determination and Solution of the Mathematical Equations

The voltage drop depends on the energy flow exchanged between two nodes as well as the impedance of the power line. To reduce the voltage drop, either the power line is replaced by a larger cable section or the exchanged energy flow is reduced according to X.L. DANG, 2014 [8]. In this study, we chose to integrate residential grid-connected PV systems to reduce the energy flow. From **Figure 1**, we determine and solve the mathematical equations with all the quantities in complex numbers.

3.1. Determination of the Induction Currents of PV Systems

First of all, we calculate the induction differences across each resistor and the current intensity by applying Kirchhoff's two laws:

From the law of nodes we have:

$$I_S + I_{G1} = I_{C1} + I_1 \quad (1.1)$$

From the law of meshes,

If we take mesh 1 we have:

$$V_1 - \frac{I_1}{Y_{L1}} - V_2 = 0 \quad (1.2)$$

This gives

$$I_1 = (V_1 - V_2) * Y_{L1} \quad (1.3)$$

We also know that

$$I_{C1} = V_1 * Y_{C1} \quad (1.4)$$

From relations (1.1), (1.2), (1.3), and (1.4) we find that:

$$I_S + I_{G1} = V_1 * (Y_{L1} + Y_{C1}) - V_2 * Y_{L1} \quad (1.5)$$

Then, again applying Kirchhoff's law for the second node, we have:

$$I_1 + I_{G2} = I_{C2} + I_2 \quad (1.6)$$

$$V_1 - V_2 = \frac{I_2}{Y_{L2}} \quad (1.7)$$

$$\rightarrow I_2 = (V_2 - V_3) * Y_{L2} \quad (1.8)$$

We also know that

$$I_{C2} = V_2 * Y_{C2} \quad (1.9)$$

and

$$I_{G2} = I_{C2} + I_2 - I_1 \quad (1.10)$$

Hence

$$I_{G2} = V_2 * Y_{C2} + (V_2 - V_3) * Y_{L2} - (V_1 - V_2) * Y_{L1} \quad (1.11)$$

$$I_{G2} = -V_1 * Y_{L1} + V_2 * (Y_{C2} + Y_{L2} + Y_{L1}) - V_3 * Y_{L2} \quad (1.12)$$

$$I_{C3} + I_3 = I_2 + I_{G3} \quad (1.13)$$

$$\rightarrow I_{G3} = I_3 + I_{C3} - I_2 \tag{1.14}$$

$$I_{G3} = V_3 * Y_{C3} - Y_{L2} * (V_2 - V_3) + Y_{L3} * (V_3 - V_4) \tag{1.15}$$

$$I_{G3} = -V_2 * Y_{L2} + V_3 * (Y_{C3} + Y_{L3} + Y_{L2}) - V_4 * Y_{L3} \tag{1.16}$$

From Equations (1.12) and (1.16), we can generalize by recurrence the equation for injection currents as a function of loads as follows:

For any natural number $n > 1$, with n the number of nodes in the network, we have Equation (1.17):

$$I_{Gn} = -V_{n-1} * Y_{Ln-1} + V_n * (Y_{Cn} + Y_{Ln} + Y_{Ln-1}) - V_{n+1} * Y_{Ln} \tag{1.17}$$

With all quantities in complex numbers.

When it is a single node the equation is as follows:

$$I_s + I_{G1} = V_1 * (Y_{L1} + Y_{C1}) - V_2 * Y_{L1} \tag{1.18}$$

From these equations (1.17) and (1.18) we get the following matrix system;

$$\begin{bmatrix} I_s + I_{G1} \\ I_{G2} \\ I_{G3} \\ \vdots \\ I_{Gn} \end{bmatrix} = \begin{bmatrix} -Y_{L1} + Y_{C1} & -Y_{L1} & 0 & 0 & 0 & 0 & \dots & 0 \\ -Y_{L1} & Y_{C2} + Y_{L2} + Y_{L1} & -Y_{L2} & 0 & 0 & 0 & \dots & 0 \\ 0 & -Y_{L2} & Y_{C3} + Y_{L3} + Y_{L2} & -Y_{L3} & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & -Y_{Ln-1} & Y_{Cn} + Y_{Ln} + Y_{Ln-1} & -Y_{Ln} \end{bmatrix} * \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{bmatrix}$$

To solve this system we can also use numerical methods to determine the voltages V_n . Other ways of solving are possible. In this paper, we determine the voltage expression by applying the same Kirchhoff's laws.

