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Atomic Spacetime Model Based on Atomic AString Functions

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Abstract

A novel model of spacetime and fields atomization based on Atomic Series over finite Atomic AString Functions is offered. Formulated Atomization Theorems allow representing polynomials, analytic functions, and solutions of field equations including General Relativity via superposition of solitonic atoms which can be associated with flexible spacetime quantum, metriants, or elementary distortions. Spacetime is conceptualized as a lattice of flexible Atomic Solitons adjusting locations to reproduce different metrics and other physical fields. It may offer the variants of unified field theory based on Atomic Solitons where, like in string theory, fields become interconnected having a common mathematical ancestor.

Keywords

Spacetime, Quantum, Atomic Function, AString, Soliton, Metriant, Unified Theory

1. Introduction and "Atomic Theory" of A. Einstein

In a 1933 lecture [1] cited below with some highlights, A. Einstein discussing some controversies of Quantum Mechanics mentioned the prospects of a novel "atomic theory" based on "mathematically simplest concepts and the link between them" to solve some "stumbling blocks" of continuous field theories to describe quantized fields.

"*The important point for us to observe is that all these constructions and the laws connecting them can be arrived at by the principle of looking for the mathematically simplest concepts and the link between them. In the limited number of the mathematically existent simple field types, and the simple equations possible between them, lies the theorist's hope of grasping the real in all its depth. Meanwhile the great stumbling-block for a field-theory of this kind lies in the* *conception of the atomic structure of matter and energy. For the theory is fundamentally non-atomic in so far as it operates exclusively with continuous functions of space, in contrast to classical mechanics, whose most important element, the material point, in itself does justice to the atomic structure of matter… I still believe in the possibility of a model of reality—that is to say, of a theory which represents things themselves and not merely the probability of their occurrence… But an atomic theory in the true sense of the word (not merely on the basis of an interpretation) without localization of particles in a mathematical model is perfectly thinkable. For instance, to account for the atomic character of electricity, the field equations need only lead to the following conclusions: A region of three-dimensional space at whose boundary electrical density vanishes everywhere always contains a total electrical charge whose size is represented by a whole number. In a continuum-theory atomic characteristics would be satisfactorily expressed by integral laws without localization of the entities which constitute the atomic structure. Not until the atomic structure has been successfully represented in such a manner would I consider the quantum-riddle solved.*"

Interestingly, some of Einstein's aspirations of a novel "atomic theory" with "simplest concepts and links between them" based on finite "regions of space" with "atomic structure" can be realized with the theory of Atomic Functions (AF) pioneered in the 1970s by V.L. Rvachev and V.A. Rvachev [2]-[12] without connections to Einstein's theories until 2017-2022 author's works on Atomic AString Functions and Atomic Solitons in spacetime physics [2] [3] [4] [5]. The motivation for this research is to demonstrate how two theories—General Relativity (GR) [13] [14] [15] [16] and Atomic AString Functions [2]-[12]—can be combined as well as to offer a novel mathematical interpretation of spacetime field as a superposition of flexible 'solitonic atoms' (Atomic Solitons). The combined Atomic Spacetime theory is based on formulated Atomization Theorems (§5) allowing representation of polynomials, analytic functions, and solutions of differential equations of mathematical physics including GR [13]-[33] via superposition of finite Atomic AString Functions resembling flexible quanta (§3, 7). It leads to novel Spacetime Atomization models and atomic metriants (§3, 7, 8) as well as offers some variants of unified field theory based on Atomic Solitons (§8, 9).

The main difficulty of integrating relatively new Atomic Functions theory [2]-[12] into GR [1] [13] [14] [15] [16] was to figure out how AFs known for their unique approximating properties of analytical functions and solutions of linear differential equations [6]-[12] can be applied for such a complex GR equation including nonlinear Ricci tensors [13] [14] [15] [16]. The breakthrough idea first intuitively envisaged in [2] [3] is based on the combination of three properties—derivatives and integrals of finite Atomic Function splines expressed via AF itself [2]-[12], the ability of Atomic AString Functions to represent polynomials and analytic functions [6]-[12], and "preservation of the analyticity" for Ricci tensor first noted here (§6). It means if smooth spacetime geometry is represented as a superposition of finite AF splines, the deformations, metric, Ricci, and Einstein curvature tensors would also be some AF combinations because derivatives and complex multiplications of AFs are expressed via AF themselves. It offers a discrete-continuous interpretation of spacetime and other fields as a complex network/lattice of shifted and stretched "solitonic atoms" resonating with A. Einstein's [1] aspirations of a "perfectly thinkable" "atomic theory" with "simplest concepts and links between them" where finite "regions of space" can have "atomic structure". The background, challenges and contributions to Atomic Functions theory are described in the historical review hereafter.

2. Brief History of Atomic and AString Functions

Theory of Atomic Functions (AF) [2]-[12] has been evolving since 1967-1971 when V.L. Rvachev[1], had envisaged finite pulse function $up(x)$ for which derivatives (also pulses) would conveniently be similar to the original pulse shifted and stretched by the factor of 2:

$$up'(x) = 2up(2x+1) - 2up(2x-1) \text{ for } |x| \le 1, \ up(x) = 0 \text{ for } |x| > 1. \tag{1.1}$$

This and other similar functions possess unique properties of infinite differentiability, smoothness, nonlinearity, nonanalyticity, finiteness, and compact support like widely-used splines. What the most significant is that other functions like polynomials, sinusoids, exponents, and other analytic functions can be represented via a converging series of shifts and stretches of AFs. So, like from "mathematical atoms" [6]-[12], smooth functions can be composed of the AF superpositions, and because of that those "atoms" have been called Atomic Functions in the 1970s.

The foundation of AF theory has been developed by V.L Rvachev and V.A. Rvachev [2] [3] [4] [5] [6] [25] [27] and enriched by many followers from different countries, notably by schools of V.F. Kravchenko [9] [10] [11] [12], B. Gotovac, H. Gotovac [26] [33], and the author [2] [3] [4] [5] [21] [22] [23], with the number of papers and books observed in [10] has grown to a few hundred. In 2017, the author noted [2] [3] [4] [5] that AF $up(x)$ (1.1) is a composite object consisting of two kink functions called AStrings [2] [3] [4] [5] making them more generic:

$$up(x) = AString(2x+1) - AString(2x-1) = AString'(x). \tag{1.2}$$

Moreover, AString is not only a "composing branch" but also an integral of $up(x)$. Mutual relationships (1.1), (1.2) imply that theories and theorems involving AFs can be reformulated via AStrings. Composing AF pulse (1.1) via kink-antikink pair (1.2) of nonlinear AStrings resembles "solitonic atoms" (or bions) from the theory of soliton dislocations [5] [29] [30]. This led to the theory

[1]Vladimir Logvinovich Rvachev (1926-2005), https://en.wikipedia.org/wiki/Vladimir_Rvachev, Academician of National Academy of Sciences of Ukraine, author of 600 papers, 18 books, mentor of 80 PhDs, 20 Doctors and Professors including the author.

of Atomic Solitons [3] [5] where AString (1.2) becomes a solitonic kink while $up(x)$ is a "solitonic atom" made of AStrings. The ability of AFs to compose polynomials and analytic functions leads to novel interpretations of spacetime and field composition from Atomic Solitons [2] [3] [4] [5].

AString possesses another important property of composing/partitioning a line and curves from a superposition of AStrings resembling the ideas of quantization of space lines, geodesics, and generally spacetime field published in 2018 [3] as an "intuition theory". It assumes the representation of spacetime and gravity as a superposition of Atomic Solitons leading to ideas of "atomization of spacetime" and supporting some A. Einstein's aspirations [1] of an "…atomic theory with mathematically simplest concepts and the link between them" to solve some "stumbling blocks" of continuous field theories to describe quantized fields.

The mathematical foundation of the "fields atomization" theory is based on the sequence of 13 *Atomization Theorems*, also proven in [23]. Starting from some known theorems extended here for recently introduced AStrings with the so-called *Atomic Series*, it is extended to new theorems for composite analytic functions, nonlinear theories, and finally nonlinear General Relativity (GR) equations. Interestingly, Atomic AString Functions can be not only introduced into GR but deduced from GR to uphold A. Einstein's ideas of finite "regions of space" with "discrete energies" [1] [23].

The Atomization Theorems are not limited to spacetime but can also be applied to many physical theories including Quantum Mechanics, electromagnetism, elasticity, heat conductivity, soliton theories, and field theories [7]-[12] [22]-[27] [29]-[50]. Unified representation of fields composed of Atomic AString Functions may offer some novel variants of unified theory based on Atomic Solitons [2] [3] [4] [5] [23] [42] where, like in string theory, fields become interconnected having a common mathematical ancestor.