3.2. Determination of Node Voltages

To improve the voltage of a given node, it is necessary to inject generation to reduce the energy flow. The solution to the above matrix system allows quantifying the current to be injected at each grid node with the PV system. This injection current influences the node voltage. Thus, to determine the variability of the voltage as a function of the generator currents, we solve the following equations:

From Equation (1.2) we find Equation (2.1)

$$\frac{I_{C1}}{Y_{C1}} - \frac{I_1}{Y_{L1}} - \frac{I_{C2}}{Y_{C2}} = 0 \tag{2.1}$$

$$\frac{I_1}{Y_{L1}} = \frac{I_{C1}}{Y_{C1}} - \frac{I_{C2}}{Y_{C2}} \tag{2.2}$$

$$I_1 = I_{C1} * \frac{Y_{L1}}{Y_{C1}} - I_{C2} * \frac{Y_{L1}}{Y_{C2}} \tag{2.3}$$

Replacing I_1 in Equation (1.1) with its expression (2.3), we obtain equation (2.4).

$$I_s + I_{G1} = I_{C1} + I_{C1} * \frac{Y_{L1}}{Y_{C1}} - I_{C2} * \frac{Y_{L1}}{Y_{C2}} \tag{2.4}$$

$$I_s + I_{G1} = I_{C1} + V_1 * Y_{L1} - \frac{Y_{L1}}{Y_{C2}} * I_{C2} \tag{2.5}$$

$$V_1 * Y_{L1} = I_s + I_{G1} + \frac{Y_{L1}}{Y_{C2}} * I_{C2} - I_{C1} \quad (2.6)$$

$$V_1 = \frac{I_s}{Y_{L1}} + \frac{I_{G1}}{Y_{L1}} + \frac{I_{C2}}{Y_{C2}} - \frac{I_{C1}}{Y_{L1}} \quad (2.7)$$

The relation (2.7) then leads to the equations of V_1 (2.8) and (2.9).

$$V_1 = \frac{1}{Y_{L1}} * (I_s + I_{G1} - I_{C1}) + \frac{I_{C2}}{Y_{C2}} \quad (2.8)$$

$$V_1 = \frac{Y_{Ls}}{Y_{L1}} * V_{Ls} + \frac{I_{G1}}{Y_{L1}} - \frac{I_{C1}}{Y_{L1}} + \frac{I_{C2}}{Y_{C2}} \quad (2.9)$$

The voltage V_1 can also be expressed as a function of the source voltage V_s according to the mesh law:

$$V_1 = V_s - \frac{I_s}{Y_{Ls}} \quad (2.10)$$

Equation (2.10) is then the initial condition.

Applying again the law of nodes and meshes for the second node we find the Equation (2.11).

$$I_1 + I_{C2} = I_2 + I_{C2} \quad (2.11)$$

$$V_{L2} + V_3 - V_2 = 0 \quad (2.12)$$

$$\frac{I_2}{Y_{L2}} + \frac{I_{C3}}{Y_{C3}} = \frac{I_{C2}}{Y_{C2}} \quad (2.13)$$

$$I_2 = Y_{L2} * \left(\frac{I_{C2}}{Y_{C2}} - \frac{I_{C3}}{Y_{C3}} \right) \quad (2.14)$$

By replacing I_2 in Equation (2.11) with its value in Equation (2.14), we determine equation (2.15).

$$I_1 + I_{G2} = I_{C2} + Y_{L2} * \left(\frac{I_{C2}}{Y_{C2}} - \frac{I_{C3}}{Y_{C3}} \right) \quad (2.15)$$

$$I_1 + I_{G2} = I_{C2} + Y_{L2} * (V_2 - V_3) \quad (2.16)$$

$$V_1 - V_{L1} - V_2 = 0 \quad (2.17)$$

$$V_1 = V_2 + V_{L1} = \frac{I_{C2}}{Y_{C2}} + \frac{I_1}{Y_{L1}} \quad (2.18)$$

$$\frac{I_1}{Y_{L1}} = V_1 - \frac{I_{C2}}{Y_{C2}} \quad (2.19)$$

$$I_1 = V_1 * Y_{L1} - \frac{Y_{L1}}{Y_{C2}} * I_{C2} \quad (2.20)$$

Similarly, by replacing I_1 in Equation (2.15) with its value (2.20), we get the combination (2.21).

$$I_{G2} + V_1 * Y_{L1} - \frac{Y_{L1}}{Y_{C2}} * I_{C2} = I_{C2} + V_2 * Y_{L2} - V_3 * Y_{L2} \quad (2.21)$$

$$V_2 * Y_{L2} = V_1 * Y_{L1} - \frac{I_{C2}}{Y_{C2}} * Y_{L1} + I_{G2} - I_{C2} + \frac{I_{C3}}{Y_{C3}} * Y_{L2} \tag{2.22}$$

$$V_2 = -\frac{I_{C2}}{Y_{L2}} + \frac{I_{G2}}{Y_{L2}} - \frac{I_{C2}}{Y_{C2}} * \frac{Y_{L1}}{Y_{L2}} + V_1 * \frac{Y_{L1}}{Y_{L2}} + \frac{I_{C3}}{Y_{C3}} \tag{2.23}$$