3. Deriving Simple AString Metriant Function

Let's consider the problem of composing a straight x and curved $\tilde{x}(x)$ spaceline via superpositions over some finite metriant functions $m(x), x \in [-1,1]$:

$$x = \sum_{k=-\infty}^{\infty} am\left(\left(x-ka\right)/a\right);\ \ \tilde{x}(x) = \sum_{k=-\infty}^{\infty} c_k m\left(\left(x-b_k\right)/a_k\right) \tag{3.1}$$

composing a spaceline from "elementary pieces" set at regular points ka resembling quanta of width $2a$ (**Figure 1**). We seek spaceline x to appear not only as a Lego-like translation (3.1) but also in "interaction zones" between quanta ($a=1$) (**Figure 1(a)**, **Figure 1(b)**):

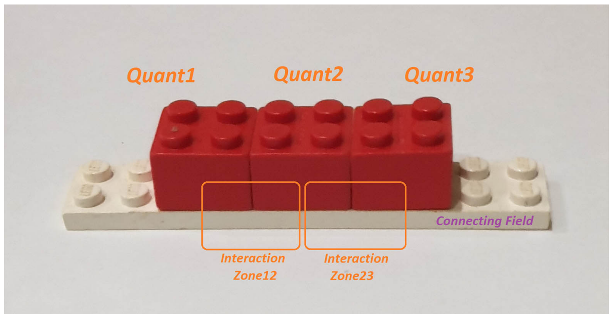
$$x \equiv \cdots + m(x-1) + m(x) + m(x+1) + \cdots;$$

$$x \equiv m\left(x-\frac{1}{2}\right) + m\left(x+\frac{1}{2}\right), x \in \left[-\frac{1}{2},\frac{1}{2}\right]. \tag{3.2}$$

Reformulated for derivatives $p(x) = m'(x)$, the problem leads to a "partition of unity" [2]-[7] to represent a constant via a series of finite pulses:

$$1 \equiv \cdots + p(x-1) + p(x) + p(x+1) + \cdots;$$
$$1 \equiv p\left(x - \frac{1}{2}\right) + p\left(x + \frac{1}{2}\right), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]. \tag{3.3}$$

It can be achieved with polynomial splines but it leads to a "polynomial trap" problem [24] imposing artificial polynomial order on spacetime models and not being able to compose a smooth curve $\tilde{x}(x)$ of arbitrary polynomial order. Instead, seeking a solution amongst finite functions for which derivatives are expressed via themselves



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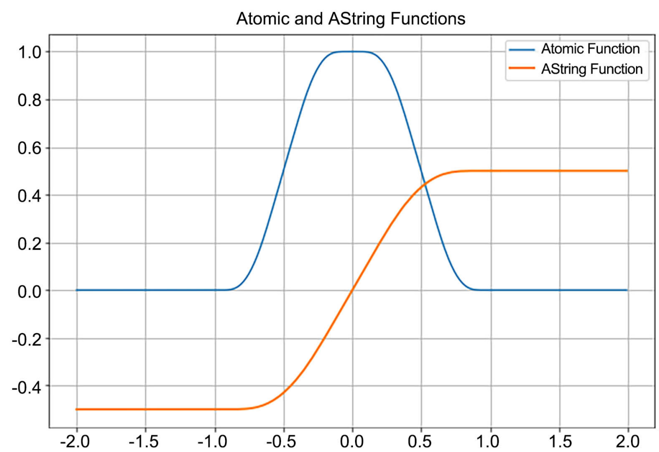

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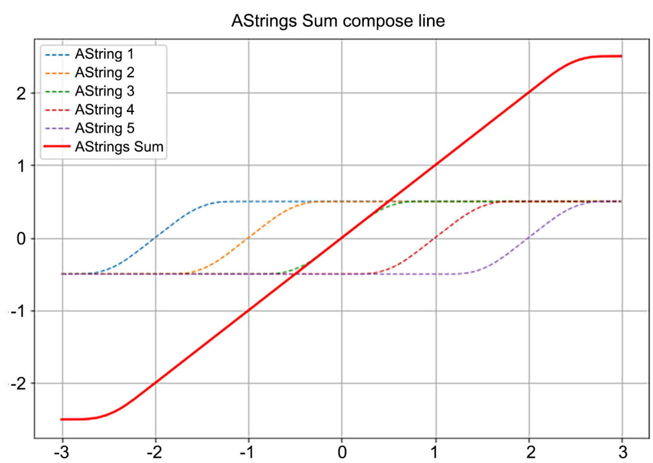

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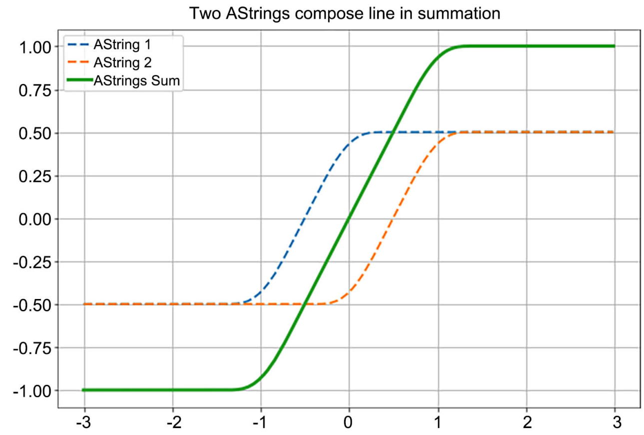


(d)

Figure 1. (a) Lego model with interaction zones; (b) desired metriant function and its derivative; (c) expansion of space by the sum of metriant functions; (d) emergence of line $y = x$ by summing two metriant functions in "interaction zone".

$$p'(x) = f\left(p(x)\right) = cp(ax+b) + dp(ax-b) \tag{3.4}$$

yields so-called atomic function (AF) $up(x)$ [2]-[12] discovered in the 1970s by V.L. Rvachev and V.A. Rvachev [6] (**Figure 1(b)**)

$$up'(x) = 2up(2x+1) - 2up(2x-1),\ p(x) = up(x). \tag{3.5}$$

The desired metriant function $m(x)$ would be the integral of $up(x)$ called *AString* in 2017 [2] [3] [4] [5]:

$$p(x) = up(x),\ m(x) = \int_0^x up(x)\mathrm{d}x = AString(x),\ x \equiv \sum_k AString(x-k). \tag{3.6}$$

AString shaped as a kink (**Figure 1**) can compose both straight and curved lines from solitary pieces offering spacetime quantization models based on Atomic and AString Functions [2] [3] [4] [5] [23] [42] described hereafter.

4. Atomic and AString Functions

Let's describe Atomic [2]-[12] and AString [2] [3] [4] [5] Functions in more detail.

4.1. Atomic Function

Atomic Function (AF) (V.L. Rvachev, V.A. Rvachev, [6], 1971) $up(x)$ is a finite compactly supported non-analytic infinitely differentiable function (**Figure 2**) with the first derivative expressible via the function itself shifted and stretched by the factor of 2:

$$up'(x) = 2up(2x+1) - 2up(2x-1) \ \text{ for } |x| \le 1, \quad up(x) = 0 \ \text{ for } |x| > 1. \tag{4.1}$$

With exact Fourier series representation [2] [3] [4] [5] [7]-[12]

$$up(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{itx} \prod_{k=1}^{\infty} \frac{\sin\left(t2^{-k}\right)}{t2^{-k}}\mathrm{d}t,\ \int_{-1}^{1} up(x)\mathrm{d}x = 1, \tag{4.2}$$

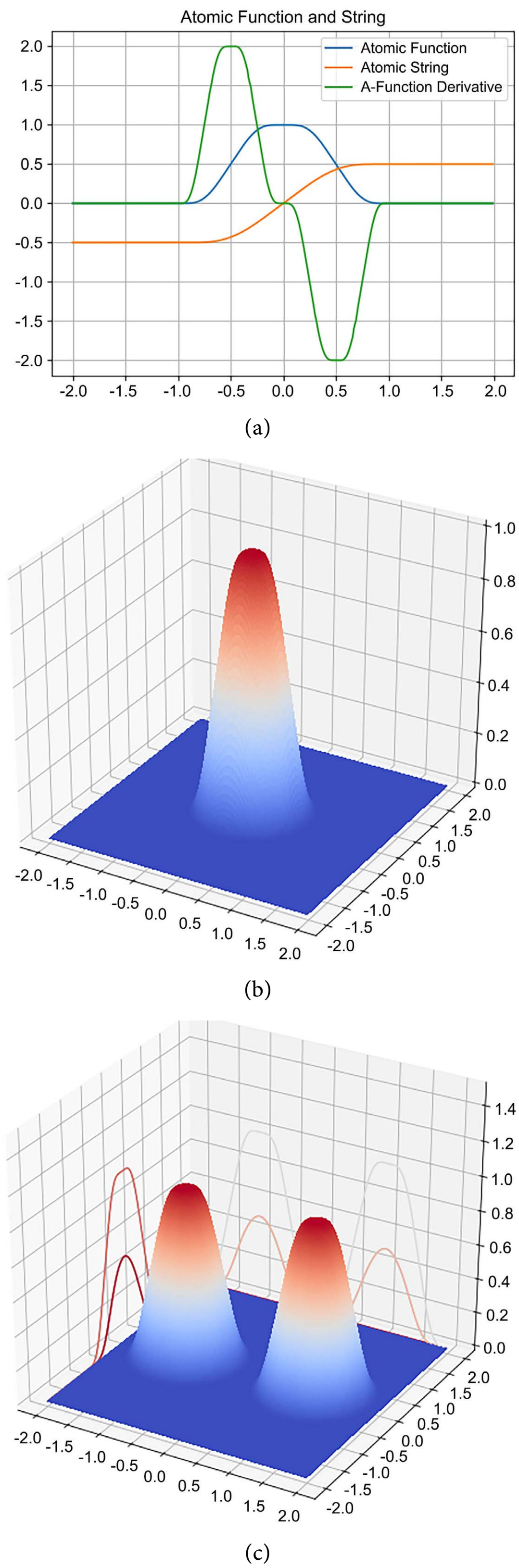


Figure 2. (a) Atomic Function pulse with its derivative and integral (AString); (b) Atomic Function pulse ("solitonic atom") in 2D; (c) TWO Atomic Function pulses ("solitonic atoms" or "atomic solitons").

the values of $up(x)$ can be calculated with computer scripts [2] [4] [9] [10] [11] [12] [43].

Higher derivatives $up^{(n)}$ and integrals I_m can also be expressed via $up(x)$ [6]-[12] [25] [26]

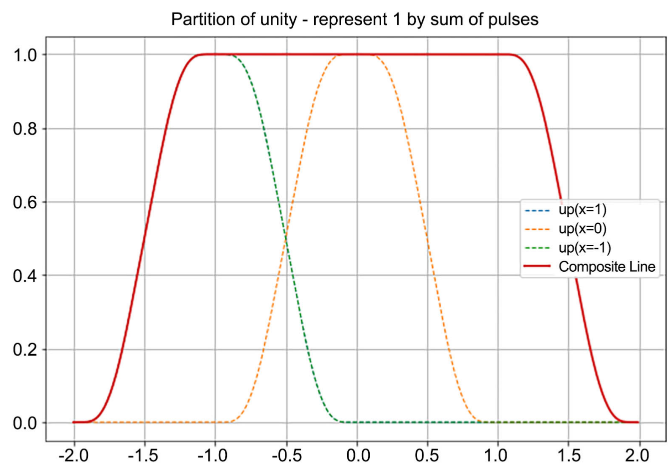
$$up^{(n)}(x) = 2^{\frac{n(n+1)}{2}} \sum_{k=1}^{2^n} \delta_k up\left(2^n x + 2^n + 1 - 2k\right), \delta_{2k} = -\delta_k, \delta_{2k-1} = \delta_k, \delta_1 = 1;$$

$$I_m(x) = 2^{C_m^2} up\left(2^{-m} x - 1 + 2^{-m}\right), x \le 1;$$

$$I_m(x) = 2^{C_m^2} up\left(2^{-m+1} - 1\right) + \frac{(x-1)^{m-1}}{(m-1)!}, x > 1;$$

$$I_1(x) = up\left(2^{-1} x - 2^{-1}\right); \ I_1'(x) = up(x). \tag{4.3}$$

AF satisfies *partition of unity* [2]-[12] to exactly represent the number 1 by summing up individual overlapping pulses set at regular points…−2, −1, 0, 1, 2… (**Figure 3(a)**):



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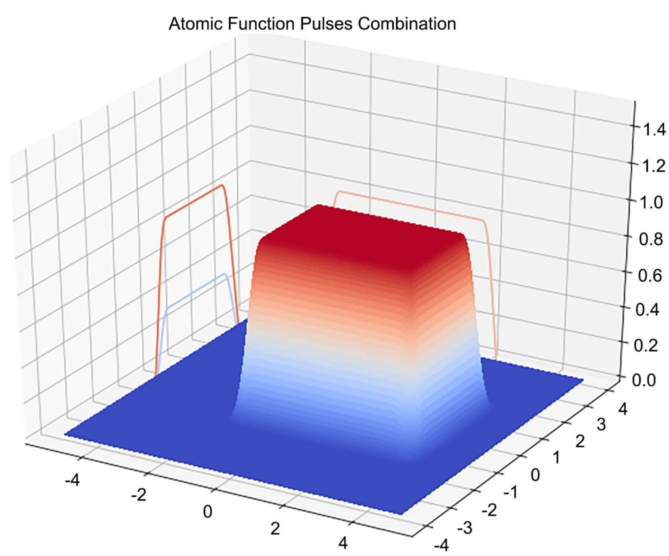

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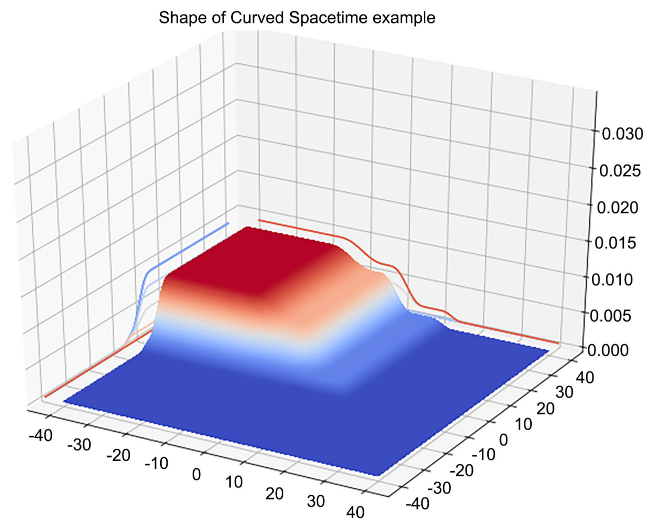

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Figure 3. (a) Partition of unity with Atomic Functions; (b) representation of flat surface via summation of Afs; (c) curved surface as a superposition of "solitonic atoms".

$$\cdots + up(x-2) + up(x-1) + up(x) + up(x+1) + up(x+2) + \cdots \equiv 1. \tag{4.4}$$

This property is related to the following double symmetry [2]-[12]:

$$up(x) = up(-x),\ x \in [-1,1];\ up(x) + up(1-x) = 1,\ x \in [0,1]. \tag{4.5}$$

Generic AF pulse of width 2*a*, height *c*, and center positions *b*, *d* has the form

$$up(x,a,b,c,d=0) = d + c * up((x-b)/a),\ \int_{-a}^{a} cup(x/a)\,\mathrm{d}x = ca. \tag{4.6}$$

Multi-dimensional atomic functions [2]-[8] [24] [27] (**Figure 3**, **Figure 4**) can be constructed as either multiplications or radial atomic functions:

$$up(x,y,z) = up(x)up(y)up(z),$$
$$up(r) = up\left(\sqrt{x^2+y^2+z^2}\right),\ \iiint cup\left(\frac{x}{a},\frac{y}{a},\frac{z}{a}\right)\mathrm{d}x\mathrm{d}y\mathrm{d}z = ca^3. \tag{4.7}$$

4.2. AString Function

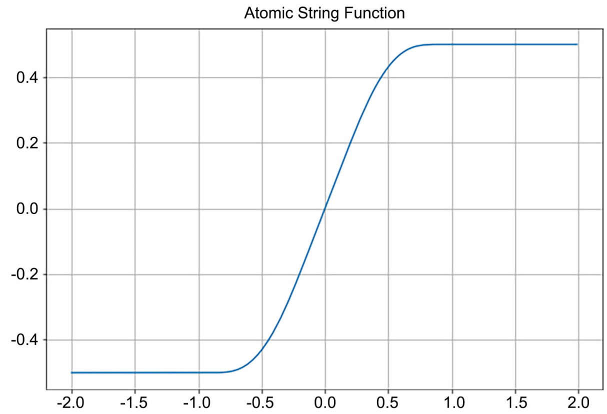
AString function (**Figure 4**) first proposed in 2018 by the author [2] [3] [4] [5] is both an integral (4.3) and "composing branch" of $up(x)$:

$$AString'(x) = AString(2x+1) - AString(2x-1) = up(x). \tag{4.8}$$

AString has a form of a solitary kink (**Figure 4(a)**) which can compose a straight line $y = x$ both between and as a translation of AString kinks leading to spacetime "atomization"/quantization ideas (§3, 7):

$$x \equiv AString\left(x-\frac{1}{2}\right) + AString\left(x+\frac{1}{2}\right),\ x \in \left[-\frac{1}{2},\frac{1}{2}\right];$$
$$x \equiv \cdots + AString(x-2) + AString(x-1) + AString(x) + AString(x+1) + AString(x+2) + \cdots \tag{4.9}$$

The Elementary AString kink function can be generalized in the form



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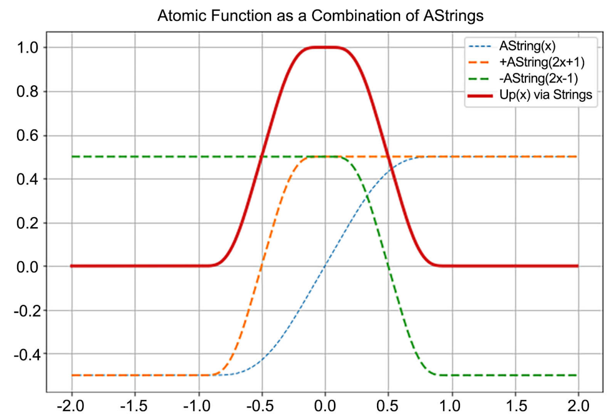

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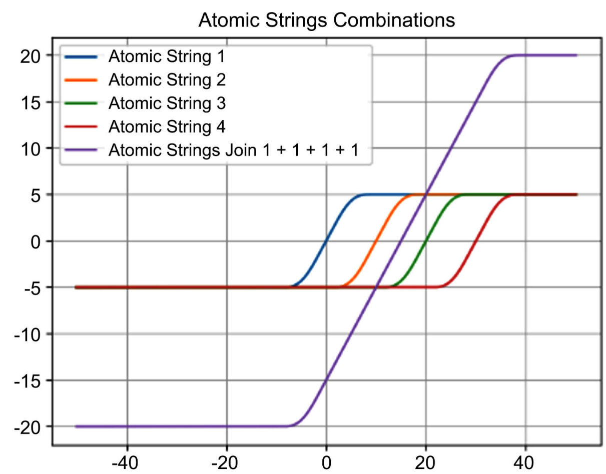

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Figure 4. (a) Atomic String Function (AString); (b) atomic function as a combination of two AStrings; (c) representation of a straight line segment by summing of AStrings.

$$AString(x,a,b,c,d=0) = d + c * AString((x-b)/a). \tag{4.10}$$

Importantly, Atomic Function pulse (4.6) can be presented as a sum of two opposite AString kinks (**Figure 4(b)**) making AStrings and AFs deeply related to each other:

$$up(x,a,b,c) = AString\left(x,\frac{a}{2},b-\frac{a}{2},c\right) + AString\left(x,\frac{a}{2},b+\frac{a}{2},-c\right). \quad (4.11)$$

4.3. Atomic Series and "Mathematical Atoms"

Atomic and AString Functions (*Atomics*) possess unique approximation properties described later in §5, 6. Like from "mathematical atoms" [6]-[12], as V.L. Rvachev called them, flat and curved smoothed surfaces/functions (**Figure 3**) can be composed of a superposition of Atomics via the so-called Generalized Taylor's Series [7] [8] [9] [24] [25] [26] [27] (or simply, *Atomic Series*) with an *exact* representation of polynomials of any order

$$\frac{1}{4}\sum_{k=-\infty}^{k=+\infty} k\,up\left(x-\frac{k}{2}\right) \equiv \sum_{k=-\infty}^{k=+\infty} AString(x-k) \equiv x;$$

$$\sum_{k=-\infty}^{k=+\infty}\left(\frac{k^2}{64}-\frac{1}{36}\right)up\left(x-\frac{k}{4}\right) \equiv x^2,$$

$$\begin{aligned} x^n &\equiv \sum_{k=-\infty}^{k=+\infty} C_k up\left(x-k2^{-n}\right) \\ &= \sum_{k=-\infty}^{k=+\infty} C_k\left(AString\left(2\left(x-k2^{-n}\right)+1\right) - AString\left(2\left(x-k2^{-n}\right)-1\right)\right). \end{aligned} \quad (4.12)$$

Notably, only a limited number of neighboring finite "atoms" are required to calculate a polynomial value at a given point.