Equation (2.24) is for the voltage V_2 .

$$V_2 = V_1 * \frac{Y_{L1}}{Y_{L2}} - \frac{I_{C2}}{Y_{L2}} * \left(\frac{Y_{L1}}{Y_{C2}} + 1 \right) + \frac{I_{G2}}{Y_{L2}} + \frac{I_{C3}}{Y_{C3}} \tag{2.24}$$

Kirchhoff's law has been reapplied to the third node. Equation (2.25) is found:

$$I_2 + I_{G3} = I_3 + I_{C3} \tag{2.25}$$

$$-V_3 + V_{L3} + V_4 = 0 \tag{2.26}$$

$$\frac{I_{C3}}{Y_{C3}} = \frac{I_3}{Y_{L3}} + \frac{I_{C4}}{Y_{C4}} \tag{2.27}$$

$$I_3 = \frac{I_{C3}}{Y_{C3}} * Y_{L3} - \frac{I_{C4}}{Y_{C4}} * Y_{L3} \tag{2.28}$$

$$I_2 + I_{G3} = I_{C3} + \frac{I_{C3}}{Y_{C3}} * Y_{L3} - \frac{I_{C4}}{Y_{C4}} * Y_{L3} \tag{2.29}$$

$$V_2 = \frac{I_2}{Y_{L2}} + \frac{I_{C3}}{Y_{C3}} \tag{2.30}$$

$$I_2 = Y_{L2} * V_2 - \frac{I_{C3}}{Y_{C3}} * Y_{L2} \tag{2.31}$$

Combining Equations (2.29) and (2.31), we end up with (2.32).

$$Y_{L2} V_2 - \frac{I_{C3}}{Y_{C3}} * Y_{L2} + I_{G3} = I_{C3} + \frac{I_{C3}}{Y_{C3}} * Y_{L3} - \frac{I_{C4}}{Y_{C4}} * Y_{L3} \tag{2.32}$$

$$V_3 Y_{L3} = Y_{L2} * V_2 - \frac{I_{C3}}{Y_{C3}} * Y_{L2} + I_{G3} - I_{C3} + \frac{I_{C4}}{Y_{C4}} * Y_{L3} \tag{2.33}$$

$$V_3 = \frac{Y_{L2}}{Y_{L3}} * V_2 - \frac{Y_{L2}}{Y_{L3}} * \frac{I_{C3}}{Y_{C3}} + \frac{I_{C4}}{Y_{C4}} + \frac{I_{G3}}{Y_{L3}} - \frac{I_{C3}}{Y_{L3}} \tag{2.34}$$

$$V_3 = \frac{Y_{L2}}{Y_{L3}} * V_2 - \frac{I_{C3}}{Y_{L3}} * \left(\frac{Y_{L2}}{Y_{C3}} + 1 \right) + \frac{I_{G3}}{Y_{L3}} + \frac{I_{C4}}{Y_{C4}} \tag{2.35}$$

V_3 is then the voltage at node 3.

By recurrence, we can generalize the expression for the voltage V_n . For n given nodes, we express the corresponding voltages through the general formula (2.36).

$$V_n = \frac{Y_{Ln-1}}{Y_{Ln}} * V_{n-1} - I_{Cn} * \left(\frac{Y_{Ln-1} + Y_{Cn}}{Y_{Ln} * Y_{Cn}} \right) + \frac{I_{Gn}}{Y_{Ln}} + \frac{I_{Cn+1}}{Y_{Cn+1}} \tag{2.36}$$

with $n > 1$.

4. Analysis of Results and Discussion

The relations (1.17) and (2.36) and the matrix system are the three results to

which we carry out the analyses and discussions.

On the one hand, the relations (1.17), (1.18), and (2.10), (2.36) can be written as systems of linear equations when we consider the positive integer n as the number of nodes of the residential PV systems connected to the power grid. These are:

$$I_{Gn} = \begin{cases} -I_S + V_n * (Y_{Ln} + Y_{Cn}) - V_{n+1} * Y_{Ln}, & si\ n = 1 \\ -V_{n-1} * Y_{Ln-1} + V_n * (Y_{Cn} + Y_{Ln} + Y_{Ln-1}) - V_{n+1} * Y_{Ln}, & si\ n > 1 \end{cases} \quad (3.1)$$

$$V_n = \begin{cases} V_S - \frac{I_S}{Y_{LS}}, & si\ n = 1 \\ \frac{Y_{Ln-1} * V_{n-1} - I_{Cn} * \left(\frac{Y_{Ln-1} + Y_{Cn}}{Y_{Ln} * Y_{Cn}} \right) + \frac{I_{Gn}}{Y_{Ln}} + \frac{I_{Cn+1}}{Y_{Cn+1}}}{Y_{Ln}}, & si\ n > 1 \end{cases} \quad (3.2)$$