It means Atomics can also represent/atomize any *analytic function* [28] (a function representable by converging Taylor's series) with known calculable coefficients:

$$\begin{aligned} y(x) &= \sum_{m=0}^{\infty}\frac{y^{(m)}(0)}{m!}x^m = \sum_{m=0}^{\infty} B_m x^m = \sum_{m=0}^{\infty}\sum_{k=-\infty}^{k=+\infty} B_m C_k up\left(x-k2^{-m}\right) \\ &= \sum_{mk=-\infty}^{\infty} c_{mk} up\left(\frac{x-b_{mk}}{a_{mk}}\right) = \sum_{l=-\infty}^{l=+\infty} AString(x,a_l,b_l,c_l). \end{aligned} \quad (4.13)$$

Analytic functions [28] represent a wide range of polynomial, trigonometric, exponential, hyperbolic, and other functions, their sums, derivatives, integrals, reciprocals, multiplications, and superpositions. Therefore, they all can be 'atomized' via superpositions of Atomic and AString Functions with any degree of precision, which is the most important property.

Instead of sums (4.12), and (4.13), we will be using short notation with localized basis atomic functions $A_k(x)$ and function values y^k at node k assuming summation over repeated indices k:

$$y(x) = A_k(x)y^k; \;\; f(x,y,z) = A_k(x,y,z)f^k. \quad (4.14)$$

4.4. Atomic Solitons

Being solutions of special kinds of nonlinear differential equations with shifted arguments (4.1), (4.8), AStrings and Atomic Functions possess some mathematical properties of lattice solitons [29] [30] [31] [32] and have been called *Atomic Solitons* [2] [3] [4] [5]. AString is a solitonic kink whose particle-like properties exhibit themselves in the composition of a line (4.9) and kink-antikink "atoms"

(4.8) (**Figure 4**). Being a composite object (4.8) made of two AStrings, AF $up(x)$ is not a true soliton but rather a *solitonic atom*, like "bions" or "dislocation atoms" [2] [4] [29] [30], as described in [2] [4].

5. Atomization Theorems

Unique properties of Atomic and AString Functions (Atomics) allow formulating *Atomization Theorems* stating how scalar, vector, tensor functions, and solutions of linear and nonlinear differential equations can be represented via a series of Atomics/Atomic Solitons leading to spacetime quantization and field unification ideas. To shorten the content, the proof would be provided only to a few theorems referring to [23] for more detailed descriptions with proof.

Theorem 1 (*Polynomial atomization theorem*). Polynomials of any order can be exactly represented/atomized via the Atomic Series of Atomic and AString Functions:

$$\left(x^n\right)^{(n)} = c\sum\nolimits_{k=-\infty}^{k=+\infty} up(x-k) \equiv c,$$

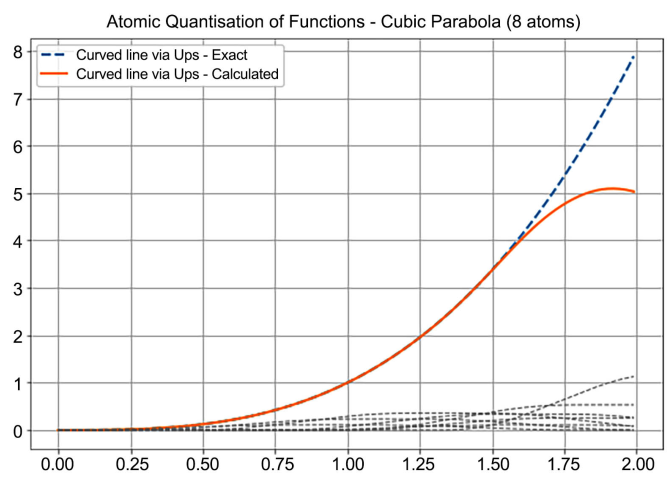
$$x^n = I_n\left(c\sum\nolimits_{k=-\infty}^{k=+\infty} up(x-k)\right) = \sum\nolimits_{k=-\infty}^{k=+\infty} C_k up\left(x-k2^{-n}\right). \tag{5.1}$$

$$\begin{aligned} P_n(x) = x^n + a_1 x^{n-1} + \cdots + a_n &\equiv \sum\nolimits_k C_k up\left(\frac{x-ka}{a}\right) \\ &= \sum\nolimits_k AString\left(x, a_k, b_k, c_k\right) = A_k(x) P_n^k. \end{aligned} \tag{5.2}$$

Based on (4.12) and finiteness, it states that only a few neighboring Atomics are required to calculate a polynomial value at a given point (**Figure 5(a)**).

Theorem 2 (*Analytic atomization theorem*). Analytic functions representable by converging Taylor's series via polynomials can be represented/atomized via converging Atomic Series of localized Atomic and AString Functions:

$$\begin{aligned} y(x) = \sum\nolimits_{m=0}^{\infty} \frac{y^{(m)}(0)}{m!} x^m &= \sum\nolimits_{m=0}^{\infty} B_m x^m = \sum\nolimits_{k=-\infty}^{k=+\infty} B_m C_k up\left(x-k2^{-m}\right) \\ &= \sum\nolimits_{mk=-\infty}^{\infty} c_{mk} up\left(\frac{x-b_{mk}}{a_{mk}}\right) = \sum\nolimits_{l=-\infty}^{l=+\infty} AString\left(x, a_l, b_l, c_l\right) = A_k(x) y^k. \end{aligned} \tag{5.3}$$



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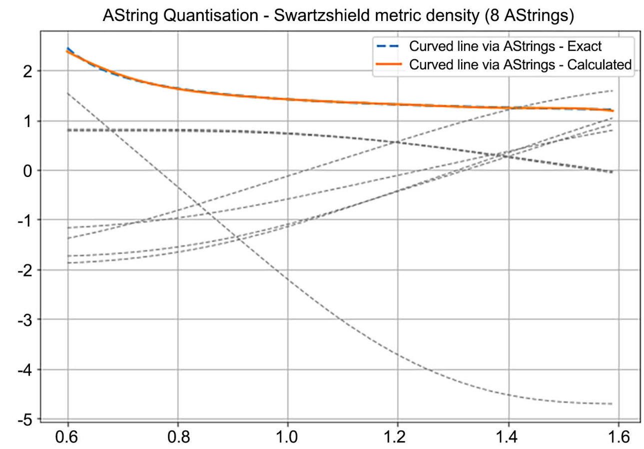

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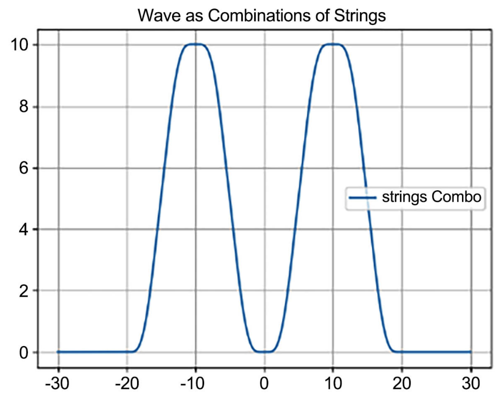
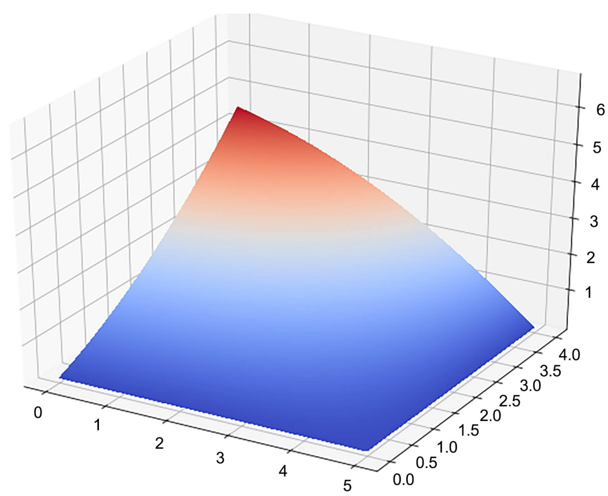


(c)



(d)

Figure 5. Representing sections of polynomials and analytic functions with AStrings and Atomic Functions (a) cubic parabola via 8 Atomic Functions; (b) Schwarzschild metric function; (c) wave-like formation; (d) 2d surface.

By definition, an analytic function [28] is representable via conversing Taylor's series by polynomials which in turn can be expressed via Atomics (5.1). It means physical fields described by exponential, trigonometric, hyperbolic, and other analytic functions are also atomizable. This theorem proven in [23] can be generalized to various combinations of analytic functions [23] [28] [45] with the following.

Theorem 3 (*Complex analytic atomization theorem*). Complex functions $y(x)$ that are sums $y = y_1 + y_2$, products $y = y_1 y_2$, reciprocals $y = 1/y_1 (y_1 \neq 0)$, inverse $y(y_1) = x$, derivatives $y = y_1'$, integrals $I(y)$, and superposition $y = y_1(y_2)$ of analytic functions $y_1(x), y_2(x)$ can be represented/atomized by Atomic Series over Atomic and AString Functions.

In general, polynomic, trigonometric, exponential, and other analytic functions are the solutions of some linear differential equations (LDE) implying that Atomization Theorems can be extended to differential equations [7]-[12], [23] [27].

Theorem 4 (*LDE atomization theorem*). Solutions of linear differential equations (LDE) with constant coefficients can be represented/atomized via series over Atomic and AString Functions:

$$L(y) = y^{(n)}(x) + a_1 y^{(n-1)}(x) + \cdots + a_{n-1} y'(x) + a_n y(x) = 0;$$
$$y(x) = A_k(x) y^k. \tag{5.4}$$

This theorem can be generalized [23] to equations with variable analytic coefficients frequently appearing in mathematical physics.

Theorem 5 (*Variable LDE atomization theorem*). Solutions of linear differential equation with variable coefficients $a_k(x)$ representable by analytic functions can be represented/atomized via Atomic Series over Atomic and AString Functions:

$$L(y, a_k(x)) = y^{(n)}(x) + a_1(x) y^{(n-1)}(x) + \cdots + a_n(x) y(x) = 0;$$
$$y(x) = A_k(x) y^k. \tag{5.5}$$

Atomization of composite functions like $\arctan(\exp(x)), \mathrm{sech}(x)$ satisfying nonlinear sine-Gordon and Schrodinger differential equations [31] [32] imply that the Atomization procedure is also applicable to some nonlinear differential equations.

Theorem 6 (*NDE atomization theorem*). Solutions of nonlinear differential equations (NDE) with linear differential operator $L(y)$ and nonlinear analytic function $f(y)$ can be represented/atomized via Atomic Series over Atomic AString Functions:

$$L(y, a_k(x)) = y^{(n)}(x) + a_1(x) y^{(n-1)}(x) + \cdots + a_n(x) y(x) = f(y);$$
$$y(x) = A_k(x) y^k. \tag{5.6}$$

The preservation of analyticity and Theorem 3 imply that compositions $y = y_1(y_2)$ of analytic functions are also analytic [23] [28] [45], so the theorems 1-6 can be Generalized to the following theorem.

Theorem 7 (*Complex NDE atomization theorem*). Solutions of nonlinear differential equations with functional-differential operator F preserving analyticity of function $y(x)$ can be represented/atomized via Atomic Series over Atomic and AString Functions.

$$F\left(y(x), \frac{\partial^m y}{\partial x^n}\right) = 0;$$

$$y(x) \equiv \sum_l up(x, a_l, b_l, c_l) = \sum_k AString(x, a_k, b_k, c_k) = A_k(x) y^k. \quad (5.7)$$

Applying Theorem 2 to the widely used Fourier Series leads to the following theorem.

Theorem 8 (*Waves atomization theorem*). Any smooth function with a *finite spectrum* [7] [8] [9] [23] can be represented/atomized via Atomic Series over Atomic and AString Functions:

$$\begin{aligned} y(x) &\equiv \sum_{i=1}^n d_i \sin(e_i x - f_i) = \sum_l up(x, a_l, b_l, c_l) \\ &= \sum_k AString(x, a_k, b_k, c_k) = A_k(x) y^k. \end{aligned} \quad (5.8)$$

Theorems 1-8 can be extended to multiple noting that multidimensional n-order polynomials in m-dimensions $P_{mn} = P_n(x_1, \cdots, x_m)$ which are some multiplications of 1D polynomials exactly representable by Atomics (Theorem 1) are also exactly representable by multiplications of Atomic Functions (multidimensional atomic functions (4.9) $\boldsymbol{UP}(a_k, b_k, c_k)$ which in turn are AStrings combinations (4.8).

Theorem 9 (3*D atomization theorem*). Representable by converging Taylor's series, multidimensional analytic functions with their sums, multiplications, reciprocals, derivatives, integrals, and superpositions can be represented/atomized via Atomic Series over localized multidimensional Atomic and AString Functions:

$$\begin{aligned} P_{mn} &= P_n(x_1, \cdots, x_m) = \prod_{i=1}^m P_n(x_i) = \sum_k \prod_{i=1}^m up_i(x_i, a_{ik}, b_{ik}, c_{ik}) \\ &= \sum_k \boldsymbol{UP}(a_k, b_k, c_k) = \sum_l \boldsymbol{AString}(a_l, b_l, c_l) = A_k(x_1, \cdots, x_m) P_{mn}^k. \end{aligned} \quad (5.9)$$

Similar to the 1D case with theorems 4, 5, the atomization procedure can be extended to multi-dimensional differential equations containing linear differential operators like Laplacian and Poisson operators widely used in mathematical physics:

$$L(y_1, \cdots, y_m)(x_1, \cdots, x_m) = a_{ijmn} \frac{\partial^m y_i}{\partial x_j^n} = 0; \nabla_i = \frac{\partial}{\partial x_i}; \Delta = \sum \frac{\partial^2}{\partial x_i^2}; \Delta + k; \Delta\Delta.$$

$$y_i(x_j) \equiv \sum_{ijjkl} up_i(x_j, a_{ijl}, b_{ijl}, c_{ijl}) = \sum_{jjkl} AString_i(x_j, a_{ijk}, b_{ijk}, c_{ijk}). \quad (5.10)$$

In summary, formulated Atomization Theorems, also proven in [23], demonstrate how polynomials, complex multi-dimensional analytical functions, and solutions of linear and nonlinear differential equations can be represented/atomized via superpositions of localized Atomic and AString Functions.

6. Atomization Theorems in General Relativity

Theorems 1-9 lead to the following theorems provided with proof and targeting

Einstein's General Relativity (GR) theory [13] [14] [15].

6.1. Atomization Theorems for Metric, Curvature, and Ricci Tensors

Considering together multidimensional Atomic Series (5.9), (5.10) and Atomization Theorems 1-6 leads to the following theorems important for General Relativity.

Theorem 10 (*Tensor's atomization theorem*). First $\partial_i = \frac{\partial}{\partial x_i}$ and second derivatives $\partial_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}$ as well as the metric tensor g_{ij} defining interval on a curved surface $\mathrm{d}s^2 = g_{ij}\left(x_n\right)\mathrm{d}x_i \mathrm{d}x_j$ preserve analyticity and being applied to analytic functions $y_k\left(x_l\right)$ lead to analytic functions representable/atomizable by Atomic Series over Atomic and AString Functions.

Proof. Being linear differential operators, both first and second derivative operators preserve analyticity because derivatives of multidimensional polynomials $B_{lm} x_l^m$ would also be polynomials exactly representable via multidimensional Atomic Functions and AStrings (5.9) using Atomic Series (5.2). For curved spacetime surfaces/geometries described by some analytic functions $\tilde{x}_i = \tilde{x}_i\left(x_j\right); \mathrm{d}\tilde{x}_i = \frac{\partial \tilde{x}_i}{\partial x_j}\mathrm{d}x_j$, the derivatives and their multiplications would also be analytic, hence representable by Atomics (Theorems 2, 3, 9). This theorem can be proved in another way by noting that all derivatives and integrals of Atomics are expressed via themselves (4.3), (4.8), and if space geometry analytic functions $\tilde{x}_i\left(x_j\right)$ are the sum of Atomics, then all derivatives and metric tensors would also be some Atomics combinations:

$$g_{ij}\left(x_n\right) = \sum\nolimits_{ijnk} up\left(x_n, a_{ijnk}, b_{ijnk}, c_{ijnk}\right) = \sum\nolimits_{ijnl} AString\left(x_n, a_{ijnl}, b_{ijnl}, c_{ijnl}\right). \quad (6.1)$$

Proof obtained. This theorem means that for analytic spacetime geometries/configurations, their deformations, curvatures, metrics, and geodesics would also be some Atomics superpositions, with a range of analytical surfaces and spacetime metrics known in GR [13] [14] [15] described later. Furthermore, due to the properties of analytic function superpositions to preserve analyticity (Theorem 3), the last theorem can be extended to nonlinear Ricci tensors important in GR [13] [14] [15].

Theorem 11 (*Ricci tensor atomization theorem*). Nonlinear Ricci tensor R_{jk} and Christoffel operators Γ_{ij}^k preserve analyticity and applied to analytic functions would yield analytic functions representable/atomizable by Atomic Series via Atomic and AString Functions.

Proof. Christoffel operators [13] [14] [15], which include multiplications of functions to their spatial derivatives, transform analytic metric tensor functions (6.1) representable by polynomials into more complex polynomials representable by Atomics via Atomic Series (5.3). Similarly, Ricci tensors are also a combination of derivatives and multiplications of Christoffel symbols [13] [14] [15] which preserve analyticity, hence representable via Atomics:

$$\Gamma_{ij}^{k} = \frac{1}{2} g^{kl} \left(\partial_i g_{jl} + \partial_j g_{il} - \partial_l g_{ij} \right); \; R_{jk} = \partial_i \Gamma_{jk}^{i} - \partial_j \Gamma_{ik}^{i} \Gamma_{ip}^{i} \Gamma_{jk}^{p} - \Gamma_{jp}^{i} \Gamma_{ik}^{p} \quad (6.2)$$

$$R_{ij}(x_n) = \sum_{ijnk} up\left(x_n, a_{ijnk}, b_{ijnk}, c_{ijnk}\right) = \sum_{ijnl} AString\left(x_n, a_{ijnl}, b_{ijnl}, c_{ijnl}\right). \quad (6.3)$$

Proof obtained. This theorem can be intuitively understood in the sense that polynomials are "hard to destroy" because their multiplications, derivatives, integrals, and superpositions would also be polynomials representable by Atomics. It also means that not only spacetime metrics but also curvature tensors can be "atomized" using shifts and stretches of finite AString and Atomic Functions which, as described later, may be associated with flexible spacetime quanta.