System (3.1) is the equation of the injection currents of the photovoltaic generators. Similarly, system (3.2) represents the equations of the voltages of the different electrical loads. Their recurrence character allows us to deduce that PV systems can be successively connected to several nodes of the LV network; this could be a solution to increase the critical lengths of the lines according to the article of Zongo *et al.*, 2020 [4]. Guinguane, 2018 [2]; Koucoï, 2017 [7] and Dang, 2014 [8] in their respective theses, have proposed methods for connecting PV systems to the grid. All these methods have allowed us to verify the influence of these systems on electricity distribution. In the same logic as these works, the matrix system and the relations (3.1) and (3.2) show that on a power distribution line, it is possible to improve the voltage at the user’s premises by connecting PV generators.

On the other hand, the matrix writing of Equations (1.17) and (1.18) can be put in the form $AX = B$ with:

$$A = \begin{bmatrix} -Y_{L1} + Y_{C1} & -Y_{L1} & 0 & 0 & 0 & 0 & \dots & 0 \\ -Y_{L1} & Y_{C2} + Y_{L2} + Y_{L1} & -Y_{L2} & 0 & 0 & 0 & \dots & 0 \\ 0 & -Y_{L2} & Y_{C3} + Y_{L3} + Y_{L2} & -Y_{L3} & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & -Y_{Ln-1} & Y_{Cn} + Y_{Ln} + Y_{Ln-1} & -Y_{Ln} \end{bmatrix}$$

$$X = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{bmatrix}; \quad B = \begin{bmatrix} I_S + I_{G1} \\ I_{G2} \\ I_{G3} \\ \vdots \\ I_{Gn} \end{bmatrix}$$

We can easily notice that A is a square matrix of order n . Also, equation (2.36) explains that matrix A is invertible since it is a solution of the matrix system. From this, we can deduce that the determinant of matrix A is non-zero, which proves that it is possible to solve the linear equation system (3.1) and find a numerical or algebraic solution.

Furthermore, the relations (3.1) and (3.2) are systems of linear equations. They

present the current to be injected and the voltage of the grid load. Similar work by Thi Minh Chau Le in his thesis, [9] was used to calculate electrical parameters for the influence and consistency of PV systems connected to the LV grid. He also determined other parameters that should be taken into account to minimize unwarranted disconnections of grid-connected PV systems to optimize their operation.

5. Conclusion

In this paper, the study models a section of the LV grid with five PV system injection nodes. This modeling has allowed me to write mathematical equations that show the proper functioning of the power grid. They allow knowing the currents to be injected into each node of the network to improve the voltage. In addition, the voltages of the nodes were calculated. The different results are recurrence relations with an initial condition. They were analyzed and discussed, which will allow us to use them for further research. Thus, for a better interpretation of the mathematical equations and the obtained solutions, we plan to carry out in our next research work a simulation under Matlab/Simulink, an experiment on an LV network, and a numerical resolution of the linear equation system (3.1).

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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The Annotations in Figure 1

I_S : the secondary current of the transformer;

$PV_1, PV_2, PV_3, PV_4, PV_5, PV_6$: the PV generators of nodes 1, 2, 3, 4, 5, 6;

$Y_{C1}, Y_{C2}, Y_{C3}, Y_{C4}, Y_{C5}, Y_{C6}$: the admittances of loads 1, 2, 3, 4, 5, 6;

$Y_{L1}, Y_{L2}, Y_{L3}, Y_{L4}, Y_{L5}, Y_{L6}$: the admittances of the line sections of nodes 1, 2, 3, 4, 5, 6;

$I_{G1}, I_{G2}, I_{G3}, I_{G4}, I_{G5}, I_{G6}$: the PV generator currents of nodes 1, 2, 3, 4, 5, 6;

$I_{C1}, I_{C2}, I_{C3}, I_{C4}, I_{C5}, I_{C6}$: the currents of the loads flowing through nodes 1, 2, 3, 4, 5, 6;

$I_1, I_2, I_3, I_4, I_5, I_6$: the outgoing currents from nodes 1, 2, 3, 4, 5, 6;

$V_1, V_2, V_3, V_4, V_5, V_6$: the load voltages at nodes 1, 2, 3, 4, 5, and 6;

V_S : the LV voltage of the source;

G_S : the synchronous generator.