6.2. Atomization Theorem for General Relativity

The sequence of theorems 1-10 converges into the theorem for Einstein's General Relativity [13] [14] [15].

Theorem 12 (*Atomic Spacetime Theorem*). For analytic manifolds, Einstein's curvature tensor preserves analyticity and yields spacetime shapes, deformations, curvatures, and matter/energy tensors representable via multi-dimensional Atomic and AString Functions superpositions. Solutions of General Relativity equations can be represented/atomized by converging Atomic Series over finite Atomic and AString Functions:

$$\begin{aligned} G_{\mu\nu} &= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \\ &= \sum_{\mu\nu i} \boldsymbol{UP}\left(x_i, a_{\mu\nu i}, b_{\mu\nu i}, c_{\mu\nu i}\right) \\ &= \sum_{\mu\nu i} \boldsymbol{AString}\left(x_i, a_{\mu\nu i}, b_{\mu\nu i}, c_{\mu\nu i}\right). \end{aligned} \quad (6.4)$$

Proof. For analytic manifolds—spacetime geometries described by analytic functions $\tilde{x}_i = \tilde{x}_i\left(x_j\right)$ representable by converging Taylor's series—the metric tensors $g_{\mu\nu}$ composed of derivatives and their multiplications would also be some analytic functions (Theorem 10). Being injected into Christoffel operators (6.2) and then Ricci tensors $R_{\mu\nu}$, they would yield another set of analytic functions (Theorem 11) representable by Taylor's series because the derivatives, multiplications, and superposition of analytic functions would also be analytic (Theorem 3). The curvature scalar $R = g^{\mu\nu} R_{\mu\nu}$ in (6.4) preserves analyticity because of the cross-multiplication of polynomials and their derivatives would also be polynomials. Injected into (6.4), those tensors produce Einstein's tensor $G_{\mu\nu}$ and energy-momentum tensor $T_{\mu\nu}$ ($\frac{8\pi G}{c^4}$ is a constant) supposedly representable by polynomials via multi-dimensional Taylor's series. Because a polynomial of any order is exactly representable/atomizable via Atomic and AString Functions (Theorem 1), the spacetime curvature, metric, and energy/momentum tensors would be the superpositions of multi-dimensional Atomic ***UP*** and ***AString*** functions, derivatives of which are expressed via themselves. Due to fundamental relation (4.8) $up(x) = AString'(x) = AString(2x+1) - AString(2x-1)$, the Atomic Function $up(x)$ is a sum of two AStrings which can be associated with a finite quantum/metriant being able, within one model, to compose straight $x=\sum_k AString(x,a,ka,a)$ and curved $\tilde{x}=\sum_k AString(x,a_k,b_k,c_k)$ lines from elementary AString pieces resembling quanta (§3, 6, 7, 8).

Proof obtained. In a nutshell, this theorem tells that the spacetime field is representable/atomizable via AStrings and Atomic Functions, the derivatives of which are expressed via themselves meaning the spacetime shape, deformations, curvatures, and energy/momentum tensors can be represented as some superposition of Atomics. Now, this idea first hypothesized in 2017 [3] is based on a set of theorems. It offers an "atomic model" of spacetime [2] [3] [4] [5] probably envisaged by A. Einstein in 1933 [1] as a "*perfectly thinkable*" "*atomic theory*" dealing with "*simplest concepts and links between them*" (§1). Let's note that Atomization is not a simple discretization of space—separation of a volume into adjacent finite elements [22] [24] [38]. Here, the "finite elements" (AStrings) are overlapping (§3, **Figure 1**) and capable to describe both expansions of space (4.9) and localized solitonic atoms $up(x)$ (4.8).

Let's note that reversing Atomization Theorems (12-1) allows deriving Atomic and AString Functions from General Relativity equations (6.4), as described in [23].

7. Atomization of Spacetime Field

7.1. Spacetime Atomization Model

Theorems 10-12 provide a theoretical foundation for atomization/quantization of spacetime field based on Atomic and AString Functions when GR equations and solution, along with Ricci, curvature, and metric tensors, can be represented via Atomic Series over multidimensional Atomic and AString Functions (4.7):

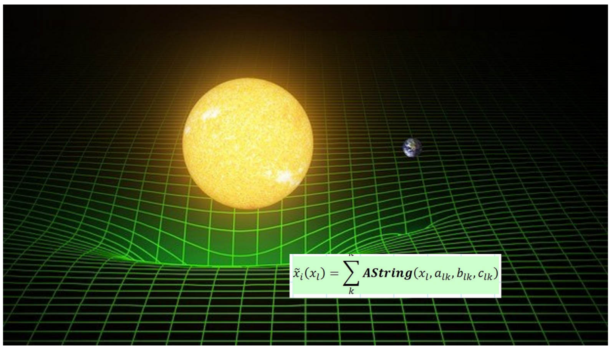
$$\begin{aligned} G_{\mu\nu} &= R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\frac{8\pi G}{c^4}T_{\mu\nu} \\ &=\sum\nolimits_{\mu\nu i}\boldsymbol{UP}\left(x_i,a_{\mu\nu i},b_{\mu\nu i},c_{\mu\nu i}\right) \\ &=\sum\nolimits_{\mu\nu i}\boldsymbol{AString}\left(x_i,a_{\mu\nu i},b_{\mu\nu i},c_{\mu\nu i}\right), \end{aligned} \tag{7.1}$$

$$\begin{aligned} R_{ij}(x_n) &=\sum\nolimits_{ijnk}\boldsymbol{UP}\left(x_n,a_{ijnk},b_{ijnk},c_{ijnk}\right) \\ &=\sum\nolimits_{ijnl}\boldsymbol{AString}\left(x_n,a_{ijnk},b_{ijnk},c_{ijnk}\right), \end{aligned} \tag{7.2}$$

$$\begin{aligned} g_{ij}(x_n) &=\sum\nolimits_{ijnk}\boldsymbol{UP}\left(x_n,a_{ijnk},b_{ijnk},c_{ijnk}\right) \\ &=\sum\nolimits_{ijnl}\boldsymbol{AString}\left(x_n,a_{ijnk},b_{ijnk},c_{ijnk}\right), \end{aligned} \tag{7.3}$$

$$\tilde{x}_i(x_l)=\sum\nolimits_k \boldsymbol{AString}\left(x_l,a_{lk},b_{lk},c_{lk}\right). \tag{7.4}$$

These formulae express the mathematical fact that it is possible to compose analytical manifolds (**Figure 6**) by adjusting parameters of localized Atomic and AString Functions, or like in Lego-game, compose a smooth shape from elementary pieces/quanta. If finite Atomics, for which derivatives are expressed via themselves, represent spacetime shape $\tilde{x}_i(x_l)$ (7.4), the series over Atomics would also describe spacetime deformations, curvatures, metrics, Ricci's, Einstein's, and energy-momentum tensors.



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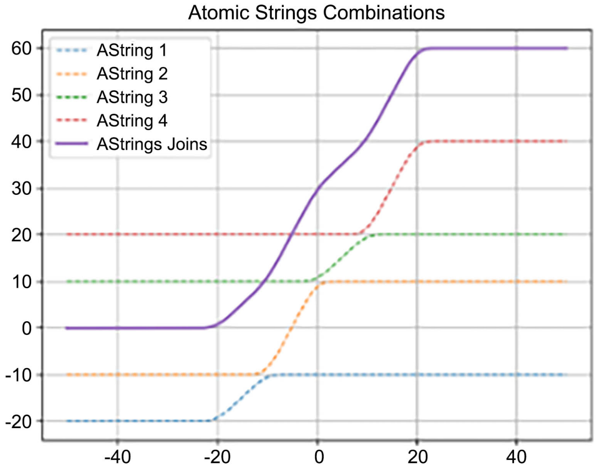

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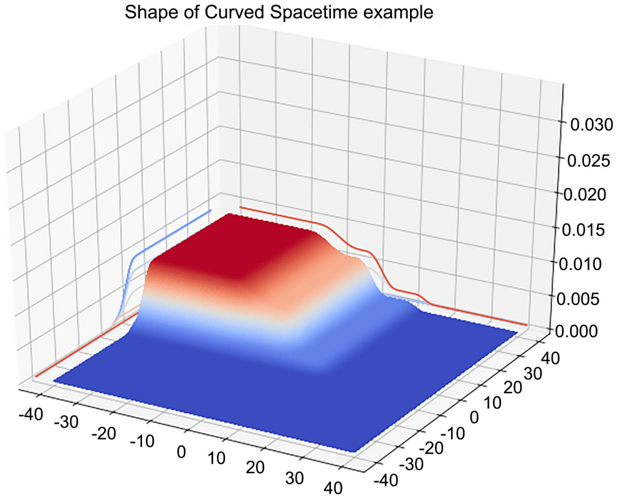


(c)

Figure 6. (a) Curved spacetime composed of AStrings; (b) joining AStrings of different heights simulates spacetime curving; (c) curved spacetime geodesics represented via joints of 3D solitonic atoms.

Because AString can compose a line and a curve from "elementary pieces" resembling quanta (§3, 6, 8) one can envisage a spacetime field as a complex network of flexible spacetime quanta (**Figure 4**, **Figure 6**). The notion of "quantum" here is not directly related to Quantum Mechanics and Quantum Gravity [18] [19] [34]-[39] but rather to the finiteness of "solitonic atoms" capable to compose shapes and fields as described later.

7.2. Atomization of Known General Relativity Solutions

The idea of Atomic Spacetime atomization/quantization can be demonstrated for known GR solutions [13] [14] [15] [16].

Einstein-Minkowski solution $T_{\mu\nu}=0,\ g_{ij}=1$ for homogeneous uniform spacetime/universe [13] [14] [15] [16] [20] is simply atomizable/quantizable via translations of identical overlapping AString quanta (§3, **Figure 3**) [2] [3] [4] [5] in vector notation:

$$\begin{aligned}&\boldsymbol{AQuantum}\left(x_1,x_2,x_3,t,a,\rho,c_l\right)\\&=AString\left(x_1,a,a,\rho a\right)\boldsymbol{e}_1+AString\left(x_2,a,a,\rho a\right)\boldsymbol{e}_2\\&\quad+AString\left(x_3,a,a,\rho a\right)\boldsymbol{e}_3+AString\left(t,a/c_l,a/c_l,\rho a/c_l\right)\boldsymbol{e}_t,\end{aligned} \tag{7.5}$$

or schematically (**Figure 3**, **Figure 4**, **Figure 6**, **Figure 7(c)**)

$$\boldsymbol{UniformSpace}\left(x_1,x_2,x_3,t\right)=\sum\nolimits_k \boldsymbol{AQuantum}\left(x_1,x_2,x_3,t,a,\rho,c_l\right). \tag{7.6}$$

Friedmann solution for expanding spatially homogeneous universe with metric [13] [14] [15] [16]

$$\mathrm{d}s^2=a\left(t\right)^2\mathrm{d}\overline{s}^2-c^2\mathrm{d}t^2;\ \mathrm{d}\overline{s}^2=\mathrm{d}r^2+S_k\left(r\right)^2\mathrm{d}\Omega^2 \tag{7.7}$$

includes analytic function $S_k\left(r\right)$ representable via Atomic Series (5.4), (5.5) as per Theorems 3,4:

$$\begin{aligned}S_k\left(r\right)&=rsinc\left(r\sqrt{k}\right)=r-\frac{kr^3}{6}+\frac{kr^5}{120}-\cdots\\&=\sum\nolimits_k c_k up\left(\frac{r-b_k}{a}\right)=\sum\nolimits_k AString\left(r,a_k,b_k,c_k\right).\end{aligned} \tag{7.8}$$

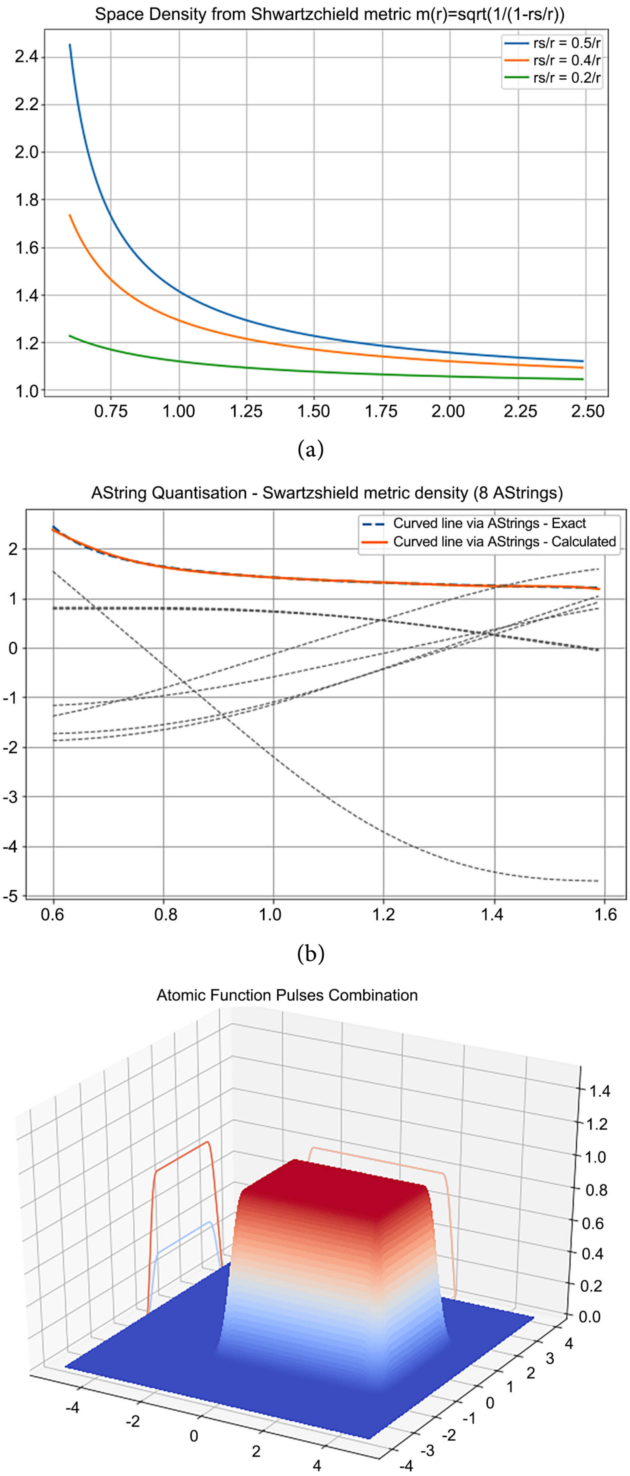
Scale factor $a\left(t\right)$ [13] [14] [15] [16] being an analytic power function [28] is also representable via Atomics:

$$a\left(t\right)=a_0t^{\frac{2}{3(w+1)}};\ a\left(t\right)\sim t^{2/3},\ w=0;\ a\left(t\right)\sim t^{1/2},\ w=1/3, \tag{7.9}$$

$$a\left(t\right)=\sum\nolimits_k up\left(t,a_k,b_k,c_k\right)=\sum\nolimits_l AString\left(t,a_l,b_l,c_l\right). \tag{7.10}$$

Schwarzschild solution (**Figure 7**) for radial bodies and black holes has spacetime metric [13] [14] [15] [16] [20]

$$\begin{aligned}&\mathrm{d}s^2=-A\left(r\right)c^2\mathrm{d}t^2+B\left(r\right)\mathrm{d}r^2+r^2\mathrm{d}\Omega;\\&A\left(r\right)=\left(1-\frac{r_s}{r}\right);\ B\left(r\right)=\left(1-\frac{r_s}{r}\right)^{-1}.\end{aligned} \tag{7.11}$$



(a) (b) (c)

Figure 7. (a) Space density function from Schwarzschild GR solution; (b) representing Schwarzschild metric via AStrings; (c) uniform spacetime field as a superposition of solitonic atoms.

Analytic (outside of singularity) function $A(r)$ and it's reciprocal $B(r)$ (also ana lytic as per Theorem 3) representable via converging Taylor's series is also representable via Atomics (5.4):

$$A(r)=\sum_k c_k up\left(\frac{r-b_k}{a}\right)=\sum_k AString\left(r,a_k,b_k,c_k\right), r\neq 0. \tag{7.12}$$

In summary, the atomization of known GR solutions confirms the main idea that analytic spacetime fields are representable via the superposition of finite AStrings and Atomic Functions.

8. Atomic Spacetime Model Interpretations

Representing the spacetime field as a superposition of Atomic and AString Functions offers the following interpretations discussed in detail in [2] [3] [4] [5] [6] [23].

1) No point in space—space is rather a superposition of finite "solitonic atoms" $cup\left((x-b)/a\right)$.

2) Spacetime is a field composed of "solitonic atoms" $cup\left((x-b)/a\right)$ on a lattice with size *a* which may be associated with Planck's length $a\sim 1.6\times 10^{-35}$ m.

3) Finite "solitonic atoms" may be associated with some flexible quanta (§3, 7).

4) Atoms/quanta interact in overlapping zones with the preservation of smoothness (§3, §4.1).

5) Spacetime "emerges" between quanta/atoms $x\equiv AString\left(x-\frac{1}{2}\right)+AString\left(x+\frac{1}{2}\right)$ (§3, 4).

6) Spacetime atomization is not a simple space discretization, due to overlapping zones.

7) Spacetime is a flexible "fabric" of solitonic atoms adjusting their locations and intensities to reproduce variable fields between atoms (§7.1, §7.2).

8) Expansion of spacetime means bigger quanta (7.10) with growing width (§7.2).

9) Space "solitonic atoms" are mathematically "made of" AStrings $up(x)=AString(2x+1)+AString(1-2x)$ (4.8).

10) AString kinks pointing in one direction model spacetime expansion $x=\sum_k AString(x-k)$ while opposite AStrings compose finite atom $up(x)=AString(2x+1)+AString(1-2x)$ (§3, 4).

11) AString models a "quantum of length" capable to represent a length function $L(x)$ as a sum of AStrings:

$$L(x)=\int_0^x \mathrm{d}\tilde{x}(x)=\int_0^x \rho(x)\mathrm{d}x=\int_0^x \sum_k up\left(x,a_k,b_k,c_k\right)\mathrm{d}x$$
$$=\sum_k AString\left(x,a_k,b_k,c_k\right).$$

12) AString may model a "metriant"—space quantum in some General Thermodynamics theories [34].

13) Spacetime has energy as a sum of discrete energies ca^3 (4.7) of multiple solitonic atoms (§7.1).

14) Spacetime may be solitonic in nature [2] [3] [4] [5] and composed of

Atomic Solitons (§4.4).

15) Spacetime field is both discrete and continuous; discrete, being composed of finite solitonic atoms, and continuous, being united with smooth connections between "atoms" (§3, §7.1).

16) Spacetime is fractalic due to fractalic properties of a "length" composed of "pieces" [2] [3] [4] [5] [23].

17) Atomic spacetime model introduces new constants, mainly lattice size a which may be associated with Planck's length $a \sim 1.6\times10^{-35}$ m, or another fractalic lattice size for macroscopic fields.

9. Fields Atomization

Atomization Theorems (§5, 6) provide the foundation for "atomizing" (presenting via Atomic Series) not only spacetime but also other fields in theoretical physics [2] [3] [4] [5].

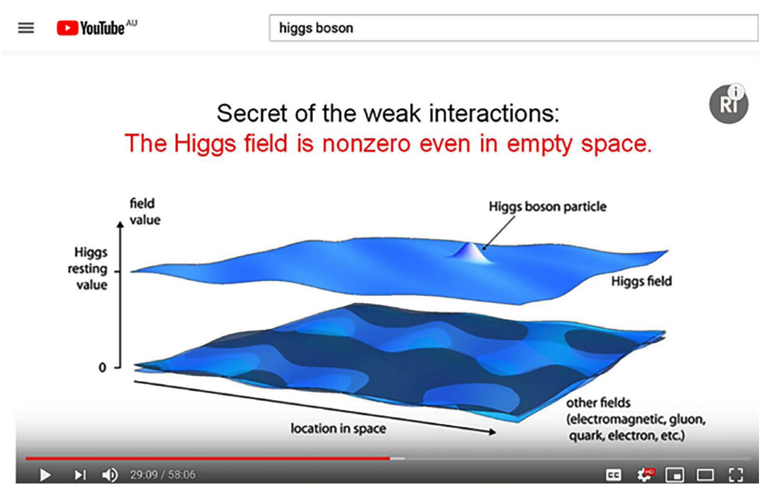
For example, a uniform Higgs field [44] [47] with a constant energy level ($Hg = 126\ \mathrm{GeV/c^2}$) (**Figure 8**) permeating space in arbitrary direction x can be represented by smooth joining of atomic pulses $up(x)$ with width s:

$$Hg\left(\cdots + up\left(\frac{x-2s}{s}\right) + up\left(\frac{x-s}{s}\right) + up\left(\frac{x}{s}\right) + up\left(\frac{x+s}{s}\right) + up\left(\frac{x+2s}{s}\right) + \cdots\right) \equiv Hg\,. \quad (9.1)$$

Uneven distribution in some direction x_i can be described by a more generic expression [2] [4]:

$$\begin{aligned} HF(x_i) &= \sum\nolimits_k up(x_i, a_k, b_k, c_k, d_k) \\ &= \sum\nolimits_l AString(x_i, a_l, b_l, c_l, d_l) \\ &= A_k(x_i)\, HF(x_i)^k\,. \end{aligned} \quad (9.2)$$

In this case an individual "atom"/field excitation $HB(r)$ in a radial direction with energy E_{hb} (**Figure 8**) may be associated with the Higgs boson to uphold Quantum Field Theory [34] [46] [47] where particles are interpreted as some excitations of fields:



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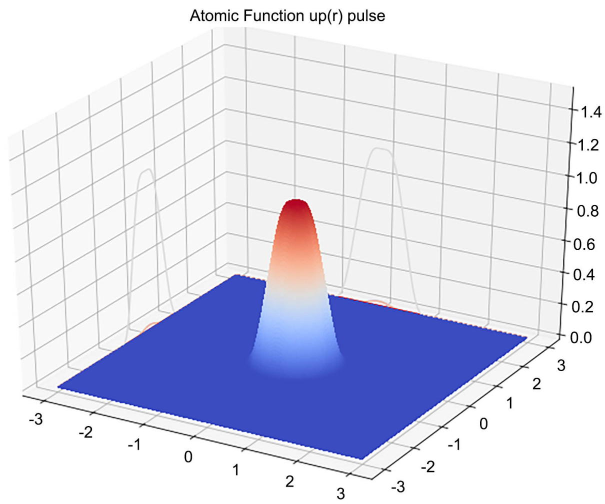

(b)

Figure 8. (a) S. Carrol's lecture https://www.youtube.com/watch?v=RwdY7Eqyguo features Higgs boson as an excitation of a field (obtained with permission) (b) Higgs boson representation via atomic soliton pulse $up(r)$.

$$HB(r) = d + c * up\left(\frac{r-b}{a}\right),$$

$$E_{hb} = \int HB(r) = \int cup\left(\frac{r}{a}\right)\mathrm{d}r = ca = \rho a^2;\ \rho = \frac{c}{a} \tag{9.3}$$

In the same "atomized" way, it is possible to describe the distributions of other macroscopic and fundamental physical fields [2] [3] [4] [5], eq quantum vacuum zero-point field [40] [41]:

$$Zpf\left(\cdots + up\left(\frac{x-2s}{s}\right) + up\left(\frac{x-s}{s}\right) + up\left(\frac{x}{s}\right) + up\left(\frac{x+s}{s}\right) + up\left(\frac{x+2s}{s}\right) + \cdots\right) \equiv Zpf. \tag{9.4}$$

10. Conclusion and Future Research Directions on an Atomic Unified Theory

Atomization theorems provide a theoretical foundation for applications of AStrings and Atomic Functions in many physical theories being researched further [2] [3] [4] [5] [42]. The common feature of these theories is the unified description of fields as superpositions of flexible overlapping solitonic atoms $cup((x-b)/a)$ made of two AStrings $up(x) = AString(2x+1) - AString(2x-1) = AString'(x)$. It means that mathematically field distributions are just complex combinations/lattices/networks of flexible AStrings. Moreover, due to properties (4.1), (4.8) of Atomics to have derivatives expressed via the functions themselves, not only the fields but their derivatives expressing fields deformations and curvatures, would also be some AStrings combinations. This invites the hypothesis raised in [2] [3] [4] [5] whether AString mathematically describes some fundamental solitonic object from which *everything* is made, and having a common "ancestor", different fields may be deeply related to each other.

The obvious candidate for a "particle of everything" is a new kind of string [2] [3] [4] [5] from string theory [15] [35] [36]. Rather than consider a string as a "linearly vibrating" filament, one can hypothesize that a string with length a may vibrate "nonlinearly" with compact $cup(x/a)$ "solitonic atom" shape composed of two AStrings (**Figure 4**) with intensity/amplitude c and energy/integral ca (ca^3 in 3D). Two neighboring strings may overlap and produce either constant or variable smooth field while continuous space x "emerges" between strings $x \equiv aAString\left(x-\frac{a}{2}\right)+aAString\left(x+\frac{a}{2}\right), x\in\left[-\frac{a}{2},\frac{a}{2}\right]$ and extends by adding more strings $x \equiv \sum_k aAString\left(\left(x-ka\right)/a\right)$ (§3, 7). Then, electron, quark, or Higgs "particles" may be the spatial excitation of strings with their own intensity, energy, and size (§9). Because these fields have a common "string ancestor", they become deeply related to each other, with the preservation of energy during exchanges.

Interestingly, this Atomic String model assumes the existence of a "string with length" a embedding the concepts of a "length", "dimensionality", and "lattice". It looks like every particle (electron, boson) apart from unique physical characteristics (eq electric charge) includes some "quanta of space", or metriants (§8), as A. Veinik [34], call them in the 1980s. Like the Higgs field giving matter "a property of mass" [44] [47], metriants "give the matter the property of size" and "order of location" [34]. AString function not only offers a mathematical model for the metriant (§3, 7) but also allows the building of "solitonic atoms" $up(x) = AString(2x+1) - AString(2x-1)$ capable to compose different fields in superposition. This concept of AString metriants as "common blocks" of fields has synonyms like "quantum of length", "elementary distortion of spacetime", and "ripple/excitation" of spacetime used by other authors [16] [20] [38] [46] [47].

Hopefully, verified by physicists and string theorists, these string and metriant models [2] [3] [4] [5] may contribute to spacetime physics, quantum field theories, and unified theories of nature [16] [34] [35] [38] [46] [47] [48] [49] [50].

11. Atomic Spacetime and "Atomic Theory" of A. Einstein

To conclude the paper let's recall again A. Einstein's 1933 paper [1] cited in §1 where he envisaged a "*perfectly thinkable*" "*atomic theory*" with "*simplest concepts and links between them*" resolving "*stumbling blocks*" of theories operating "…*exclusively with continuous functions of space*" and explaining how a finite region of space can have quantized energy levels. The described Atomic Spacetime theory may be that theory sought by A. Einstein. Indeed, the theory is "atomic" assuming atomizing/quantizing spacetime field with finite Atomic String Functions. Atomic Functions, as "solitonic atoms" made of two AStrings (4.8), are "*simple concepts*". "*Links between them*", in the form of overlapping superpositions (4.9), (4.12), (7.1), allow describing flat and curved spacetime and other physical fields. The theory also overcomes "*stumbling blocks*" of theories dealing "…*exclusively with continuous functions of space*" *[1]*; here, atomized spacetime is both discrete and continuous (§7, 8). Also, Einstein's "…*region of three-dimensional space at whose boundary electrical density vanishes everywhere*" [1] naturally leads to a *finite* Atomic Function (**Figure 2**) with discrete energy (integral (4.7)) levels ca^3. Atomic Spacetime theory seems to support another Einstein's quote: "*I have deep faith that the principle of the universe will be beautiful and simple*". This "principle" may be realized with the simple model of spacetime and field composition from Atomic Solitons composed of simple AString metriant functions.

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The theory described here expands the 50 years of history of Atomic Functions pioneered by the author's teacher Academician NAS of Ukraine V.L. Rvachev to new domains of theoretical physics, and this paper is dedicated to his genius.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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